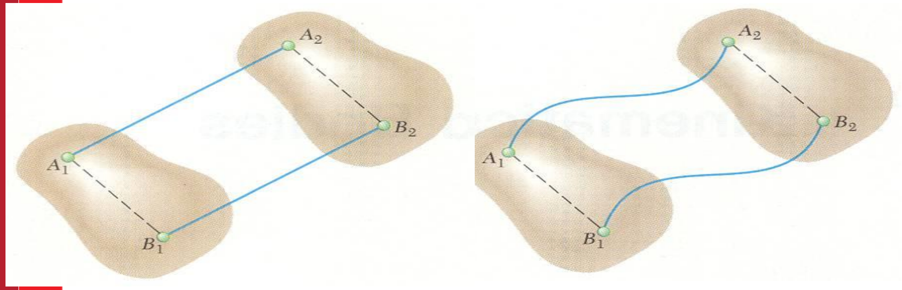
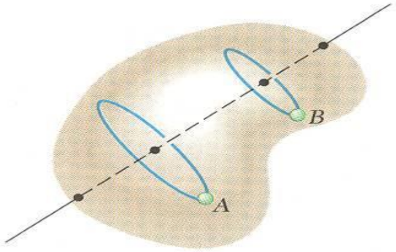
# K J SOMAIYA COLLEGE OF ENGINEERING, MUMBAI-77

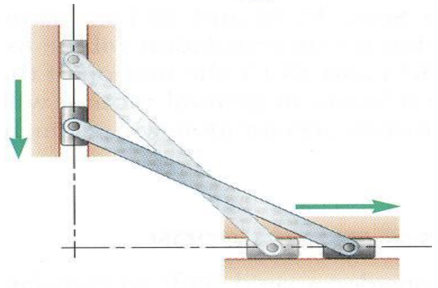
## (CONSTITUENT COLLEGE OF SOMAIYA VIDYAVIHAR UNIVERSITY)

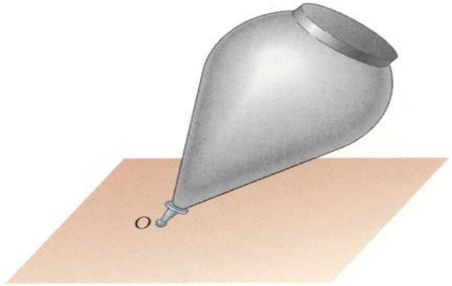
**Presented by:**
**Prof. R. B. Pansare**


# Introduction



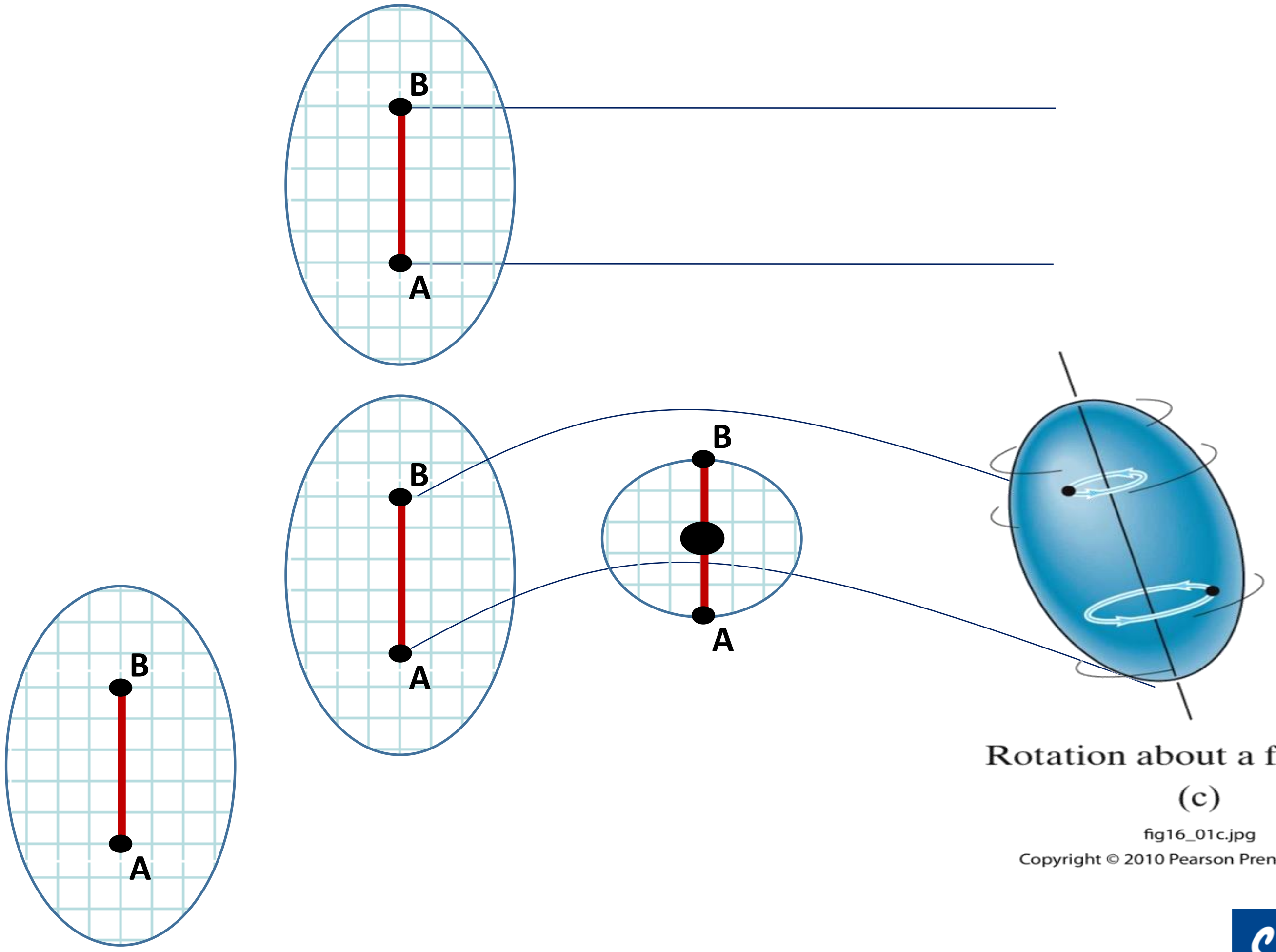



- Kinematics of rigid bodies: relations between time and the positions, velocities, and accelerations of the particles forming a rigid body.

- Classification of rigid body motions:
  - translation:
    - rectilinear translation
    - curvilinear translation
  - rotation about a fixed axis
  - general plane motion
  - motion about a fixed point
  - general motion

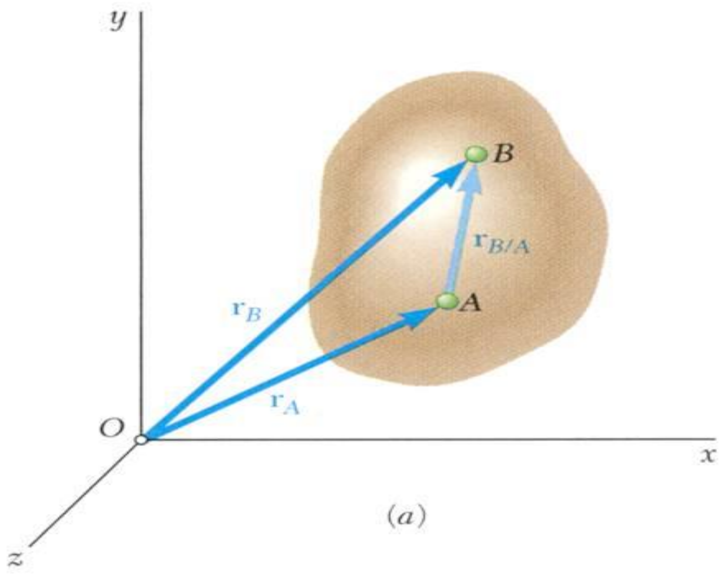
# Types of rigid body motion

- Kinematically speaking…
  - Translation
    - –Orientation of AB constant
  - Rotation
    - –All particles rotate about fixed axis
  - General Plane Motion (both)
    - –Combination of both types of motion

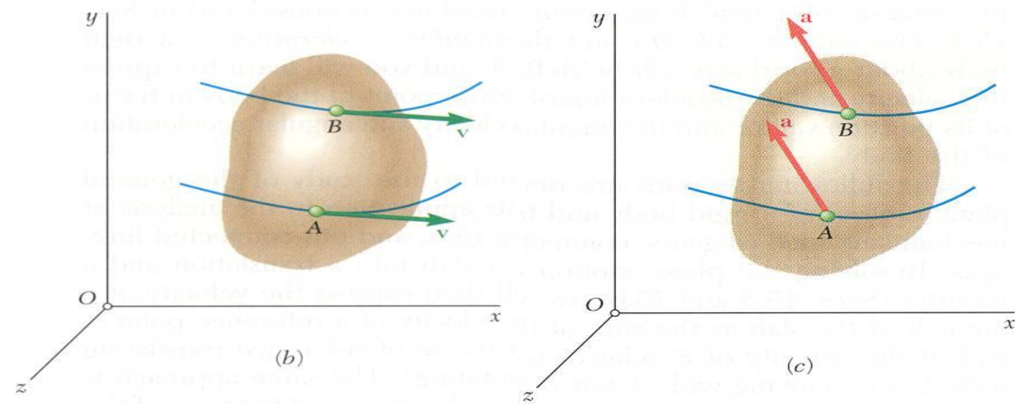



Rotation about a fixed axis
(c)
fig16_01c.jpg

# Translation




(a)




- Consider rigid body in translation:
  - direction of any straight line inside the body is constant,
  - all particles forming the body move in parallel lines.

- For any two particles in the body,

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

- Differentiating with respect to time,

$$\dot{\vec{r}}_B = \dot{\vec{r}}_A + \dot{\vec{r}}_{B/A} = \dot{\vec{r}}_A$$

$$\vec{v}_B = \vec{v}_A$$

All particles have the same velocity.

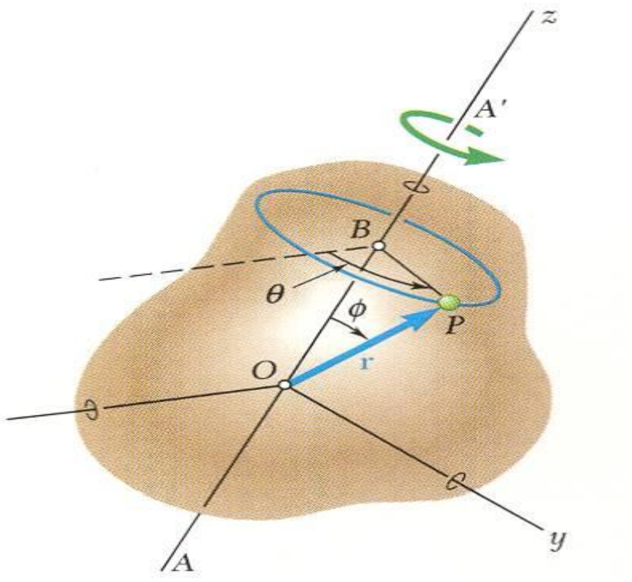
- Differentiating with respect to time again,

$$\ddot{\vec{r}}_B = \ddot{\vec{r}}_A + \ddot{\vec{r}}_{B/A} = \ddot{\vec{r}}_A$$

$$\vec{a}_B = \vec{a}_A$$

All particles have the same acceleration.

# Rotation About a Fixed Axis.  Velocity

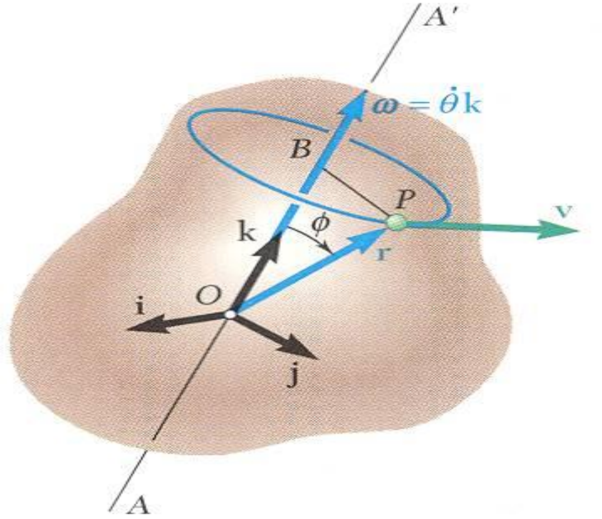



- Consider rotation of rigid body about a fixed axis *AA'*

- Velocity vector $\vec{v} = d\vec{r}/dt$ of the particle *P* is tangent to the path with magnitude $v = ds/dt$

$$\Delta s = (BP)\Delta\theta = (r\sin\phi)\Delta\theta$$

$$v = \frac{ds}{dt} = \lim_{\Delta t \to 0}(r\sin\phi)\frac{\Delta\theta}{\Delta t} = r\dot{\theta}\sin\phi$$

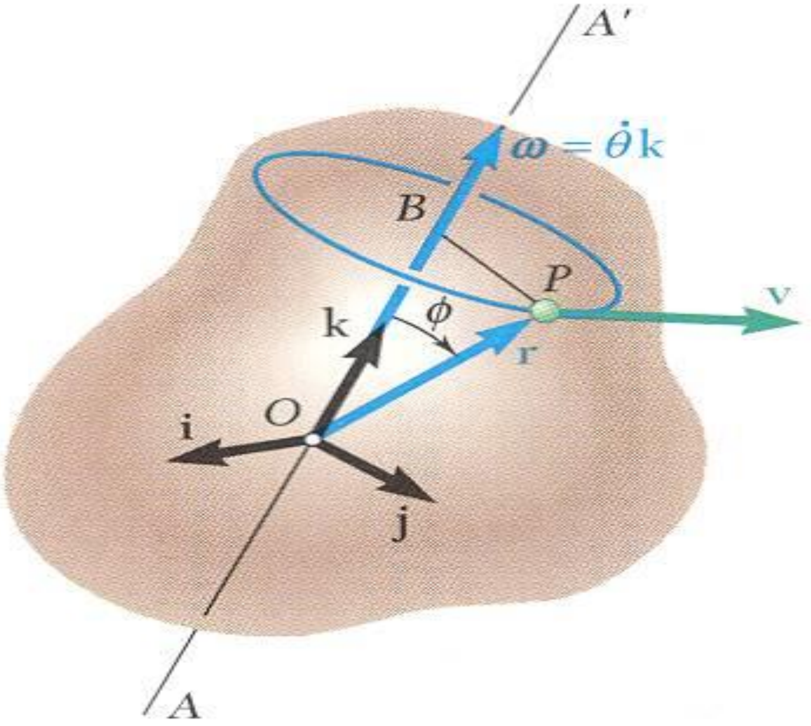



- The same result is obtained from

$$\vec{v} = \frac{d\vec{r}}{dt} = \vec{\omega}\times\vec{r}$$

$$\vec{\omega} = \omega\vec{k} = \dot{\theta}\vec{k} = angular\ velocity$$

# Rotation About a Fixed Axis. Acceleration




- Differentiating to determine the acceleration,

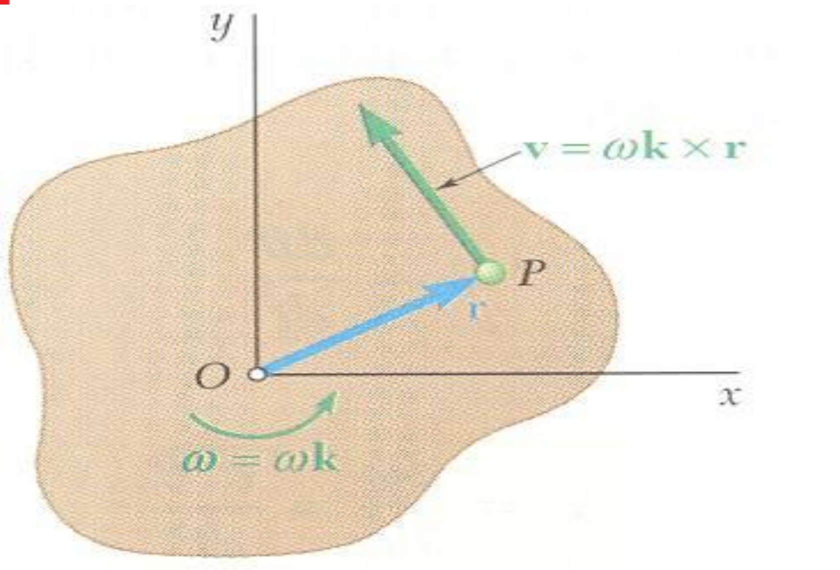
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r})$$
$$= \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$$
$$= \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \vec{v}$$

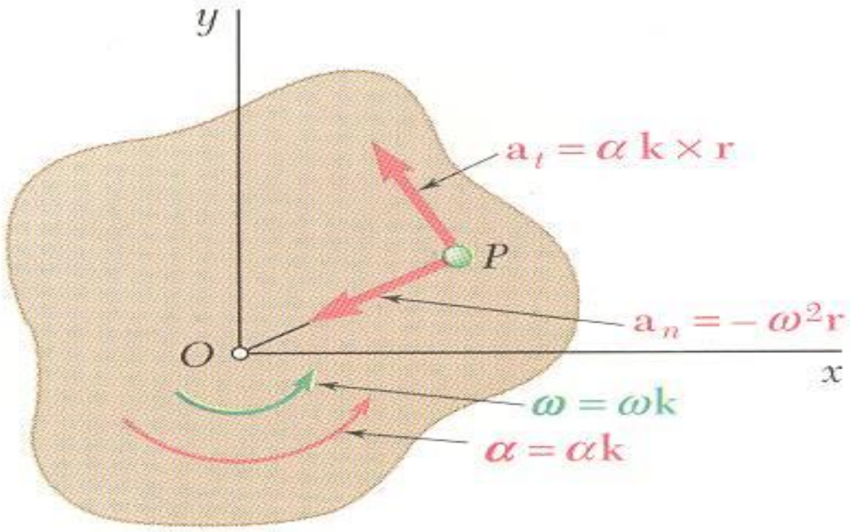
- $$\frac{d\vec{\omega}}{dt} = \vec{\alpha} = angular\ acceleration$$
$$= \alpha\vec{k} = \dot{\omega}\vec{k} = \ddot{\theta}\vec{k}$$

- Acceleration of *P* is combination of two vectors,

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{\omega} \times \vec{r}$$
$$\vec{\alpha} \times \vec{r} = \text{tangential acceleration component}$$
$$\vec{\omega} \times \vec{\omega} \times \vec{r} = \text{radial acceleration component}$$

# Rotation About a Fixed Axis.  Representative Slab




- Consider the motion of a representative slab in a plane perpendicular to the axis of rotation.

- Velocity of any point *P* of the slab,

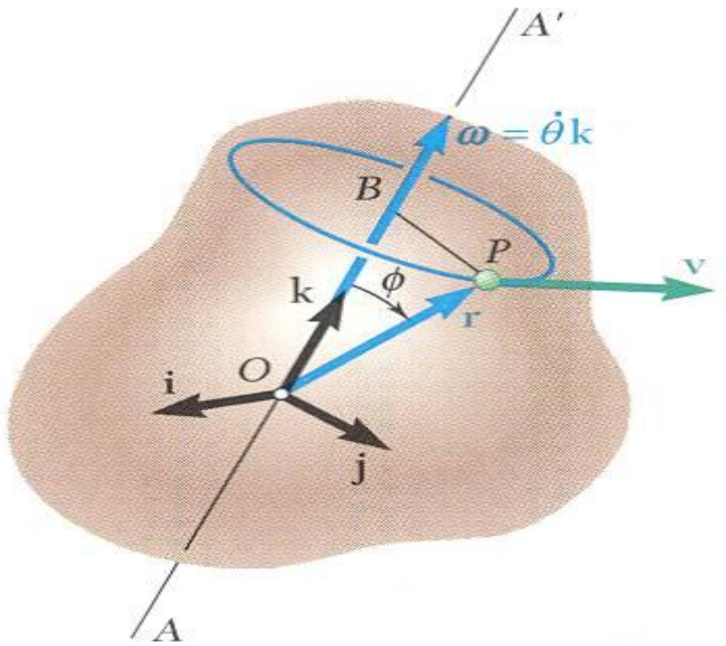
$$\vec{v} = \vec{\omega} \times \vec{r} = \omega \vec{k} \times \vec{r}$$
$$v = r\omega$$

- Acceleration of any point *P* of the slab,

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{\omega} \times \vec{r}$$
$$= \alpha \vec{k} \times \vec{r} - \omega^2 \vec{r}$$

- Resolving the acceleration into tangential and normal components,

$$\vec{a}_t = \alpha \vec{k} \times \vec{r} \qquad a_t = r\alpha$$
$$\vec{a}_n = -\omega^2 \vec{r} \qquad a_n = r\omega^2$$




- Motion of a rigid body rotating around a fixed axis is often specified by the type of angular acceleration.

- Recall $\omega = \dfrac{d\theta}{dt}$ or $dt = \dfrac{d\theta}{\omega}$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \omega\frac{d\omega}{d\theta}$$

- *Uniform Rotation*, $\alpha = 0$:

$$\theta = \theta_0 + \omega t$$

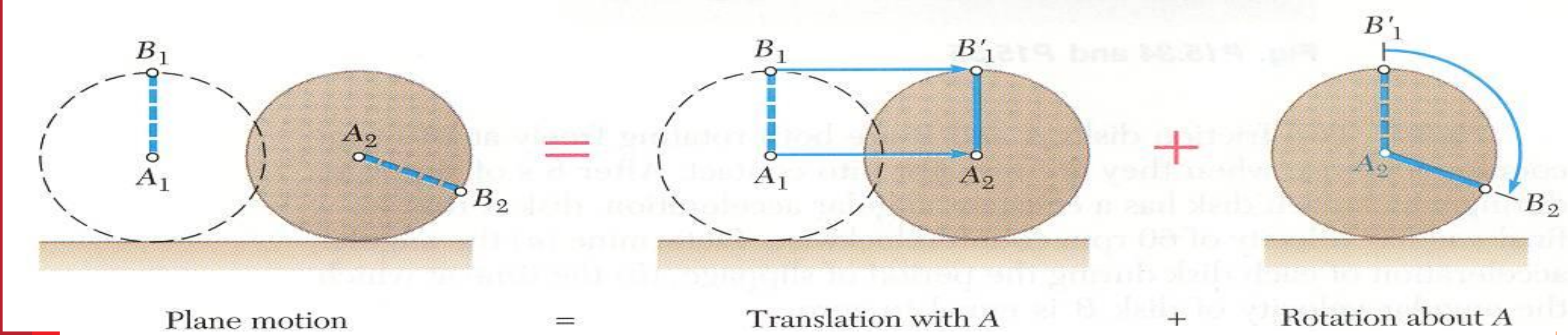
- *Uniformly Accelerated Rotation*, $\alpha$ = constant:

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$

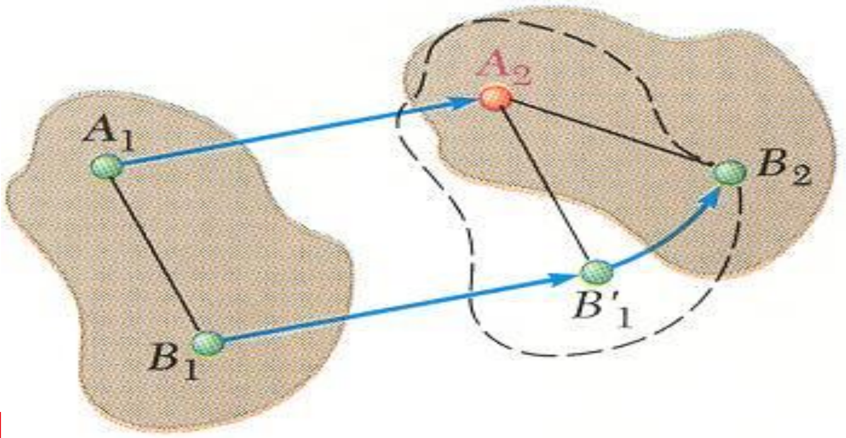
$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

# General Plane Motion



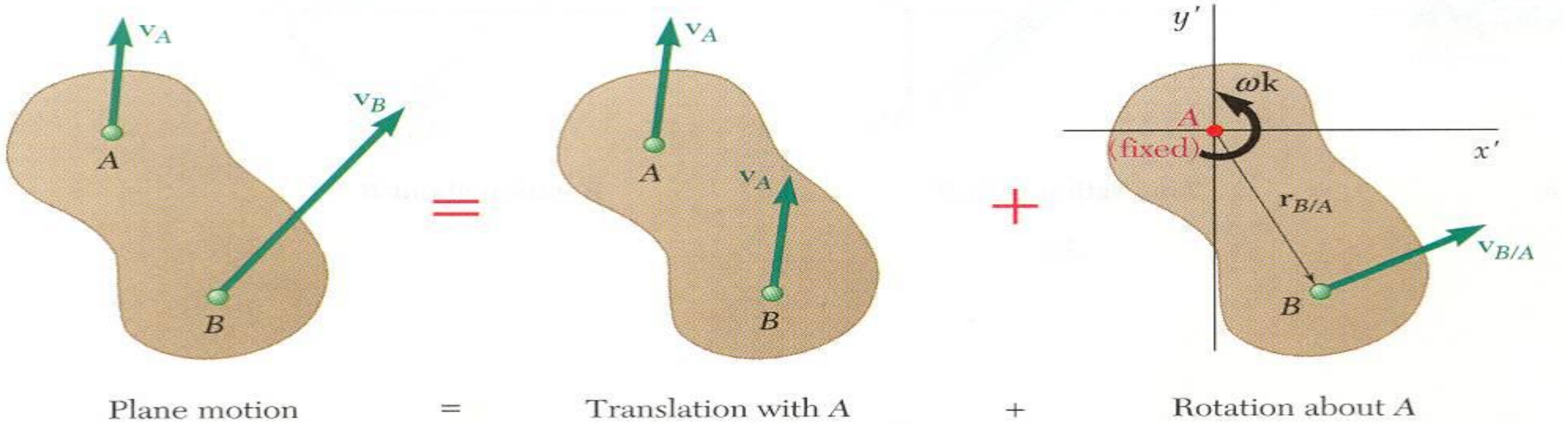

- *General plane motion* is neither a translation nor a rotation.

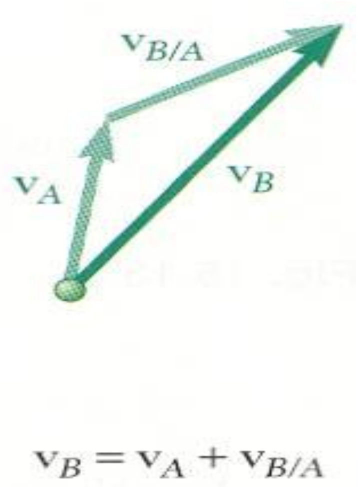
- General plane motion can be considered as the *sum* of a translation and rotation.




- Displacement of particles $A$ and $B$ to $A_2$ and $B_2$ can be divided into two parts:
  - translation to $A_2$ and $B_1'$
  - rotation of $B_1'$ about $A_2$ to $B_2$

# Absolute and Relative Velocity in Plane Motion



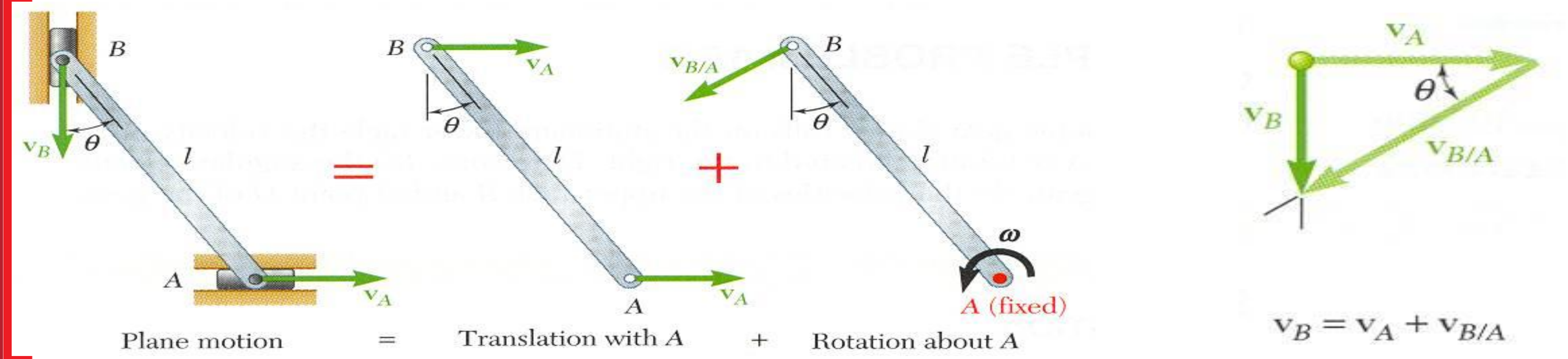

- Any plane motion can be replaced by a translation of an arbitrary reference point *A* and a simultaneous rotation about *A*.

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{v}_{B/A} = \omega\vec{k} \times \vec{r}_{B/A} \qquad v_{B/A} = r\omega$$

$$\vec{v}_B = \vec{v}_A + \omega\vec{k} \times \vec{r}_{B/A}$$

# Absolute and Relative Velocity in Plane




- Assuming that the velocity $v_A$ of end $A$ is known, wish to determine the velocity $v_B$ of end $B$ and the angular velocity ω in terms of $v_A$, $l$, and θ.
- The direction of $v_B$ and $v_{B/A}$ are known. Complete the velocity diagram.

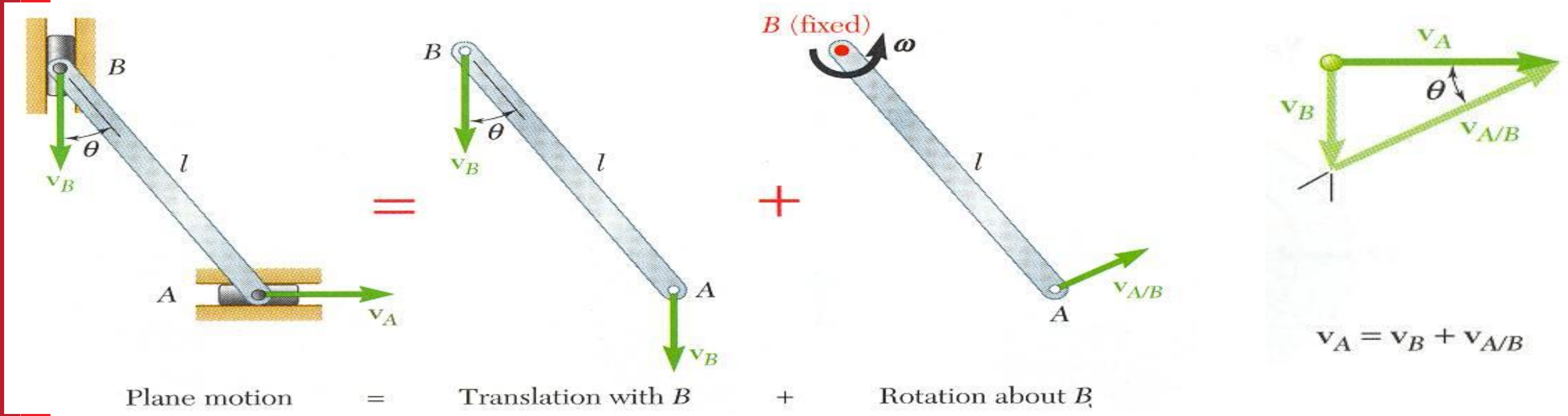
$$\frac{v_B}{v_A} = \tan\theta$$

$$v_B = v_A \tan\theta$$

$$\frac{v_A}{v_{B/A}} = \frac{v_A}{l\omega} = \cos\theta$$

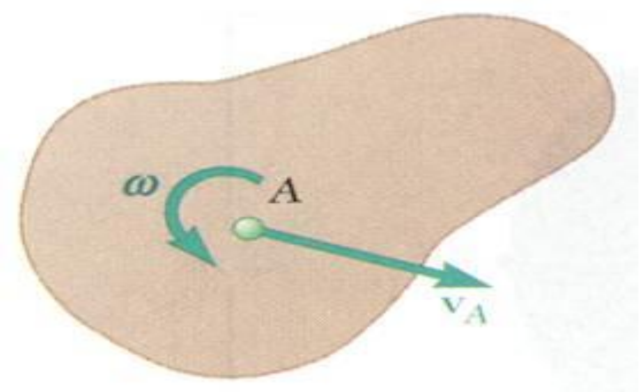
$$\omega = \frac{v_A}{l\cos\theta}$$

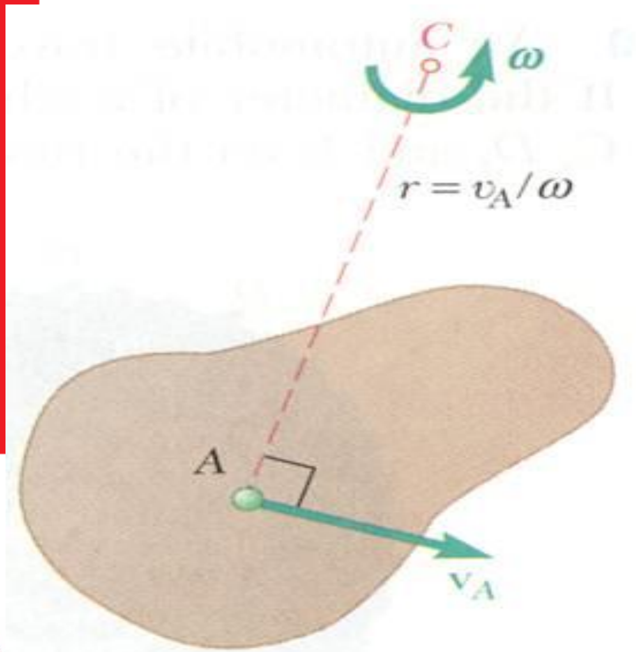
# Absolute and Relative Velocity in Plane




- Selecting point $B$ as the reference point and solving for the velocity $v_A$ of end $A$ and the angular velocity $\omega$ leads to an equivalent velocity triangle.
- $v_{A/B}$ has the same magnitude but opposite sense of $v_{B/A}$. The sense of the relative velocity is dependent on the choice of reference point.
- Angular velocity $\omega$ of the rod in its rotation about $B$ is the same as its rotation about $A$. Angular velocity is not dependent on the choice of reference point.

# Instantaneous Center of Rotation in Plane Motion



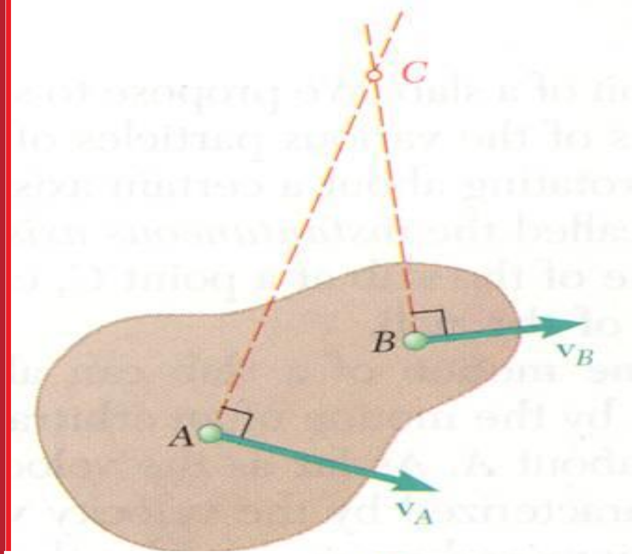

- Plane motion of all particles in a slab can always be replaced by the translation of an arbitrary point *A* and a rotation about *A* with an angular velocity that is independent of the choice of *A*.

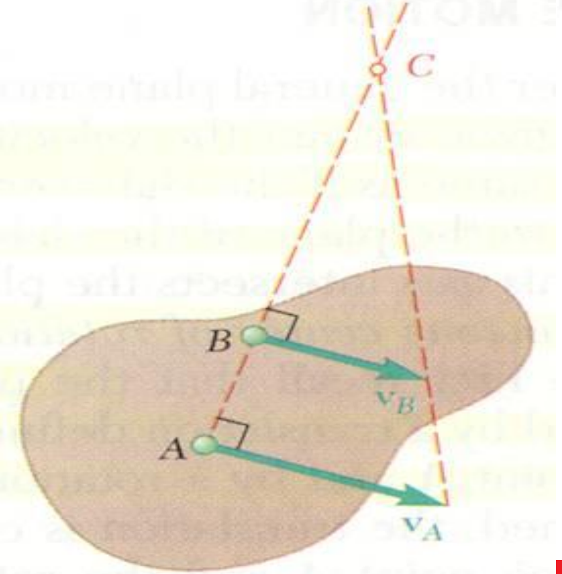
- The same translational and rotational velocities at *A* are obtained by allowing the slab to rotate with the same angular velocity about the point *C* on a perpendicular to the velocity at *A*.

- The velocity of all other particles in the slab are the same as originally defined since the angular velocity and translational velocity at *A* are equivalent.

- As far as the velocities are concerned, the slab seems to rotate about the *instantaneous center of rotation C*.

# Instantaneous Center of Rotation in Plane Motion



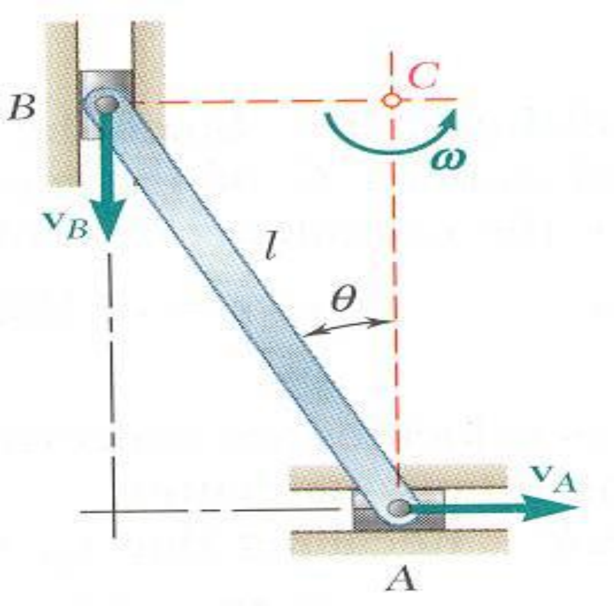

- If the velocity at two points *A* and *B* are known, the instantaneous center of rotation lies at the intersection of the perpendiculars to the velocity vectors through *A* and *B* .

- If the velocity vectors are parallel, the instantaneous center of rotation is at infinity and the angular velocity is zero.

- If the velocity vectors at *A* and *B* are perpendicular to the line *AB*, the instantaneous center of rotation lies at the intersection of the line *AB* with the line joining the extremities of the velocity vectors at *A* and *B*.

- If the velocity magnitudes are equal, the instantaneous center of rotation is at infinity and the angular velocity is zero.

# Instantaneous Center of Rotation in Plane Motion

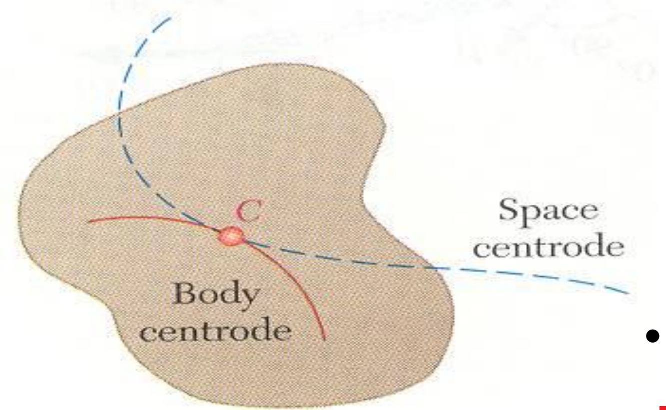



- The instantaneous center of rotation lies at the intersection of the perpendiculars to the velocity vectors through $A$ and $B$ .

$$\omega = \frac{v_A}{AC} = \frac{v_A}{l\cos\theta} \qquad v_B = (BC)\omega = (l\sin\theta)\frac{v_A}{l\cos\theta}$$
$$= v_A \tan\theta$$

- The velocities of all particles on the rod are as if they were rotated about $C$.
- The particle at the center of rotation has zero velocity.
- The particle coinciding with the center of rotation changes with time and the acceleration of the particle at the instantaneous center of rotation is not zero.

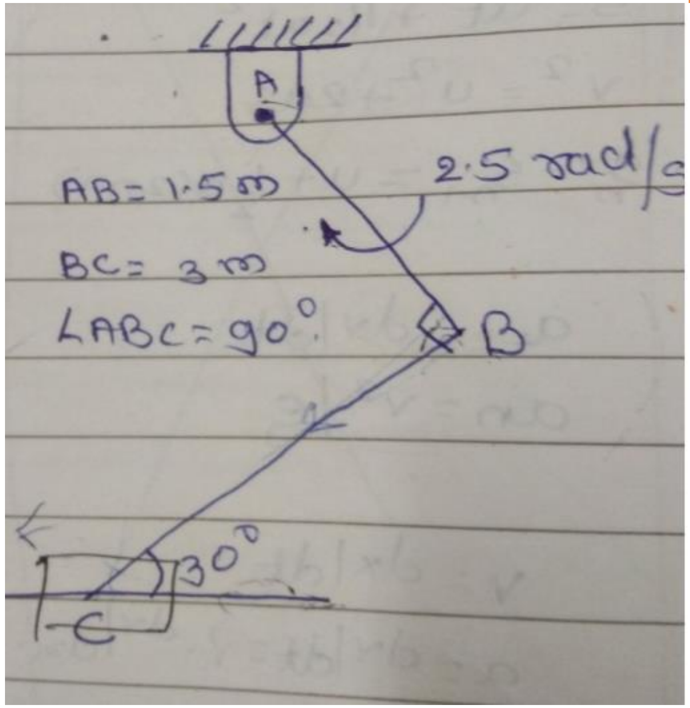
The acceleration of the particles in the slab cannot be determined as if the slab were simply rotating about $C$.



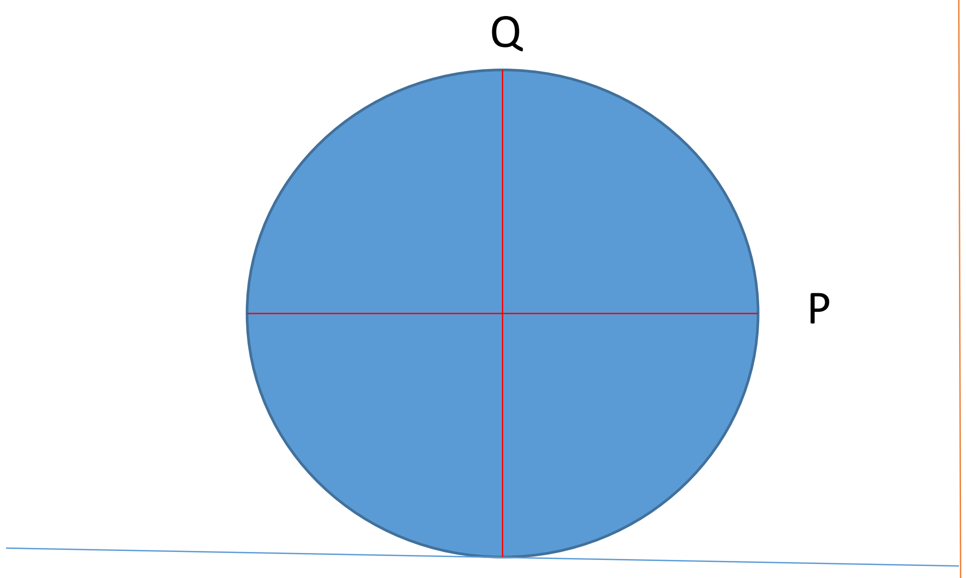

- The trace of the locus of the center of rotation on the body is the body centrode and in space is the space centrode.

- A rod AB 26 m long leans against a vertical wall. The end A on the floor is drawn away from the wall at a rate of 24 m/s. When the end A of the rod is 10 m from the wall, determine the velocity of B sliding down vertically and the angular velocity of the rod.

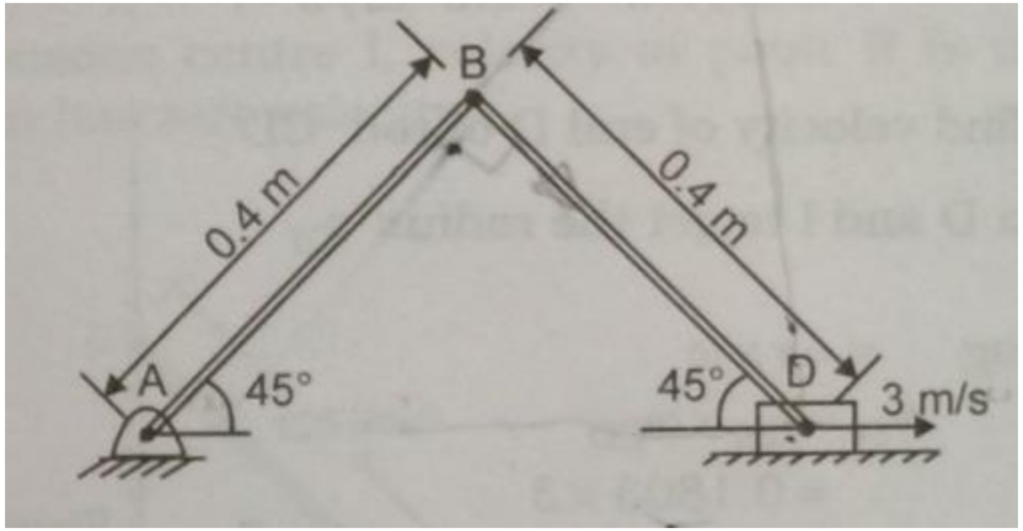
- At the instant shown in figure, the rod AB is rotating clockwise at 2.5 rad/sec. If the end C of the rod BC is free to move on horizontal surface, find the angular velocity of the point C.



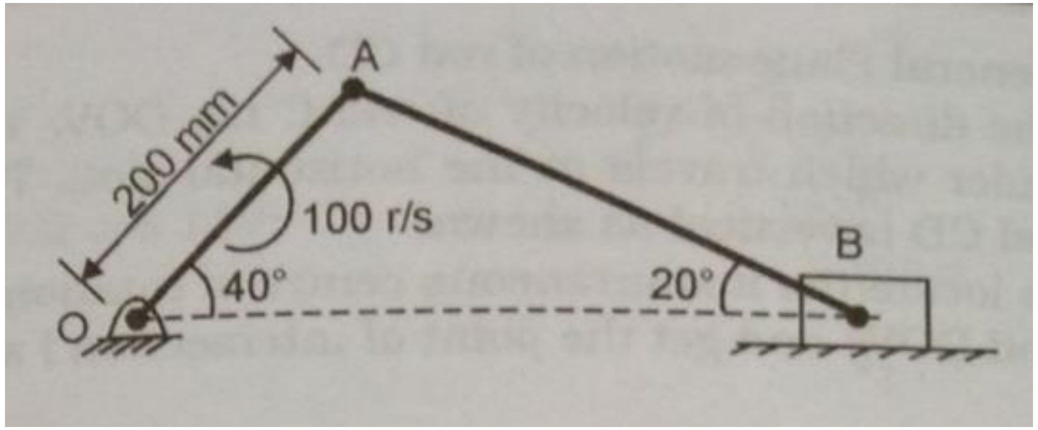
- A wheel of radius o.75 m rolls without slipping on a horizontal surface to right. Determine the velocities of the points P and Q shown in figure when the velocity of the wheel is 10 m/s towards right.



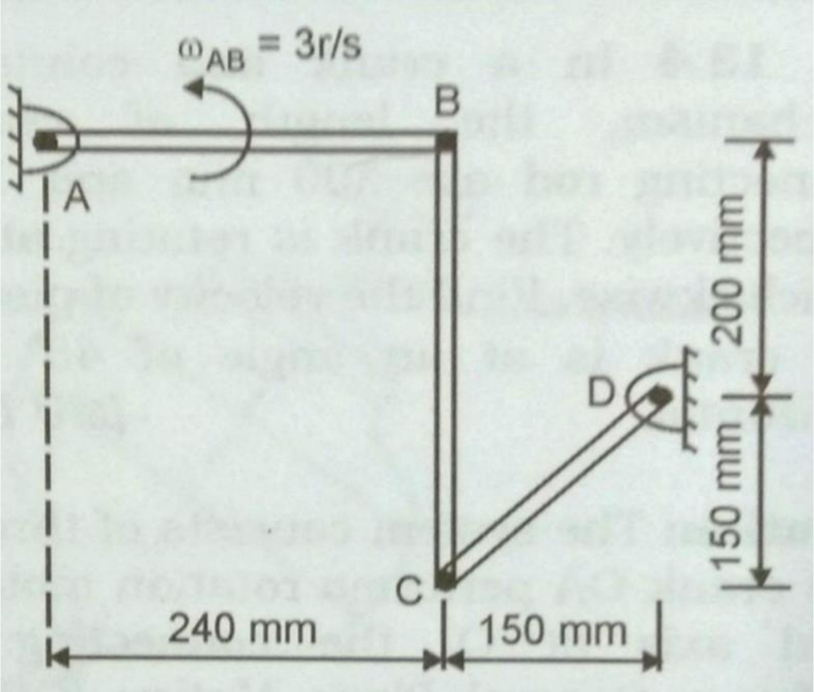
- Block D shown in figure moves with a speed of 3 m/s. Determine the angular velocities of link BD and AB and the velocity of point B at the instant shown.



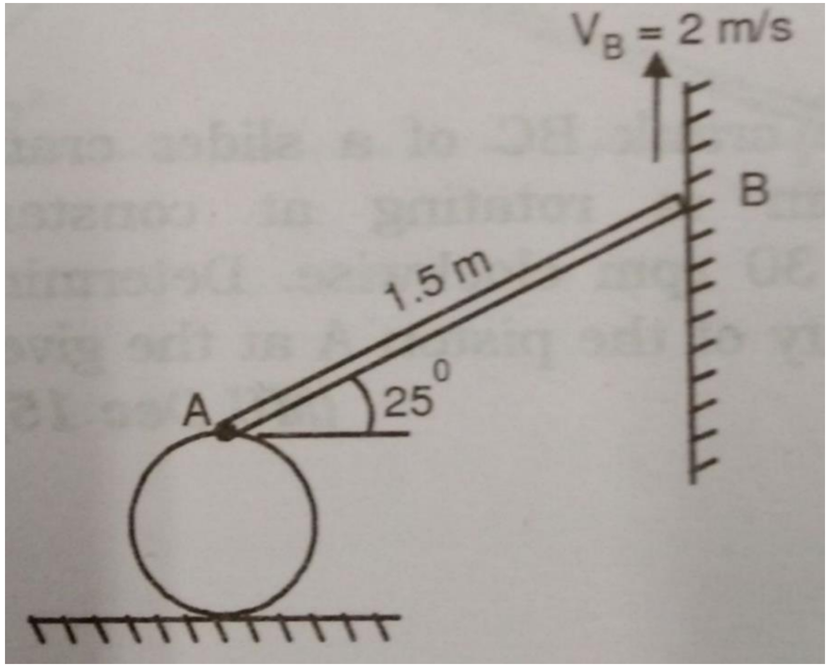
- A slider crank mechanism is shown in the figure. The crank OA rotates anticlockwise at 100 rad/sec. Find the angular velocity of the rod AB and the velocity of the slider B.



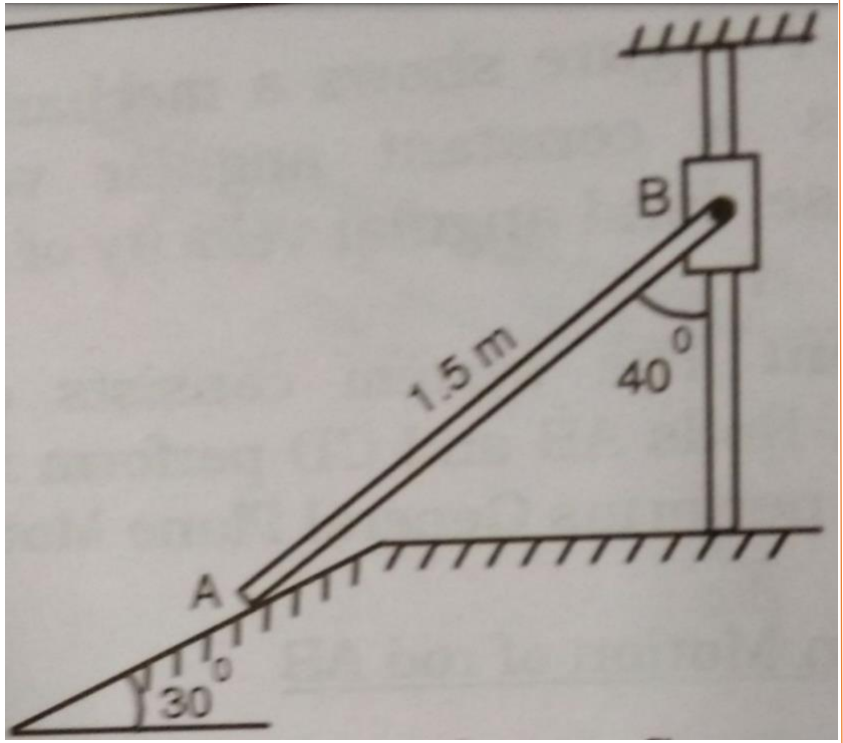
- In the position shown, bar AB has constant angular velocity of 3 rad/sec anticlockwise, determine the angular velocity of bar CD.



- One end of rod AB is pinned to the cylinder of diameter 0.5 m while the other end slides vertically up the wall with a uniform speed 2 m/s. For the instant, when the end A is vertically over the center of the cylinder, find the angular velocity of the cylinder, assuming it to roll without slip.



- Figure shows a collar B which moves up with constant velocity of 2 m/s. To the collar is pinned a rod AB, the end A of which slides freely against a 30° sloping ground. For this instant, determine the angular velocity of the rod and velocity of end A of the rod.



- Locate the Instantaneous center of rotation for the link ABC and determine the velocity of points B and C. Angular velocity of rod OA is 15 rad/sec counter clock wise. Length of OA is 200 mm, AB is 400 mm and BC is 150 mm.