K J SOMAIYA COLLEGE OF ENGINEERING, MUMBAI-77

(CONSTITUENT COLLEGE OF SOMAIYA VIDYAVIHAR UNIVERSITY)

Module 3.1 : Centroid

Presented by:

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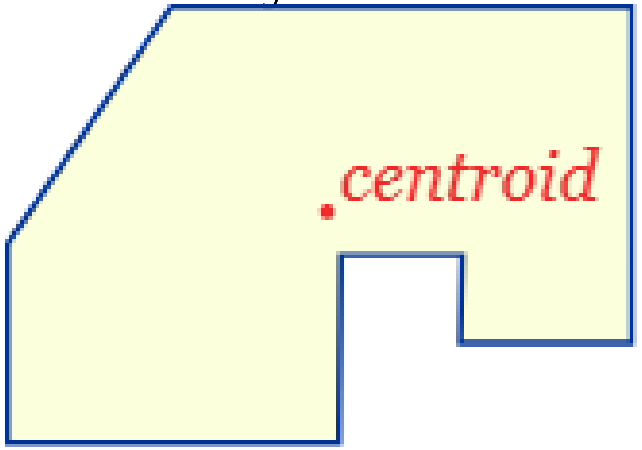
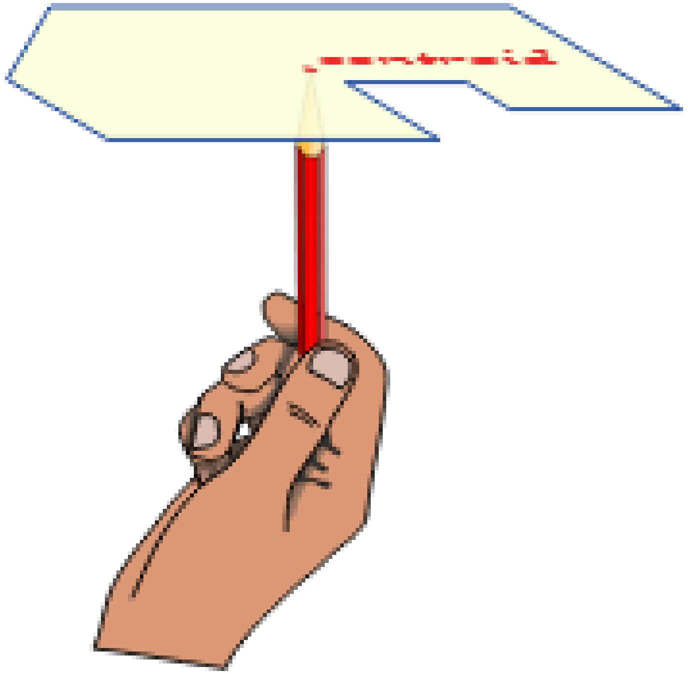
Centroid of wires/rods, Centroid of plane laminas: Plane lamina consisting of primitive geometrical shapes, Centre of gravity of solids: Solids consisting of primitive Solids

Centroid

- The **centroid** is also known as the geometric center of the object.
- The **Center of Gravity** is the same as the centroid when the density is the same throughout.
- **Center of gravity**, **center of mass** and **centroid** are all the same for simple solids.
- They are often marked by a **cross** or **dot** and sometimes the letters **CG** or just **G**





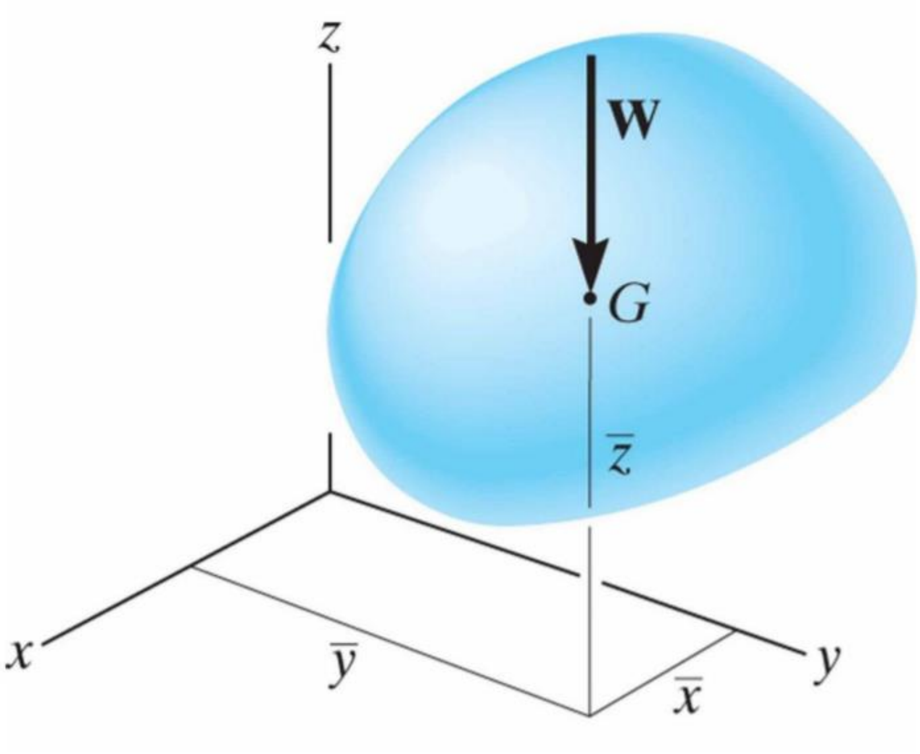
Centroid and its location

- The centroid of an area is analogous to the center of gravity of a body.
- The concept of the first moment of an area is used to locate the centroid.
- Varignon’s theorem can be applied.
- Location of centroid for

❑ Plane areas

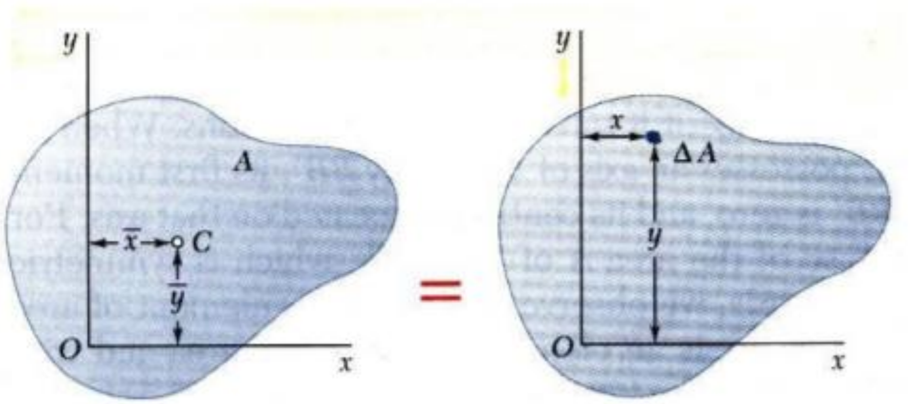
❑ Wires

❑ Solids



Location of centroid

- Centroid of an area



$$\bar{x}W = \int x\,dW$$

$$\bar{x}gM = g\int x\,dM$$

$$M = \rho V = \rho(tA)$$

$$dM = \rho dV = \rho t dA$$

$$\bar{x}(\gamma A t) = \int x(\gamma t)dA$$

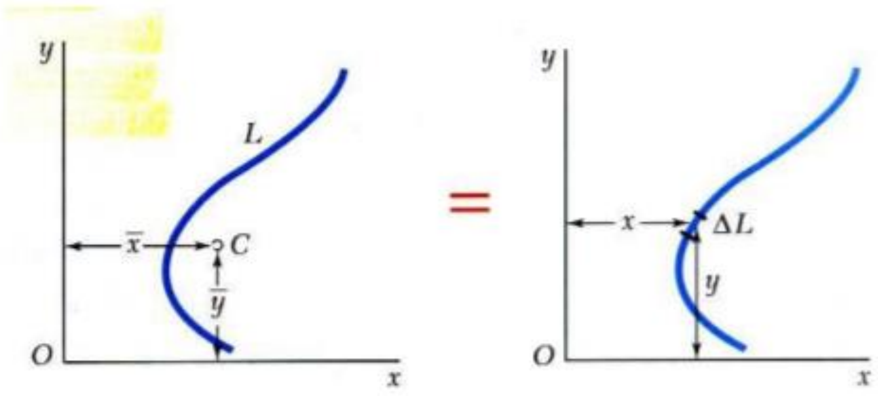
$$\bar{x}A = \int x\,dA = Q_y$$

= first moment with respect to y

$$\bar{y}A = \int y\,dA = Q_x$$

= first moment with respect to x

- Centroid of a line



$$\bar{x}W = \int x\,dW$$

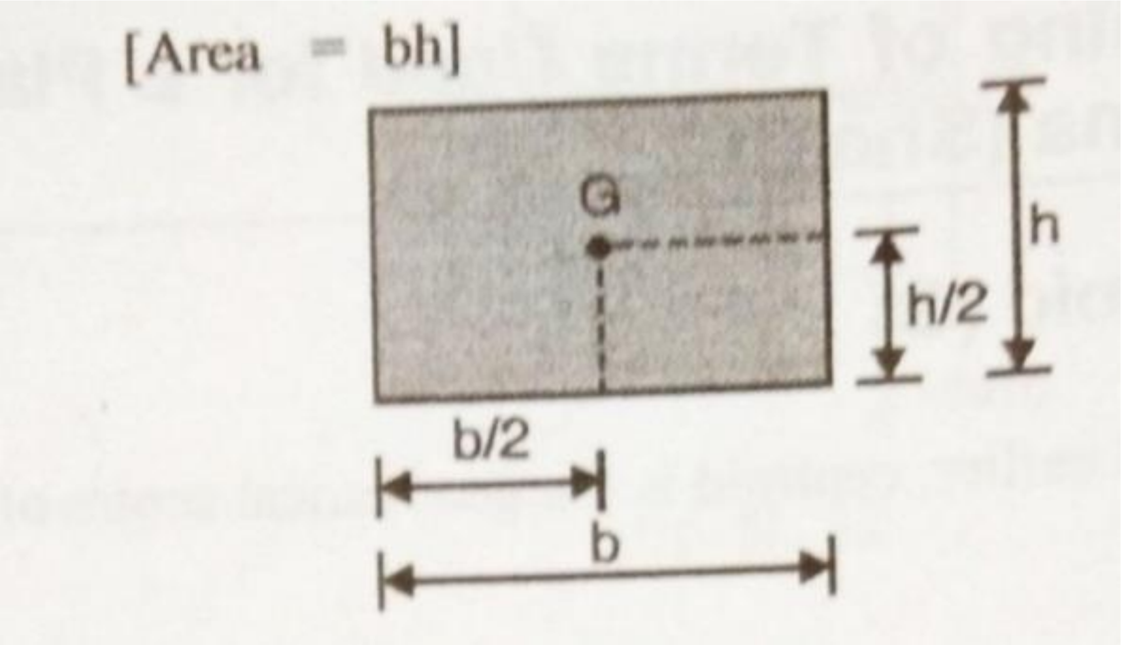
$$\bar{x}(\gamma L a) = \int x(\gamma a)dL$$

$$\bar{x}L = \int x\,dL$$

$$\bar{y}L = \int y\,dL$$

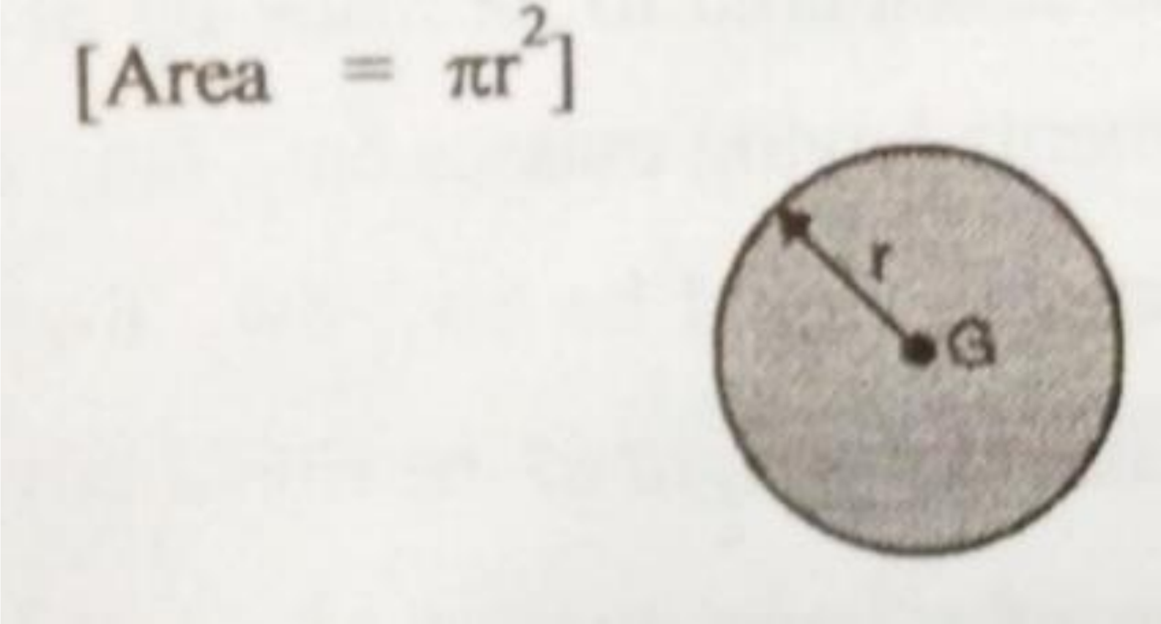
Centroid of common areas

1. Rectangle



[Area = bh]

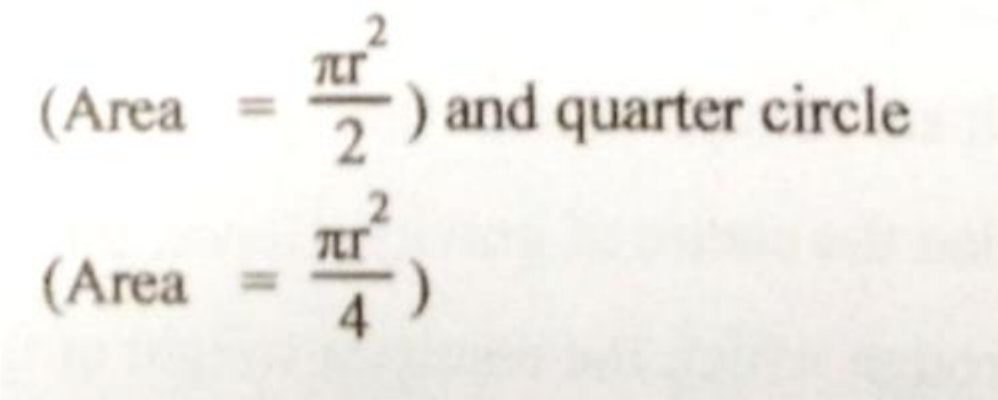
2. Circle



[Area = πr^2]

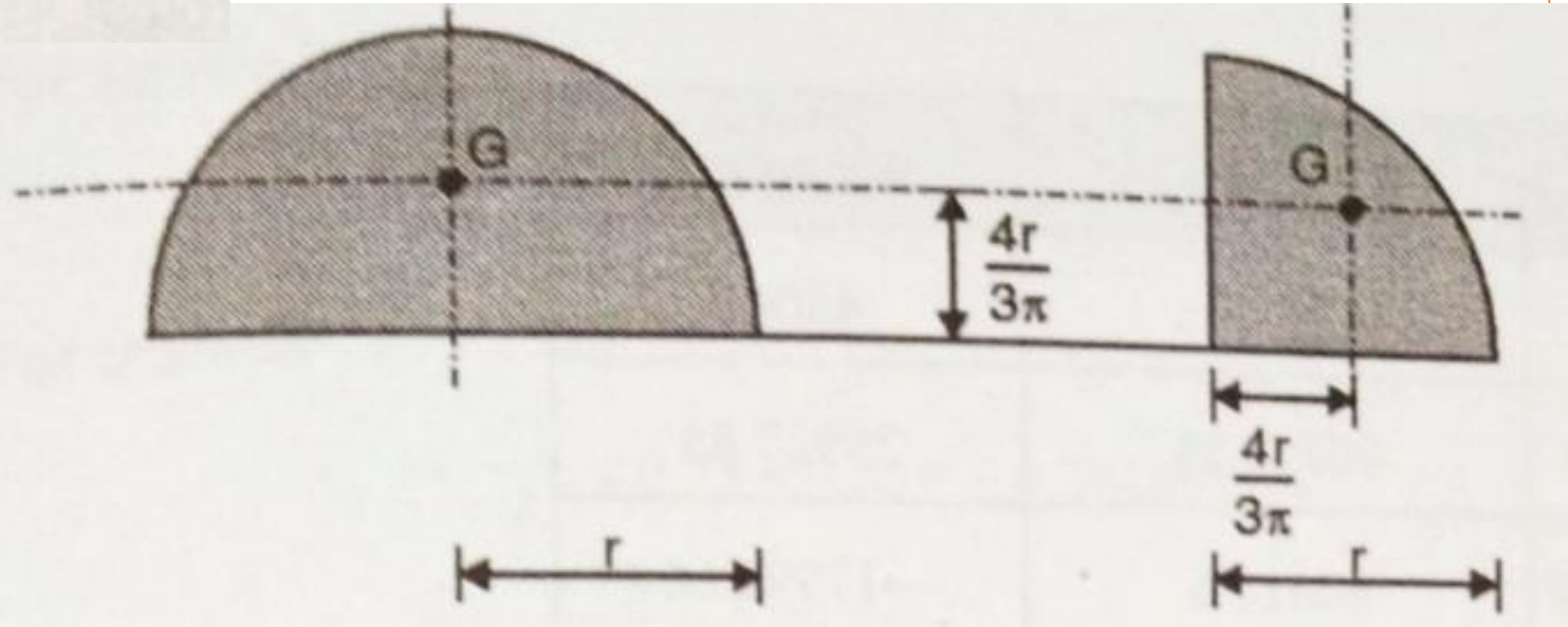
Centroid of common areas

3. Semicircle and Quarter circle



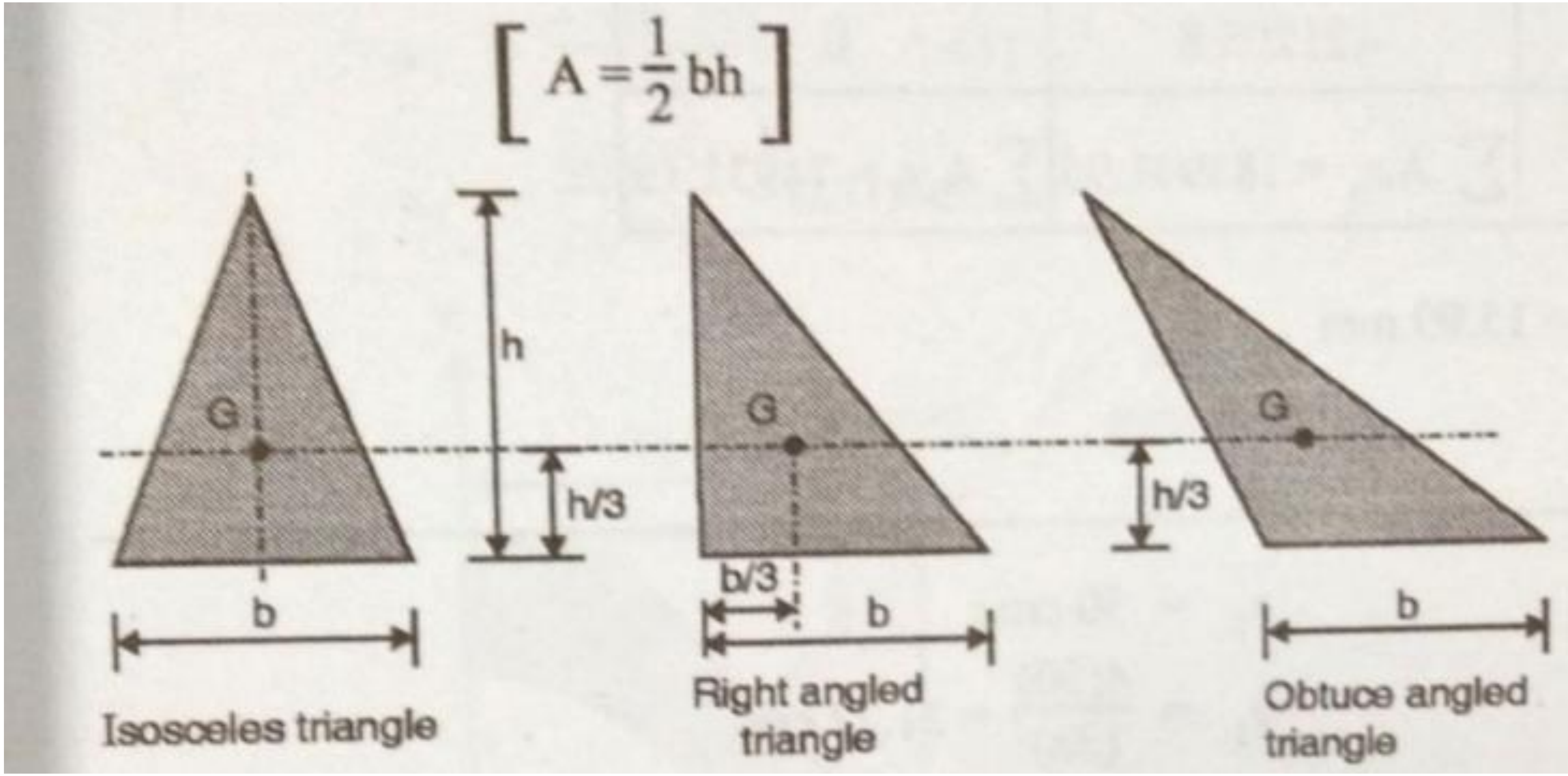
(Area $= \frac{\pi r^2}{2}$) and quarter circle

(Area $= \frac{\pi r^2}{4}$)



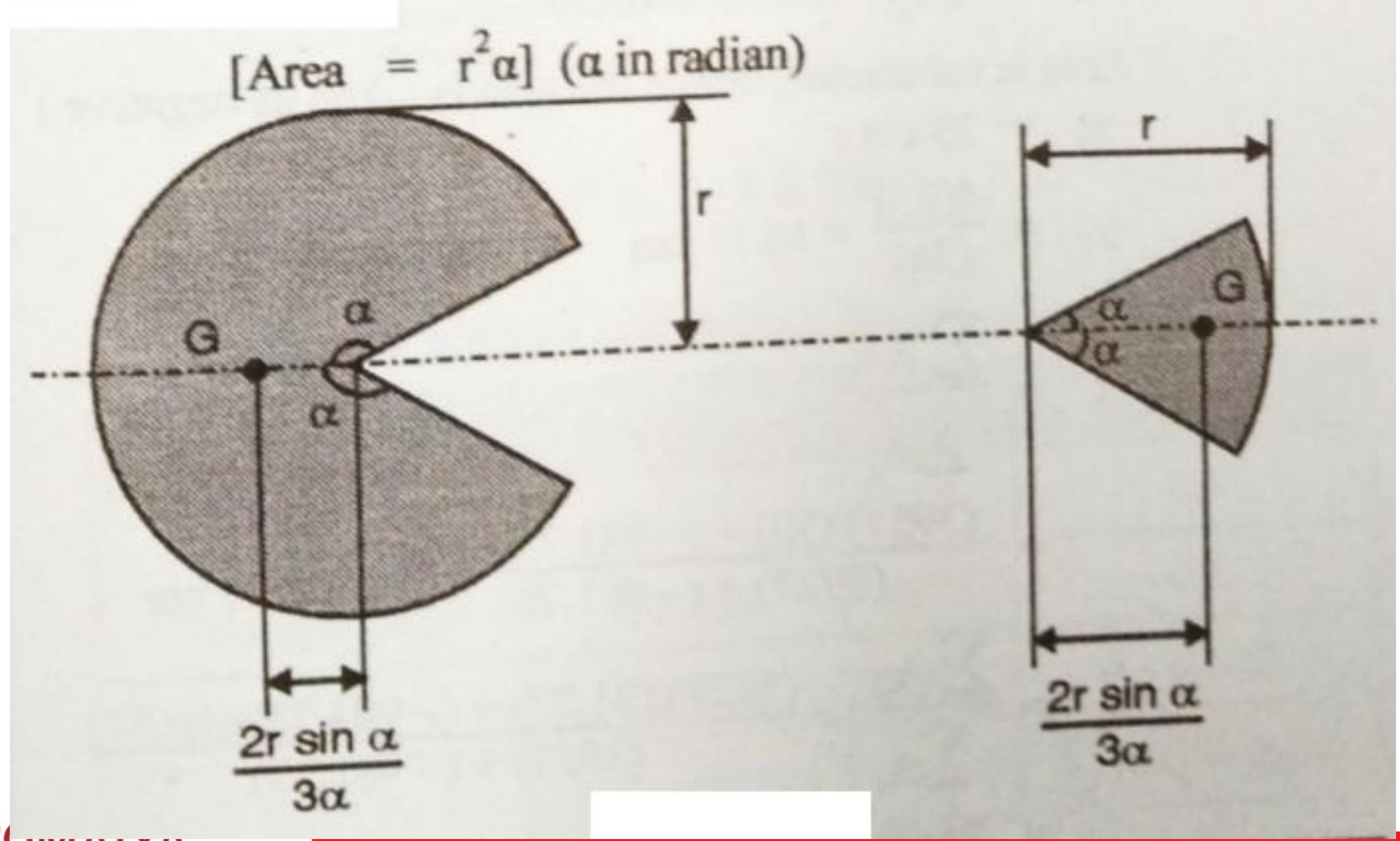
Centroid of common areas

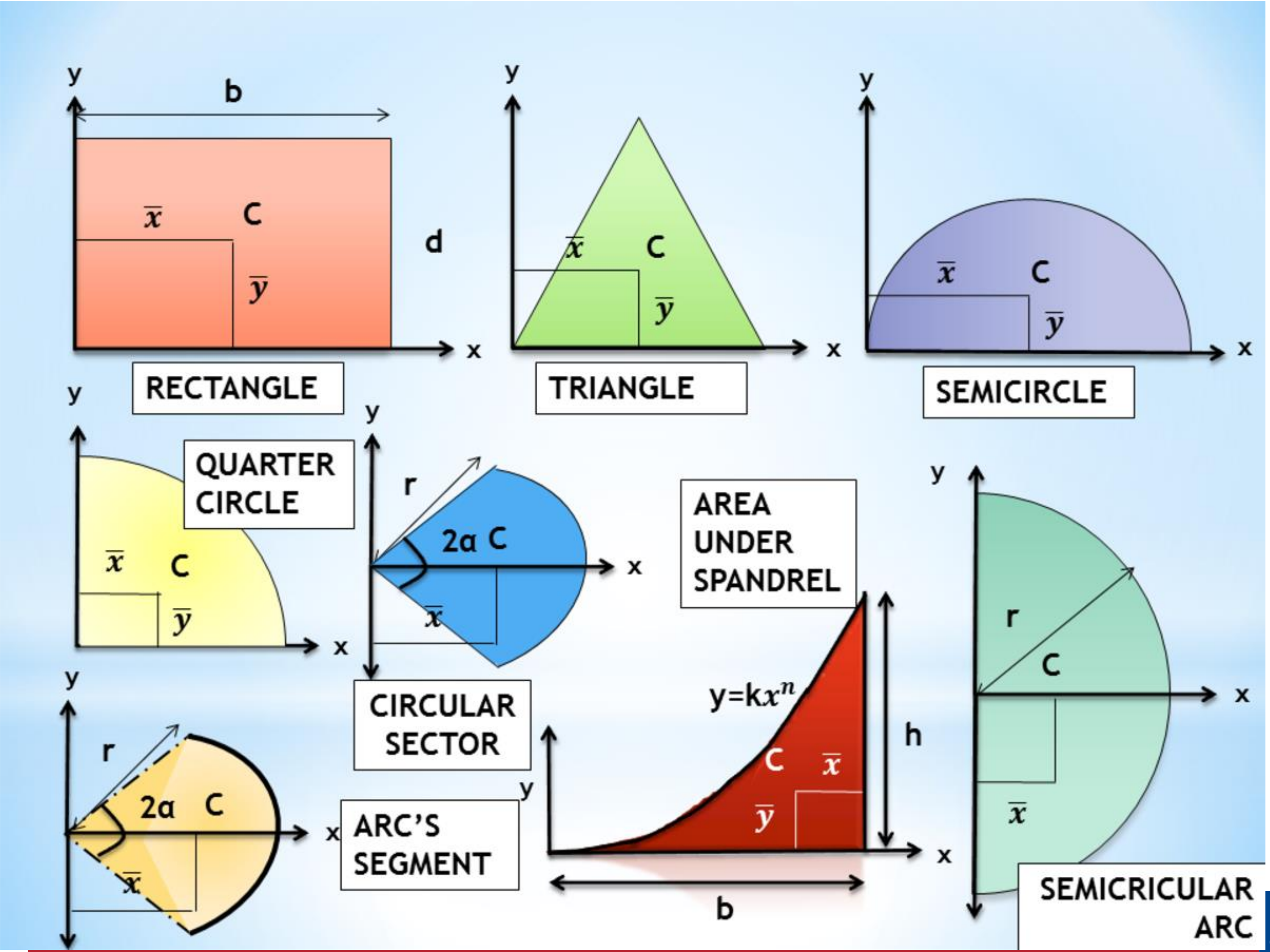
4. Triangle



Centroid of common areas

5. Sector of circle


[Area = $r^2\alpha$] (α in radian)

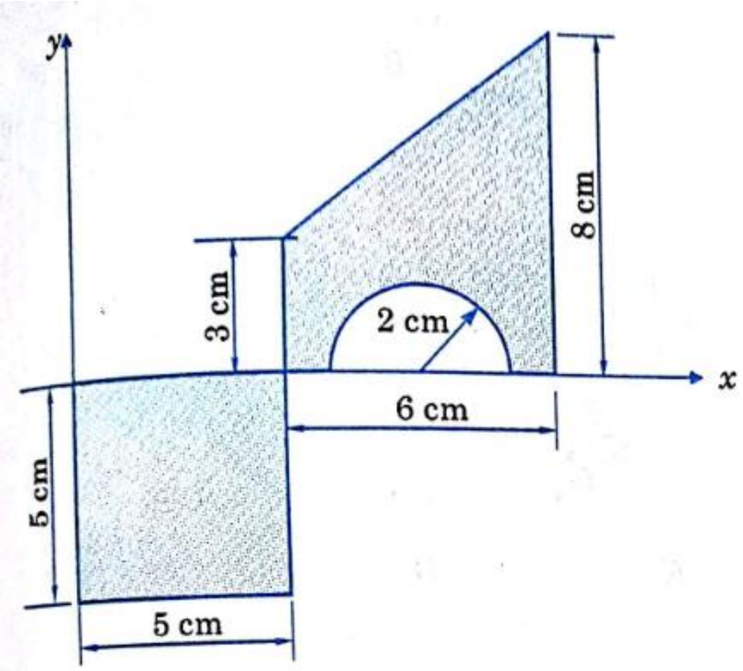

RECTANGLE
y
b
d
x
C
$\bar{x}$
$\bar{y}$
TRIANGLE
SEMICIRCLE
QUARTER CIRCLE
r
2α C
CIRCULAR SECTOR
AREA UNDER SPANDREL
y=kx^n
h
b
ARC'S SEGMENT
SEMICRICULAR ARC

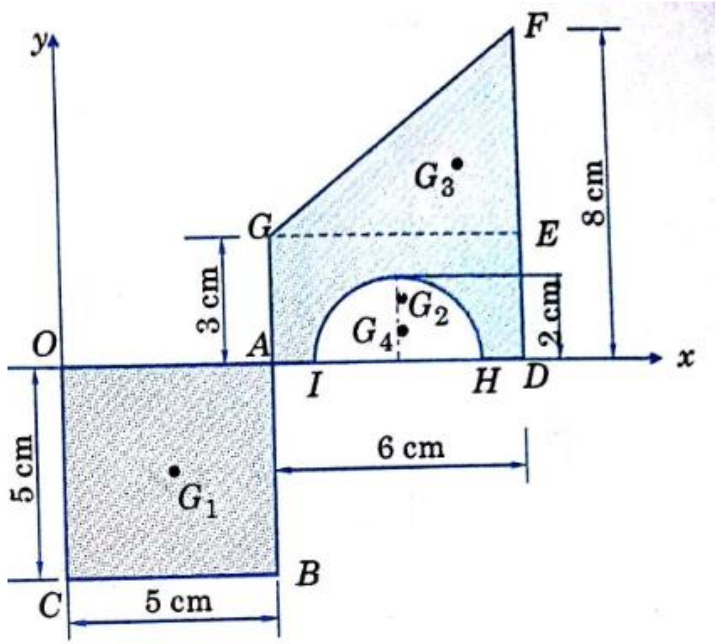
Problem 1

- Locate the centroid of the shaded lamina as shown in figure below.

Problem 2

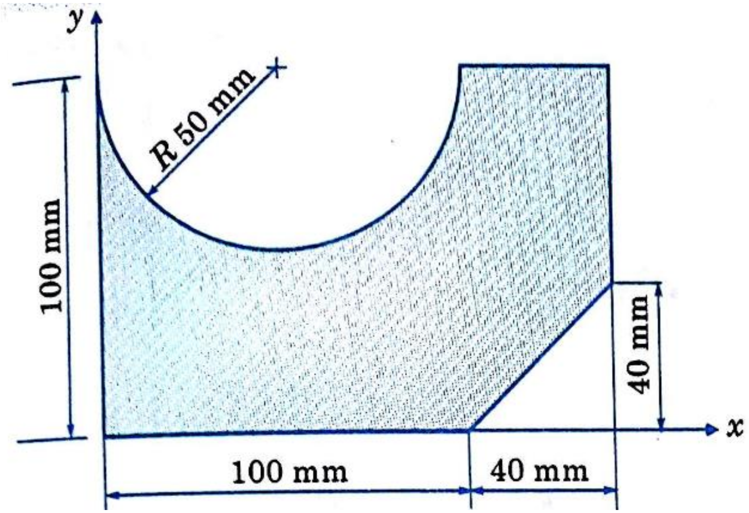
- Locate the centroid of the shaded lamina as shown in figure below.





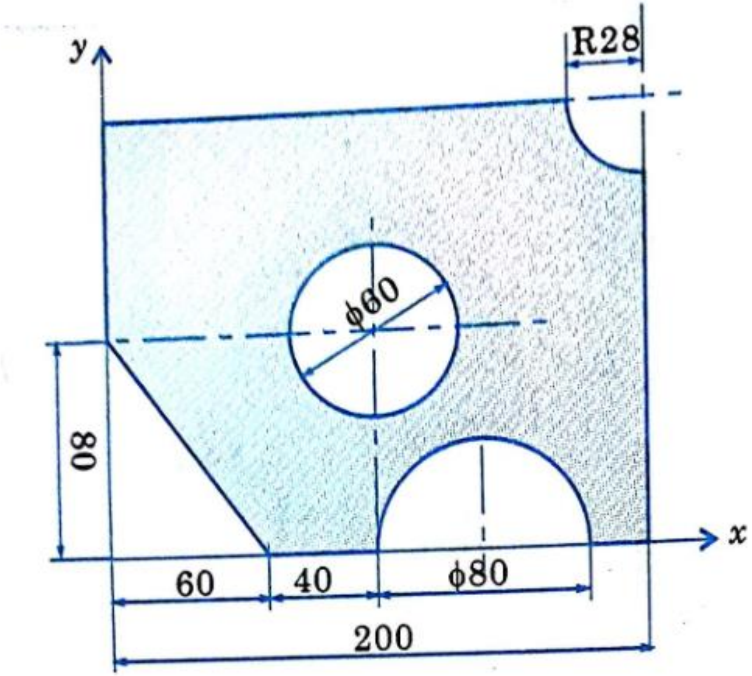
Problem 3

- Locate the centroid of the shaded lamina as shown in figure below.



Problem 4

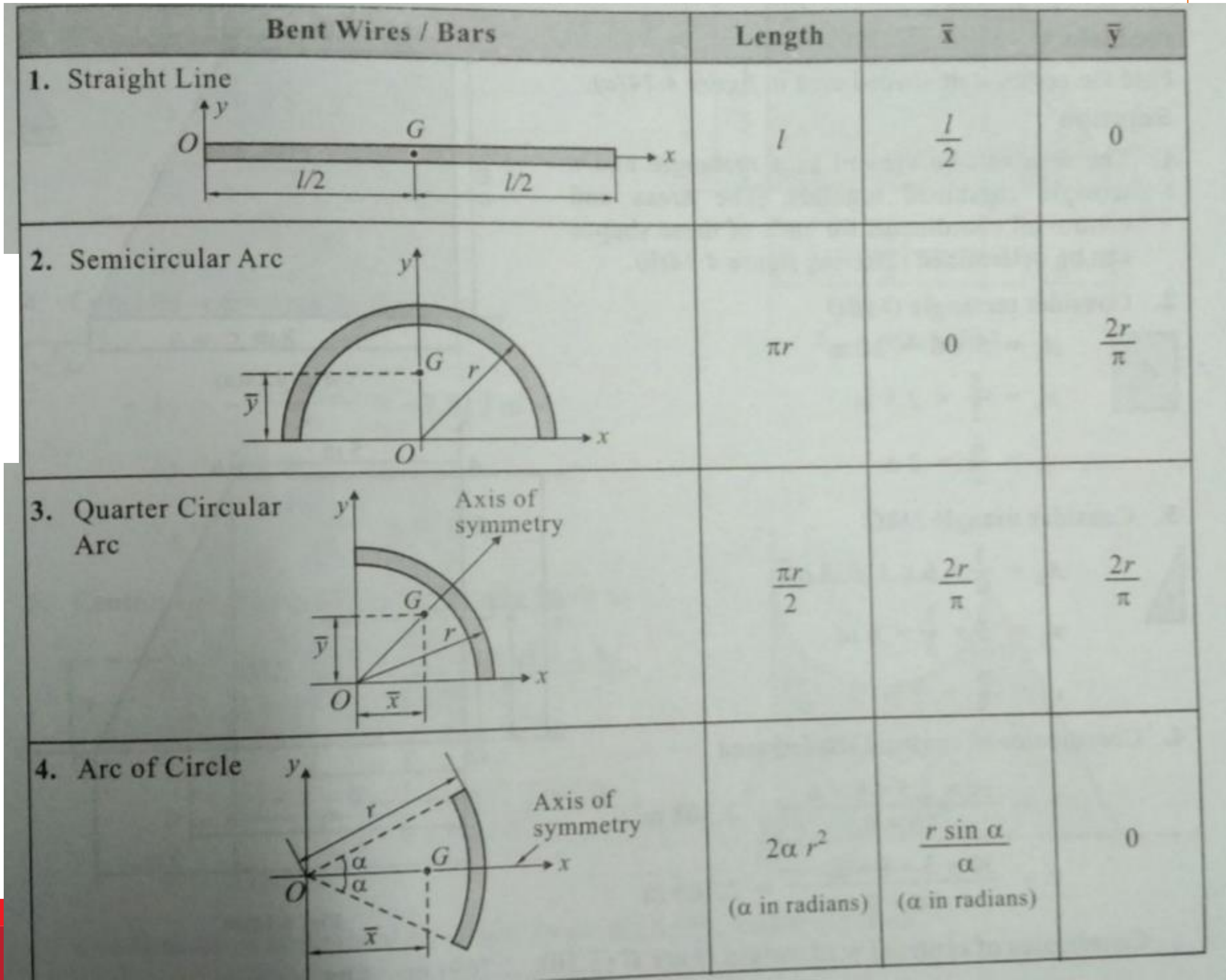
- Locate the centroid of the shaded lamina as shown in figure below.



Centroid of Wires

$$\bar{x} = \frac{l_1 x_1 + l_2 x_2 + \dots + l_n x_n}{l_1 + l_2 + l_3 + \cdots l_n} = \frac{\sum l_i x_i}{\sum l}$$

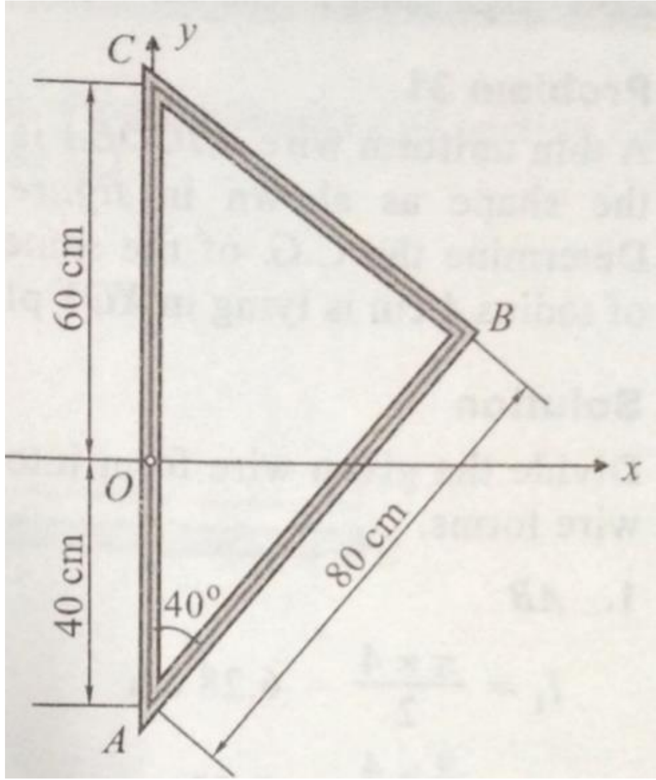
$$\bar{y} = \frac{\sum l_i y_i}{\sum l}$$

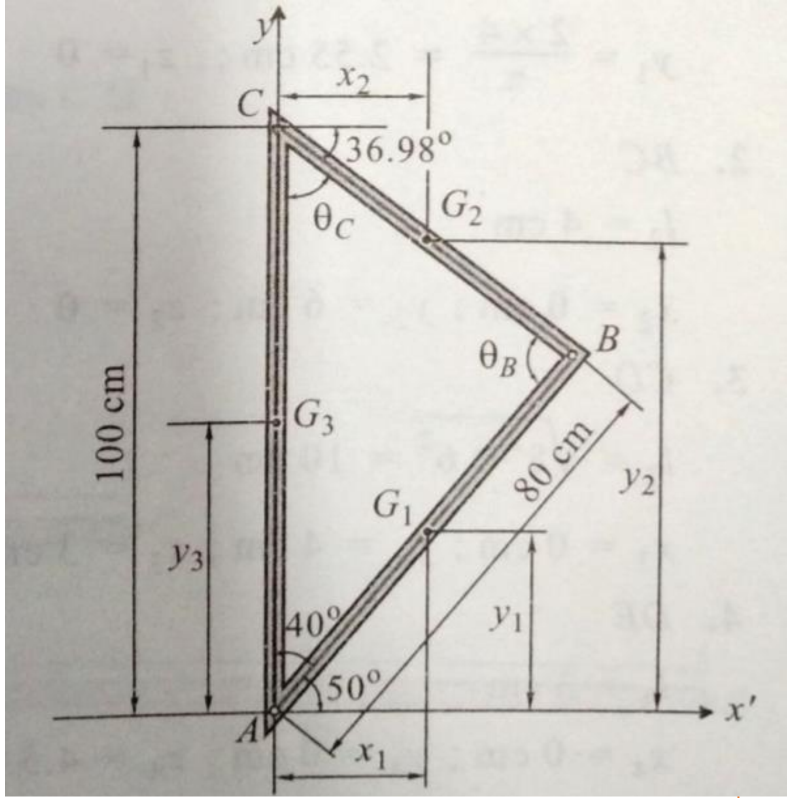


Bent Wires / Bars	Length	$\bar{x}$	$\bar{y}$
1. Straight Line	l	$\frac{l}{2}$	0
2. Semicircular Arc	πr	0	$\frac{2r}{\pi}$
3. Quarter Circular Arc	$\frac{\pi r}{2}$	$\frac{2r}{\pi}$	$\frac{2r}{\pi}$
4. Arc of Circle	$2\alpha r^2$ (α in radians)	$\frac{r \sin \alpha}{\alpha}$ (α in radians)	0

Problem 5

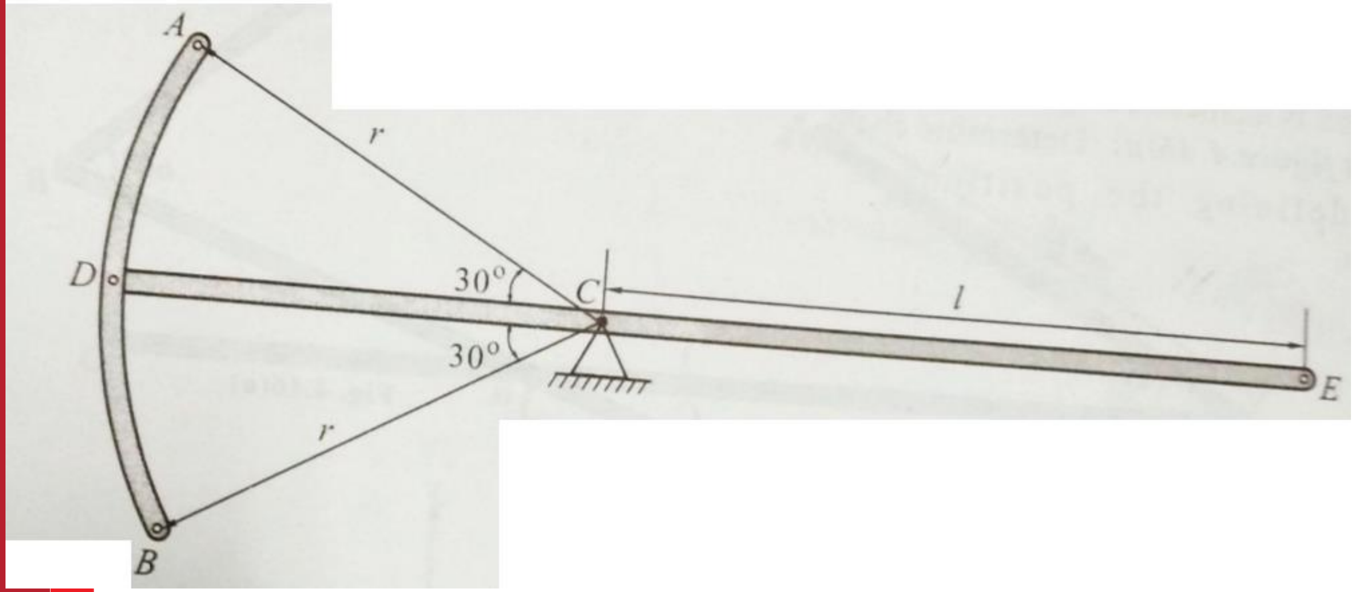
- Locate the centre of gravity of bent wire ABCA as shown in figure with respect to given x and y axis.





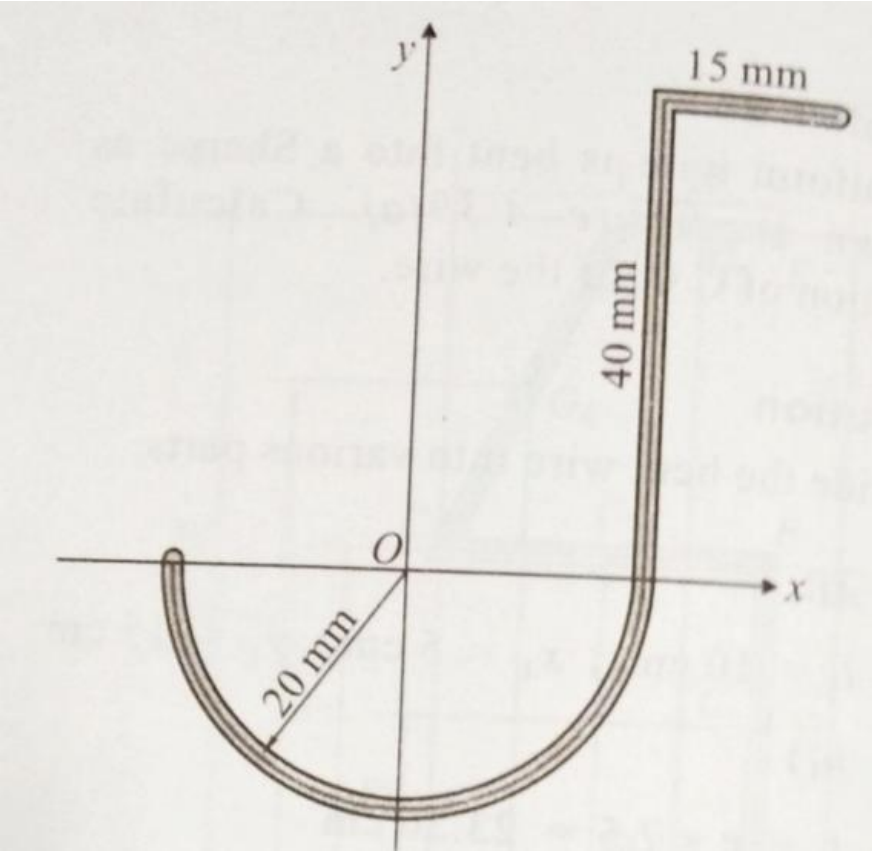
Problem 6

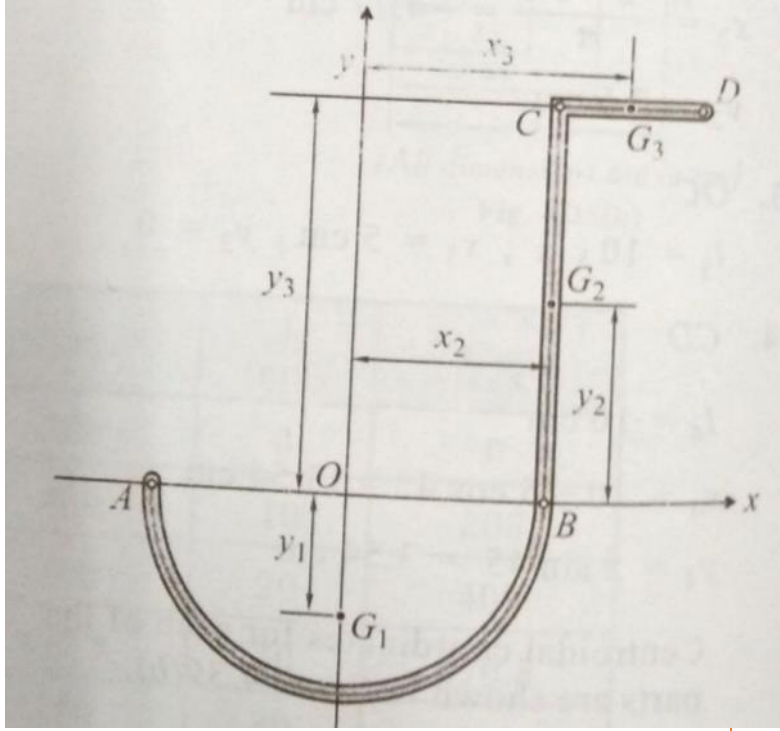
- The figure below is formed of a thin homogeneous wire. Find the length l of portion CE of bent wire for which the centre of gravity of the figure is located at ‘C’.



Problem 7

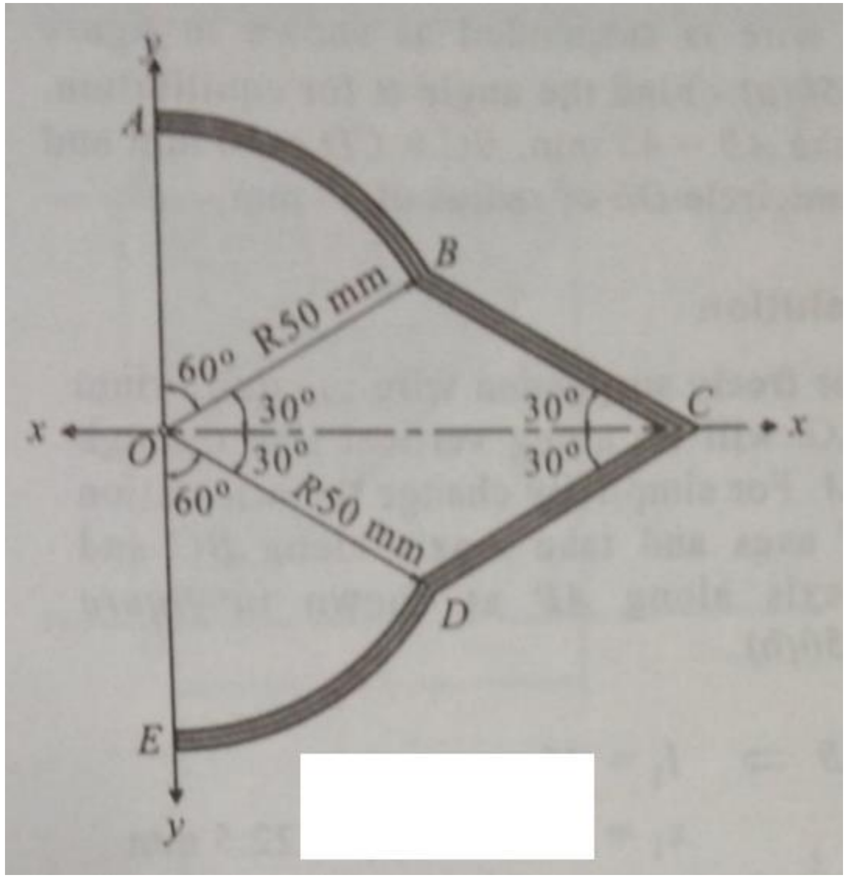
- A thin rod is bent into shape as shown in figure. Determine the centroid of the bent rod.

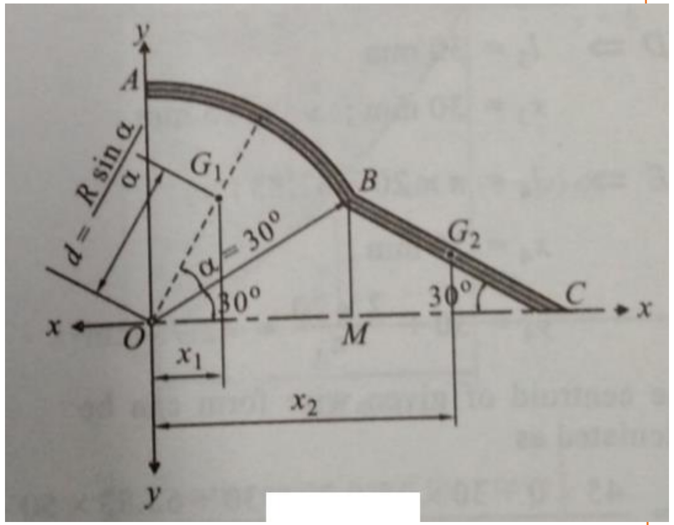




Problem 8

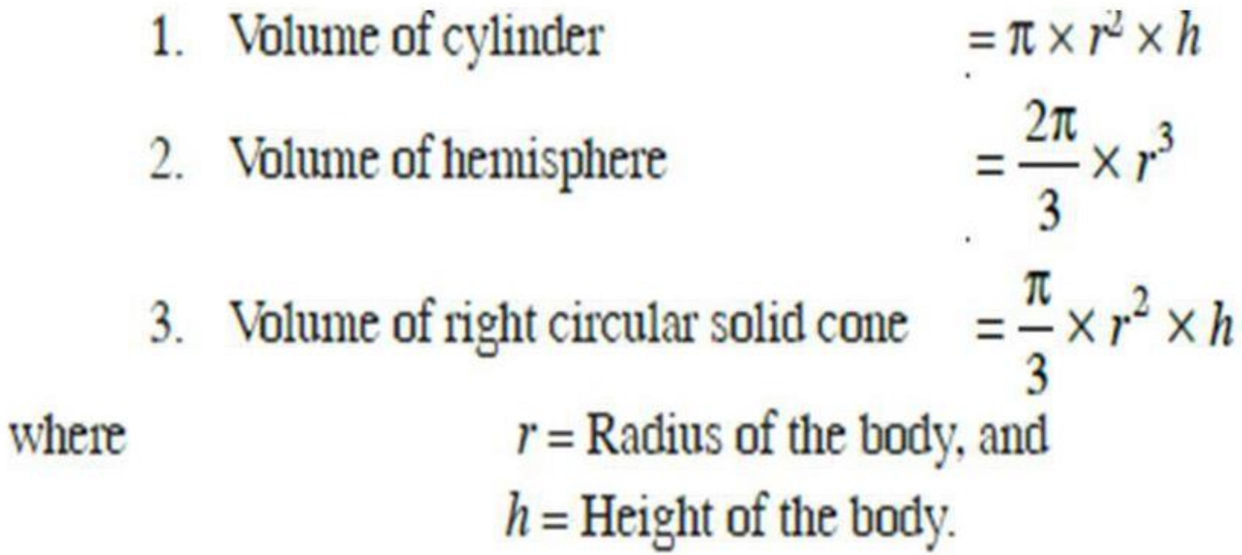
- Determine the centre of gravity of the wire ABCDE of uniform weight of 2 KN/m bent as shown in figure.





Centroid of Solids

- The centre of gravity of solid bodies (such as hemispheres, cylinders, right circular solid cones etc.) is found out in the same way as that of plane figures.
- The only difference, between the plane figures and solid bodies, is that in the case of solid bodies, we calculate volumes instead of areas.
- The volumes of few solid bodies are given below :

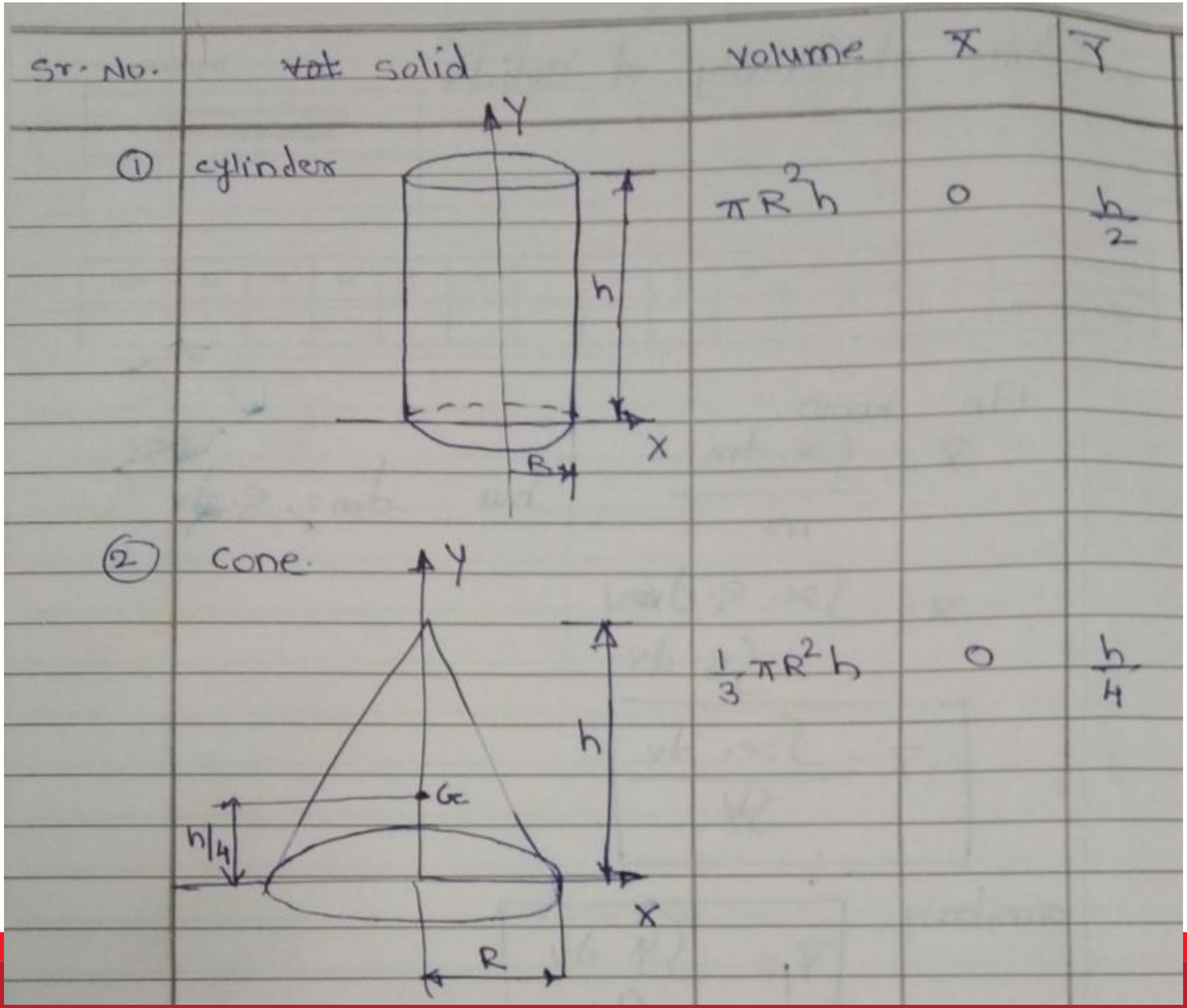


1. Volume of cylinder $= \pi \times r^2 \times h$
2. Volume of hemisphere $= \frac{2\pi}{3} \times r^3$
3. Volume of right circular solid cone $= \frac{\pi}{3} \times r^2 \times h$

where $r =$ Radius of the body, and

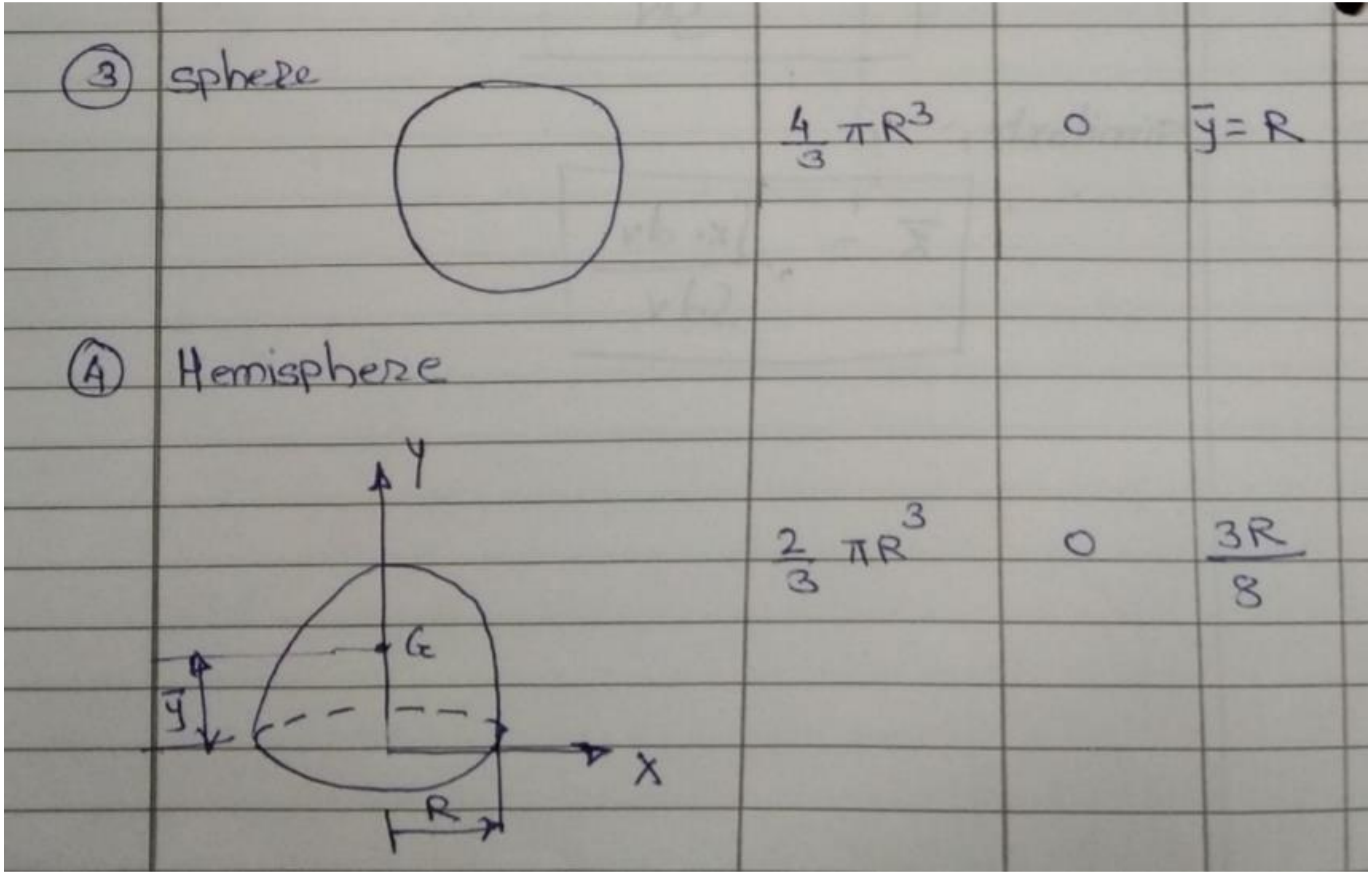
$h =$ Height of the body.

Primitive volumes



Sr. No.	~~Vol~~ solid	Volume	$\bar{x}$	$\bar{y}$
①	cylinder	$\pi R^2 h$	0	$\frac{h}{2}$
②	cone.	$\frac{1}{3}\pi R^2 h$	0	$\frac{h}{4}$

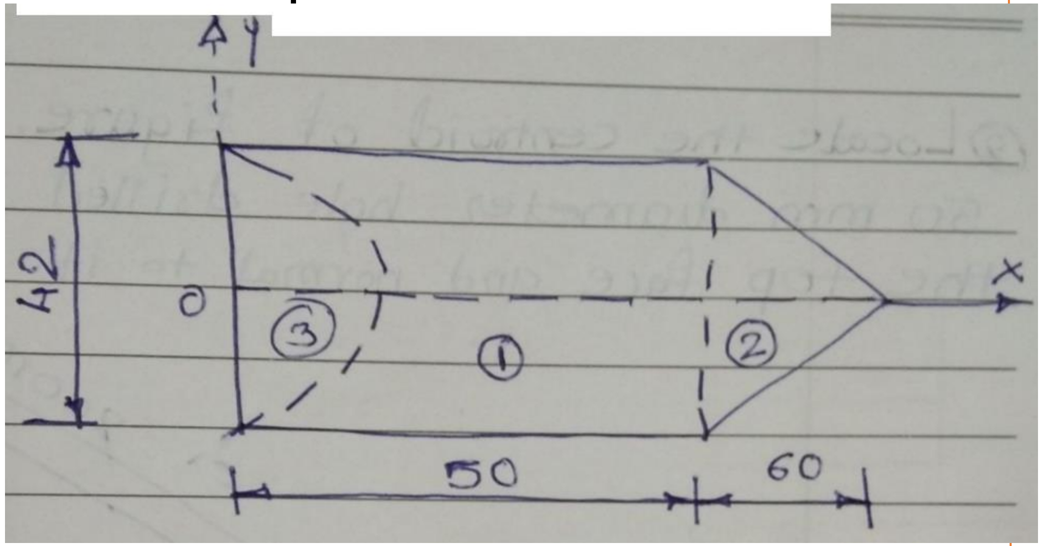
Primitive volumes



③ sphere	$\frac{4}{3}\pi R^3$	0	$\bar{y} = R$
④ Hemisphere	$\frac{2}{3}\pi R^3$	0	$\frac{3R}{8}$

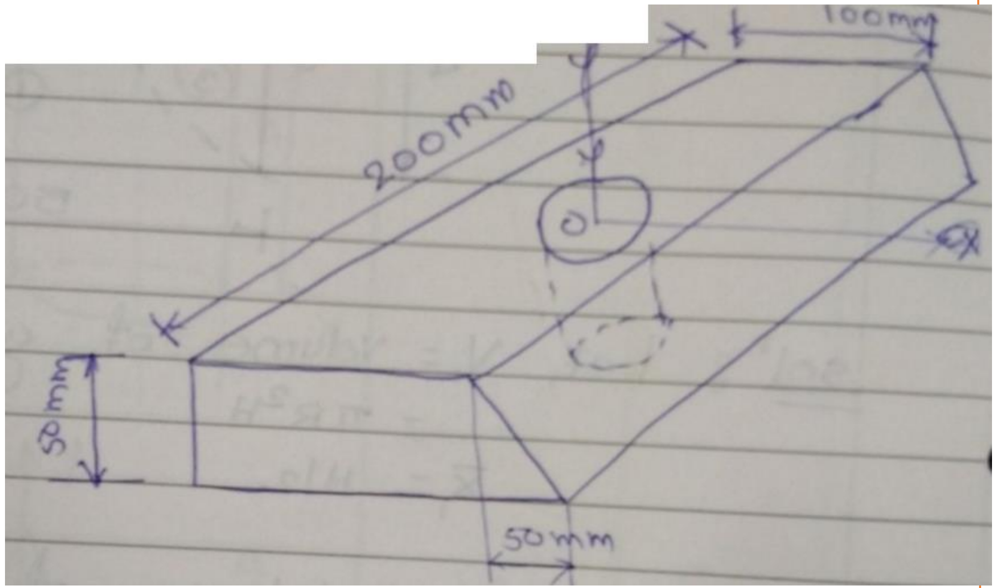
Problem 9

- A cylinder with a hemispherical cavity and a conical cap is shown in figure. All dimensions are in centimetre. Find the centroid of the composite volume.



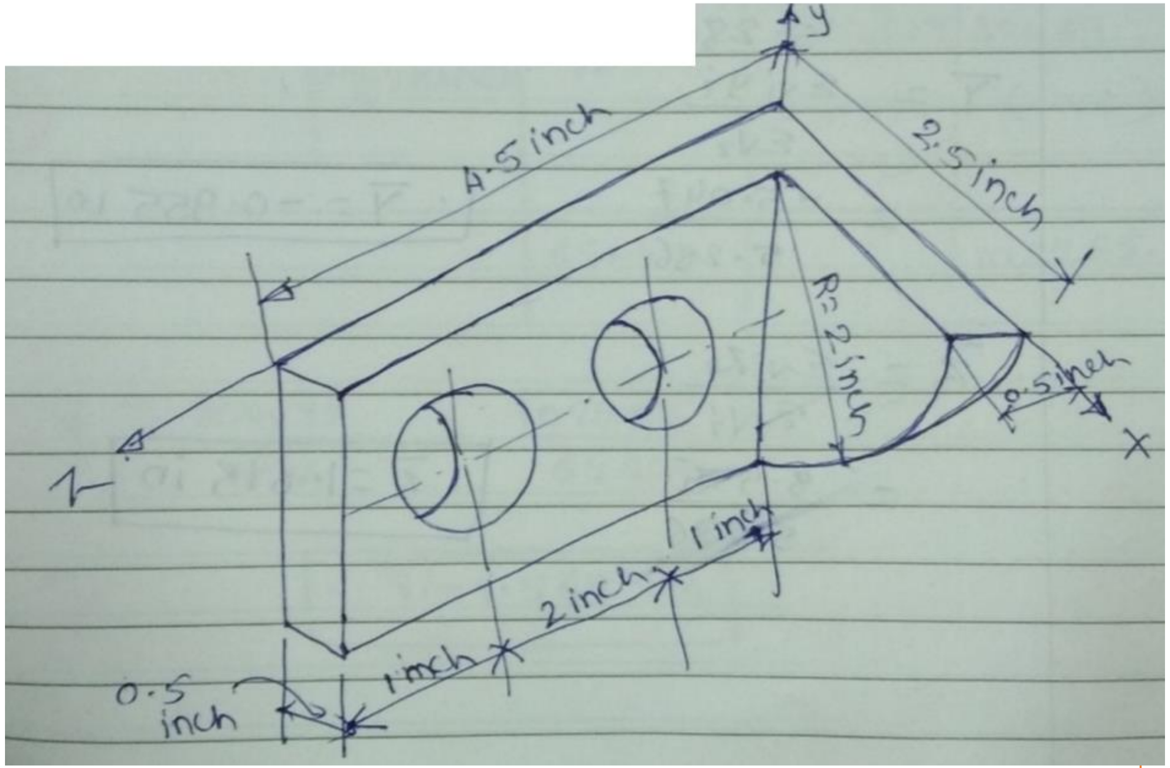
Problem 10

- Locate the centroid of figure shown below. The 50 mm diameter hole drilled in the centre of the top face and normal to it.



Problem 11

- Locate the centroid of the steel machine element shown in the figure. Both holes are of 1 inch diameter.



Problem 12

- A homogeneous composite body is shown in the figure. Determine the coordinates of the centroid.