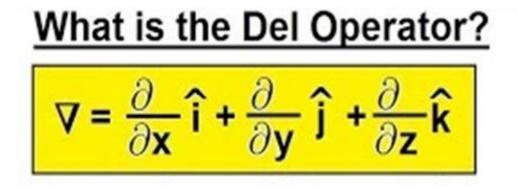
## **Electrodynamics**

4	Electi	omagnetism	08	CO 4
	4.1	Vector operators:		
		Gradient, divergence, curl and their physical		
		interpretation, fundamental theorems of vector calculus		
	4.2	Electrostatics and electromagnetic induction:		
		Electric charge density, electric field, electric potential		
		and their interrelations, Coulomb's and Gauss' law,		
		Gauss' and Faraday's laws in integral and differential		
		forms		
	4.3	Magnetostatics:		
		Biot-savart's and Ampere's law, absence of magnetic		
		monopoles, Ampere's law in integral and differential form		
	4.4	Electromagnetic wave propagation:		
		Continuity equation, Maxwell's correction to Ampere's		
		law, Maxwell's equations, electromagnetic waves in		
		vacuum, speed of light, energy density of electromagnetic		
		waves		

**Concept of Field** 

- A region of space where some physical quantity takes different values at different points.
- A scalar field is something that has a particular value at every point in space.
  - Example: Temperature.....
- A vector field is having a value and direction at every point in space.
- Example: Electric Field.....

### **Vector Differential Operator**



del operator = differential operator by itself it has no spcific use (like  $\frac{d}{dx}$  by itself)

" " " " " " <u>curl</u> of a <u>vector</u>

#### GRADIENT OF A SCALAR FIELD

- The gradient of a scalar function f(x<sub>1</sub>, x<sub>2</sub>, x<sub>3</sub>, ..., x<sub>n</sub>) is denoted by ∇f or where ∇ (the nabla symbol) denotes the vector differential operator, del. The notation "grad(f)" is also commonly used for the gradient.
  - The gradient of f is defined as the unique vector field whose dot product with any vector v at each point x is the directional derivative of f along v. That is,

$$(\nabla f(x)) \cdot \mathbf{v} = D_{\mathbf{v}} f(x).$$

 In 3-dimensional cartesian coordinate system it is denoted by:

$$\nabla f = \mathbf{i} \frac{\partial f}{\partial x} + \mathbf{j} \frac{\partial f}{\partial y} + \mathbf{k} \frac{\partial f}{\partial z}$$
$$= \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} + \frac{\partial f}{\partial z} \mathbf{k}$$

#### PHYSICAL INTERPRETATION OF GRADIENT

- Using the language of vector fields, we may restate this as follows: For the given function f(x, y), gravitational force defines a vector field F over the corresponding surface z = f(x, y), and the initial velocity of an object at a point (x, y) is given mathematically by – ∇f(x, y).
  - The gradient also describes directions of maximum change in other contexts. For example, if we think of f as describing the temperature at a point (x, y), then the gradient gives the direction in which the temperature is increasing most rapidly.



#### If A and B are two scalars ,then

# 1) $\nabla(A \pm B) = \nabla A \pm \nabla B$

## 2) $\nabla(AB) = A(\nabla B) + B(\nabla A)$

# **Directional Derivative**

#### Directional derivative of $\phi$ in the direction of <u>a</u> is

$$\frac{d\phi}{ds} = \hat{a}.grad\phi$$

where,

$$\hat{a} = \frac{dr}{|dr|}$$

Which is a unit vector in the direction of dr.

## DIVERGENCE

- In vector calculus, divergence is a vector operator that measures the magnitude of a vector field's source or sink at a given point, in terms of a signed scalar.
- More technically, the divergence represents the volume density of the outward flux of a vector field from an infinitesimal volume around a given point.

## **Divergence of a vector**

If  $A = a_x^{\hat{i}} + a_y^{\hat{j}} + a_z^{\hat{k}}$ , the divergence of A is defined as

 $divA = \nabla A$ =  $\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot \left(a_x\hat{i} + a_y\hat{j} + a_z\hat{k}\right)$  $\Rightarrow \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}.$ 

## **Curl of a vector**

If  $A = a_x^{\hat{i}} + a_y^{\hat{j}} + a_z^{\hat{k}}$ , the curl of A is defined by

$$curlA = \nabla \times A$$

$$= \left( \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) \times \left( a_x \hat{i} + a_y \hat{j} + a_z \hat{k} \right)$$

$$\Rightarrow curlA = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a_x & a_y & a_z \end{vmatrix}$$

#### CURL

- In vector calculus, the curl is a vector operator that describes the infinitesimal rotation of a 3dimensional vector field.
- At every point in that field, the curl of that point is represented by a vector.
- The attributes of this vector (length and direction) characterize the rotation at that point.
- The direction of the curl is the axis of rotation, as determined by the right hand rule, and the magnitude of the curl is the magnitude of that rotation.

# SOLENOIDAL AND IRROTATIONAL FIELDS

- The with null divergence is called solenoidal, and the field with null-curl is called irrotational field.
- The divergence of the curl of any vector field A must be zero, i.e.

 Which shows that a solenoidal field can be expressed in terms of the curl of another vector field or that a curly field must be a solenoidal field.

#### POINTS TO BE NOTED:

If curl F=0 then F is called an irrotational vector.

- If F is irrotational, then there exists a scalar point function φ such that F=∇φ where φ is called the scalar potential of F.
- The work done in moving an object from point P to Q in an irrotational field is

$$[\Phi]_{p} = \phi(Q) - \phi(P).$$

 The curl signifies the angular velocity or rotation of the body.