NLPP

(1) NLPP with no constraints
(2) NLPP with linear equality constraint
(3) NLPP with linear inequality constraint.

* **NLPP with no constraints:**

The Object function is of the form:

$$z = a_{11}x_1^2 + a_{22}x_2^2 + \cdots + a_{nn}x_n^2 + a_{12}x_1x_2 + a_{13}x_1x_3 \cdots + a_{1n}x_1x_n + c_1x_1 + c_2x_2 + \cdots c_nx_n$$

put $\frac{\partial f}{\partial x_i} = 0, \quad 1 \le i \le n$ and find $X_0 = (x_1, x_2 \ldots x_n)$

Define a Hessian matrix as follows:

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \frac{\partial^2 f}{\partial x_2 \partial x_3} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} & & & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$$

If:

(i) All the principele minors of H at x_0: are positive and X_0 is point of minimum.

(ii) If the principle minors D_1, D_3, D_5 are negative and D_2, D_4, D_6 are +ve then X_0 is point of maxima.

(iii) In general, if H is indefinite then x_0 is a saddle point (no point of maxima or minima)

Example:

Q Obtain maxima or minima of the function:

$$z = x_1 + 2x_3 + x_2 x_3 - x_1^2 - x_2^2 - x_3^2$$

$$z = f(x_1, x_2, x_3)$$

ans: $\frac{\partial f}{\partial x_1} = 0 \Rightarrow 1 - 2x_1 = 0$

$\Rightarrow x_1 = 1/2$

$\frac{\partial f}{\partial x_2} = 0 \Rightarrow x_3 - 2x_2 = 0$

$\frac{\partial f}{\partial x_3} = 0 \Rightarrow 2 + x_2 - 2x_3 = 0$

$\Rightarrow x_2 = 2/3$, $x_3 = 4/3$

$\therefore x_0 = (1/2, 2/3, 4/3)$ is a stationary point.

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \frac{\partial^2 f}{\partial x_2 \partial x_3} \\ \frac{\partial^2 f}{\partial x_3 \partial x_1} & \frac{\partial^2 f}{\partial x_3 \partial x_2} & \frac{\partial^2 f}{\partial x_3^2} \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix} \text{ at } X_0$$

$D_1 = |-2| = -2$

$D_2 = \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix} = 4$

$D_3 = |H| = -6$

$\therefore$ D_1 & D_3 are -ve and D_2 is +ve

$\therefore$ X_0 is a point of maxima.

$= f(1/2, \frac{2}{3}, 4/3)$

$\therefore f_{max} = \frac{1}{2} + \frac{8}{3} + \frac{8}{9} - \frac{1}{4} - \frac{4}{9} - \frac{16}{9}$

$= \frac{19}{12}$ //

* **NLPP with one equality constraint:**

→ Consider the following form of NLPP:

$z = f(x_1, x_2 \ldots x_n)$

subject to $g(x_1, x_2 \ldots x_n) = b$

i.e $z = f(x_1, x_2, \ldots x_m)$

s.t $g(x_1, x_2 \ldots x_n) - b = 0 \Rightarrow h(x_1, x_2 \ldots x_n) = 0$

$x_i \geq 0, \quad 1 \leq i \leq n$

→ Construct a new function lagrangian function using multiplier called lagrangian multiplier as:

$$L(x_1, x_2, \ldots x_n, \lambda) = f(x_1, x_2, x_3 \ldots x_n) - \lambda h(x_1, x_2 \ldots x_n) \quad -(1)$$

→ The necessary conditions for maxima or minima subject to the constraints $h(x_1, x_2 \ldots x_n) = 0$ are:

$$\frac{\partial L}{\partial x_1} = 0, \frac{\partial L}{\partial x_2} = 0 \cdots \frac{\partial L}{\partial x_n} = 0, \frac{\partial L}{\partial \lambda} = 0 \quad -(2)$$

from (1) we get,

$$\frac{\partial L}{\partial x_1} = \frac{\partial f}{\partial x_1} - \lambda \frac{\partial h}{\partial x_1}$$

$$\frac{\partial L}{\partial x_2} = \frac{\partial f}{\partial x_2} - \lambda \frac{\partial h}{\partial x_2}$$

lastly, $\frac{\partial L}{\partial x_n} = \frac{\partial f}{\partial x_n} - \lambda \frac{\partial h}{\partial x_n}$

$$\frac{\partial L}{\partial \lambda} = -h$$

using (2) we get following $n+1$ conditions:

$$\frac{\partial f}{\partial x_1} = \lambda \frac{\partial h}{\partial x_1},$$

$\frac{\partial f}{\partial x_2} = \lambda \frac{\partial h}{\partial x_2}$ lastly, $\frac{\partial f}{\partial x_n} = \lambda \frac{\partial h}{\partial x_n}$

and $-h = 0 \therefore h = 0$

$\therefore h(x_1, x_2 \cdots x_n) = 0$

- Solving these $n+1$ conditions we can find $x_1, x_2, \cdots x_n$ and λ. Thus the point of maxima or minima can be obtained

→ Now, find the value of following determinant of order $n+1$ at x_0 :

$$\Delta_{n+1} = \begin{vmatrix} 0 & \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} & \cdots & \frac{\partial h}{\partial x_n} \\ \frac{\partial h}{\partial x_1} & \frac{\partial f}{\partial x_1^2} - \lambda\frac{\partial^2 h}{\partial x_1^2} & \cdots & & \frac{\partial^2 f}{\partial x_1 \partial x_n} - \lambda\frac{\partial^2 h}{\partial x_1 \partial x_n} \\ \frac{\partial h}{\partial x_2} & \frac{\partial f}{\partial x_2 \partial x_1} - \lambda\frac{\partial^2 h}{\partial x_2 \partial x_1} & \cdots & & \frac{\partial^2 f}{\partial x_2 \partial x_n} - \lambda\frac{\partial^2 h}{\partial x_2 \partial x_n} \\ \vdots & & & & \\ \frac{\partial h}{\partial x_n} & \frac{\partial f}{\partial x_n \partial x_1} - \lambda\frac{\partial^2 h}{\partial x_n \partial x_1} & \cdots & & \frac{\partial^2 f}{\partial x_n^2} - \lambda\frac{\partial^2 h}{\partial x_n^2} \end{vmatrix}$$

→ If the signs of the principle minors $\Delta_3, \Delta_4, \Delta_5$ are alternatively +ve and -ve then the point x_0 is a point of maxima.

→ If all the principle minors, $\Delta_3, \Delta_4, \Delta_5 \cdots \Delta_{n+1}$ are -ve then the point x_0 is a point of minima.

This method is called lagrangian's multiplier method.

Q Use the method of lagrangian's multiplies to solve following NLP:

$$z = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$$

subject to $x_1 + x_2 + x_3 = 20$ where $x_1, x_2, x_3 \geq 0$

ans: consider $L(x_1, x_2, x_3, \lambda) = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100 - \lambda[x_1 + x_2 + x_3 - 20)$

$\frac{\partial L}{\partial x_1} = 0 \Rightarrow 4x_1 + 10 - \lambda = 0$ —(1)

$\frac{\partial L}{\partial x_3} = 0 \Rightarrow 6x_3 + 6 - \lambda = 0$ —(2)

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$$\frac{\partial L}{\partial x_3} = 0 \Rightarrow 6x_3 + 6 - \lambda = 0 \quad -(3)$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow -(x_1 + x_2 + x_3 - 20) = 0 \quad -(4)$$

$\Rightarrow (1)\times 3 + (2)\times 6 + (3)\times 2$

$\Rightarrow 12(x_1+x_2+x_3) + 906 - 11\lambda = 0$

from eq^n (4):

$\Rightarrow 12\times 20 + 90 - 11\lambda = 0$

$\Rightarrow \lambda = \frac{330}{11} = 30$

$\therefore (1) \Rightarrow 4x_1 + 10 = 30 \quad \therefore x_1 = 5$

$(2) \Rightarrow 2x_2 + 8 = 30 \quad \therefore x_2 = 11$

$(3) \Rightarrow 6x_3 + 6 = 30 \quad \therefore x_3 = 4$

$\therefore h(x_1, x_2, x_3) = x_1 + x_2 + x_3 - 20$

$\therefore X_0 = (5, 11, 4)$ is a stationary point

$$\Delta_4 = \begin{vmatrix} 0 & \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} & \frac{\partial h}{\partial x_3} \\ \frac{\partial h}{\partial x_1} & \frac{\partial^2 f}{\partial x_1^2} - \lambda\frac{\partial^2 h}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 x_2} - \lambda\frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} - \lambda\frac{\partial^2 f}{\partial x_1 \partial x_3} \\ \frac{\partial h}{\partial x_2} & \frac{\partial^2 f}{\partial x_2 x_1} - \lambda\frac{\partial^2 h}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} - \lambda\frac{\partial^2 f}{\partial x_2^2} & \frac{\partial^2 f}{\partial x_2 \partial x_3} - \lambda\frac{\partial^2 f}{\partial x_2 \partial x_3} \\ \frac{\partial h}{\partial x_3} & \frac{\partial^2 f}{\partial x_3 x_1} - \lambda\frac{\partial^2 h}{\partial x_3 \partial x_1} & \frac{\partial^2 f}{\partial x_3 \partial x_2} - \lambda\frac{\partial^2 f}{\partial x_3 \partial x_2} & \frac{\partial^2 f}{\partial x_3^2} - \lambda\frac{\partial^2 f}{\partial^2 x_3} \end{vmatrix}$$

$$f = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$$

$$= \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 4 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 6 \end{vmatrix}$$

$\therefore$ at x_0

~~$\Delta_1 \Rightarrow 1$~~

$= C_2 - C_4,\ C_3 - C_4 \Rightarrow \begin{vmatrix} 0 & 0 & 0 & 1 \\ 1 & 4 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & -6 & -6 & 6 \end{vmatrix}$

-8
$+32$

$$= -1 \begin{vmatrix} 1 & 4 & 0 \\ 1 & 0 & 2 \\ 1 & -6 & -6 \end{vmatrix}$$

$$= -1[1(+12) - 4(-6-2)] = \underline{\underline{-44}}$$

$$\Delta_3 = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 4 & 0 \\ 1 & 0 & 2 \end{vmatrix} = \underline{-6}$$

Since $\Delta_3, \Delta_4 \ldots$ is $-ve$, x_0 is point of minima.

~~* CNLPP with one inequality constraint~~

Q Optimize $z = 12x_1 + 8x_2 + 6x_3 - x_1^2 - x_2^2 - x_3^2 - 23$ subject to $x_1 + x_2 + x_3 = 10$ and $x_1, x_2, x_3 \geq 0$

Ans: $x_1 = 5$ $\quad \Delta_3 = 4$ $\quad \Rightarrow$ maxima

$x_2 = 3$ $\quad \Delta_4 = -12$

$x_3 = 2$

$\therefore Z_{max} = 35$

* NLPP with one inequality constraint :

→ Necessary conditions for maximization :

$$\frac{\partial f}{\partial x_1} - \lambda \frac{\partial h}{\partial x_1} = 0$$

$\cdots$

$$\frac{\partial f}{\partial x_m} - \lambda \frac{\partial h}{\partial x_n} = 0$$

$\lambda h(x_1, x_2 \cdots x_n) = 0$

$h(x_1, x_2 \cdots x_n) \leq 0$ $\qquad \lambda \geq 0$

for the following problem :

$\max z = f(x_1, x_2, x_3 \cdots x_n)$

subject to $g(x_1, x_2 \cdots x_n) \leq b$ $x_1, x_2 \cdots x_n \geq 0$

here $h(x_1, x_2 \cdots x_n) = g(x_1, x_2 \cdots n) - b$.

→ Consider the following problem minimize $z = f(x_1 \cdots x_n)$ such that $g(x_1, x_2 \cdots x_n) \geq b$
$x_1, x_2 \cdots x_n \geq 0$

The necessary conditions are :

$$\frac{\partial f}{\partial x_1} - \lambda \frac{\partial h}{\partial x_1} = 0$$

$\cdots$

$$\frac{\partial h}{\partial x_m} - \lambda \frac{\partial h}{\partial n} = 0$$

$\lambda h(x_1, x_2 \cdots x_n) = 0$

$h(x_1, x_2 \cdots x_n) \geq 0$ $\qquad \lambda \geq 0$

→ These conditions are called Kuhn-Tucker conditions for optimization

Q. use Kuhn-Tucker ~~co~~ (KT) conditions to solve the following NLPP:

$$\max z = 2x_1^2 - 7x_2^2 + 12x_1x_2 \text{ subject to } 2x_1 + 5x_2 \leq 98$$

where $x_1, x_2 \geq 0$

ans: $f(x_1, x_2) = 2x_1^2 - 7x_2^2 + 12x_1x_2$

$h(x_1, x_2) = 2x_1 + 5x_2 - 98$

Kuhn-Tucker conditions are:

(i) $\Rightarrow \frac{\partial f}{\partial x_1} - \lambda \frac{\partial h}{\partial x_1} = 0$

$\Rightarrow 4x_1 + 12x_2 - 2\lambda = 0 \quad -(1)$

(ii) $\Rightarrow \frac{\partial f}{\partial x_2} - \lambda \frac{\partial h}{\partial x_2} = 0$

$\Rightarrow -14x_2 + 12x_1 - 5\lambda = 0 \quad -(2)$

(iii) $\lambda(2x_1 + 5x_2 - 98) = 0 \quad -(3)$

(iv) $2x_1 + 5x_2 - 98 \leq 0 \quad -(4)$

(v) $\lambda \geq 0 \quad -(5)$

(3) $\Rightarrow$ either $\lambda = 0$ or $2x_1 + 5x_2 = 98$

case (a): If $\lambda = 0$:

then (1) $\Rightarrow 4x_1 + 12x_2 = 0$

(2) $\Rightarrow -14x_2 + 12x_1 = 0$

$\Rightarrow x_1 = 0, \; x_2 = 0$

$\therefore z = 0$

which is not a feasible solution $\therefore$ we reject this case!

Case (b): If $\lambda \neq 0$:

then $2x_1 + 5x_2 = 98$

(1) $\times 5$ − (2) $\times 2$

$\Rightarrow 88x_2 - 4x_1 = 0$

$\Rightarrow x_1 = 22x_2$

put $x_1 = 22x_2$ in $2x_1 + 5x_2 = 98$

$\therefore x_2 = 2, \; x_1 = 44.$

put in (1)

$\therefore 4(44) + 12(2) - 2\lambda = 0$

$176 + 24 = 2\lambda$

$\therefore \lambda = 100$

$\therefore$ All Kuhn-Tucker conditions are satisfied. Hence, this is the solution.

$\therefore Z_{max} = 4900$

Q Use KT conditions to solve max $Z = 8x_1 + 10x_2 - x_1^2 - x_2^2$ subject to $3x_1 + 2x_2 \leq 6$, $x_1, x_2 \geq 0$

Ans: $f(x_1, x_2) = 8x_1 + 10x_2 - x_1^2 - x_2^2$

$h(x_1, x_2) = 3x_1 + 2x_2 - 6$

Kuhn-Tucker conditions are

(i) $\frac{\partial f}{\partial x_1} - \lambda \frac{\partial h}{\partial x_1} = 0 \Rightarrow 8 - 2x_1 - 3\lambda = 0$ — (1)

(ii) $\frac{\partial f}{\partial x_2} - \lambda \frac{\partial h}{\partial x_2} = 0 \Rightarrow 10 - 2x_2 - 2\lambda = 0$ — (2)

(iii) $\lambda(3x_1 + 2x_2 - 6) = 0$ — (3)

(iv) $3x_1 + 2x_2 - 6 \le 0$ — (4)

(v) $\lambda \geq 0$ — (5)

(3) $\Rightarrow \lambda = 0$ or $3x_1 + 2x_2 - 6 = 0$

case (a): $\lambda = 0$

then (1) $\Rightarrow 8 - 2x_1 = 0$

(2) $\Rightarrow 10 - 2x_2 = 0$ $\Big\}$ $x_1 = 4$, $x_2 = 5$

$\therefore Z_{max} = 41$

All conditions are not satisfied $\therefore$ we reject this case

case (b): $\lambda \neq 0$

$3x_1 + 2x_2 - 6 = 0$

66
12

2
13 - 8
8
104
94

(1) × 2 − (2) × 3

$\Rightarrow -14 - 4x_1 + 6x_2 = 0$ — (6)

− (6) + $(3x_1 + 2x_2 - 6) \times 3$

$$\Rightarrow \begin{array}{r} 9x_1 + \cancel{6x_2} = 18 \\ - 4x_1 + \cancel{6x_2} = 14 \\ \hline 13x_1 \quad = 4 \end{array}$$

$\therefore x_1 = \frac{4}{13}$ and $x_2 = \frac{33}{13}$

put in (4)

$= 3\left(\frac{4}{13}\right) + 2\left(\frac{33}{13}\right) - 6 \Rightarrow +ve$ condition satisfied.

$3\lambda = 8 - 2\left(\frac{4}{13}\right) = \frac{96}{13} > 0$ ✓

32

Since all conditions are satisfied. we accept this case.

$$\therefore\ Z_{max} = 8\left(\frac{4}{13}\right) + 10\left(\frac{33}{13}\right) - \left(\frac{4}{13}\right)^2 - \left(\frac{33}{13}\right)^2$$

$$= 21.31$$

* Linear Programming

→ defⁿ: the variable $x_1, x_2 \dots x_n$ which enter into the problem are called decision variables.

→ The function which is to be optimized is called objective function

→ The restrictions imposed on the relationship between the variables in the form of equalities or inequalities are called constraints.

→ Any set of values $x_1, x_2 \dots x_n$ which satisfy the constraints is called solution of LPP.

→ Any solution which satisfy non-negativety restrictions is called feasible solⁿ.

→ The region determined by the constraints and the axes in the 1st quadrant is called feasible region.

→ Any feasible solⁿ which optimizes the objective func is called optimum feasible solution.

→ Any solution in which one or more of the variables become 0 is called degenerate solution

* Canonical and standard form of LPP:

① **Canonical:**

$$\text{Max } z = \sum_{i=0}^{n} c_i x_i$$

subject to $\sum_{j=1}^{n} a_{ij} x_j \le b_i \quad , \quad i = 1, 2 \ldots m$

$$x_j \geq 0 \quad , \quad j = 1, 2 \ldots n$$

→ The characteristics of this form are:

(1) If the objective function is of minimization type then we must have greater than or equal to type inequality.

If the objective func is of maximization type then we must have less than or equal to inequality type.

(2) If the constraint is in the form of a eqⁿ then express it as an inequality.

eg: $a_1 x_1 + a_2 x_2 = b$
$\Rightarrow a_1 x_1 + a_2 x_2 \le b$ & $a_1 x_1 + a_2 x_2 \geq b$

(3) We should have $x_i \geq 0$. If any variable is unrestricted. For ex: if x_j is unrestricted then we write x_j as:

$$x_j = x_j' - x_j''$$ where both x_j'' and x_j' are non-negative.

(2) <u>Standard form</u>:

→ In the standard form we introduce slack variable and express the objective function as well as constraints in the form of equalities i.e

$$\text{Max } z = c_1x_1 + c_2x_2 \cdots + 0S_1 + 0S_2 \cdots + 0S_m$$

$(n+m)$ variables.

where $S_1, S_2 \ldots S_m$ are called slack variables.

subject to,

$$a_{11}x_1 + a_{12}x_2 \cdots + a_{1n}x_n + S_1 + 0S_2 + 0S_3 \cdots + 0S_m = b_1 \quad -(i)$$

$$a_{21}x_1 + a_{22}x_2 \cdots + a_{2n}x_n + 0S_1 + S_2 + 0S_3 \cdots + 0S_m = b_1$$

$$\cdots\cdots$$

$$a_{n1}x + a_{n2}x_2 \cdots + a_{nn}x_n + 0S_1 + 0S_2 \cdots + S_n = b_1$$

where, $x_1, x_2 \ldots x_n$, $S_1, S_2 \ldots S_n \geq 0$

Characteristics:

(1) All constraints are expressed in the form of equality eq^n using slack variables.

(2) RHS of all constraints are non-negative

Ex: $2x_1 + 3x_2 - S_3 = -4$

then, $-2x_1 + 3x_2 + S_3 = 4$

(3) Objective func should be of maximization type.

(4) All decision variables $(x_1, x_2 \ldots x_n)$ & slack variables are non-negative.

Q Convert the following LPP into std form:

$\min -Z = -3x_1+2x_2-x_3$ subject to $x_1-3x_2+2x_3 \geq -6$

$3x_1+4x_3 \leq 3$

$-3x_1+5x_2 \leq 4$

$x_1, x_2 \geq 0$, x_3 - unrestricted

ans: $x_3 = x_3' - x_3''$, $x_3' \,\&\, x_3'' \geq 0$

$\max z' = -Z = 3x_1 - 2x_2 + (x_3' - x_3'' + 0s_1 + 0s_2 + 0s_3)$

subject to $-x_1 + 3x_2 - 2(x_3' - x_3'') + s_1 + 0s_2 + 0s_3 = 6$

$3x_1 + 4(x_3' - x_3'') + 0s_1 + s_2 + 0s_3 = 3$

$3x_1 + 5x_2 + 0s_1 + 0s_2 + s_3 = 4$

$x_1, x_2, x_3', x_3'', s_1, s_2, s_3 \geq 0$

Defin:

(1) Basic solution is obtained by putting any variable out of n+m variables to zero and obtain values of remaining m variables

(2) These m variables (0 or non-zero) are called basic variables and other n-zero valued variables are called non-basic variables.

(3) Basic feasible solution: which statisfies non-negativity conditions is called as basic feasible solution.

(i) Non degenerate BFS: If all the values of basic feasible solution are positive, then it is called non-degenerate BFS.

(ii) Degenerate BFS: If one or more values of BFS are zero that is called degenerate BFS.

Q find all basic solutions of the following system which of these are BFS, non degenerate solution, Infeasible solution, Optimum basic feasible solution

$$\max z = x_1 + 3x_2 + 3x_3$$

such that $x_1 + 2x_2 + 3x_3 = 4$

$2x_1 + 3x_2 + 5x_3 = 7$

ans no. of variables = 3

no of constraints = 2

we put 3-2 = 1 var to 0

no. of BS	non-Basic variables =0	Basic variables	Eq^s & value of BV		Is sol^n feasible	I sol^n degen	value of z	Is sol^n Optimal
1	$x_3 = 0$	x_1, x_2	$x_1 + 2x_2 = 4$ $2x_1 + 3x_2 = 4$	$x_2 = 2$ $x_1 = 1$	Yes	No	$z = 5$	Yes
2	$x_2 = 0$	x_1, x_3	$x_1 + 3x_3 = 4$ $2x_1 + 5x_3 = 7$	$x_1 = 1$ $x_3 = 1$	Yes	No	$z = 4$	
3	$x_1 = 0$	x_2, x_3	$2x_2 + 3x_3 = 4$ $3x_2 + 5x_3 = 7$	$x_2 = -1$ $x_3 = 2$	No		–	

SIMPLEX METHOD

Q Solve using simplex method:

minimize $z = x_1 - 3x_2 + 3x_3$ subject to:

$3x_1 - x_2 + 2x_3 \leq 7$

$2x_1 + 4x_2 \geq -12$ $\qquad x_1, x_2, x_3 \geq 0$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

ans: max, $z' = -x_1 + 3x_2 - 3x_3 - 3x_3 + 0s_1 + 0s_2 + 0s_3$

subject to $3x_1 - x_2 + 2x_3 + s_1 + 0s_2 + 0s_3 = 7$

$-2x_1 - 4x_2 + 0x_3 + 0s_1 + s_2 + 0s_3 = 12$

$-4x_1 + 3x_2 + 8x_3 + 0s_1 + 0s_2 + s_3 = 10$

where $x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

$\boxed{\text{UNIQUE SOL}^{n}}$

no. of variable = 6 (n) $\Rightarrow 6 - 3 = 3$

no. of constraints = 3 (c)

The initial BFS is $x_1, x_2, x_3 = 0$ $\therefore z' = 0$

$\therefore s_1 = 7,\ s_2 = 12,\ s_3 = 10$

Co-eff of:

Ite No	Basic Variables	x_1	x_2 ↓	x_3	s_1	s_2	s_3	RHS soln	Ratio
0	z'	1	-3	3	0	0	0	0	
x_2 enters	s_1	3	-1	2	1	0	0	7	$7/-1 = -7$
s_3 leaves	s_2	-2	-4	0	0	1	0	12	$12/-4 = -3$
	s_3	-4	3*	8	0	0	1	10	$10/3 \rightarrow$
	z'	-3	0	11	0	0	1	10	
x_1 enters	s_1	5/3*	0	14/3	1	0	1/3	31/3	$31/5 \longrightarrow$
s_1 leaves	s_2	-22/3	0	32/3	0	1	4/3	76/3	$-76/22$
	x_2	-4/3	1	8/3	0	0	1/3	10/3	$-10/4$

Iteno	BV	x_1	x_2	x_3	s_1	s_2	s_3	RHS	Ratio
	z'	0	0	97/5	9/5	0	8/5	143/5	
	x_1	1	0	14/5	3/5	0	1/5	31/5	
	s_2	0	0	468/15	66/15	1	42/15	1062/15	
	x_2	0	1	96/15	12/15	0	5/15	174/15	

Since all the entries in z' row are non negative. Optimal basic feasible soln is obtained which is given by:

$x_1 = 31/5$ $\quad s_1 = 0$

$x_2 = 174/15$ $\quad s_2 = 1062/15$

$x_3 = 0$ $\quad s_3 = 0$

$z' = 143/5 \quad \therefore z = -143/5$

Q Maximize $z = 107x_1 + x_2 + 2x_3$

subject to:

$14x_1 + x_2 - 6x_3 + 3x_4 = 7$

$16x_1 + \left(\frac{1}{2}\right)x_2 - 6x_3 \leq 5$ $\quad x_1, x_2, x_3, x_4 \geq 0$

$3x_1 - x_2 - x_3 \leq 0$

ans: Standard form:

$z' = \cancel{107x_1 - x_2 - 2x_3 + 0s_1 + 0s_2 + 0s_3}$

max, $z - 107x_1 - x_2 - 2x_3 = 0$

subject to: $\frac{14}{3}x_1 + \frac{x_2}{3} - \frac{6}{3}x_3 + x_4 = \frac{7}{3}$

$16x_1 + \frac{1}{2}x_2 - 6x_3 + s_1 = 5$ $\quad x_1, x_2, x_3, x_4, s_1, s_2 \geq 0$

$3x_1 - x_2 - x_3 + s_2 = 0$