Derivation for Clausius-Mosotti Equation

$$E \rightarrow E_a + E_i \quad , \quad E_i = \frac{\gamma P}{\epsilon_0}$$

$$P = \epsilon_0 (K-1) E \longleftrightarrow P = \alpha N E$$

$$P = \epsilon_0 (K-1) E_a \quad \text{—— (1)}$$

$$P = \alpha N (E_a + E_i)$$

$$\downarrow$$

$$P = \alpha N \left(E_a + \frac{\gamma P}{\epsilon_0}\right)$$

$\uparrow$ use (1) at this step

$$\epsilon_0 (K-1) E_a = \alpha N \left[E_a + \frac{\gamma}{\epsilon_0} \epsilon_0 (K-1) E_a\right]$$

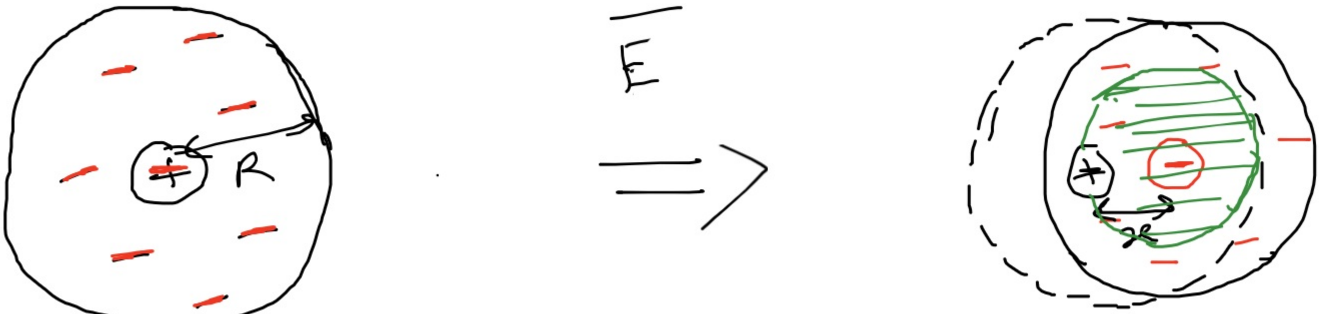
$$\epsilon_0 (K-1) = \alpha N \left[1 + \gamma (K-1)\right]$$

$\nu \simeq \frac{1}{3}$ for most of the solids

$$G_0(K-1) = \alpha N \left[1 + \frac{1}{3}(K-1)\right]$$

$$\Rightarrow \alpha = \frac{3 G_0 (K-1)}{N(K+2)}$$

Derivation for electronic polarizability



$Q_N = +Ze$

$Q_{e\text{-cloud}} = -Ze$

Q_x : e^- charge present in sphere of radius (x) shown in green

electric dipole moment $\mu = |Q|x$

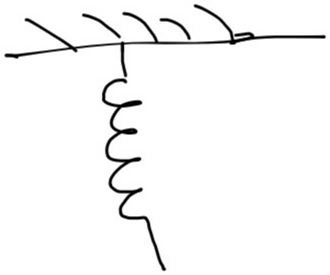
$= Ze \cdot x$ ——— (1)

$$\left.\begin{aligned} \frac{4}{3}\pi R^3 &= -Ze \\ \frac{4}{3}\pi x^3 &= ? \end{aligned}\right\} \quad -\frac{Zex^3}{R^3} = Q_x$$

Two forces : $\bar{F}_a = Q\bar{E} = -Ze\bar{E}$ — (2)

(applied force)

restoring force between nucleus and part of e^- cloud contained sphere of radius x



$$\bar{F}_r = \frac{1}{4\pi\epsilon_0} \frac{Q_N Q_x}{x^2} \hat{i}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{(+Ze)\left(-\frac{Zex^3}{R^3}\right)}{x^2} \hat{i}$$

$$= \frac{1}{4\pi\epsilon_0} \left(-Z^2e^2 \frac{x}{R^3}\right) \hat{i} \quad \text{— (3)}$$

at equilibrium, $F_a = F_r$

$$-ZeE = \frac{1}{4\pi\epsilon_0}\left(\frac{-Z^2e^2x}{R^3}\right)$$

$$x = \frac{4\pi\epsilon_0 R^3 \cdot E}{Ze}$$

sub in (1)

$$\mu = Ze \cdot \left[\frac{4\pi\epsilon_0 R^3 E}{Ze}\right] = 4\pi\epsilon_0 R^3 E$$

compare with $\mu_e = \alpha_e E \Rightarrow \boxed{\alpha_e = 4\pi\epsilon_0 R^3}$

Numericals (6-10)

6) H → $N = 9.8 \times 10^{26} / m^3$, $\epsilon_0 = 8.85 \times 10^{-12}$ F/m

↳ $R = 0.53$ Å $= 0.53 \times 10^{-10}$ m

$$\alpha_e = 4\pi\epsilon_0 R^3 = 4 \times 3.14 \times 8.85 \times 10^{-12} \times (0.53 \times 10^{-10})^3$$

$$= \underline{1.65} \times 10^{-41} \text{ F-m}^2$$

$$\alpha = \frac{\epsilon_0 (K-1)}{N} \Rightarrow K = \frac{\alpha N}{\epsilon_0} + 1$$

$$= \frac{1.65 \times 10^{-41} \times 9.8 \times 10^{26}}{8.85 \times 10^{-12}} + 1$$

$$= 1.001871$$

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$K = 1.00043$, $N = 2.7 \times 10^{25}/m^3$

$$\alpha = \frac{\epsilon_0 (K-1)}{N} = \frac{8.85 \times 10^{-12}}{2.7 \times 10^{25}} \times (1.00043 - 1)$$

$$= 1.41 \times 10^{-40}\ F\text{-}m^2$$

α values typically 10^{-40} or 10^{-41}

$\alpha = 10^{-3}$

check

8 $\quad K = 1.000074, \quad E = 8 \times 10^{4}\ V/m, \quad N = 2.7 \times 10^{25}/m^3$

$$\alpha = \frac{\epsilon_0 (K-1)}{N} = \frac{8.85 \times 10^{-12} \times (1.000074 - 1)}{2.7 \times 10^{25}}$$

$$= 2.42 \times 10^{-41}\ F\text{-}m^2$$

F - V - m

C ×V

Q

$$\mu = \alpha E = 2.42 \times 10^{-41} \times 8 \times 10^{4}$$

$$= 1.94 \times 10^{-36}\ C\text{-}m$$

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$K = 2.87, \quad N = 3\times 10^{28}/m^3, \quad E = 5000\, V/m$

$$\alpha = \frac{3\epsilon_0 (K-1)}{N(K+2)} = \frac{3\times 8.85\times 10^{-12}(2.87-1)}{3\times 10^{28}\times(2.87+2)}$$

$$= \underline{3.4}\times 10^{-40}\ F\text{-}m^2$$

$\times E$

$$\mu = \alpha E = 1.7\times 10^{-36}\ C\text{-}m$$

(10) $K = 1.000134$, $R = 0.735\,\mathring{A}$, $A = 20$ gm/mol

Ne

$$\alpha = 4\pi\epsilon_0 R^3$$

20×10^{-3} kg/mol

$$= 4 \times 3.14 \times 8.85 \times 10^{-12} \times (0.735 \times 10^{-10})^3$$

$$= 4.41 \times 10^{-41}\ F\text{-}m^2$$

To find atomic density: $\alpha = \dfrac{\epsilon_0 (K-1)}{N}$

$$\Rightarrow N = \frac{\epsilon_0 (K-1)}{\alpha} = \frac{8.85 \times 10^{-12} \times (1.000134 - 1)}{4.41 \times 10^{-41}}$$

$$= 2.69 \times 10^{25} / m^3$$