① Find the gradient of scalar function $f(x,y) = x + y^2$

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\vec{\nabla} f = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$$

$$\vec{\nabla} f = \hat{i}\frac{\partial}{\partial x}(x+y^2) + \hat{j}\frac{\partial}{\partial y}(x+y^2) + \hat{k}\underbrace{\frac{\partial}{\partial z}(x+y^2)}_{0}$$

$$\vec{\nabla} f = \hat{i}(1+0) + \hat{j}(0+2y) + 0$$

$$\vec{\nabla} f = \underline{\underline{\hat{i} + 2y\hat{j}}}$$

② find the gradient of scalar field $f(x,y) = 2x + 3z$

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\vec{\nabla} f = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$$

$$\vec{\nabla} f = \hat{i}\frac{\partial}{\partial x}(2x+3z) + \hat{j}\underbrace{\frac{\partial}{\partial y}(2x+3z)}_{0} + \hat{k}\frac{\partial}{\partial z}(2x+3z)$$

$$\vec{\nabla} f = \hat{i}(2+0) + 0 + \hat{k}(0+3)$$

$$\vec{\nabla} f = \underline{\underline{2\hat{i} + 3\hat{k}}}$$

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3. If $\phi = x^2yz^3 + xy^2z^2$. Find $\vec{\nabla}\phi$ at $(1,3,2)$

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial y} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\vec{\nabla}\phi = \hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}$$

$$\frac{\partial\phi}{\partial x} = \frac{\partial}{\partial x}(x^2yz^3 + xy^2z^2)$$

$$\frac{\partial\phi}{\partial x} = 2xyz^3 + y^2z^2 \longrightarrow (1)$$

$$\frac{\partial\phi}{\partial y} = \frac{\partial}{\partial y}(x^2yz^3 + xy^2z^2)$$

$$\frac{\partial\phi}{\partial y} = x^2z^3 + 2xyz^2 \longrightarrow (2)$$

$$\frac{\partial\phi}{\partial z} = \frac{\partial}{\partial z}(x^2yz^3 + xy^2z^2)$$

$$\frac{\partial\phi}{\partial z} = 3x^2yz^2 + 2xy^2z \longrightarrow (3)$$

$$\therefore \vec{\nabla}\phi = \hat{i}(2xyz^3 + y^2z^2) + \hat{j}(x^2z^3 + 2xyz^2) + \hat{k}(3x^2yz^2 + 2xy^2z)$$

at $(1,3,2)$

$$\vec{\nabla}\phi = 84\hat{i} + 32\hat{j} + 72\hat{k}$$

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4. find the gradient of scalar field $\phi = x + 2y^2$

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\vec{\nabla}\phi = \hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}$$

$$\vec{\nabla}\phi = \hat{i}\frac{\partial}{\partial x}(x+2y^2) + \hat{j}\frac{\partial}{\partial y}(x+2y^2) + \hat{k}\underbrace{\frac{\partial}{\partial z}(x+2y^2)}_{0}$$

$$\vec{\nabla}\phi = \hat{i}(1+0) + \hat{j}(0+4y) + 0$$

$$\vec{\nabla}\phi = \hat{i} + 4y\,\hat{j}$$

5. find the gradient of scalar field

$$\phi = \sqrt{y^2+z^2}$$

$$\phi = (y^2+z^2)^{1/2}$$

$$\vec{\nabla}\phi = \hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}$$

$$\frac{\partial \phi}{\partial x} = \frac{\partial}{\partial x}(y^2+z^2)^{1/2}$$

$$\frac{\partial \phi}{\partial x} = 0 \longrightarrow (1)$$

$$\frac{\partial \phi}{\partial y} = \frac{\partial}{\partial y}(y^2+z^2)^{1/2}$$

$$\frac{\partial \phi}{\partial y} = \frac{1}{2}(y^2+z^2)^{-1/2} \cdot \frac{\partial}{\partial y}(y^2)$$

$$\frac{\partial \phi}{\partial y} = \left(\frac{1}{2}\right)\frac{2y}{(y^2+z^2)^{1/2}} = \frac{y}{\sqrt{y^2+z^2}} \longrightarrow (2)$$

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$$\frac{\partial\phi}{\partial z} = \frac{\partial}{\partial z}(y^2+z^2)^{1/2}$$

$$\frac{\partial\phi}{\partial z} = \left(\frac{1}{2}\right)(y^2+z^2)^{-1/2} \cdot \frac{\partial}{\partial z}(z^2)$$

$$\frac{\partial\phi}{\partial z} = \left(\frac{1}{2}\right)(y^2+z^2)^{-1/2} \cdot (2z)$$

$$\frac{\partial\phi}{\partial z} = \frac{z}{(y^2+z^2)^{1/2}} \longrightarrow (3)$$

$$\nabla\phi = \hat{j}\left[\frac{y}{(y^2+z^2)^{1/2}}\right] + \hat{k}\left[\frac{z}{(y^2+z^2)^{1/2}}\right]$$

(6) Calculate the directional derivative of $\phi = x^2z + 2xy^2 + yz^2$ at the point $(1,2,-1)$ in the direction of the vector $\vec{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$.

The directional derivative

$$\frac{d\phi}{ds} = \hat{A}\cdot(\nabla\phi) \longrightarrow (1)$$

$$\nabla\phi = \hat{i}\frac{\partial}{\partial x}(x^2z+2xy^2+yz^2) + \hat{j}\frac{\partial}{\partial y}(x^2z+2xy^2+yz^2) + \hat{k}\frac{\partial}{\partial z}(x^2z+2xy^2+yz^2)$$

$$\nabla\phi = \hat{i}(2xz+2y^2) + \hat{j}(4xy+z^2) + \hat{k}(x^2+2yz)$$

at $(1,2,-1)$

$$\nabla\phi = 6\hat{i} + 9\hat{j} - 3\hat{k} \longrightarrow (2)$$

$$|\vec{A}| = \sqrt{4+9+16} = \sqrt{29}$$

$$\therefore \hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{1}{\sqrt{29}}(2\hat{i}+3\hat{j}-4\hat{k}) \longrightarrow (3)$$

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from ①, ② and ③, we get

$$\frac{d\phi}{ds} = \left(\frac{1}{\sqrt{29}}\right)(2\hat{i}+3\hat{j}-4\hat{k})\cdot(6\hat{i}+9\hat{j}-3\hat{k})$$

$$\frac{d\phi}{ds} = \frac{51}{\sqrt{29}}$$

⑦ Find the directional derivative of the function $f = 3x^2 - 3y^2$ at the point $(1,2,3)$ along x-dir^n.

$$\vec{\nabla} f = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$$

$$\vec{\nabla} f = \hat{i}\frac{\partial}{\partial x}(3x^2-3y^2) + \hat{j}\frac{\partial}{\partial y}(3x^2-3y^2) + \hat{k}\frac{\partial}{\partial z}(3x^2-3y^2)$$

$$\vec{\nabla} f = \hat{i}(6x-0) + \hat{j}(0-6y) + \hat{k}(0-0)$$

$$\vec{\nabla} f = 6x\,\hat{i} - 6y\,\hat{j} \longrightarrow ①$$

∴ The directional derivative in the x- direction is

$$\hat{i}\cdot\vec{\nabla} f = \hat{i}\cdot[(6x)\hat{i} - (6y)\hat{j}]$$

$$\hat{i}\cdot\vec{\nabla} f = 6x$$

at $(1,2,3)$

$$\hat{i}\cdot\vec{\nabla} f = \underline{\underline{6}}$$

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(8) If $\vec{A} = x^2 y\,\hat{i} - xyz\,\hat{j} + yz^2\,\hat{k}$, determine $\vec{\nabla}\cdot\vec{A}$ at (1, 1, 2)

$$\nabla\cdot\vec{A} = \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]\cdot\left[\hat{i}A_x + \hat{j}A_y + \hat{k}A_z\right)$$

$$\vec{\nabla}\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla}\cdot\vec{A} = \frac{\partial}{\partial x}(x^2 y) + \frac{\partial}{\partial y}(-xyz) + \frac{\partial}{\partial z}(yz^2)$$

$$\vec{\nabla}\cdot\vec{A} = 2xy - xz + 2yz$$

at (1, 1, 2)

$$\vec{\nabla}\cdot\vec{A} = (2)(1)(1) - (1)(2) + (2)(1)(2)$$

$$\vec{\nabla}\cdot\vec{A} = (2) - (2) + (4)$$

$$\vec{\nabla}\cdot\vec{A} = \underline{\underline{4}}$$

(9) find the divergence of $\vec{V} = x\hat{i} + 2y^2\hat{j} + z\hat{k}$ at (3, 1, 2)

$$\vec{\nabla}\cdot\vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

$$\vec{\nabla}\cdot\vec{V} = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(2y^2) + \frac{\partial}{\partial z}(z)$$

$$\vec{\nabla}\cdot\vec{V} = 1 + 4y + 1$$

$$\vec{\nabla}\cdot\vec{V} = 4\times 1 = \underline{\underline{4}}$$

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(10) Find divergence of $\vec{F} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ at $(2,2,-3)$

$$\vec{\nabla}\cdot\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\vec{\nabla}\cdot\vec{F} = \frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial y}(y^2) + \frac{\partial}{\partial z}(z^2)$$

$$= 2x + 2y + 2z$$

$$= 2(x+y+z)$$

$$\vec{\nabla}\cdot\vec{F} = 2(2+2-3)$$

$$\vec{\nabla}\cdot\vec{F} = 2$$

(11) If $\vec{A} = (y^4 - x^2z^2)\hat{i} + (x^2+y^2)\hat{j} - x^2yz\hat{k}$

find $\vec{\nabla}\times\vec{A}$ at $(1,3,-2)$

$$\vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (y^4 - x^2z^2) & (x^2+y^2) & -x^2yz \end{vmatrix}$$

$$\vec{\nabla}\times\vec{A} = -x^2z\hat{i} - (-2xyz + 2x^2z)\hat{j} + (2x - 4y^3)\hat{k}$$

At $(1,3,-2)$

$$\vec{\nabla}\times\vec{A} = 2\hat{i} - 8\hat{j} - 106\hat{k}$$

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(12) Find curl of $\vec{F} = (x^2 - y)\hat{i} + 4z\hat{j} + x^2\hat{k}$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (x^2 - y) & 4z & x^2 \end{vmatrix}$$

$$\vec{\nabla} \times \vec{F} = (0-4)\hat{i} - (2x-0)\hat{j} + (0+1)\hat{k}$$

$$\vec{\nabla} \times \vec{F} = -4\hat{i} - 2x\hat{j} + \hat{k}$$

(13) Find curl of $\vec{F} = 3x^2\hat{i} + 2z\hat{j} - x\hat{k}$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2 & 2z & -x \end{vmatrix}$$

$$\vec{\nabla} \times \vec{F} = (0-2)\hat{i} - (-1-0)\hat{j} + (0-0)\hat{k}$$

$$\vec{\nabla} \times \vec{F} = -2\hat{i} + \hat{j}$$

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