# K J Somaiya College of Engineering, Mumbai-77

## (Constituent college of Somaiya Vidyavihar University)

**Presented by:**

**Prof. R. B. Pansare**


- **Kinetics of particle**

| | |
|---|---|
| 5.3 | Impulse and Momentum: Principle of linear impulse and momentum, law of conservation of momentum, impact and collision, direct central and oblique central impact. |

# Momentum

- Momentum: The motion of inertia

$$\text{momentum} = \text{mass x velocity}$$

$$\text{momentum} = mv$$

  - Units: kg·m/s
  - Objects which aren't moving have no velocity, and therefore have no momentum

# Impulse

- <u>Impulse:</u> The change in momentum of an object due to a force that is applied during a period of time

$$\text{Impulse} = \text{force} \times \text{time}$$

$$\text{Impulse} = Ft$$

  - Units: N·s

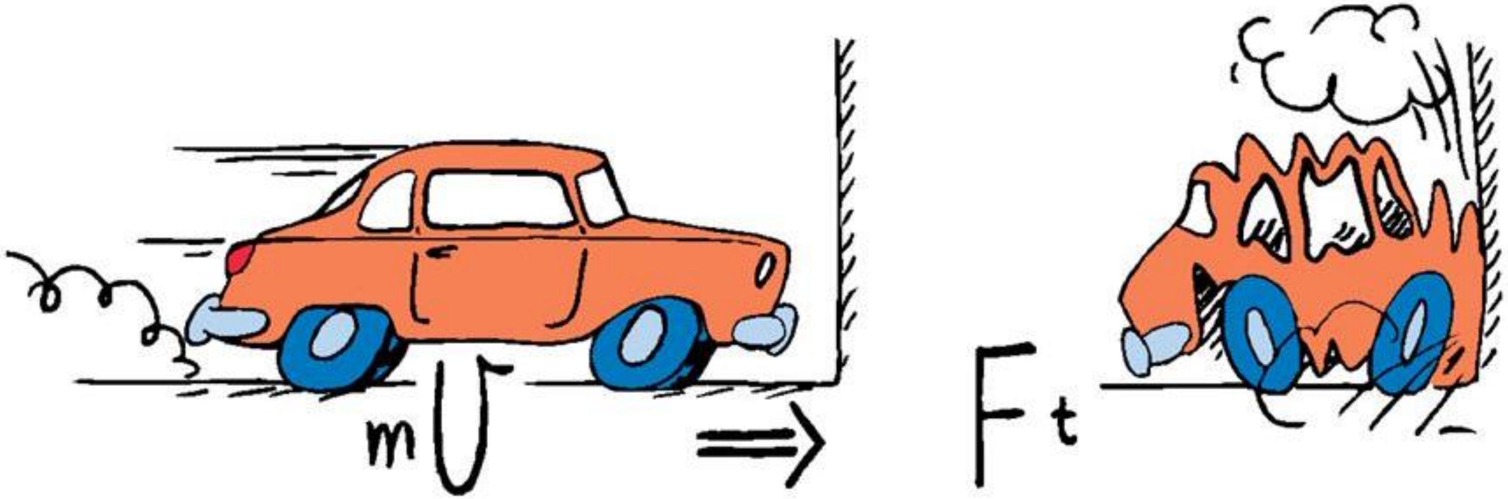
- Impact- The time duration in which a change in momentum is occurring.
- Units- seconds

# Impulse-Momentum Theorem

- Impulse is equal to a change in momentum

## Impulse Changes Momentum

- If the change in momentum occurs over a short time, the force of impact is large.



# Impulse-Momentum Principle

$$F = ma \qquad \text{But acceleration } a = \frac{dv}{dt}$$

$$\therefore \; F = m\frac{dv}{dt}$$

$$F\,dt = m\,dv \qquad \text{Integrating both sides}$$

$$\int_{t_1}^{t_2} F\,dt = \int_{v_1}^{v_2} m\,dv$$

$$\therefore \int_{t_1}^{t_2} F\,dt = m[v_2 - v_1] \qquad \text{... (I)}$$

The term $\int_{t_1}^{t_2} F\,dt$ is known as impulse and is expressed in N.sec.

$t_1$

The term $\int_{t_1}^{t_2} F\,dt$ is known as impulse and is expressed in N.sec.

The term $m\,[v_2 - v_1]$ gives change in momentum.

Thus equation (I) can be expressed as

$$\text{Impulse} = \text{Change in momentum}$$

$$\text{or Impulse} = \text{Final momentum} - \text{Initial momentum}$$

If the force is constant during the time interval $t_1$ to $t_2$, then impulse will be

$$I = F\,[t_2 - t_1] = F \times t$$

Since the velocity is a vector quantity impulse is also a vector quantity.

In component form, impulse momentum theorem can be expressed as

$$\therefore \int_{t_1}^{t_2} F_x\,dt = m\left[(v_x)_2 - (v_x)_1\right]$$

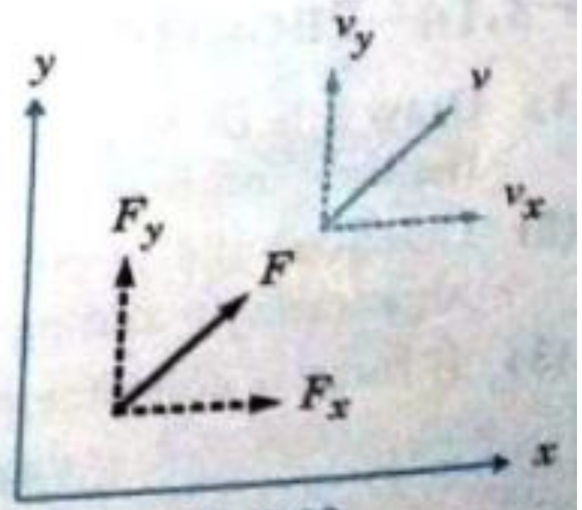


Fig. 3.13

# Law of Conservation of Momentum

- Law of conservation of momentum states that

For two or more bodies in an isolated system acting upon each other, their total momentum remains constant unless an external force is applied. Therefore, momentum can neither be created nor destroyed.

- Hence, the sum of the momenta of two bodies before a collision is the same as the sum of their momenta after a collision.

$$p_{1(i)} + p_{2(i)} = p_{1(f)} + p_{2(f)}$$

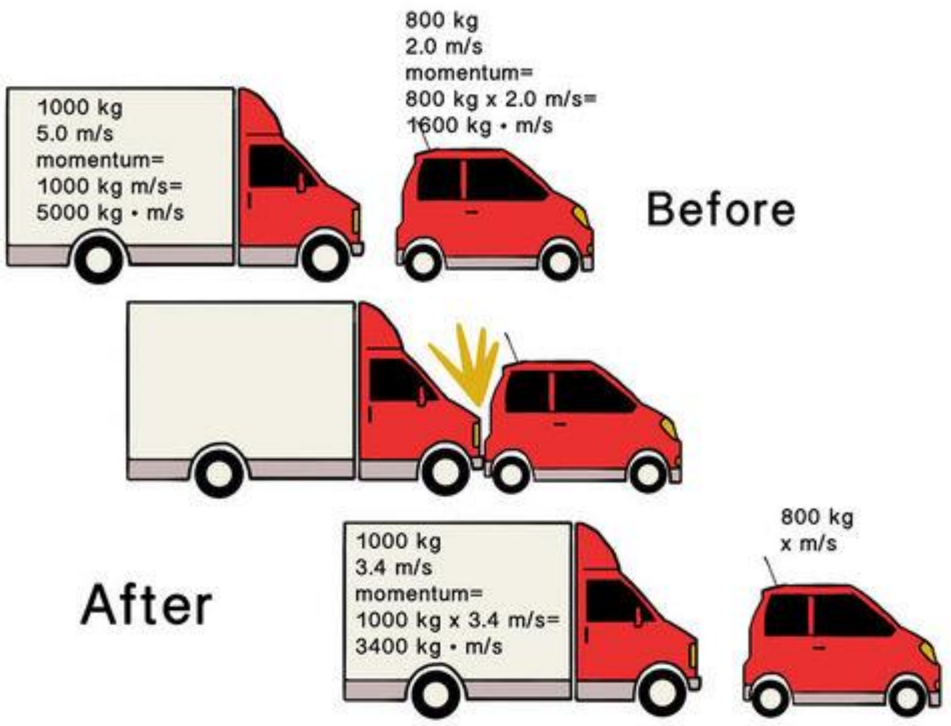
or

$$m_1 v_{1(i)} + m_2 v_{2(i)} = m_1 v_{1(f)} + m_2 v_{2(f)}$$

- It is most simply written as:

$$p_{before} = p_{after}$$

- Conservation of Momentum is true for a closed system where all the forces are internal.



# Derivation of Conservation of Momentum

- Newton's third law states that for a force applied by an object A on object B, object B exerts back an equal force in magnitude, but opposite in direction.
- This idea was used by Newton to derive the law of conservation of momentum.
- Consider two colliding particles A and B whose masses are $m_1$ and $m_2$ with initial and final velocities as $u_1$ and $v_1$ of A and $u_2$ and $v_2$ of B. The time of contact between two particles is given as t.

$$A = m_1(v_1 - u_1) \text{ (change in momentum of particle A)}$$

$$B = m_2(v_2 - u_2) \text{ (change in momentum of particle B)}$$

$$F_{BA} = -F_{AB} \text{ (from third law of motion)}$$

$$F_{BA} = m_2 * a_2 = \frac{m_2(v_2 - u_2)}{t}$$

$$F_{AB} = m_1 * a_1 = \frac{m_1(v_1 - u_1)}{t}$$

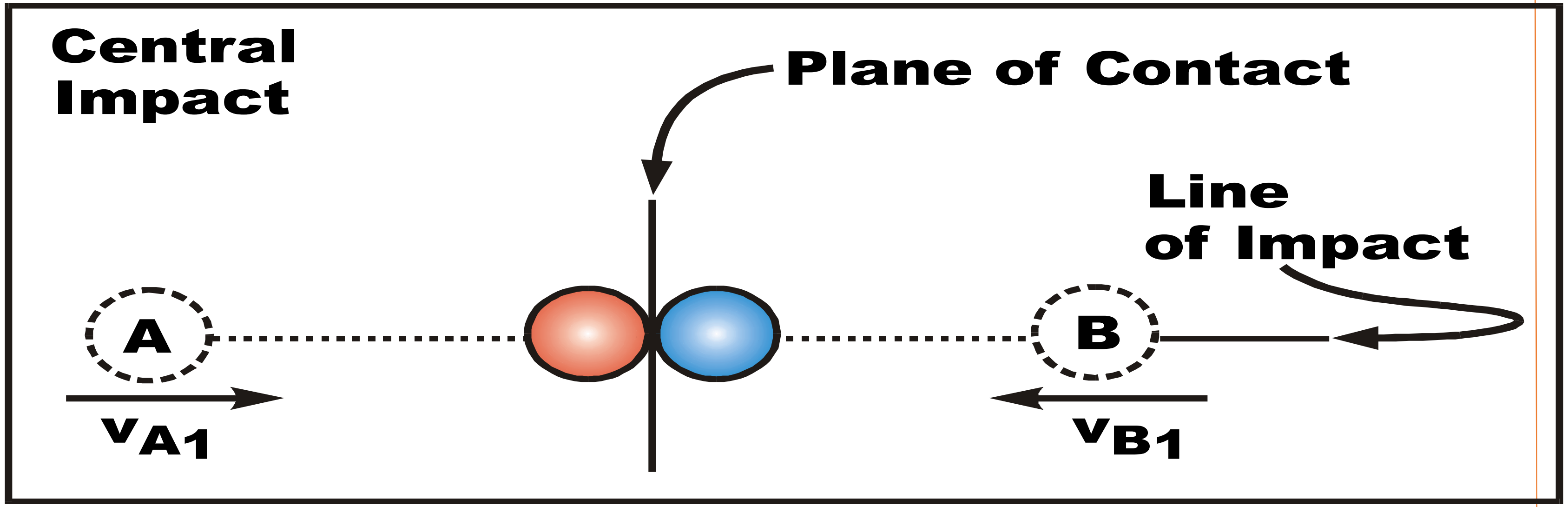
$$\frac{m_2(v_2 - u_2)}{t} = \frac{-m_1(v_1 - u_1)}{t}$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

# Impact for Particles

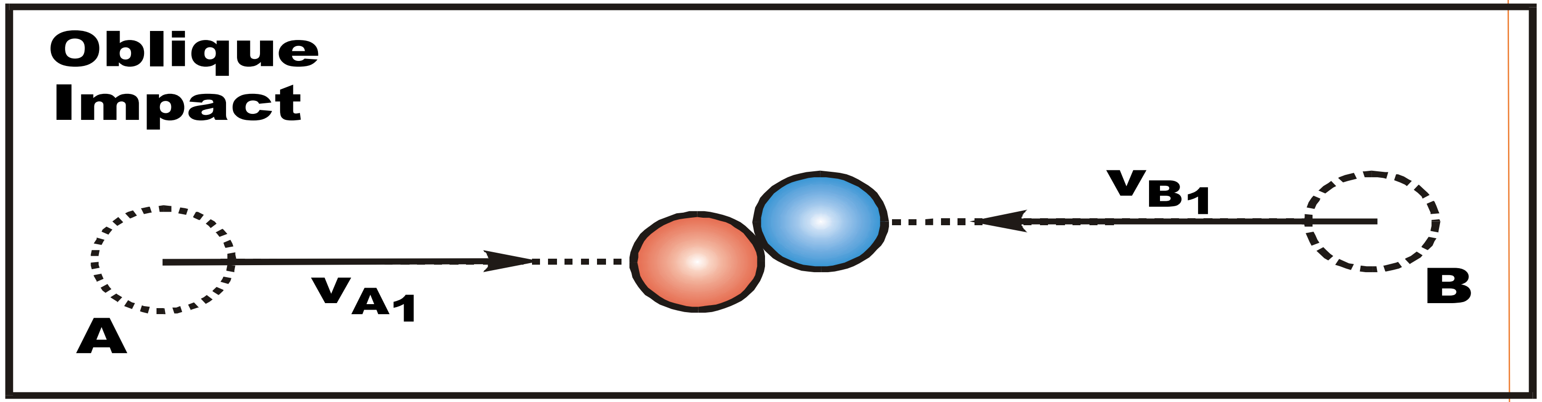
Terminology:

1. Direct Central Impact: The incident and departure velocities of the two particles are collinear. (After impact, A and B move along same line.)




2. Oblique Impact: Initial velocities of A and B are NOT collinear.

Particles A and B strike a glancing blow and their departure velocities (at least one, A or B) are at angles to their initial velocities.



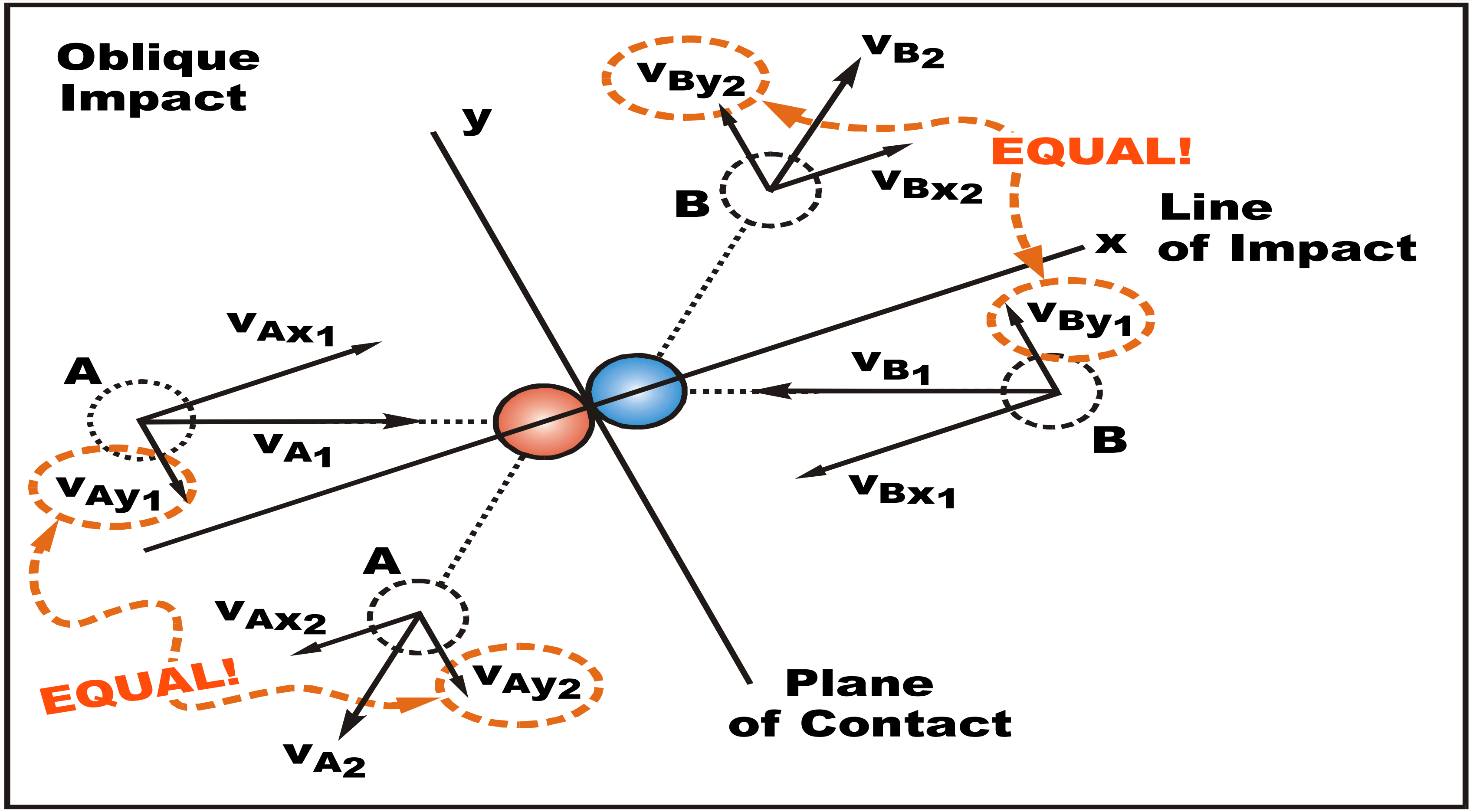
Important!!

You must draw and label the plane of contact and line of impact.

Assume no friction impulse along the plane of contact, thus:

$v_{Ay2} = v_{Ay1}$

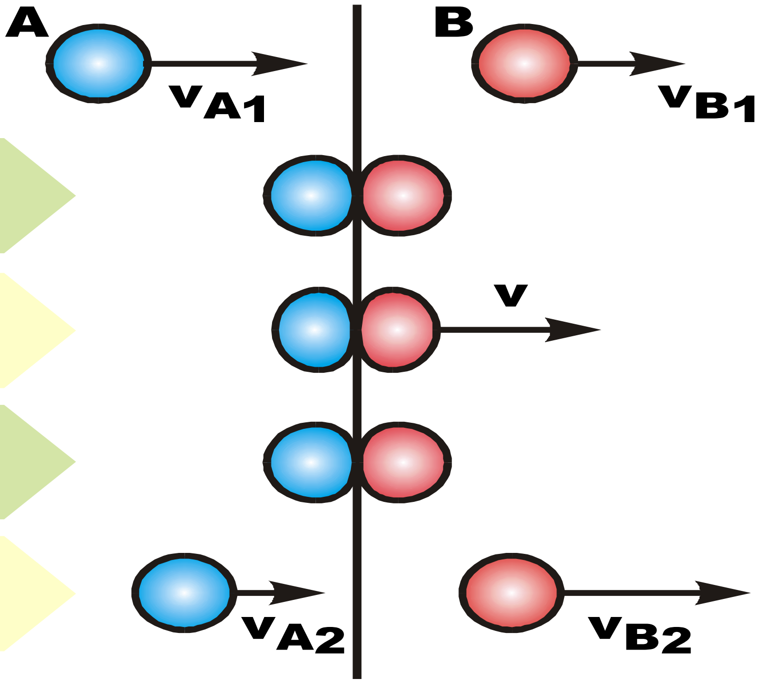
$v_{By2} = v_{By1}$



# Details of Impact; Coefficient of Restitution

What happens during impact? An Up-Close View:

| Description of Phase of Impact | A Close View of Impact of Particles A and B |
|---|---|
| Before impact, relative velocity $v_{B1/A1} = v_{B1} - v_{A1}$ | |
| Deformation phase: Kinetic energy stored as deformation | |
| Max deformation: A and B momentarily share same velocity, v. | |
| Restitution phase: Energy stored as deformation converted back to KE. | |
| After impact: If e < 1, some KE lost. Rel Velocity: $v_{B2/A2} = v_{B2} - v_{A2}$ | |




Coefficient of Restitution

$$e = \frac{-v_{B2/A2}}{v_{B1/A1}} = \frac{-(v_{B2} - v_{A2})}{(v_{B1} - v_{A1})} = \frac{(v_{B2} - v_{A2})}{(v_{A1} - v_{B1})}$$

# Details of Impact; Coefficient of Restitution

Coefficient of Restitution, e: (Abbreviated as "COR")

COR, e, is a measure of the energy stored in deformation during impact which is recovered back to kinetic energy.

More precisely…. (see your text for a derivation…)

$$e = \frac{\text{Relative Departure Velocity}}{\text{Relative Incident Velocity}} = \frac{-v_{B2/A2}}{v_{B1/A1}} = \frac{-(v_{B2} - v_{A2})}{(v_{B1} - v_{A1})} = \frac{(v_{B2} - v_{A2})}{(v_{A1} - v_{B1})}$$

Cases: e = 1 "Perfectly Elastic"

Rel Departure Velocity = Rel Incident Velocity

e = 0 "Perfectly Plastic" (Particles stick together…)

Rel Departure Velocity = 0

0 < e < 1 Range for e is between zero and one.

Typical values: .5 to .8 for balls

# More on Coefficient of Restitution (COR)

Kinetic Energy recovered after impact is approx $e^2$ .

A COR (e) of 0.8 sounds high. But the KE after impact is $(.8)^2 = 64\%$ of the original KE, meaning 36% of KE was lost!

Applications:

Golf Drivers: The USGA limits the COR of a driver's face to be no greater than COR = 0.83. They have on-site testing facilities to test compliance (if requested).

High school and college metal baseball bats: Using metal bats saves money because wood bats break. But metal bats have a higher COR (batted balls have a 5-10% higher velocity off of a metal bat) than wooden bats. Batting and slugging averages are inflated because of this. Pitchers are also subject to injury.

# Equations for Impact Problems

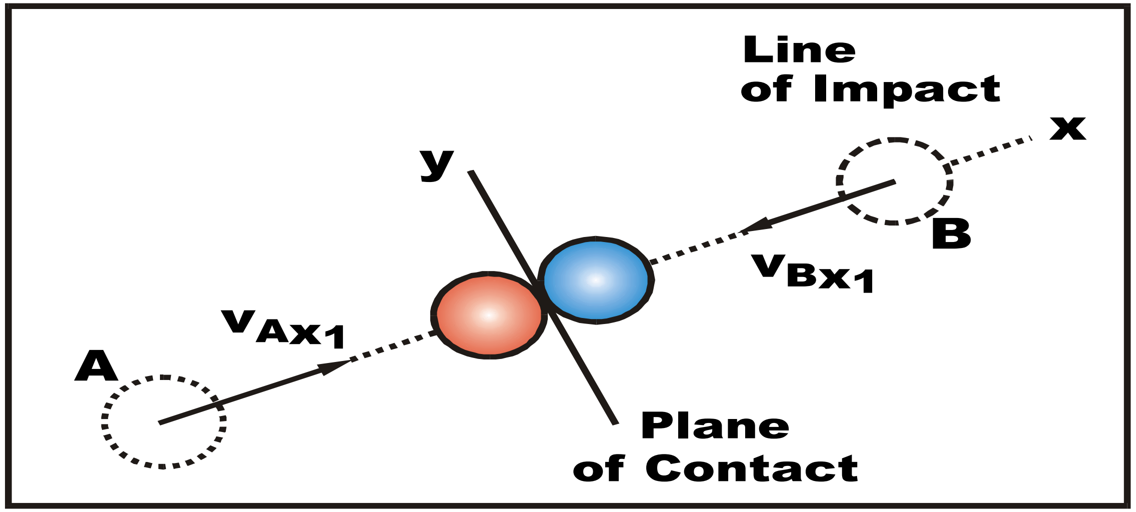
(Let x be the Line of Impact, y be the Plane of Contact)

Along the Plane of Contact:
(Assumes no friction impulse along this plane….)

$$v_{Ay2} = v_{Ay1}$$
$$v_{By2} = v_{By1}$$

Along the Line of Impact (Conservation of Momentum:

$$m_A v_{Ax1} + m_B v_{Bx1} = m_A v_{Ax2} + m_B v_{Bx2}$$




Also along the Line of Impact:

$$e = \frac{(v_{Bx2} - v_{Ax2})}{(v_{Ax1} - v_{Bx1})}$$

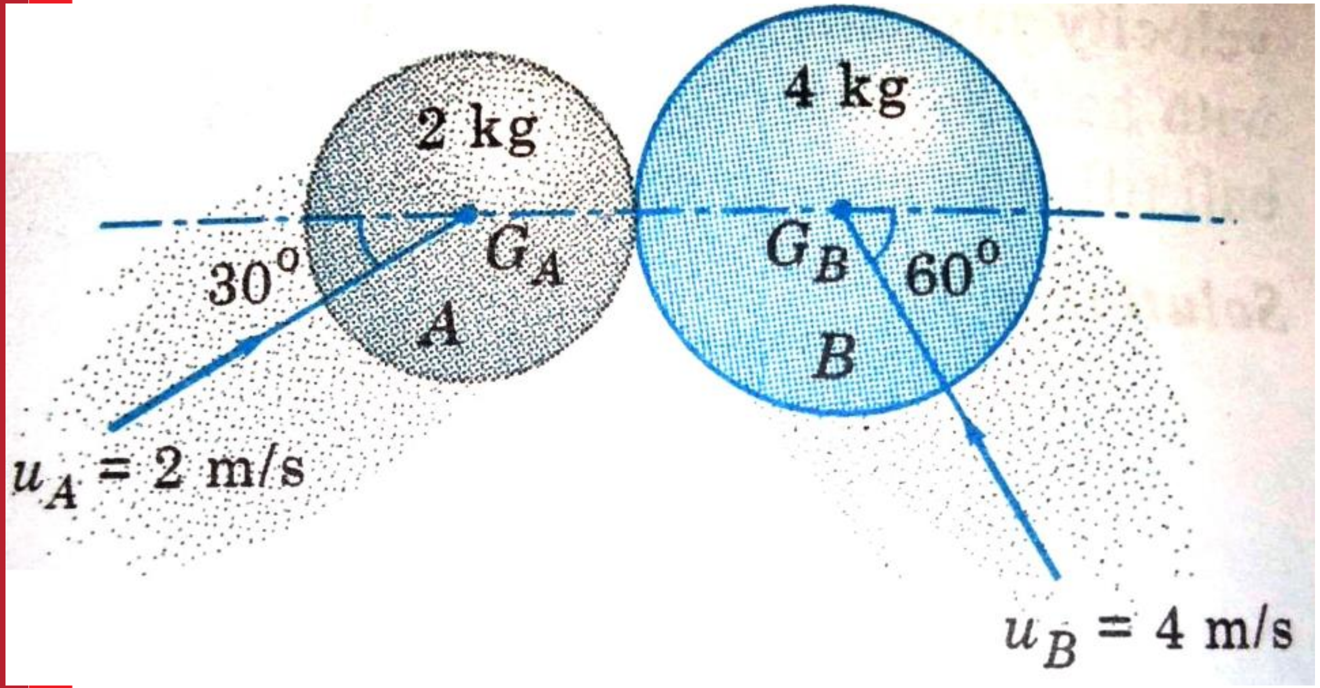
Use a consistent sign convention for v's in these equations.

**Problem No. 1:**

A ball of mass 1 kg is moving with a velocity of 2 m/s to the right collides with another ball of mass 2 kg moving with a velocity of 3 m/s to the left. Determine the velocities of the balls after impact and corresponding percentage loss in kinetic energy when, (i) the impact is elastic, (ii) the impact is plastic, (iii) the impact is such that e = 0.5.

## Problem No. 2:

Two smooth spheres A and B having a mass of 2 kg and 4 kg respectively collide with initial velocities as shown in figure. If the coefficient of restitution for the spheres is e = 0.8, determine the velocities of each sphere after collision.



## Problem No. 3:

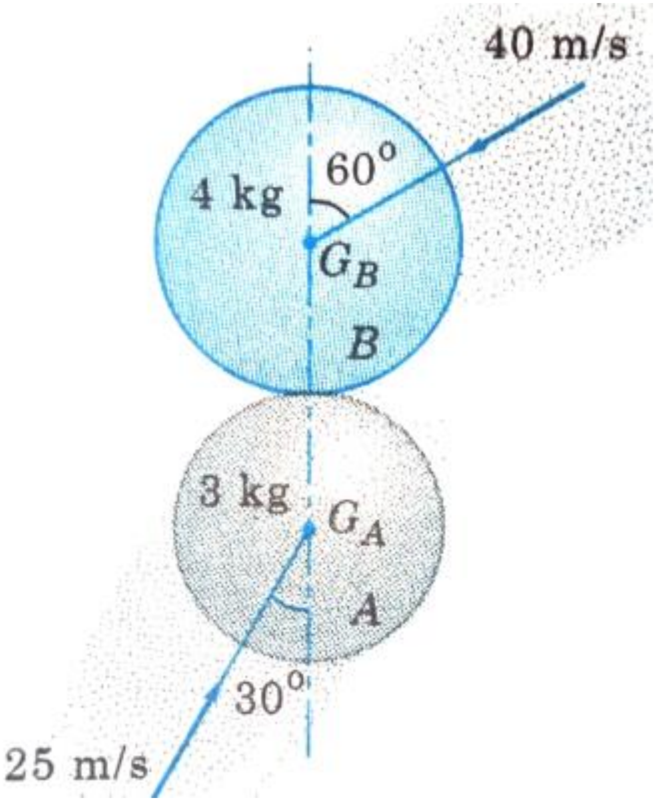
A 50 gm ball is dropped from a height of 600 mm on a small plate. It rebounds to a height of 400 mm when the plate is directly rests on the ground and to a height of 250 mm when a foam rubber mat is placed between the plate and the ground. Determine (i) the coefficient of restitution between the plate and the ground (ii) mass of the plate.

## Problem No. 4:

A ball falls from a height of 1m hits the ground and rebounds with half its velocity just before impact. Then after rising it falls and hits the ground and again rebounds with half its velocity just before impact, and so on. Determine total distance travelled by the ball till it comes to rest on the ground.

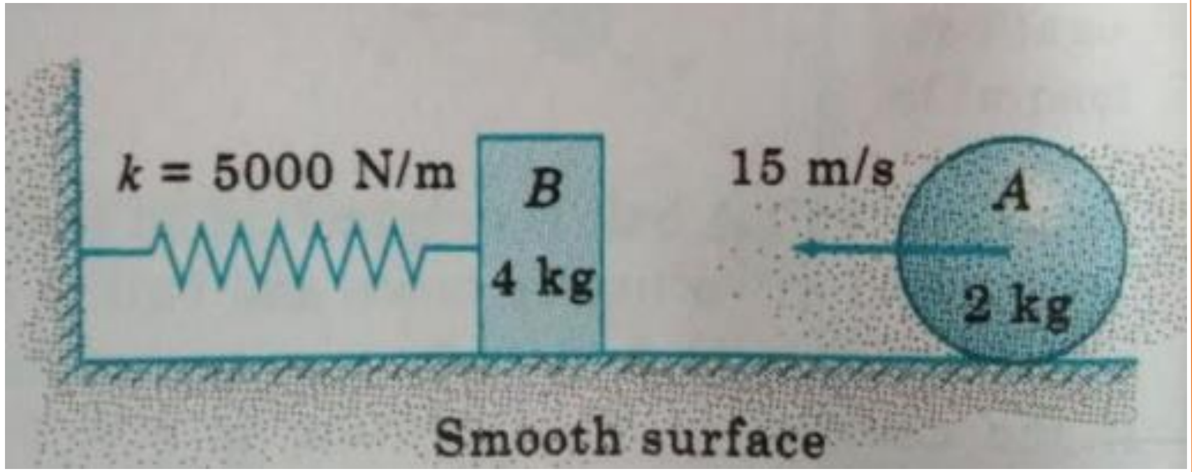
## Problem No. 5:

**Two smooth balls of mass A 3kg and ball B of mass 4kg are moving with velocities 25m/s and 40 m/s respectively at an angle of 30º and 60ºwith the vertical as shown in figure. If the coefficient of restitution between two balls is 0.8, find the magnitude and direction of velocities of these balls after impact.**



- **Problem No. 6 (Mixed type)**

A 2 kg sphere is moving with a velocity of 15 m/s when it strikes the vertical face of 4 kg block which is at rest. The block is attached by spring of constant k = 5000 N/m. If e = 0.75 between the block and the sphere and there is no friction between the block and the surface and between sphere and the surface, determine the maximum compression of the spring due to impact.



- **Problem No. 7 (Mixed type)**

The bullet travelling horizontally with a velocity of 600 m/s and weighing 0.25 N strikes a wooden block weighing 50 N resting on a rough horizontal floor. The coefficient of kinetic friction between floor and 50 N block is 0.5. Find the distance through which the block is displaced from its initial position. Given that the bullet after striking remains buried in the block.