Module 3

QUANTUM MECHANICS – NUMERICAL PROBLEMS

Class Work

- Elementary charge q = 1.6 x 10^{-19} C
- Speed of light c in vacuum = 3 x 10^{8} m/s
- Mass of electron m_e = 9.1 x 10^{-31} kg
- Mass of proton m_p =1.67 x 10^{-27} kg
- Planck's constant h = 6.63 x 10^{-34} J-s
- Reduced Planck's constant ħ = h/2π = 1.05 x 10^{-34} J-s

1. Calculate de' Broglie wavelengths of (i) cricket ball of mass 150 gm thrown at a speed of 150 km/hr and (ii) electron orbiting in hydrogen atom at a speed of 10^{6} m/s. Comment on your results.
2. What is de' Broglie wavelength of a neutron having energy 1 MeV. Use $m_n = m_p$. By how much potential difference a proton has to be accelerated in order to have the same de' Broglie wavelength?
3. Find kinetic energy of an electron whose de' Broglie wavelength is the same as that of 100 keV photon.
4. An electron and a proton have the same kinetic energies. Compare their de' Broglie wavelengths. Given $m_p = 1800\ m_e$.
5. Estimate de'Broglie wavelength of an electron and hence its speed in the 1^{st} Bohr orbit. Given radius of 1^{st} Bohr orbit a_0 = 0.5 Å.
6. Calculate uncertainty in the determination of momentum of an electron confined to a quantum well of size 1 nm. What is the percentage uncertainty in the momentum if its mean speed is 10^{6} m/s? In a similar way, determine uncertainty in the measurement of momentum of a marble of mass 10 gm confined to a box of dimensions 50 cm. What is the percentage uncertainty in the momentum if it is moving with a speed of 20 cm/s. Is it significant as compared to the result of electron in earlier case? What can you say about the measurement?
7. Find minimum energy possessed by an electron in an atom.
8. Calculate the percentage uncertainty in the measurement of momentum of a neutron having energy 20 MeV confined to a region of width equal to (i) 10 nuclei (ii) 10 atoms. Comment on the results.
9. The frequency of radiation emitted from any source is never sharp at a singular value but it has a small spread. Using uncertainty principle, show that this spread of typically a few megahertz.
10. Uncertainty principle indicates that electrons cannot pre-exist in the nucleus like protons or neutrons. But, in radioactive elements during β-decay, electrons do come out from the nucleus during the process described by neutron disintegration process: $\mathrm{n}_0^1 \rightarrow \mathrm{p}_{+1}^1 + \mathrm{e}_{-1}^0 + \bar{\nu}\,_0^0$. Show using the uncertainty principle again that electron comes out of the nucleus almost spontaneously as soon as it is generated and doesn't stay there.

11. The phase velocity of ocean waves is given by $v_{ph} = \sqrt{\frac{g\lambda}{2\pi}}$, where g is gravitational acceleration. Calculate their group velocity.
12. The wave function of a particle is given by $\varphi(x) = \sqrt{\frac{\pi}{2}}\, x;\ \ 0 \le x \le 1$. Find the probability that the particle can be found between x = 0.45 to x = 0.55.
13. Find the probability that a particle confined to an infinite square well of size "a" can be found within 0.3a to 0.7a in its ground state. Its wave function is $\varphi(x) = \sqrt{\frac{2}{a}} \sin\frac{n\pi x}{a};\ -\frac{a}{2} < x < +\frac{a}{2}$.
14. Calculate the energy, momentum and de' Broglie wavelength of an electron trapped in a one-dimensional quantum well of size 10 Å in its ground state.
15. Solid-state blue lasers are made using lower band gap $In_xGa_{1-x}N$ layers sandwiched with higher band gap GaN layers. A typical commercial blue laser diode emits light at 445 nm wavelength. The thin $In_xGa_{1-x}N$ layers act as quantum well. Treat it to be one-dimensional and determine width of a $In_xGa_{1-x}N$ layer. (Assume transition between lowest allowed energy levels).
