Ex (1) Max $z = 2x_1 + x_2 + x_3$

subject to

$$x_1 + x_2 + x_3 \geq 6$$
$$3x_1 - 2x_2 + 3x_3 = 3$$
$$-4x_1 + x_3 \leq 10$$
$$x_1, x_3 \geq 0, \quad x_2 - \text{unrestricted.}$$

Solun: The canonical form

Let $x_2 = x_2' - x_2''$

$\therefore$ Max $z = 2x_1 + x_2 + x_3$.

subject to

$$-x_1 - x_2 - x_3 \leq -6$$
$$3x_1 - 2x_2 + 3x_3 \leq 3$$
$$-3x_1 + 2x_2 - 3x_3 \leq -3$$
$$-4x_1 + x_3 \leq 10 \quad \text{put } x_2 = x_2' - x_2''.$$

ie max $z = 2x_1 + x_2' - x_2'' + x_3$

subject to

$$-x_1 - x_2' + x_2'' - x_3 \leq -6$$
$$3x_1 - 2x_2' + 2x_2'' + 3x_3 \leq 3$$
$$-3x_1 + 2x_2' - 2x_2'' - 3x_3 \leq -3$$
$$-4x_1 + x_3 \leq 10$$
$$x_1, x_2', x_2'', x_3 \geq 0$$

Let y_1, y_2', y_2'', y_3 be the dual variables

$\therefore$ The dual LPP is.

min $W = -6y_1 + 3y_2' - 3y_2'' + 10y_3$

subject to

$$-y_1 + 3y_2' - 3y_2'' - 4y_3 \geq 2$$
$$-y_1 - 2y_2' + 2y_2'' + 0y_3 \geq 1$$
$$y_1 + 2y_2' - 2y_2'' + 0y_3 \geq -1$$
$$-y_1 + 3y_2' - 3y_2'' + y_3 \geq 1$$

and $y_1, y_2', y_2'', y_3 \geq 0$.

$\therefore$ The dual LPP becomes.

Min $W = -6y_1 + 3y_2' - 3y_2'' + 10y_3$

subject to

$$-y_1 + 3y_2' - 3y_2'' - 4y_3 \geq 2$$
$$-y_1 - 2y_2' + 2y_2'' = 1$$
$$-y_1 + 3y_2' - 3y_2'' + y_3 \geq 1$$
$$y_1, y_2', y_2'', y_3 \geq 0.$$

Let $y_2' - y_2'' = y_2$ ∴ y_2 is unrestricted

∴ Dual LPP is

Min $w = -6y_1 + 3y_2 + 10y_3$

subject to

$-y_1 + 3y_2 - 4y_3 \geq 2$

$-y_1 - 2y_2 = 1$

$-y_1 + 3y_2 + y_3 \geq 1$

$y_1, y_3 \geq 0$ ∴ y_2 is unrestricted

The given LPP is

Max $Z = x_1 + 3x_2 - 2x_3 + 5x_4$

Subject to
$$3x_1 - x_2 + x_3 - 4x_4 = 6$$
$$5x_1 + 3x_2 - x_3 - 2x_4 = 4$$
$$x_1, x_2 \geq 0, \quad x_3, x_4 \text{ unrestricted}$$

The canonical form.

let $x_3 = x_3' - x_3''$ & $x_4 = x_4' - x_4''$

Max $Z = x_1 + 3x_2 - 2x_3' + 2x_3'' + 5x_4' - 5x_4''$

subject to
$$3x_1 - x_2 + x_3' - x_3'' - 4x_4' + 4x_4'' \leq 6$$
$$-3x_1 + x_2 - x_3' + x_3'' + 4x_4' - 4x_4'' \leq -6$$
$$5x_1 + 3x_2 - x_3' + x_3'' - 2x_4' + 2x_4'' \leq 4$$
$$-5x_1 - 3x_2 + x_3' - x_3'' + 2x_4' - 2x_4'' \leq -4$$
$$x_1, x_2, x_3', x_3'', x_4'', x_4' \geq 0.$$

let y_1, y_2, y_3, y_4 be the dual variables (# of constraints in primal = # of variables in dual.)

The dual LPP is

primal	Dual
no. of variable	no. of variable
no. of constraint	no. of constraint

Min $W = 6y_1 - 6y_2 + 4y_3 - 4y_4$

Subject to
$$3y_1 - 3y_2 + 5y_3 - 5y_4 \geq 1$$
$$-y_1 + y_2 + 3y_3 - 3y_4 \geq 3$$
$$y_1 - y_2 - y_3 + y_4 \geq -2$$
$$-y_1 + y_2 + y_3 - y_4 \geq 2$$
$$-4y_1 + 4y_2 - 2y_3 + 2y_4 \geq 5$$
$$4y_1 - 4y_2 + 2y_3 - 2y_4 \geq -5$$

and $y_1, y_2, y_3, y_4 \geq 0$

$y_1 - y_2 - y_3 + y_4 \geq -2$ & $-y_1 + y_2 + y_3 - y_4 \geq 2$

is equivalent to $\boxed{-y_1 + y_2 + y_3 - y_4 = 2}$

Similarly $-4y_1 + 4y_2 - 2y_3 + 2y_4 \geq 5$ & $4y_1 - 4y_2 + 2y_3 - 2y_4 \geq -5$

is equivalent to $\boxed{-4y_1 + 4y_2 - 2y_3 + 2y_4 = 5}$

The Dual LPP becomes

$$\text{Min } W = 6y_1 - 6y_2 + 4y_3 - 4y_4$$

Subject to

$$3y_1 - 3y_2 + 5y_3 - 5y_4 \geq 1$$
$$-y_1 + y_2 + 3y_3 - 3y_4 \geq 3$$
$$-y_1 + y_2 + y_3 - y_4 = 2$$
$$-4y_1 + 4y_2 - 2y_3 + 2y_4 = 5$$
$$y_1, y_2, y_3, y_4 \geq 0.$$

Let $y_1 - y_2 = w_1$ & $y_3 - y_4 = w_2$

$\therefore$ w_1 & w_2 are unrestricted

$\therefore$ Dual LPP is.

$$\text{Min } W = 6w_1 + 4w_2$$

Subject to

$$3w_1 + 5w_2 \geq 1$$
$$-w_1 + 3w_2 \geq 3$$
$$-w_1 + w_2 = 2$$
$$-4w_1 - 2w_2 = 5$$

w_1 & w_2 are unrestricted. //

In primal, the variable which is unrestricted. the corresponding constraint in dual is equation and vice versa

In primal, the constraint which is equality the corresponding variable in dual is unrestricted.