Simulation Examples

(1)

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K. J. SOMAIYA COLLEGE OF ENGINEERING

(AFFILIATED TO THE UNIVERSITY OF MUMBAI)

Candidate Roll No. ______________

(In figures)

Test Exam.

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Date : ______________ 20

Examination : ______________ Branch/Semester ______________

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Simulation of Single channel Queue (Manual Simulation)

Assumptions

→ A grocery store has only one checkout counter

→ Customers arrive at this checkout counter at random from 1 to 8 minutes apart. Each possible value of interarrival time has the same probability of occurrence.

→ The service times vary from 1 to 6 minutes with the probabilities shown in table below.

* The Problem is to analyze the system by simulating the arrival and service of 20 customers

* Distribution of Time between Arrivals.

Time between Arrivals (Minutes)	Probability	Cumulative Probability	Random-digit Assignment
1	0.125	0.125	001 - 125
2	0.125	0.250	126 - 250
3	0.125	0.375	251 - 375
4	0.125	0.500	376 - 500
5	0.125	0.625	501 - 625
6	0.125	0.750	626 - 750
7	0.125	0.875	751 - 875
8	0.125	1.000	876 - 000.

Service Time Distribution

Service Time (min)	Probability	Cumulative Probability	Random-digit Assignment.
1	0.10	0.10.	01 – 10
2	0.20	0.30	11 – 30
3	0.30	0.60	31 – 60
4	0.25	0.85	61 – 85
5	0.10	0.95	86 – 95
6	0.05	1.00.	96 – 00.

→ A simulation of a grocery store that starts with an empty system is not realistic unless the intention is to model the system from start-up or to model until steady state operation is reached.

→ A set of uniformly distributed random numbers is needed to generate the arrivals at checkout counter. Random nos have following Properties

1) The set of random nos is uniformly distributed between 0 and 1.

2) Successive random nos are independent.

→ Random digits are converted to random nos by placing a decimal Point appropriately.

Time Between Arrivals Determination

customers	Random digit	Time between Arrivals. (Minutes)
1	–	–
2	913	8
3	727	6
4	015	1
5	948	8
6	309	3
7	922	8
8	753	7
9	235	2
10	302	3
11	109	1
12	093	1
13	607	5
14	738	6
15	359	3
16	888	8
17	106	1
18	212	2
19	493	4
20	535	5

Service Times Generated

Customer	Random digit	Service Time (Minutes)
1	84	4
2	10	1
3	74	4
4	53	3
5	17	2
6	79	4
7	91	5
8	67	4
9	89	5
10	38	3
11	32	3
12	94	5
13	79	4
14	05	1
15	79	5
16	84	4
17	52	3
18	55	3
19	30	2
20	50	3

The essence of manual simulation is simulation table

A Customer	B Time since last arrival (Min)	C Arrival Time	D Service Time (Min)	E Service Time begins	F Time customer waits in queue (Min)	G Service Time Ends	H Time customer spends in system (Min)	I Idle time of server (Min)
1	—	0	4	0	0	4	4	0
2	8	8	1	8	0	9	1	4
3	6	14	4	14	0	18	4	5
4	1	15	3	18	3	21	6	0
5	8	23	2	23	0	25	2	2
6	3	26	4	26	0	30	4	1
7	8	34	5	34	0	39	5	4
8	7	41	4	41	0	45	4	2
9	2	43	5	45	2	50	7	0
10	3	46	3	50	4	53	7	0
11	1	47	3	53	6	56	9	0
12	1	48	5	56	8	61	13	0
13	5	53	4	61	8	65	12	0
14	6	59	1	65	6	66	7	0
15	3	62	5	66	4	71	9	0

16	8	70	4	71	1	75	5	0
17	1	71	3	75	4	78	7	0
18	2	73	3	78	5	81	8	0
19	4	77	2	81	4	83	6	0
20	5	82	3	83	1	86	4	0
			68		56		124	18

1) The average waiting time for a customer

$= \dfrac{\text{total time customers wait in queue}}{\text{total no of customers}} = \dfrac{56}{20}$

$= 2.8$ min

2) The Probability that a customer has to wait in queue $=$ Probability (wait) $= \dfrac{\text{no of customers who wait}}{\text{total no of customers}}$

$= 13/20 = 0.65$

3) The fraction of idle time of server.

Probability of idle server $= \dfrac{\text{total idle time of server}}{\text{total run time of simulation}}$

$= 18/86 = 0.21$

4) The average service time

$= \dfrac{\text{total service time}}{\text{total no of customers}} = 68/20 = 3.4$ min

5) Average waiting time $= \dfrac{\text{total time customers wait in queue}}{\text{total no of customers who wait}}$

$= 56/13 = 4.3$ (min)

6) The average time a customer spends in the system.

$= \dfrac{\text{total time customers spend in system}}{\text{total no of customers}}$

$= 124/20 = 6.2$ (min).

$\therefore$ Avg time customer spends in system $=$ Avg time customer spends waiting $+$ Avg time customer spends in service $= 2.8 + 3.4$

$= 6.2$ min

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Simulation Examples

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1) Students arrive at library with inter arrival time distribution as follows

Time between arrivals:	3	4	5	6
Probability:	0.3	0.5	0.15	0.05

The time for transaction at counter is given by following distribution table,

RD: 55, 25, 78, 89, 02, 67, 34, 21, 18, 93.

Transaction time:	2	3	4	5
Probability	0.2	0.4	0.25	0.15

RD: 34, 78, 65, 11, 23, 28, 69, 89, 56, 28.

If more than two students are in queue, arriving student goes away w/o joining the queue. Based on simulation of 10 students determine balking rate (leaving the queue)

Ans →

I.A. Time Distribⁿ

Time	Prob	Cumulative Prob	R.D
3	0.3	0.3	0-29
4	0.5	0.8	30-79
5	0.15	0.95	80-94
6	0.05	1	95-99

Transⁿ Time Distribution

Time	Prob	Cumulative Prob.	R-D
2	0.2	0.2	0-19
3	0.4	0.6	20-59
4	0.25	0.85	60-84
5	0.15	1	85-99.

⇛

Student	RD Inter Arrival	Inter Arrival Time	Arrival Time	RD Transⁿ	Transⁿ Time	Transⁿ Begins	Transⁿ Ends	Wait time	Queue
1	—	—	0	34	3	0	3	0	—
2	55	4	4	78	4	4	8	0	—
3	25	3	7	65	4	8	12	1	3 in queue
4	78	4	11	11	2	12	14	1	4 in queue.
5	89	5	16	23	2	16	19	0	—
6	02	03	19	28	3	19	22	0	—
7	67	04	23	69	04	23	27	0	—
8	34	04	27	89	05	27	32	0	—
9	21	03	30	56	03	32	35	2	9 in queue.
10	18	03	33	28	03	35	38	2	10 in queue.

⇛ it can be seen that from table no of Students in queue ~~was~~ is not greater than 2 Thus no Student left the queue Thus Balking rate = 0.

2). A bank has one drive in teller and room for one additional customer to wait. Customers arriving when the queue is full, park and go inside the bank to transact business. The times between arrivals and service time distribution follow:—

Time between Arrivals (Min)	Prob	Service time (Minutes)	Probability
0	0.09	1	0.20
1	0.17	2	0.40
2	0.27	3	0.28
3	0.20	4	0.12
4	0.15		
5	0.12		

Time in system
3
4
5
3
3
3
4
5
5
5

Q21- continue.

Simulate the operation of drive in teller for 10 new customers. The first of 10 new customers arrives at a time determined at random. Start the simulation with one customer being served leaving at time 3, and one in the queue. How many customers went into the bank to transact business?

⇒ Solution

Cust	RD for Arrival	IAT	AT	RD for Service	S.T	No in queue	S.T Begins	S.T Ends	Going Bank
1	30	2	2	27	2	1	-	.	✓
2	46	2	4	26	2	0	4	6	
3	39	2	6	99	4	0	6	10	
4	86	4	10	72	3	0	10	13	
5	63	3	13	12	1	0	13	14	
6	83	4	17	17	1	0	17	18	
7	07	0	17	78	3	1	18	21	
8	37	2	19	91	4	1	-	-	✓
9	69	3	22	42	3	0	22	25	
10	78	4	26	62	3	0	26	29	

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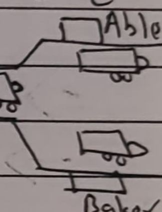
Two Server Problem

* Simulating a Queue with two Servers *

→ This Queuing example has a queue with two servers that have different service times.

Example → A drive in restaurant where carhops take orders and bring food to the car.



Assumptions

1) Two carhops Able and Baker - Able is better able to do the job and works a bit faster than Baker.
2) The distribution of their service times is given.
3) A simplifying rule is that Able gets the customer if both carhops are idle.
4) If both are busy the customer begins service with first server to become free.
5) To estimate the system measures of performance a simulation of 1 hour of operation is made.
6) The problem is to find how well the current arrangement is working.

⟹ Interarrival Distribution of Cars

Time betwn Arrivals (Min)		Prob	Cumulative Prob	R-D Assignment
1	0.25	0.25		01-25
2	0.40	0.65		26-65
3	0.20	0.85		66-85
4	0.15	1.00		86-00

→ Service distribution of Able

S.T (Minutes)	Prob	Cumulative Prob	R-D Assignment
2	0.30	0.30	01–30
3	0.28	0.58	31–58
4	0.25	0.83	59–83
5	0.17	1.00	84–00

⇒ Service Distribution of Baker

S.T (Min)	Prob	Cumulative Prob	R-D Assignment
3	0.35	0.35	01–35
4	0.25	0.60	36–60
5	0.20	0.80	61–80
6	0.20	1.00	81–00

* Simulation Table

A Customer No	B R-D Arrival	C IAT	D A.T	E R-D Service	F Able S.T Begins	G Able S.T	H Able S.T Ends	I Baker S.T Begins	J Baker S.T	K Baker S.T Ends	L Time in Queue
1	-	-	0	95	0	5	5				0
2	26	2	2	21				2	3	5	0
3	98	4	6	51	6	3	9				0
4	90	4	10	92	10	5	15				0
5	26	2	12	89				12	6	18	0
6	42	2	14	38	15	3	18				1
7	74	3	17	13	18	2	20				1
8	80	3	20	61	20	4	24				0
9	68	3	23	50				23	4	27	0
10	22	1	24	49	24	3	27				0
11	48	2	26	39	27	3	30				1
12	34	2	28	53				28	4	32	0
13	45	2	30	88	30	5	35				0
14	24	1	31	01				32	3	35	1
15	34	2	33	81	35	4	39				2

A	B	C	D	E	F	G	H	I	J	K	L
16	63	2	35	53				35	4	39	0
17	38	2	37	81	39	4	43				2
18	80	3	40	64				40	5	45	0
19	42	2	42	01	43	2	45				1
20	56	2	44	67	45	4	49				1
21	89	4	48	01				48	03	51	0
22	18	1	49	47	49	3	52				0
23	51	2	51	75				51	5	56	0
24	71	3	54	57	54	3	57				0
25	16	1	55	87				56	6	62	1
26	92	4	59	47	59	3	62				0
						56			43		11
						Able S.T total			Baker S.T. total		

⇒ The analysis of above table:-

→ Over the 62 minute Period able was busy 90% of the time.
i.e. 56/62 → Able S.T. / Total S.T
∴ 0.90 ∴ 90%.

→ Baker was busy only 69% of the time. The Seniority rule keeps baker less busy
∴ 43/62 = 0.69 ∴ 69%.

→ Nine of the 26 arrivals (about 35%) had to wait.
i.e 9/26 i.e. 34.61 i.e 35%.

→ The average waiting time for all customers was only about 0.42 minute (25 seconds), which is very small.
i.e. 11/26 i.e 0.42 min

→ Those Nine who have to wait only waited an average of 1.22 min, which is quite low.

i.e 11/9 i.e 1.22 min

→ In summary this system seems well balanced. one server cannot handle all the diners, and three servers would probably be too many. Adding an additional server would surely reduce the waiting time to nearly zero. However the cost of waiting would have to be quite high to justify an additional server.

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* The news Dealer's Problem *

→ A classical inventory Problem concerns the Purchase and sale of newspapers. The newsstand buys the Papers for 33 cents each and sells them for 50 cents each. Newspapers not sold at the end of the day are sold as scrap for 5 cents each.

→ Newspapers can be purchased in bundles of 10. Thus the newsstand can buy 50, 60 and so on.

→ There are three types of news days: "good", "fair" and "Poor", they have Probabilities 0.35, 0.45 and 0.20 respectively.

→ The distribution of newspapers demanded on each of these days is given in table below.

→ The Problem is to compute the optimal number of Papers the newsstand should purchase. This will be accomplished by simulating demands for 20 days and recording Profits from sales each day.

→ Distribution of NewsPapers Demanded Per day

Demand	Demand Probability Distrib^n Good	Fair	Poor	R-D Assignment Good	Fair	Poor
40	0.03	0.10	0.44	01-03		
50	0.05	0.18	0.22			
60	0.15	0.40	0.16			
70	0.20	0.20	0.12			
80	0.35	0.08	0.06			
90	0.15	0.04	0.00			
100	0.07	0.00	0.00			

⇒ The profits are given by following relationship.

Profit = { revenue from sales } − { cost of newspapers } − { lost profit from excess demand } + { salvage from sale of scrap papers }

⇒ From the problem statement the revenue from sales is 50 cents for each paper sold. The cost of newspapers is 33 cents for each paper purchased. The lost profit from excess demand is 17 cents for each paper demanded that could not be provided.

→ Such a shortage cost is somewhat controversial, but makes the problem much more interesting. The salvage value of scrap papers is 5 cents each.

⇒ Random Digit Assignment for type of Newsday

Type of Newsday	Prob	Cumulative Prob	R-D Assignment
Good	0.35	0.35	01-35
Fair	0.45	0.80	36-80
Poor	0.20	1.00	81-00

⇒ Random Number for Newspapers Demanded

Demand	Cumulative Distribution Good	Fair	Poor	R-D Assignment Good	Fair	Poor
40	0.03	0.10	0.44	01-03	01-10	01-44
50	0.08	0.28	0.66	04-08	11-28	45-66
60	0.23	0.68	0.82	09-23	29-68	67-82
70	0.43	0.88	0.94	24-43	69-88	83-94
80	0.78	0.96	1.00	44-78	89-96	95-00
90	0.93	1.00	1.00	79-93	97-00	
100	1.00	1.00	1.00	94-00		

Simulation Table for Purchase of 70 Newspapers

Day	R-D for Newsday type	Type of Newsday	R-D for Demand	Demand	Revenue from Sales	Lost Profit from Excess Demand	Salvage from Sale of Scrap	Daily Profit
1	58	Fair	93	80	$35.00	$1.70	–	$10.20
2	17	Good	63	80	35.00	1.70	–	10.20
3	21	Good	31	70	35.00	–	–	11.90
4	45	Fair	19	50	25.00	–	1.00	2.90
5	43	Fair	91	80	35.00	1.70	–	10.20
6	36	Fair	75	70	35.00	–	–	11.90
7	27	Good	84	90	35.00	3.40	–	8.50
8	73	Fair	37	60	30.00	–	0.50	7.40
9	86	Poor	23	40	20.00	–	1.50	-1.60
10	19	Good	02	40	20.00	–	1.50	-1.60
11	93	Poor	53	50	25.00	–	1.00	2.90
12	45	Fair	96	80	35.00	1.70	–	10.20
13	47	Fair	33	60	30.00	–	0.50	7.40
14	30	Good	86	90	35.00	3.40	–	8.50
15	12	Good	16	60	30.00	–	0.50	7.40
16	41	Fair	07	40	20.00	–	1.50	-1.60
17	65	Fair	64	60	30.00	–	0.50	7.40
18	57	Fair	94	80	35.00	1.70	–	10.20
19	18	Good	55	80	35.00	1.70	–	10.20
20	98	Poor	13	40	20.00	–	1.50	-1.60
					$600.00	$17.00	$10.00	$131.00

→ On day 1, the demand is for 80 newspapers, but only 70 newspapers are available. The revenue from the sale of 70 newspapers is $35.00. The lost profit for excess demand of 10 newspapers is $1.70

∴ The profit is

Profit = 35.00 − 23.10 − 1.70 + 0 = 10.20

→ On the fourth day, the demand is less than the supply. The revenue from sales of 50 newspapers is 25.00. Twenty newspapers are sold for scrap at 0.05 each yielding 1.00. The daily profit is.

$$\text{Profit} = 25.00 - 23.10 - 0 + 1 = 2.90.$$

→ The profit for 20-day period is sum of daily profits 131.00, it can also be computed from the totals for 20 days of simulation as follows,

$$\text{Total Profit} = 600.00 - 462.00 - 17.00 + 10.00$$
$$= \underline{131.00}$$

where cost of newspapers for 20 days is

$$(20 \times 0.33 \times 70) = \underline{462.00}.$$

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Inventory Simulation

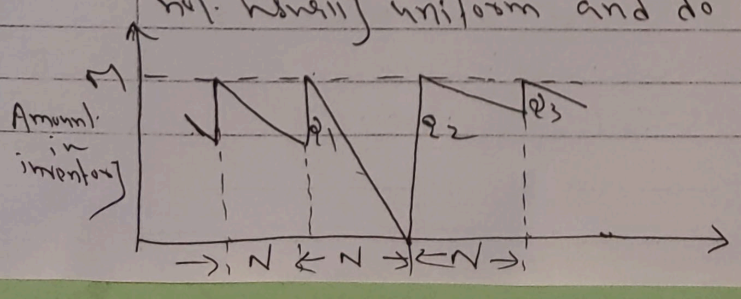
→ An important class of simulation problems involves inventory systems. The simple inventory system shown in fig is called an (M,N) system, which has an inventory review every N time periods and maximum inventory level M.

→ At each N time units, the inventory level is checked and an order is made to bring the inventory up to the level M.

→ For simplicity, the lead time (i.e, the length of time between the placement and receipt of an order) is shown as zero in fig. orders arrive immediately after being placed.

→ In reality lead time may vary greatly and would need to be modeled, as a known constant or as a random variable.

→ Demands are not usually known with certainty, so the order quantities are modeled by a probability distribution. In the fig, demand is shown as being uniform over the time period. In ~~actually~~ reality, demands are not usually uniform and do fluctuate over time.



Probabilistic order level inventory system

→ At the end of first review period, an order quantity, Q_1 is placed to bring the inventory level up to M. This is repeated at every inventory review.

→ In fig in the second cycle the amount in inventory drops below zero, indicating a shortage these units are backordered. When the order arrives, the demand for backordered items is satisfied first. To avoid shortages, a buffer or safety stock would need to be carried. In some inventory systems a portion of sales may be lost rather than backordered when inventory runs out.

→ Inventory systems have a number of potential sources of revenue and costs:-

1) Revenue from sales
2) Cost of items sold.
3) Carrying cost, the cost of carrying stock in inventory
4) Ordering cost the cost associated with placing orders.
5) Cost of lost sales or backorders.
6) Cost due to scrap.
7) Salvage revenue from scrap or other damaged goods.

→ The carrying cost can be attributed to the interest paid on funds borrowed to buy the items (or equivalently, the loss from not having the funds available for other investment purposes). Carrying cost also include the cost for handling and storage space, hiring of guards and other similar costs. Ordering cost may include shipping and transportation and discounts depending on order quantity.

⇒ The total cost (or total profit) of an inventory system is the measure of performance. This can be affected by policy alternatives., for e.g the decision maker can control the maximum inventory level M and length of cycle N.

⇒ In a model of an (M,N) inventory system, the events that may occur are the demand for items in the inventory, the review of the inventory position and resulting decision to place a replenishment order, and the arrival of replenishment stock. When the lead time is assumed to be a zero, the last two events occur at the same time.

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Simulation of (M,N) inventory system

→ This example follows the pattern of Probabilistic order level inventory system.

→ Suppose that the maximum inventory level M, is 11 units and review period, N is 5 days.

→ The Problem is to estimate by simulation the average ending units in inventory and no of days when shortage condition occurs.

→ The distribution of number of units demanded per day is shown in table.

→ In this example, lead time is a random variable.

→ Assume that orders are placed at the close of business and are received for inventory at the beginning of business as determined by lead time.

→ In this example, only 5 cycles are shown.

→ R-D Assignment for daily demand

Demand	Prob	Cumulative Prob.	R-D Assignment
0	0.10	0.10	01-10
1	0.25	0.35	11-35
2	0.35	0.70	36-70
3	0.21	0.91	71-91
4	0.09	1.00	92-00

R-D Assignment for lead time

Lead time (Days)	Prob	Cumulative Prob.	R-D Assignment
1	0.6	0.6	1-6
2	0.3	0.9	7-9
3	0.1	1	0

Simulation Table

Cycle	Day	Beginning Inventory	R-D for demand	Demand	Ending Inventory	Shortage Quantity	Order Quantity	R-D for lead time	Days until order Arrives
1	1	3	24	1	2	0	—	—	1
	2	2	35	1	1	0	—	—	0
	3	9	65	2	7	0	—	—	—
	4	7	81	3	4	0	—	—	—
	5	4	54	2	2	0	9	5	1
2	1	2	03	0	2	0	—	—	0
	2	11	87	3	8	0	—	—	—
	3	8	27	1	7	0	—	—	—
	4	7	73	3	4	0	—	—	—
	5	4	70	2	2	0	9	0	3
3	1	2	47	2	0	0	—	—	2
	2	0	45	2	0	2	—	—	1
	3	0	48	2	0	4	—	—	0
	4	9	17	1	4	0	—	—	—
	5	4	09	0	4	0	7	3	1
4	1	4	42	2	2	0	—	—	0
	2	9	87	3	6	0	—	—	—
	3	6	26	1	5	0	—	—	—
	4	5	36	2	3	0	—	—	—
	5	3	40	2	1	0	10	4	1
5	1	1	07	0	1	0	—	—	0
	2	11	63	2	9	0	—	—	—
	3	9	19	1	8	0	—	—	—
	4	8	88	3	5	0	—	—	—
	5	5	94	4	1	0	10	8	2
					88				

→ The simulation has been started with the inventory level at 3 units and an order of 8 units scheduled to arrive in 2 days time.

→ The order for 8 units is available on the morning of the third day of first cycle, raising the inventory level from 1 unit to 9 units.

→ Demands during the remainder of first cycle reduced the ending inventory level to 2 units on the fifth day. Thus an order for 9 units was placed. The lead time for this order was 1 day. The order of 9 units was added to inventory on the morning of day 2 of cycle 2.

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Test Exam.

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Question No.	1	2	3	4	5	6	7	8	9	10	11	12	Total
Marks Obtained													

→ A baker is trying to determine how many dozens of bagels to bake each day. The Probability distribution of number of bagel customers / day is given as.

No of customers / day :-	8	10	12	14
Probability :-	0.35	0.30	0.25	0.10

Customers order 1, 2, 3 or 4 dozen bagels according to probability distribution given in table below.

No of dozen ordered / customer.	1	2	3	4
Probability	0.4	0.3	0.2	0.1

Bagels sell for R.s 5.40 per dozen. They cost R.s 3.80 per dozen to make. All bagels not sold at the end of the day are sold at half price to a local grocery store. Based on 5 days of simulation how many dozen bagels should be backed each day?

Random digit for bagel customers / day and dozens ordered / customer is given below.

R.D for customers / day 44 33 95 77 10

R.D for Dozens ordered / customer 8 2 4 8 1 6 3 0 2 0 9 1 0 0 6 3 8 5 1 3 5 0 7
1 9 2 6 7 5 3 4 0 1 0 2 9 8 6 3 4 0 7
4 5 9 6 3 9 0 2 8 7 5.

Solution

Dozens ordered	Probability	Cumulative Probability	R-D Assignment
1	0.4	0.4	1-4
2	0.3	0.7	5-7
3	0.2	0.9	8-9
4	0.1	1.00	0

No of customers / day	Probability	Cumulative Probability	R-D Assignment
8	0.35	0.35	01-35
10	0.30	0.65	36-65
12	0.25	0.90	66-90
14	0.10	1.00	91-00

→ Here, the problem is to find out the optimal dozens of bagels that should be baked on each day. We can solve the optimal solution by simulating the demand for 5 days and recording the profits from sale on each day. The profit is given by

Profit = (Revenue from sale) − (cost of bagels baked) + (Revenue from grocery store sale) − (lost profit)

Let Q = no of dozens baked / day.

$S = \sum_i Q_i$, where Q_i = order quantity in dozens for ith customer.

if $Q > S$ then $Q - S$ and it represents the grocery store sales in dozens else $S - Q$ and it represents dozens of excess demand.

$$\text{Profit} = (5.40 \times \min(S, Q)) - (3.80 \times Q) + (2.70 \times (Q - S)) - (1.60 \times (S - Q))$$

Pre analysis

→ $E(\text{No of customers}) = 0.35(8) + 0.30(10) + 0.25(12) + 0.10(14) = 10.20$

→ $E(\text{Dozens ordered}) = 0.4(1) + 0.3(2) + 0.2(3) + 0.1(4) = 2.0$

→ $E(\text{Dozens sold}) = \bar{S} = (10.20)(2) = 20.4$

→ $E(\text{Profit}) = 5.40\,\text{Min}(\bar{S}, Q) - 3.80\,Q + 2.70(Q - \bar{S}) - 1.60(\bar{S} - Q)$

$= 5.40\,\text{Min}(20.4, Q) - 3.80Q + 2.70(Q - 20.4) - 1.60(20.4 - Q)$

$E(\text{Profit} \mid Q = 0) = 0 - 0 + 1.60(20.4) = -32.64$

$E(\text{Profit} \mid Q = 10) = 5.40(10) - 3.80(10) + 0 - 1.60(20.4 - 10) = -0.64$

$E(\text{Profit} \mid Q = 20) = 5.40(20) - 3.80(20) + 0 - 1.60(20.4 - 20) = \text{Rs } 31.36$

$E(\text{Profit} \mid Q = 30) = 5.40(20.4) - 3.80(30) + 2.70(30 - 20.4) - 0 = \text{Rs } 22.08$

$E(\text{Profit} \mid Q = 40) = 5.40(20.4) - 3.80(40) + 2.70(40 - 20.4) - 0 = \text{Rs } 11.08$

$E(\text{Profit} \mid Q = 50) = 5.40(20.4) - 3.80(50) + 2.70(50 - 20.40) - 0 = \text{Rs } 0.08$

The pre analysis based on expectation only indicates that simulation of the policies $Q = 20, 30, 40$ and 50 should be sufficient to determine the policy. The simulation should begin with $Q = 20$, then proceed to $Q = 30$, then most likely to $Q = 40$ and $Q = 50$.

Initially conduct a simulation for $Q = 20, 30, 40$ and 50. If profit is maximized when $Q = 20$, it will become the policy recommendation.

→ The problem requests that the simulation for each policy should run for 5 days. This is very short run length to make a policy decision.

→ Here we are simulating this exp for $Q = 40$ dozens and it is given as below

Given that cost price of 1 dozen Bagel = Rs 3.80

Selling price of 1 dozen Bagel = Rs 5.40

Cost price of 40 dozens Bagel = Rs 40×3.80

= Rs 152.00

Profit = (Revenue from sale) − (152.00) + (Revenue from grocery store sale) − (Lost profit).

S.No	Days	RD	No of customer	RD	No of customers/day	Demand	Dozen ordered	Total no of Dozens sold on each day	Revenue from sale	Revenue from grocery store sale (Rs)

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Question No.	1	2	3	4	5	6	7	8	9	10	11	12	Total
Marks Obtained													

Sr. No	Day	R.D. Customer	No of Customers / Day	R.D. Demand	No of Dozen ordered	Total no of Dozen sold on each day	Revenue from Sale (R.s)	Revenue from Grocery Store Sale (R.s)	Loss Profit	Daily Profit
1	1	44	1.0	8	3					
2				2	1					
3				4	1					
4				8	3	21	113.90	51.30	0	12.70
5				1	1					
6				6	2					
7				3	1					
8				0	4					
9				2	1					
10.				0	4					
11	2	33.	8	9	3					
12				1	1					
13.				0	4	20	108	54.00	0	10.00
14				0	4					
15				6	2					
16				3	1					
17				8	3					
18				5	2					

19	3	95	14	1	1					
20				3	1					
21				5	2					
22				0	4	27	145.8	35.10	0	28.90
23				7	2					
24				1	1					
25				9	3					
26				2	1					
27				6	2					
28				7	2					
29				5	2					
30				3	1					
31				4	1					
32				0	4					
33	4	77	12	1	1					
34				0	4					
35				2	1					
36				9	3	27	145.8	35.10	0	28.90
37				8	3					
38				6	2					
39				3	1					
40				4	1					
41				0	4					
42				7	2					
43				5	2					
44				9	3					
	5	10	8	6	2					
				3	1					
				9	3	18	97.20	59.40	0	4.60
				0	4					
				2	1					
				8	3					
				7	2					
				5	2					

→ Total Profit = 12.70 + 10.00 + 28.90 + 28.90 + 4.60
= R.S 85.10

→ Average No. of Dozens Bagels Sold / Day =
113/5 = 23

→ The comparative simulation table of baking the bagels for different values of Q is given in table below.

Day	R.D (cust)	No of cust	cust. No	R.D Demand	Demand in Dozen	Total no of Dozen sold	Revenue from sale: No of bagels baked 20	30	40	50	Sale from grocery store 20	30	40	50	Loss profit 20	30	40	50	Daily profit 20	30	40	50
1	~~44~~	10	1	8	3	21	108	113.4	113.4	113.4	0	24.3	51.3	78.3	1.60	0	0	0	30.4	23.7	12.7	1.7
			2	2	1																	
			3	4	1																	
			4	8	3																	
			5	1	1																	
			6	6	2																	
			7	3	1																	
			8	0	4																	
			9	2	1																	
			10	0	4																	
2	33	8	1	9	3	20	108	108	108	108	0	27	54	81	0	0	0	0	32	21	10	-1
			2	1	1																	
			3	0	4																	
			4	0	4																	
			5	6	2																	
			6	3	1																	
			7	8	3																	
			8	5	2																	
⋮																						

Total Profit of all | 130.6, 140.1, 85.1, 30

Conclusion

At the end of 5th day it is observed that we are getting maximum profit when $Q = 30$ hence we conclude that the baker should baked 30 dozens bagels on each day. But the Pre-analysis recommended that expected value of $Q = 20$. So, to get actual value same as that of expected one, we have to run simulation for more no of days.

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Test Exam.

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Question No.	1	2	3	4	5	6	7	8	9	10	11	12	Total
Marks Obtained													

A Reliability problem

→ A large manufacturing machine has three different bearings that fail in service.

→ The distribution function of life of each bearing is identical.

→ When a bearing fails, the machine stops working so a repairperson is called, and a new bearing is installed.

→ The delay time of repairperson arriving at the manufacturing machine is also a random variable.

→ Downtime for machine is estimated at Rs x per minute.

→ The direct on site cost of the repairperson is Rs y per hour.

→ It takes 'n_1' minutes to change one bearing 'n_2' min to change two bearing and 'n_3' min to change three bearings, where $n_1 < n_2 < n_3$.

→ The bearings cost is Rs Z each

Current Method

→ If a bearing fails only that bearing is replaced which means that only one bearing is changed at any breakdown.

Proposed Method

→ If a bearing fails, replace all three bearings.

→ Management needs an evaluation of this Proposed method.

→ This is the Problem where we are comparing two systems and the best one will be implemented.

Example: A large manufacturing machine has three diff bearings that fail in service. The distribution function of life of each bearing is identical which is given below.

Bearing life (Hours)	1100	1200	1300	1400	1500
Probability	0.10	0.25	0.15	0.30	0.20

Delay time (Min)	5	10	15
Probability	0.4	0.5	0.1

Downtime for machine is estimated at Rs 10 per min. The direct on site cost of repairperson is Rs 25 per hour. It takes 10 min to change one bearing, 20 min to change two bearings and 30 min to change three bearings. The cost of one bearing is Rs 20

There are two Methods
a) Current Method
b) Proposed Method

Management needs an evaluation of Proposed method. Run the simulation for 10,000 hrs of operation.

Solution

⇒ Using the Probability of life of bearing we can assign the random digit to bearing life and is given in table below. Similarly for delay tim

⇒

Bearing life (Min)	Prob	Cumulative Prob	R-D Assignment
1100	0.10	.10	01-10
1200	0.25	0.35	11-35
1300	0.15	0.50	36-50
1400	0.30	0.80	51-80
1500	0.20	1.00	81-00

Delay time (Min)	Prob	Cum Prob	R-D
5	0.4	0.4	1-4
10	0.5	0.9	5-9
15	0.1	1.0	0

Current Method :- As the accumulated life time of all three bearings reach to 10,000 hrs or more, then we stop the simulation. Also it is mentioned we replace one bearing at a time in case of current method.

	Bearing 1					Bearing 2					Bearing 3				
Sr. N.	RD life	life (Hrs)	Accu life Hrs	RD Delay	Delay (Min)	RD life	life (Hrs)	Accu life (Hrs)	RD Delay	Delay (Min)	RD life	life (Hrs)	Accum life (Hrs)	RD Delay	Delay (Min)
1	33	1200	1200	3	5	76	1400	1400	5	10	88	1500	1500	8	10
2	62	1400.	2600	5	10	34	1200	2600	2	5	50	1300	2800	5	10
3	1	1100	3700.	2	5	21	1200	3800	8	10	12	1200	4000	0	15
4	56	1400	5100.	4	5	87	1500	5300	1	5	1	1100	5100	3	5
5	78	1400	6500	0	15	0	1500	6800	6	10	41	1300	6400	4	5
6	45	1300	7800	6	10	91	1500	8300	3	5	29	1200	7600	2	5
7	23	1200	9000	1	5	45	1300	9600	9	10	35	1200.	8800	6	10
8	89	1500	10500	9	10	67	1400	11000	0	15	9	1100	9900	7	5
9	71					59					53	1400	11300.	1	5
10	66				Σ 65	11				Σ 70	7.60.				Σ 75

→ At the end of simulation for 10,000 hrs, 25 bearings are replaced.

→ The total cost of current method is computed as follows.
cost of bearings = 25 bearings × R.s 20/bearing = R.s 500

→ cost of delay = (65+70+75) min × R.s 10/min = R.s 2100

→ cost of downtime during repair = 25 bearings × 10 min/bearing × R.s 10/min = R.s 2500

→ cost of repair person = 25 bearing × 10 min/bear × R.s 25/60 min = R.s 104.17

Total cost = R.s (500 + 2100 + 2500 + 104.17)
= R.s 5204.17

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Question No.	1	2	3	4	5	6	7	8	9	10	11	12	Total
Marks Obtained													

Proposed Method

→ In this method the bearing life time is same as that of used in current method. As proposed method requires more bearings than the current method, the simulation requires a new set of random digits for generating the additional life times of bearings. Additional life times required at 9th replacement of bearing number 1 and 2.

→ In this method we replace all three bearings at a time when first failure occured. The repair person delay generated independently using diff sets of random digits.

Sr. No	Bearing 1 life (Hrs)	Bearing 2 life (Hrs)	Bearing 3 life (Hrs)	First failure (Hrs)	Accumulated life (Hrs)	RD Delay	Delay (Min)
1	1200	1400	1500	1200	1200	1	5
2	1400	1200	1300	1200	2400	4	5
3	1100	1200	1200	1100	3500	3	5
4	1400	1500	1100	1100	4600	7	10
5	1400	1500	1300	1300	5900	8	10
6	1300	1500	1200	1200	7100	2	5
7	1200	1300	1200	1200	8300	6	10
8	1500	1400	1100	1100	9400	9	10
9	71 | 1400	59 | 1400	1400	1400	10800	0	15
							Σ 75

→ At the end of simulation for 10,000 hrs 9 sets of bearings are replaced.

→ The total cost of proposed method is computed as follows.

→ cost of bearing = 27 bearings × R.s 20/bearing
= R.s 540

→ cost of delay = 75 min × R.s 10/min
= R.s 750

→ cost of down time during repair = 9 sets × 30 min/set × R.s 10/min
= R.s 2700

→ cost of repair person = 9 sets × 30 min/set × R.s 25/60 min
= R.s 112.5

∴ Total cost = R.s (540 + 750 + 2700 + 112.5)
= R.s 4102.5

∴ The proposed method has a saving of R.s 1101.67 over a period of 10,000 hrs simulation.

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Question No.	1	2	3	4	5	6	7	8	9	10	11	12	Total
Marks Obtained													

Q1). A Small-town Taxi operates one vehicle during the 9:00 AM to 5:00 P.m period. currently Consideration is being given to the addition of a second vehicle to the fleet. The demand for taxis follows the distribution shown.

Time between calls (Minutes) :-	15	20	25	30	35
Probability :-	0.14	0.22	0.43	0.17	0.04

The distribution of time to complete a service as follows.

Service time (Minutes)	5	15	25	35	45
Probability	0.12	0.35	0.43	0.06	0.04

Simulate 5 individual days of opern of current system and the system with an additional taxi cab. compare the two systems w.r.t to the waiting times of the customers and any other measures that might shed light on the situation.

Solution ⇒

Time between calls	Prob	C.Prob	R.D
15	0.14	0.14	01-14
20	0.22	0.36	15-36
25	0.43	0.79	37-79
30	0.17	0.96	80-96
35	0.04	1.00	97-00

S.T	Prob	C.Prob	R-D
5	0.12	0.12	01-12
15	0.35	0.47	13-47
25	0.43	0.90	48-90
35	0.06	0.96	91-96
45	0.04	1.00	97-00

Simulation of One Taxi

Day	Call	RD. Time between calls.	Time betw calls.	Call Time	R.D S.T	S.T	S.T begins.	Time customer waits.	S.T ends.	Time customer in system	idle time of Taxi
	1	15	–	0	01	5	0	0	5	5	0
1	2	01	~~20~~ 15	~~20~~ 15	53	25	15	0	40	25	10
	3	14	15	30	62	25	40	10	65	35	

This will continue for 5 days

→ The taxi operates from 9:00 AM to 5:00 P.M. i.e for 8 hrs. i.e 60x8 = 480 mins
i.e we have to stop simulation when S.T ends column cross 480 mins

Simulation of 2 taxis

Day	Call	Time betw call	Call Time	S.T	Taxi 1: S.T begins	Taxi 1: S.T	Taxi 1: S.T ends	Taxi 2: S.T begins	Taxi 2: S.T	Taxi 2: S.T ends	Time customer waits.	Time customer in system	idle Time Taxi 1	Idle Time Taxi 2
1	1	–	0	5	0	5	5				0	5		
	2	15	15	25	15	25	40				0	25	10.	
	3	15	30	25				30	25	55	0	25		30.

Typical Parameters we can calculate from the simulation are.

1) Idle time for taxi 1
2) Idle time for taxi 2
3) Total idle time
4) Average idle time per call
5) Total time customer wait
6) Average waiting time per customer.
7) No of customers that wait.

Exercise Problems

*2) An elevator in a manufacturing plant carries exactly 400 k.g.s of material. There are three kinds of material which arrive in boxes of known weight. These materials and their distributions of time between arrivals are as follows.

Material	Weight (k.g.s)	Interarrival Time (Min)
A	200	5 ± 2 (uniform).
B	100	6 (constant).
C	50	P(2) = 0.33 P(3) = 0.67

→ It takes the elevator 1 minute to go up to second floor 2 minutes to unload and 1 min to return to the first floor. The elevator does not leave the first floor unless it has a full load; Simulate 1 hr of operation of system. What is avg transit time for a box of material A (time from its arrival until it is unloaded)? What is avg waiting time for box of material B? How many boxes of material made the trip in 1 hour.

Solution

⇒ Material A (200 K.g / box)

Interarrival time	Prob	Cumu Prob	RD
3	0.2	0.2	1-2
4	0.2	0.4	3-4
5	0.2	0.6	5-6
6	0.2	0.8	7-8
7	0.2	1.0	9-0.

Box	RD interarrival time	Interarrival time	clk time
1	1	3	3
2	4	4	7
3	8	6	13
4	3	4	17
⋮		⋮	⋮
14	4		60.

→ Material B (100kg/box).

Box	1	2	3	---	10.
Clk time	6	12	18	---	60.

→ Material C (50kg/box)

interarrival time	Prob	Cumu Prob	RD
2	0.33	0.33	01-33
3	0.67	1.00.	34-00.

Box	RD interarrival time	interarrival time	Clk time
1	58	3	3
2	92	3	6
3	87	3	9
4	31	2	11
⋮	⋮	⋮	⋮
22	62	3	60.

Clk time	A Arrival	B Arrival	C Arrival
3	1		1
6		1	2
7	2		
9			3
11			4
12		2	
⋮			