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A4 Batch (S4 - Comps)

LINEAR PROGRAMMING - LPP: TUTORIAL

Q.5 Maximise $z = 2x_1 - 2x_2 + 4x_3 - 5x_4$
subject to $x_1 + 4x_2 - 2x_3 + 8x_4 \leq 2$
$-x_1 + 2x_2 + 3x_3 + 4x_4 \leq 1$
$x_1, x_2, x_3, x_4 \geq 0$

Ans- $A = \begin{bmatrix} 1 & 4 & -2 & 8 \\ -1 & 2 & 3 & 4 \end{bmatrix}$ $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ $b = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

① Putting $x_3 = 0$ $x_4 = 0$

$$\begin{bmatrix} 1 & 4 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

By $R_2 + R_1$ $\begin{bmatrix} 1 & 4 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$x_1 + 4x_2 = 2$ $\therefore x_2 = \frac{1}{2}$ $x_1 = 0$
$6x_2 = 3$

② Putting $x_2 = 0$, $x_4 = 0$

$$\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

By $R_1 + R_2$ $\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$x_1 - 2x_3 = 2$ $x_3 = 3$ $x_1 = 8$

③ Putting $x_1 = 0$, $x_4 = 0$

$$\begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

By $R_2 - (1/2)R_1$ $\begin{bmatrix} 4 & -2 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$4x_2 - 2x_3 = 2$ $x_3 = 0$ $x_2 = 1/2$

④ Putting $x_2 = 0, x_3 = 0$

$$\begin{bmatrix} 1 & 8 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

By $R_2 + R_1$ $$\begin{bmatrix} 1 & 8 \\ 0 & 12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$x_1 + 8x_4 = 2$

$\therefore 12x_4 = 3$

$x_4 = 1/4, x_1 = 0$

⑤ Putting $x_1 = 0, x_3 = 0$

$$\begin{bmatrix} 4 & 8 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

By $R_2 - (1/2)R_1$ $$\begin{bmatrix} 4 & 8 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$\therefore 4x_2 + 8x_4 = 2$

$0x_2 + 0x_4 = 1$

Solution is unbounded.

⑥ Putting $x_1 = 0, x_2 = 0$

$-2x_3 + 8x_4 = 2$

$3x_3 + 4x_4 = 1$

$$\begin{bmatrix} -2 & 8 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

By $R_2 - (1/2)R_1$ $$\begin{bmatrix} -2 & 8 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$-2x_3 + 8x_4 = 2$

$4x_3 = 0 \quad x_4 = 1/4$

Hence,

① $x_3 = 0, x_4 = 0, x_1 = 0, x_2 = 1/2, z = -3/2$

② $x_2 = 0, x_4 = 0, x_1 = 8, x_3 = 3, z = 28$

③ $x_1 = 0, x_4 = 0, x_2 = 1/2, x_3 = 0, z = -1$

$x_2=0, x_3=0, x_1=0, x_4=1/4, Z=-5/4$

④ ~~$x_2=0, x_3=0$, unbounded solution~~

⑤ $x_1=0, x_3=0$, unbounded solution

⑥ $x_1=0, x_2=0, x_3=0, x_4=1/4, Z=-5/4$

Hence, there are six basic solutions.
All the solutions except ⑤ are feasible basic solutions.
The solution ② is the optimal basic solution.

Q.11 Maximise $Z=3x_1+2x_2+5x_3$
subject to $x_1+2x_2+x_3 \leq 430$
(simplex Method) $3x_1+2x_3 \leq 460$
$x_1+4x_2 \leq 420$
$x_1, x_2, x_3 \geq 0$

Ans- $Z-3x_1-2x_2-5x_3+0s_1+0s_2+0s_3=0$
$x_1+2x_2+x_3+s_1+0s_2+0s_3=430$
$3x_1+0x_2+2x_3+0s_1+s_2+0s_3=460$
$x_1+4x_2+0x_3+0s_1+0s_2+s_3=420$

④

SIMPLEX TABLE

Iteration Number	Basic var.	x_1	x_2	x_3	s_1	s_2	s_3	RHS	Ratio
0	Z	-3	-2	-5	0	0	0	0	
s_2 leaves	s_1	1	2	1	1	0	0	430	430
x_3 enters	s_2	3	0	2	0	1	0	460	230
	s_3	1	4	1	0	0	1	420	∞
1	Z	$9/2$	-2	0	0	$5/2$	0	1150	
s_1 leaves	s_1	$-1/2$	2	0	1	$-1/2$	0	200	100
x_2 enters	x_3	$3/2$	0	1	0	$1/2$	0	230	∞
	s_3	1	4	0	0	0	0	420	105
2	Z	4	0	0	1	0	0	1350	
	x_2	$-1/4$	1	0	$1/2$	$-1/4$	0	100	
	x_3	$3/2$	0	1	0	$1/2$	0	230	
	s_3	2	0	0	-2	1	0	20	

$x_1 = 0,\ x_2 = 100,\ x_3 = 230,\ Z_{max} = 1350$

Q.13 Maximise $Z = 100x_1 + 50x_2 + 50x_3$

Subject to (Simplex Method)

$4x_1 + 3x_2 + 2x_3 \le 10$

$3x_1 + 8x_2 + x_3 \le 8$

$4x_1 + 2x_2 + x_3 \le 6$

$x_1, x_2, x_3 \ge 0$

Ans- $Z - 100x_1 - 50x_2 - 50x_3 + 0s_1 + 0s_2 + 0s_3 = 0$

$4x_1 + 3x_2 + 2x_3 + s_1 + 0s_2 + 0s_3 = 10$

$3x_1 + 8x_2 + x_3 + 0s_1 + s_2 + 0s_3 = 8$

$4x_1 + 2x_2 + x_3 + 0s_1 + 0s_2 + s_3 = 6$

SIMPLEX TABLE

Iteration No.	Basic Var.	x_1	x_2	x_3	s_1	s_2	s_3	RHS	Ratio
0	Z	-100	-50	-50	0	0	0	0	
s_3 leaves	s_1	4	3	2	1	0	0	10	2.5
x_1 enters	s_2	3	8	1	0	1	0	8	2.67
	s_3	4	2	1	0	0	1	6	1.5
1	Z	0	0	-25	0	0	25	150	
s_1 leaves	s_1	0	1	1	1	0	-1	4	4
x_3 enters	s_2	0	13/2	1/4	0	1	-3/4	7/2	14
	x_1	1	1/2	1/4	0	0	1/4	3/2	6
2	Z	0	25	0	25	0	0	250	
	x_3	0	1	1	1	0	-1	4	
	s_2	0	25/4	0	-1/4	1	-1/2	5/2	
	x_1	1	1/4	0	-1/4	0	1/2	1/2	

$x_1 = \frac{1}{2}$ $x_2 = 0$ $x_3 = 4$ $Z_{max} = 250$

Q.23 Maximise $Z = 5x_1 - 2x_2 + 3x_3$

Subject to $2x_1 + 2x_2 - x_3 \geq 2$

$3x_1 - 4x_2 \leq 3$

$x_2 + 3x_2 \leq 5$

$x_1, x_2, x_3 \geq 0$

Ans - $Z = 5x_1 - 2x_2 + 3x_3 - 0s_1 - 0s_2 - 0s_3 - MA_1$

$2x_1 + 2x_2 - x_3 - s_1 + 0s_2 + 0s_3 + A_1 = 2$

$3x_1 - 4x_2 - 0x_3 + 0s_1 + s_2 + 0s_3 + 0A_1 = 3$

$0x_1 + x_2 + 3x_3 + 0s_1 + 0s_2 + s_3 + 0A_1 = 5$

$Z = 5x_1 - 2x_2 + 3x_3 + 2Mx_1 + 2Mx_2 - Mx_3 - Ms_1 - 0s_2$
$- 0s_3 + 0A_1 - 2M$

$Z - 5x_1 - 2Mx_1 + 2x_2 - 2Mx_2 - 3x_3 + Mx_3 + Ms_1 + 0s_2$
$+ 0s_3 + 0A_1 = -2M$

SIMPLEX TABLE

Iteration No.	Basic var.	x_1	x_2	x_3	s_1	s_2	s_3	A_1	RHS	Ratio
0	Z	$-5-2M$	$2-2M$	$-3+M$	M	0	0	0	$-2M$	
A_1 leaves	A_1	2	2	-1	-1	0	0	1	2	1
x_1 enters	s_2	3	-4	0	0	1	0	0	3	1
	s_3	0	0	3	0	0	1	0	5	∞
1	Z	0	7	$-1/2$	$-5/2$	0	0		5	
s_2 leaves	x_1	1	1	$-1/2$	$-1/2$	0	0		1	-2
x_3 enters	s_2	0	-7	$3/2$	$3/2$	1	0		0	0
	s_3	0	1	3	0	0	1		5	$5/3$
2	Z	0	$-56/3$	0	3	$1/3$	0		5	
s_3 leaves	x_1	1	$-4/3$	0	0	$1/3$	0		1	$-3/5$
x_2 enters	x_3	0	$-14/3$	1	1	$2/3$	0		0	0
	s_3	0	15	0	-3	-2	1		5	$1/3$
3	Z	0	0	0	$-11/5$	$53/45$	$52/45$		$10/9$	
x_3 leaves	x_1	1	0	1	$-4/15$	$7/45$	$4/45$		$13/9$	∞
s_1 enters	x_3	0	0	1	$1/15$	$3/45$	$14/45$		$14/9$	$70/3$
	x_2	0	1	0	$-1/5$	$-2/15$	$1/15$		$1/3$	∞
4	Z	0	0	11	0	$5/3$	$42/9$		$85/3$	
	x_1	1	0	4	0	$1/3$	$4/3$		$23/3$	
	s_1	0	0	15	1	$2/3$	$14/3$		$70/3$	
	x_2	0	1	3	0	0	1		5	

$x_1 = \frac{23}{3}$ $\quad x_2 = 5$ $\quad x_3 = 0$ $\quad z_{max} = \frac{85}{3}$

Q.24 Minimise $z = x_1 + 2x_2 + x_3$

subject to $x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3 \leq 1$

(Big M Method) $\frac{3}{2}x_1 + 2x_2 + x_3 \geq 8$

$x_1, x_2, x_3 \geq 0$

Ans- Maximize $z' = -z = -x_1 - 2x_2 - x_3$

We have

Maximise $z' = -x - 2x_2 - x_3 - 0s_1 - 0s_2 - MA_2$ — (1)

subject to $x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3 + s_1 + 0s_2 + 0A_2 = 1$ — (2)

$\frac{3}{2}x_1 + 2x_2 + x_3 + 0s_1 - s_2 + A_2 = 8$ — (3)

Multiply (3) by M and add to (1)

Maximise

$z' = \left(-1 + \frac{3M}{2}\right)x_1 + (-2 + 2M)x_2 + (-1 + M)x_3 + 0s_1 - Ms_2 + 0A_2 - 8M$

$\therefore z' + \left(1 - \frac{3M}{2}\right)x_1 + (2 - 2M)x_2 + (1 - M)x_2 + 0s_1 + Ms_2 + 0s_3 + 0A_2 = -8M$

(8)

SIMPLEX TABLE

Iteration No.	Basic var.	x_1	x_2	x_3	S_1	S_2	A_2	RHS	Ratio
0	z'	$1-3/2M$	$2-2M$	$1-M$	0	M	0	$-8M$	
S_1 leaves	S_1	1	$1/2$	$1/2$	1	0	0	1	2
x_2 enters	A_2	$3/2$	2	1	0	-1	1	4	4
1	z'	$-3\,5/2M$	0	$-1+M$	$-4+4M$	M	0	$-4-4M$	
	x_2	2	1	1	2	0	0	2	
	A_2	$-5/2$	0	-1	-4	-1	1	0	

Since all entries in the row of z are positive A_2 appears with a positive values. The given problem has no feasible solution.