CSM Module 4.

→ The following data were available for Past 10 Years on demand and lead time. Estimate correlation and covariance.

Lead Time :- 6.5, 4.3, 6.9, 6.0, 6.9, 6.9, 5.8, 7.3, 4.5, 6.3

~~Covariance~~ Demand :- 103, 83, 116, 97, 112, 104, 106, 109, 92, 96.

⇒ Solution: Let X_1 represent Lead time and X_2 represents annual demand.

$$\overline{x}_1 = \frac{\left(\sum_{j=1}^{10} X_{1j}\right)}{10} = 61.4/10 = \underline{6.14}.$$

$$\overline{X}_2 = \frac{\left(\sum_{j=1}^{10} X_{2j}\right)}{10} = 1018/10 = \underline{101.8}$$

$$\Rightarrow s_1^2 = \frac{\left(\sum_{j=1}^{10} x_{1j}^2 - 10\,\overline{x}_1^2\right)}{(10-1)}.$$

$$= \frac{\left\{\left(6.5^2 + 4.3^2 + 6.9^2 \ldots\ldots 6.3^2\right) - 10\,(6.14)^2\right\}}{9}.$$

$$= \frac{386.44 - 376.996}{9}$$

$$= 1.049$$

$$\therefore \hat{\sigma}_1 = S_1 = 1.024$$

Similarly, $s_2^2 = \frac{\left(\sum_{j=1}^{10} x_{2j}^2 - 10\bar{x}_2^2\right)}{(10-1)}$

$$= \frac{104{,}520 - 103{,}623.4}{9}$$

$$= \underline{98.62}$$

$$\therefore \hat{\sigma}_2 = s_2 = \underline{9.93}$$

$$\therefore \underline{\hat{cov}} = \frac{1}{10-1}\left(\sum_{j=1}^{10} x_{1j}x_{2j} - 10\bar{x}_1\bar{x}_2\right)$$

$$= \frac{\{(6.5*103 + 4.3*83 + 6.9*116 \dots) - 10*6.14*101.8\}}{9}$$

$$= \frac{6328.5 - 6250.52}{9} = \underline{8.66}$$

and $\underline{\text{correlation}}$ $\hat{\rho} = \frac{\hat{cov}(X_1, X_2)}{\hat{\sigma}_1 \hat{\sigma}_2}$

$$= \frac{8.66}{1.02 * 9.93}$$

$$= \underline{0.86}$$

Since $\hat{\rho} = 0.86$ is closer to 1, hence we can say that Lead time and demand are strongly dependent.

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Q1- The stock broker had recorded the following data of customer buy and sell orders (in seconds)
1.95, 1.58, 1.28, 1.04, 0.84, 0.68, 11.98, 9.71, 12.62,
10.22. Find correlation and covariance and how would you use it to model • EAR(1) Process.

⟹ Solution.

$$\hat{\mu} = \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = (1.9 + 1.58 + \cdots 10.22)/10$$
$$= \underline{5.19}$$

→ Now Calculate Lag-1 auto correlation $= \hat{\phi}$.

$$\hat{\phi} = \hat{\rho} = \frac{\hat{cov}(x_t, x_{t+1})}{\hat{\sigma}_2}$$

Where $\hat{cov}(x_t, x_{t+1}) = \frac{1}{n-1}\left[\sum_{t=1}^{n-1}(x_t, x_{t+1}) - (n-1)(\bar{x})\right]$

$$= \frac{1}{9}\left[\sum_{t=1}^{9}(x_t, x_{t+1}) - 9(5.19)^2\right]$$

$$= \frac{1}{9}\left[\{(1.95 \times 1.58) + (1.58 \times 1.28) + \cdots (12.62 \times 10.22)\} - 242.42\right]$$

$$= \frac{1}{9}\left[383.86 - 242.42\right] = \underline{15.71}.$$

$$\Rightarrow \sigma_2^2 = s^2 = \frac{\left(\sum_{i=1}^{n} x_i^2\right) - n(\bar{x})^2}{n-1}$$

$$= \left(\{(1.95)^2 + (1.58)^2 + \cdots (10.22)^2\} - 10(5.19)^2\right)/9.$$

$$= \underline{21.92}.$$

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$$\therefore \hat{\phi} = \hat{\rho} = \frac{15.71}{26.92} = \underline{0.583}$$

$$\hat{\sigma}_{\varepsilon}^2 = \sigma_x^2 (1 - \hat{\phi}^2)$$
$$= 26.92(1 - (0.58)^2)$$
$$= \underline{17.86}$$

For AR(1) Model Parameters.

$\hat{\mu}$, $\hat{\phi}$ and $\hat{\sigma}_{\varepsilon}^2$ are to be estimated.

For EAR(1) Model. $\hat{\lambda} = \frac{1}{\bar{x}} = \frac{1}{5.19} = \underline{0.19}$

and $\hat{\phi} = \underline{\underline{0.58}}$

Q1- An NGO collected the records of monthly no of job related accidental injuries at an underground coal mine for study. The records for past 100 months are as follows.

Accidental Injuries per Month	Freqn of occurrence
0	35
1	40
2	13
3	6
4	4
5	1
6	1

Apply chi square test to these data to test the hypothesis that underlying distribution is Poisson. Use a level of significance $\alpha = 0.05$.

⇒ Assume following hypothesis.

H_0: The underlying distribn is Poisson.

H_1: The underlying distribn is not Poisson.

→ Estimator of Poisson distn $\hat{\lambda} = \bar{X}$.

$$\text{where } \bar{X} = \frac{\sum f_i x_i}{n} = \frac{111}{100} = 1.11$$

$$\therefore \hat{\lambda} = \bar{X} = 1.11$$

→ PMF of Poisson

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2 \ldots$$

$$= 0 \quad \text{otherwise}$$

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The PMF and test statistics is computed in following table

x_i	O_i	$P_i = \frac{e^{-1.11}(1.11)^{x_i}}{x_i!}$		$E_i = nP_i$		$\frac{(O_i-E_i)^2}{E_i}$
0	35	0.3296		32.96		0.126
1	40	0.3658		36.58		0.320
2	13	0.2030		20.30		2.627
3	6	0.0751	0.10136	7.51	10.136	0.343
4	4	0.0208		2.08		
5	1	0.0046		0.46		
6	1	0.00086		0.086		

→ We combined the last four classes because the expected frequency E_i of them is less than 5, which reduces the number of classes to 4.

∴ The test statistics –

$$\chi_0^2 = \sum_{i=1}^{n} (O_i - E_i)^2 / E_i$$

$$= \underline{3.416}$$

→ The critical value for specified significance level α 0.05 with degree of freedom 2 is <u>5.99</u>.

$$\therefore \chi_0^2 = 3.416 < 5.99.$$

∴ <u>Ho is not rejected.</u>

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