Engineering Mechanics

Module 2.1 – **Kinematics of Particles and Rigid Bodies**

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1	**System of forces**		7	CO 1
	1.1	System of coplanar forces: Resultant of concurrent forces, parallel forces, non-concurrent non parallel system of forces, moment of force about a point, couples, Varignon's theorem, Principle of transmissibility of forces		
	1.2	Resultant of forces in space		
2	**Kinematics of Particles and Rigid Bodies**		11	CO 2
	2.1	Variable motion, motion curves (a-t, v-t, s-t) (acceleration curves restricted to linear acceleration only), motion along plane curved path, velocity & acceleration in terms of rectangular components, tangential & normal component of acceleration, relative velocities.		
	2.2	Introduction to general plane motion, problems based on ICR method for general plane motion of bodies (up to 2 linkage mechanism and no relative velocity method)		

Brief Contents of module 2.1

- Variable motion, motion curves (a-t, v-t, s-t)
  (acceleration curves restricted to linear acceleration only)
- Motion along plane curved path,
- Velocity & acceleration in terms of rectangular components,
- Tangential & normal component of acceleration,
- Relative velocities

Introduction

Dynamics includes:

- ***Kinematics***: study of the geometry of motion. Relates displacement, velocity, acceleration, and time ***without reference*** to the cause of motion. (i.e. *regardless of forces*).
- *Kinema* means movement in Greek
- Mathematical description of motion
  - Position
  - Time Interval
  - Displacement
  - Velocity; absolute value: speed
  - Acceleration
  - Averages of the latter two quantities.
- ***Kinetics***: study of the relations existing between the forces acting on a body, the mass of the body, and the motion of the body. Kinetics is used to predict the motion caused by given forces or to determine the forces required to produce a given motion

Introduction (Cont..)

Particle kinetics includes:

- ***Rectilinear motion***: position, velocity, and acceleration of a particle as it moves along a straight line.
- ***Curvilinear motion***: position, velocity, and acceleration of a particle as it moves along a curved line in two or three dimensions.
- Please Recall
  1. Newton's three laws of motion
  2. Position, Displacement, velocity, acceleration
  3. Horizontal motion
  4. Vertical motion

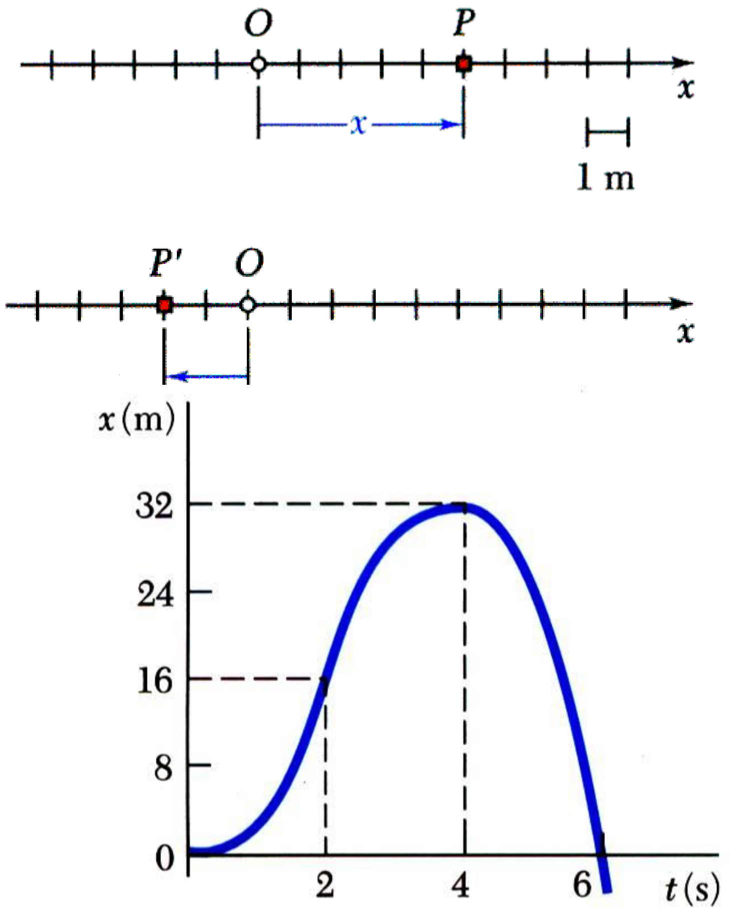
Introduction (Cont..)

- Rectilinear Motion: Position, Velocity & Acceleration

  - ***Rectilinear motion:*** particle moving along a straight line
  - ***Position coordinate:*** defined by positive or negative distance from a fixed origin on the line.

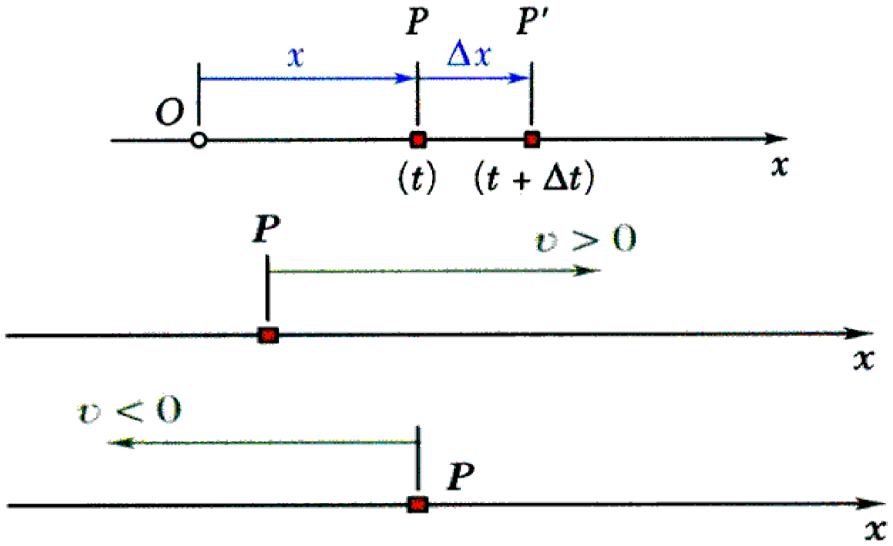
- The ***motion*** of a particle is known if the position coordinate for particle is known for every value of time *t*.
- or in the form of a graph *x* vs. *t*.
- May be expressed in the form of a function, e.g.,

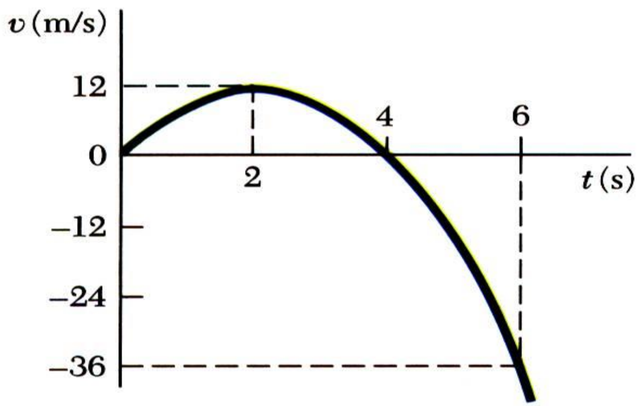
$$x = 6t^2 - t^3$$



Introduction (Cont..)

- Rectilinear Motion: Position, Velocity & Acceleration





- Consider particle which occupies position *P* at time *t* and *P'* at *t* + *Δt*,

$$\text{Average velocity} = \frac{\Delta x}{\Delta t}$$

$$\text{Instantaneous velocity} = v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

  - Instantaneous velocity may be positive or negative. Magnitude of velocity is referred to as *particle speed*.

  - From the definition of a derivative,

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

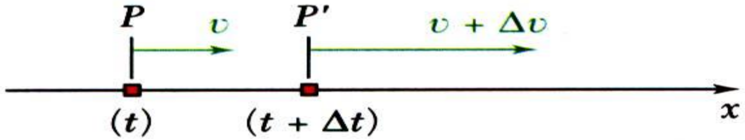
e.g.,

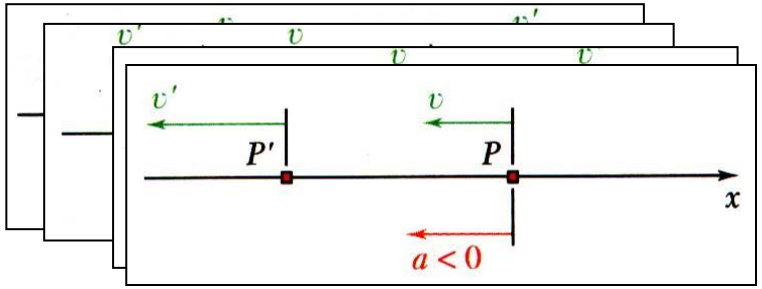
$$x = 6t^2 - t^3$$

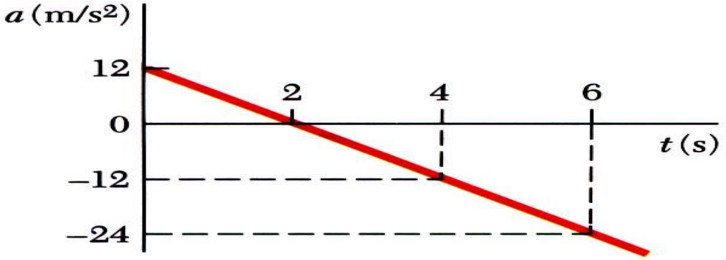
$$v = \frac{dx}{dt} = 12t - 3t^2$$

Introduction (Cont..)

- Rectilinear Motion: Position, Velocity & Acceleration







- Consider particle with velocity *v* at time *t* and *v’* at *t*+*Δt*,

$$Instantaneous\ acceleration = a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

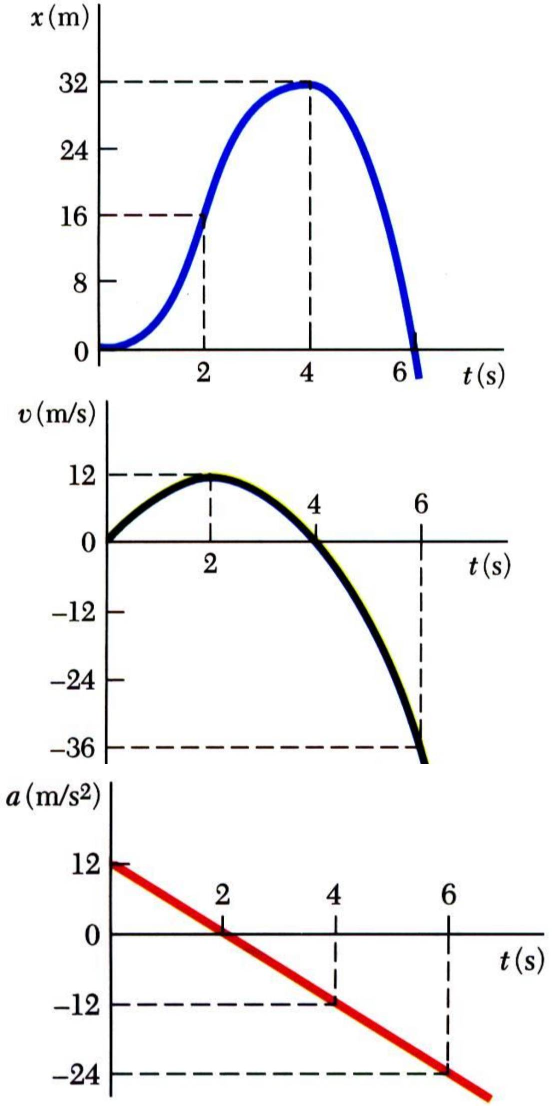
- Instantaneous acceleration may be:
  - positive: increasing positive velocity or decreasing negative velocity
  - negative: decreasing positive velocity or increasing negative velocity.
- From the definition of a derivative,

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$\text{e.g.} \quad v = 12t - 3t^2$$

$$a = \frac{dv}{dt} = 12 - 6t$$

Rectilinear Motion: Position, Velocity & Acceleration



- From our example,

$$x = 6t^2 - t^3$$

$$v = \frac{dx}{dt} = 12t - 3t^2$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = 12 - 6t$$

- What are *x*, *v*, and *a* at *t* = 2 s ?

Ans: at $t = 2$ s, $x = 16$ m, $v = v_{max} = 12$ m/s, $a = 0$

- Note that v_{max} occurs when a=0, and that the slope of the velocity curve is zero at this point.

- What are *x*, *v*, and *a* at *t* = 4 s ?

Ans: at $t = 4$ s, $x = x_{max} = 32$ m, $v = 0$, $a = -12$ m/s^2

One minute break

- **What is true about the kinematics of a particle?**

a) The velocity of a particle is always positive
b) The velocity of a particle is equal to the slope of the position-time graph
c) If the position of a particle is zero, then the velocity must zero
d) If the velocity of a particle is zero, then its acceleration must be zero

Determination of the Motion of a Particle

- **Generally we have three classes of motion**
  - acceleration given as a function of *time*, $a = f(t)$
  - acceleration given as a function of *position*, $a = f(x)$
  - acceleration given as a function of *velocity*, $a = f(v)$

- **If the acceleration is given, we can determine velocity and position by two successive integrations.**

$$v = \frac{dx}{dt} \qquad a = \frac{dv}{dt} \qquad a = \frac{d^2x}{dt^2} \qquad a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

Rectilinear motion

For a particle in uniform rectilinear motion, the acceleration is zero and the velocity is constant.

$$\frac{dx}{dt} = v = \text{constant}$$

$$\int_{x_0}^{x} dx = v \int_{0}^{t} dt$$

$$x - x_0 = vt$$

$$x = x_0 + vt$$

If forces applied to a body are constant (and in a constant direction), then you have uniformly accelerated rectilinear motion.

Uniformly Accelerated Rectilinear Motion

For a particle in uniformly accelerated rectilinear motion, the acceleration of the particle is constant. You may recognize these constant acceleration equations from Physics courses.

$$\frac{dv}{dt} = a = \text{constant} \qquad \int_{v_0}^{v} dv = a\int_{0}^{t} dt \qquad v = v_0 + at$$

$$\frac{dx}{dt} = v_0 + at \qquad \int_{x_0}^{x} dx = \int_{0}^{t} \left(v_0 + at\right) dt \qquad x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$v\frac{dv}{dx} = a = \text{constant} \qquad \int_{v_0}^{v} v\,dv = a\int_{x_0}^{x} dx \qquad v^2 = v_0^2 + 2a\left(x - x_0\right)$$

Rectilinear Motion

- Velocity as a Function of Time

Integrate

ac = dv/dt, assuming that initially v= v0 when t = 0.

$$\int_0^v dv = \int_0^t a_c dt$$

$$v = v_0 + a_c t$$

Constant acceleration

- Position as a Function of Time

Integrate

v = ds/dt = v0 + act, assuming that initially *s = s0 when t = 0*

$$\int_0^s ds = \int_0^t (v_0 + a_c t) dt$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

Constant acceleration

- Velocity as a Function of Position

Integrate

v dv = a_c ds, assuming that initially v = v_0 at s = s_0

$$\int_{v_0}^{v} v dv = \int_{s_0}^{s} a_c ds$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

Constant acceleration

Motion with variable acceleration:

- The governing equations are

$$V = dx/dt, \quad a = dv/dt, \quad a = V.dv/dx$$

- ~~Motion under gravity~~

1. ~~Motion in vertical direction is influenced by gravitational force~~
2. ~~Acceleration of particle remains constant and equal to g (gravitational force)~~
3. ~~Acceleration due to gravity is directed towards centre of earth~~
4. ~~It is taken as negative (-ve)~~
5. ~~It is a special case of uniformly accelerated motion hence equation of UAM are used with a = -g and s = y~~

Summary

Procedure:

1. Establish a coordinate system & specify an origin
2. Remember: x, v, a, t are related by:

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt}$$

$$a = \frac{d^2x}{dt^2}$$

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

3. When integrating, either use limits (if known) or add a constant of integration

Problems

- The velocity of the particle is defined as $v = t^3 - 5t^2 + 3t + 4$ where v is in m/s and t is in seconds.

Assuming initial displacement of the particle to be 2 m, find (a) initial velocity, (b) initial acceleration, (c) time interval at which acceleration will be zero, (d) displacement in first 4 seconds, (e) displacement in 6^{th} second.

Problems

- motion of a particle is given by $x= t^4 -3t^2-t$ where x is in meter, t in seconds. Find position, velocity, acceleration at t = 3 seconds.

Steps are:

1. Differentiate the given displacement equation find velocity
2. Differentiate the velocity equation and find acceleration

Answer: (x = 51meter, v = 89 m/sec., a = 102m/sec. square)

Problems

Q the motion of particle is governed by $a = t^3 - 2t^2 + 7$. It moves in straight line at t=1 second, v=3.5 m/sec. and x = 9.30 m. Find displacement, velocity, acceleration when t = 2 seconds.

Steps are:

1. $a = t^3 - 2t^2 + 7 = dv/dt$ hence $dv = (t^3 - 2t^2 + 7)\,dt$
2. Integrate it find equation for v and value of C_1
3. Now $v = dx/dt$ hence $dx = (t^4/4 - 2t^3/3 + 7t - 3)dt$
4. Integrate it and find equation for x and value of C_2
5. Answer x =15.93m, v = 9.67m/sec., a = 7m/sec. square,
6. Displacement = (15.93-9.00) = 6.93m

- The acceleration of the particle starting from rest from initial position x = 0 is given by (-6t + 180) m/s^2 where t is in seconds. Determine the distance of the particle in the interval (a) 0 to 10 seconds, (b) 0 to 70 seconds, (c) maximum velocity attained by the particle.

- A sphere is fired downward into a medium with an initial speed of 27 m/s. Sphere experiences a deceleration $a = -6t \text{ m/s}^2$ where t is in seconds, determine the distance travelled before it comes to rest.

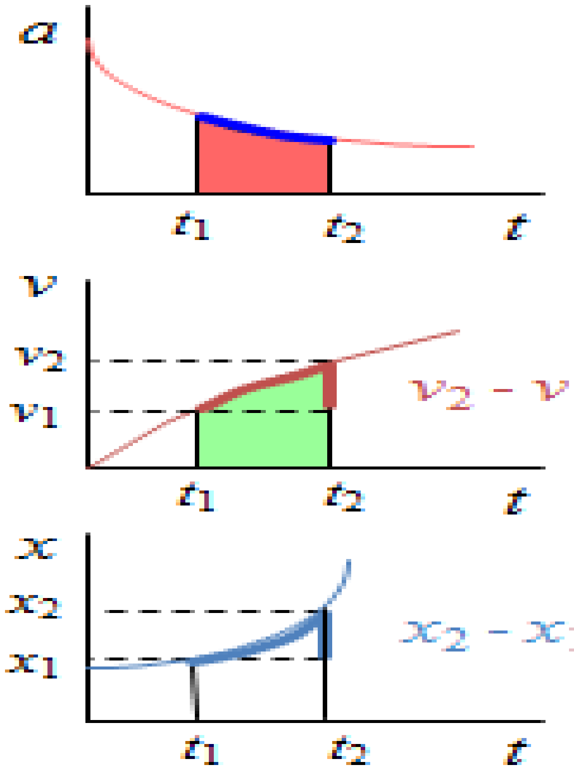
- When particle's motion is erratic, it is best described graphically using a series of curves that can be generated experimentally from computer output.

- A graph can be established describing the relationship with any two of the variables, a, v, s, t

- using the kinematics equations $a = dv/dt$, $v = ds/dt$, $a\,ds = v\,dv$

Motion Diagrams

Sometimes it is convenient to use a *graphical solution* for problems involving rectilinear motion of a particle. The graphical solution most commonly involves x - t, v - t, and a - t curves.



$$v_2 - v_1 = \int_{t_1}^{t_2} a\,dt$$

$$x_2 - x_1 = \int_{t_1}^{t_2} v\,dt$$

At any given time t,

v = slope of x - t curve
a = slope of v - t curve

while over any given time interval t_1 to t_2,

$v_2 - v_1$ = area under a - t curve
$x_2 - x_1$ = area under v - t curve

Displacement – Time diagram

Given the *s-t Graph, construct the v-t Graph*

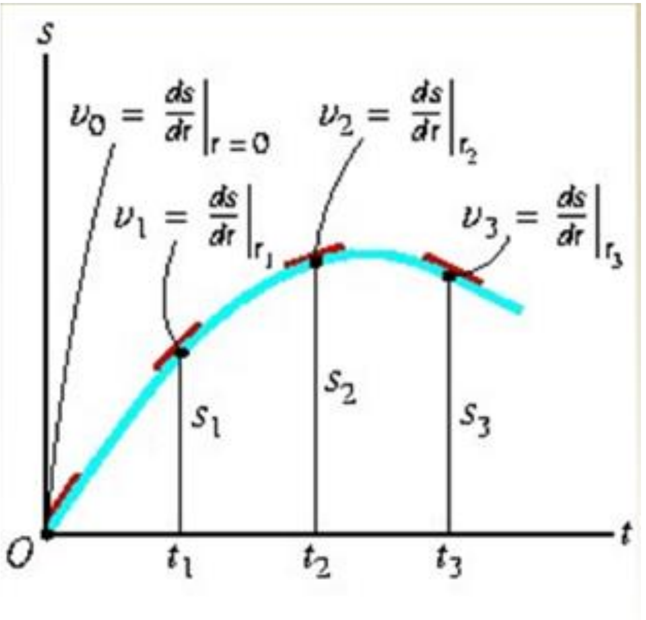
•The *s-t graph can be plotted if the position of the* particle can be *determined experimentally during a* period of time *t.*

•To determine the particle's velocity as a function of time, the *v-t Graph, use v = ds/dt*

•Velocity as any instant is determined by measuring the slope of the *s-t graph*

When displacement of particle is maximum or minimum velocity of particle is zero.

$$\frac{ds}{dt} = v$$



(a)

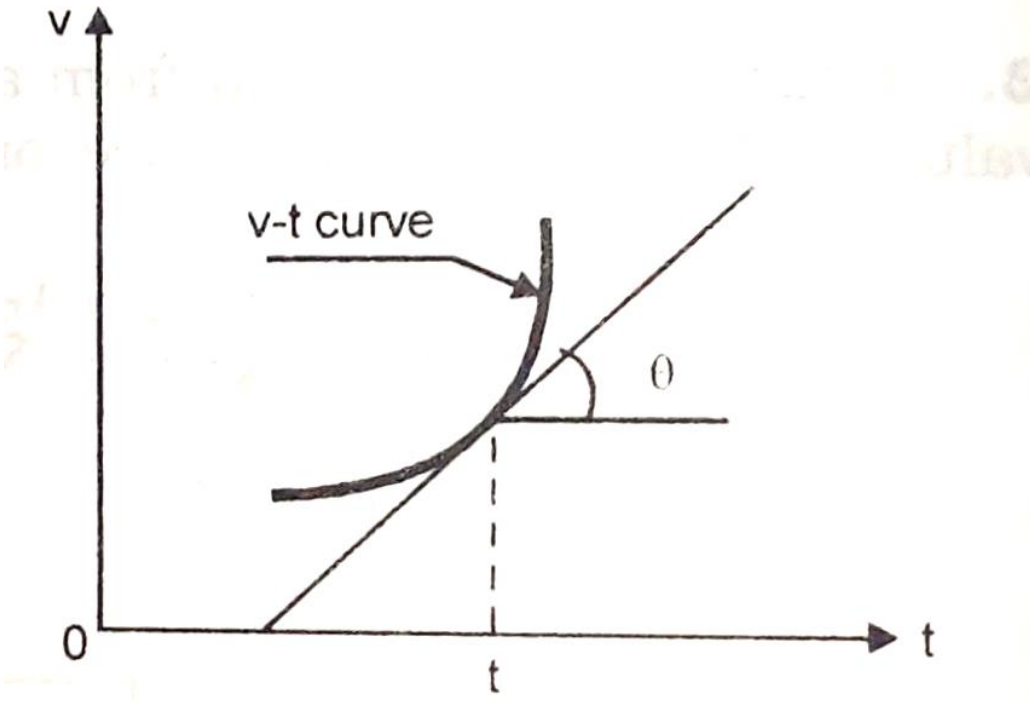
Slope of *s-t graph=velocity*

Velocity Time Diagram

- This is drawn with velocity on y axis and time on x axis.
- As $a = dv/dt$, slope of v-t curve gives acceleration of particle at that instant.
- Now $v = dx/dt$

so $dx = vdt$

When velocity of particle is maximum or minimum acceleration of particle is zero.

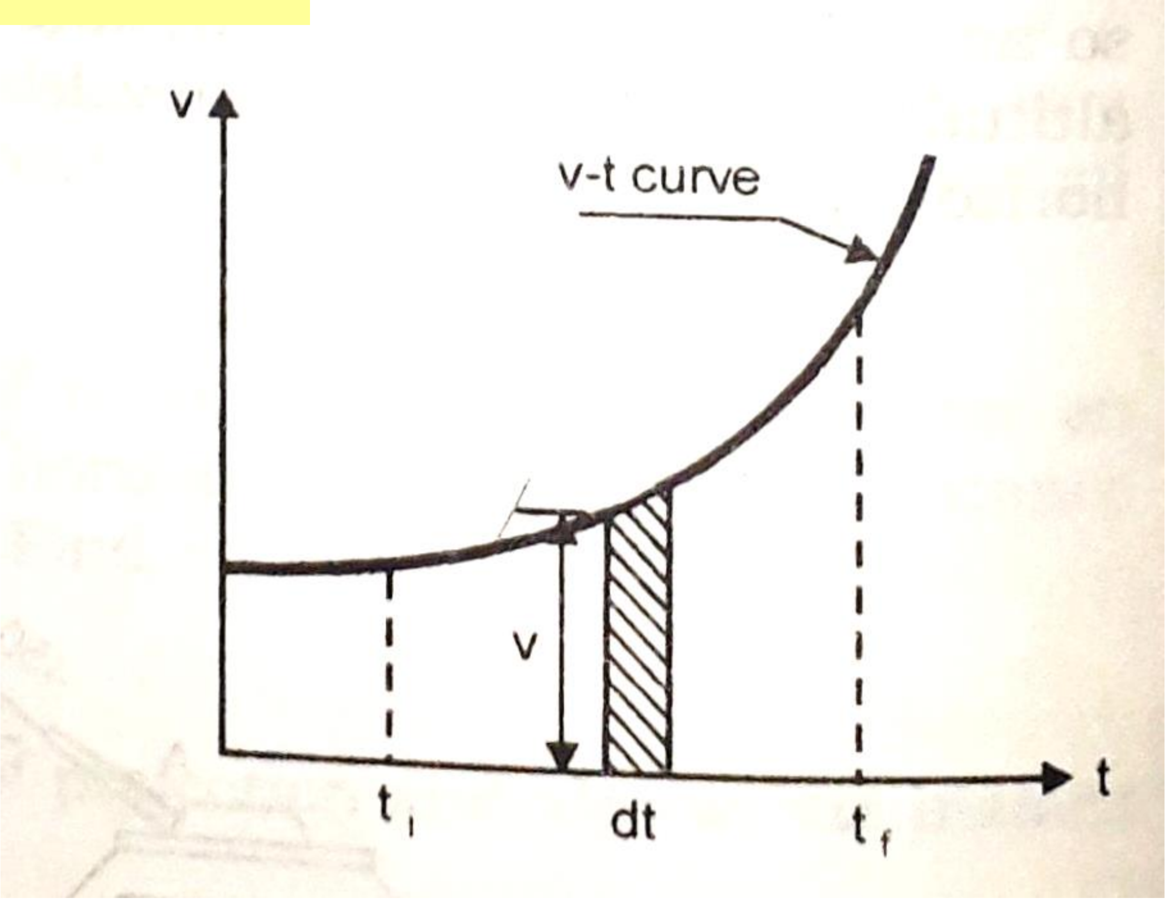


Velocity Time Diagram

$$v = \frac{dx}{dt} \quad \Rightarrow dx = vdt \quad \Rightarrow \int_{x_0}^{x} dx = \int_0^t vdt \quad \Rightarrow x - x_0 = \int_0^t vdt$$

Or

$x-x_0$ = area under v-t curve

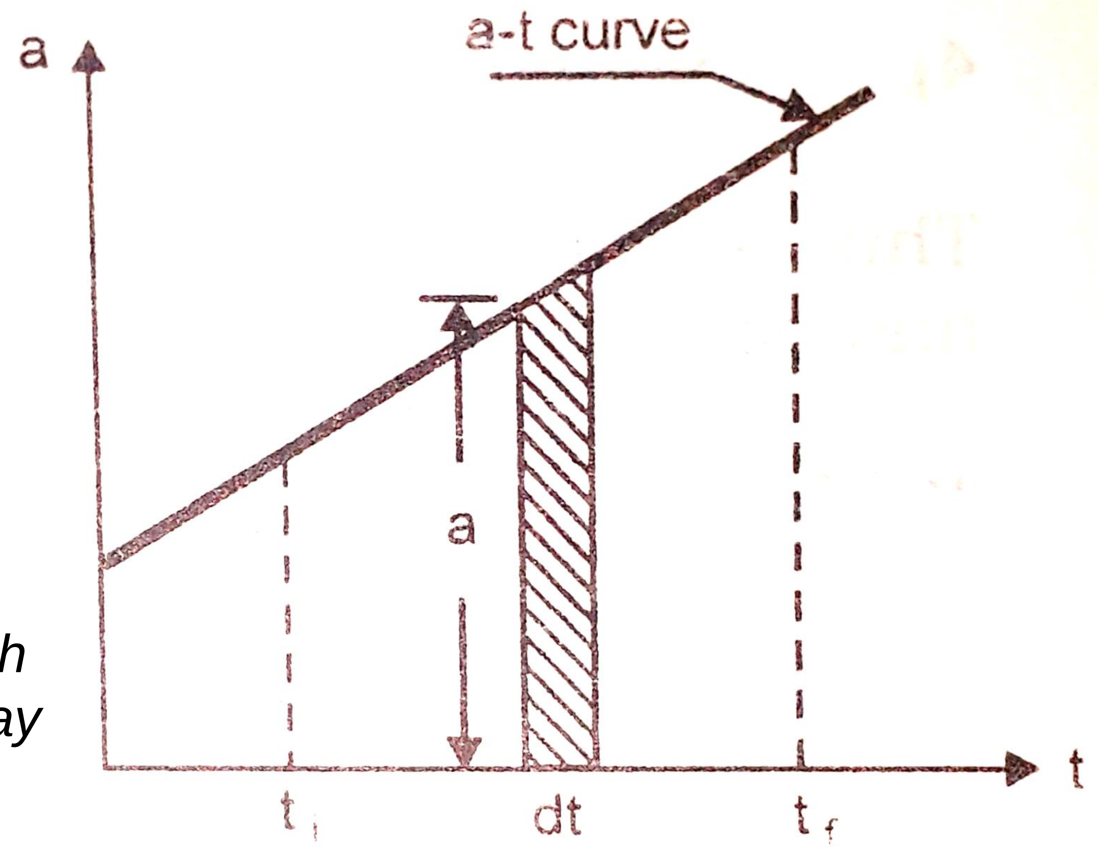


Acceleration Time Diagram

- Given the *a-t Graph, construct the v-t Graph*
- • When the *a-t graph is known, the v-t graph may* be constructed using *a = dv/dt*

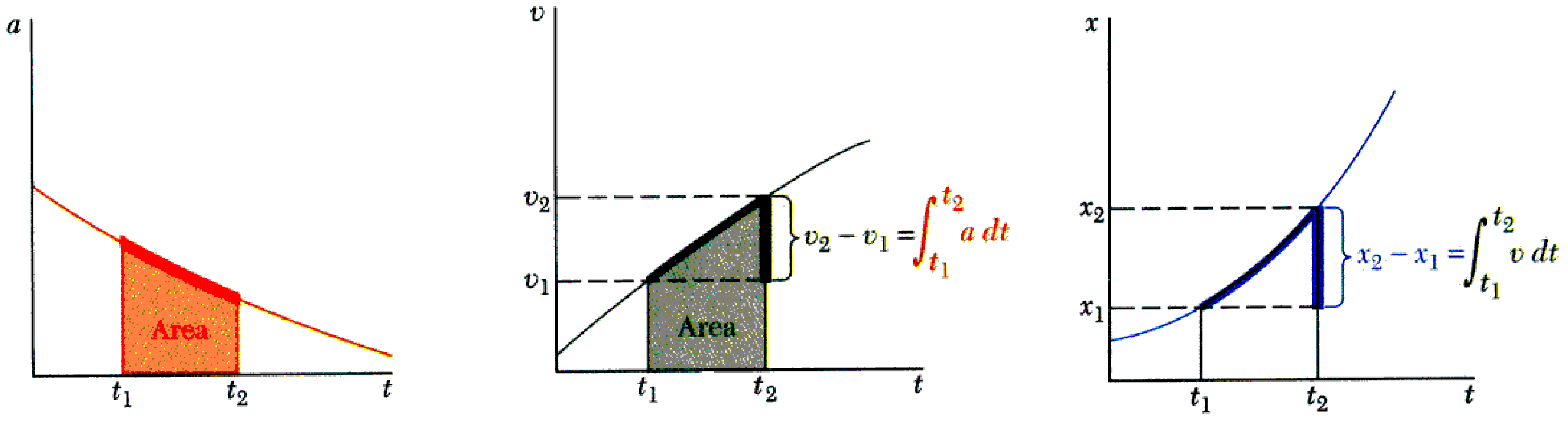
$$\Delta v = \int a\, dt$$

Change in velocity = Area under *a-t* graph



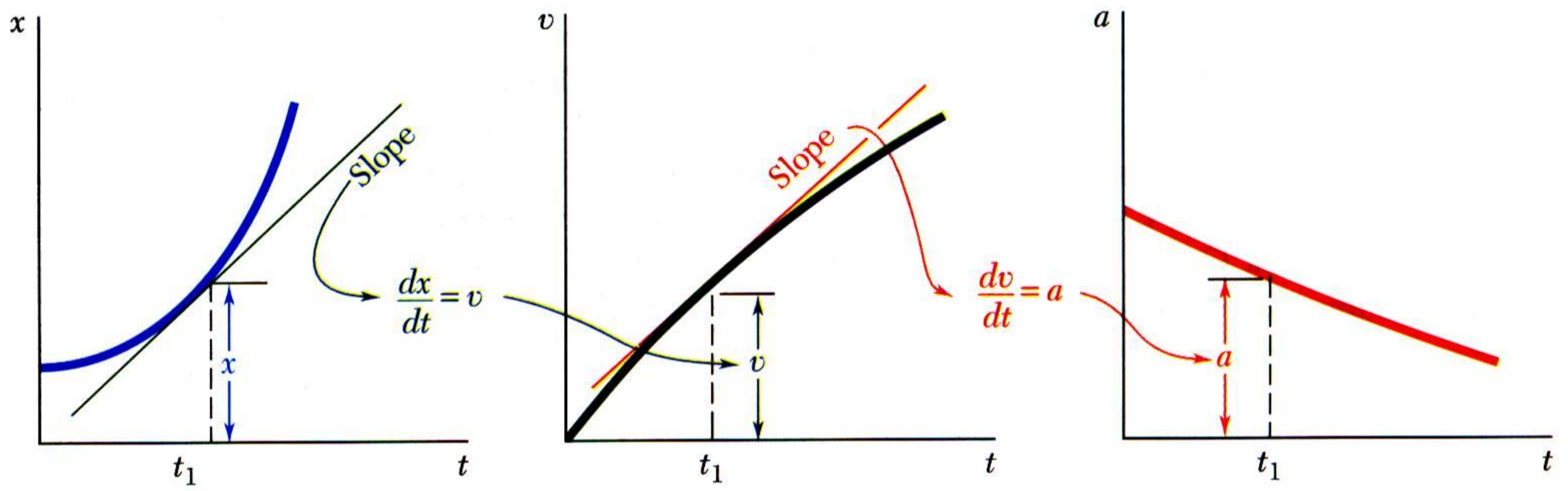
• Knowing particle's initial velocity *v0, and add to this small increments* of area (*Δv*)
• Successive points *v1 = v0 + Δv, for* the *v-t graph*
• Each eqn. for each segment of the *a-t graph may be integrated to yield* eqns. for corresponding segments of the *v-t graph*

Graphical Solution of Rectilinear-Motion Problems



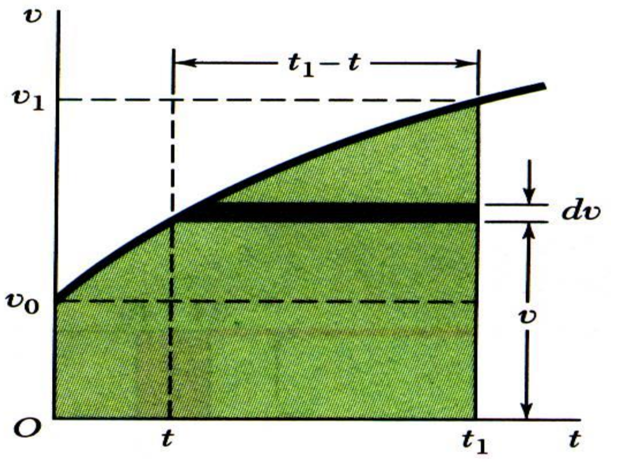
- Given the *a*-*t* curve, the change in velocity between t_1 and t_2 is equal to the area under the *a*-*t* curve between t_1 and t_2.
- Given the *v*-*t* curve, the change in position between t_1 and t_2 is equal to the area under the *v*-*t* curve between t_1 and t_2.

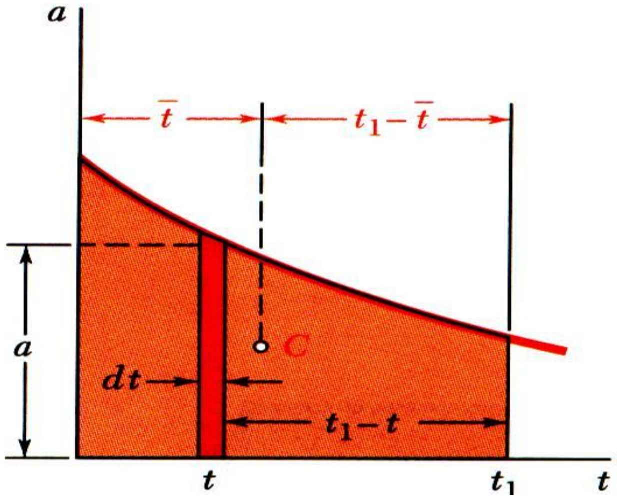
Graphical Solution of Rectilinear-Motion Problems



- Given the *x*-*t* curve, the *v*-*t* curve is equal to the slope of *x*-*t* curve
- Given the *v*-*t* curve, the *a*-*t* curve is equal to the slope *v*-*t* curve

Other Graphical Methods





- *Moment-area method* to determine particle position at time *t* directly from the *a-t* curve:

$$x_1 - x_0 = \text{area under } v - t \text{ curve}$$

$$= v_0 t_1 + \int_{v_0}^{v_1} (t_1 - t)\, dv$$

using $dv = a\,dt$,

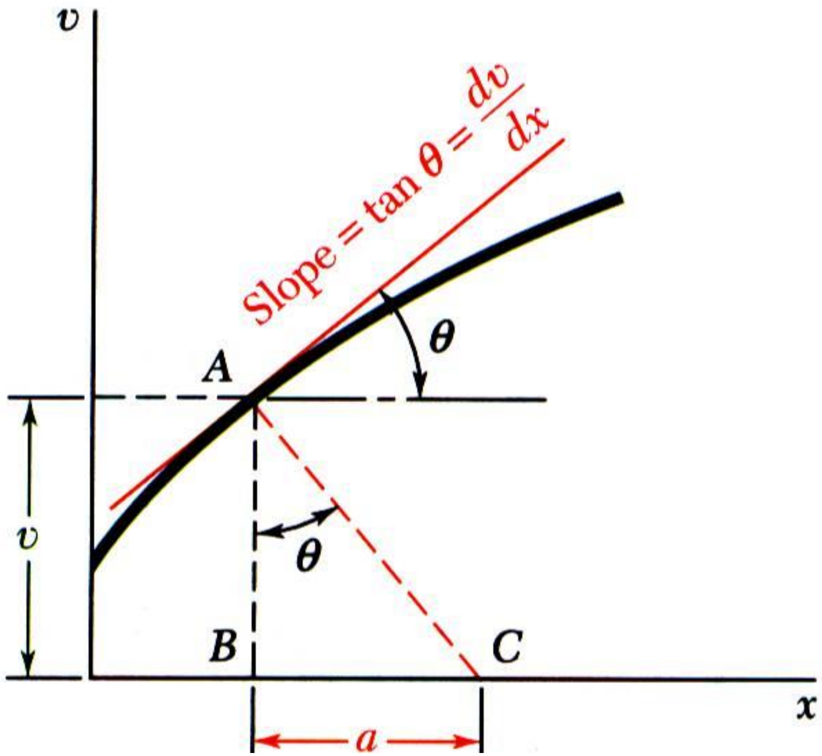
$$x_1 - x_0 = v_0 t_1 + \int_{v_0}^{v_1} (t_1 - t)\, a\, dt$$

$$\int_{v_0}^{v_1} (t_1 - t)\, a\, dt = $$ first moment of area under *a-t* curve with respect to $t = t_1$ line.

$$x_1 = x_0 + v_0 t_1 + (\text{area under } a\text{-}t \text{ curve})(t_1 - \bar{t})$$

$$\bar{t} = \text{abscissa of centroid } C$$

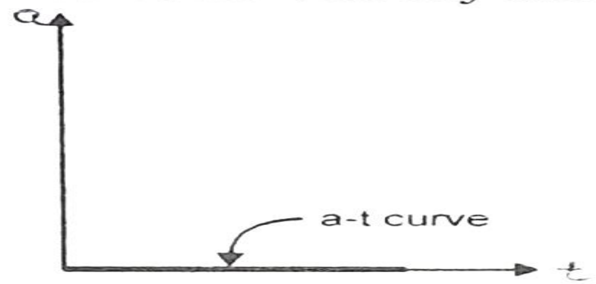
Other Graphical Methods



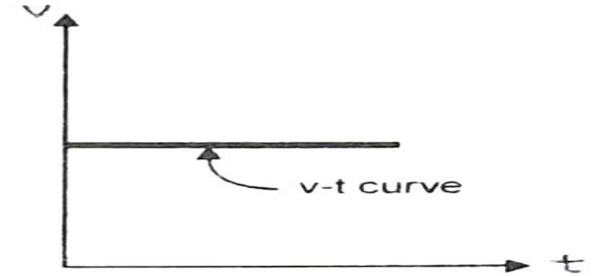
- Method to determine particle acceleration from *v*-*x* curve:

$$a = v\frac{dv}{dx}$$
$$= AB\tan\theta$$
$$= BC = \textit{subnormal} \text{ to } v\text{-}x \text{ curve}$$

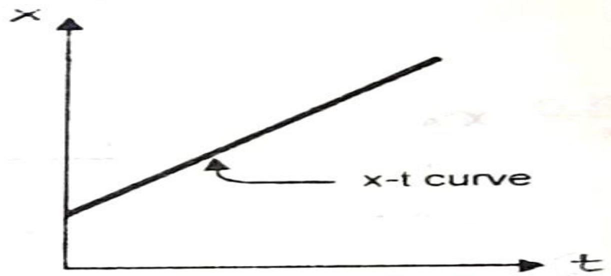
a) Uniform Velocity Motion curves



Straight horizontal curve on the acceleration axis

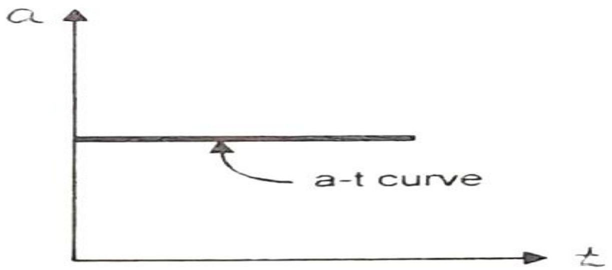


Straight horizontal curve parallel to velocity axis

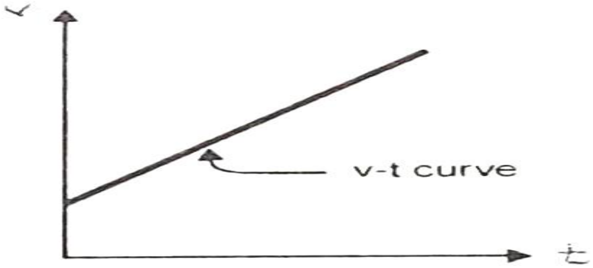


Straight inclined curve

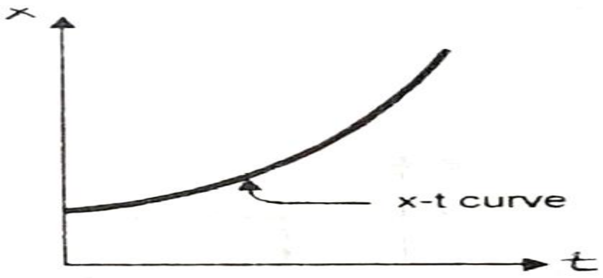
b) Uniform Acceleration Motion curves



Straight horizontal curve parallel to acceleration axis

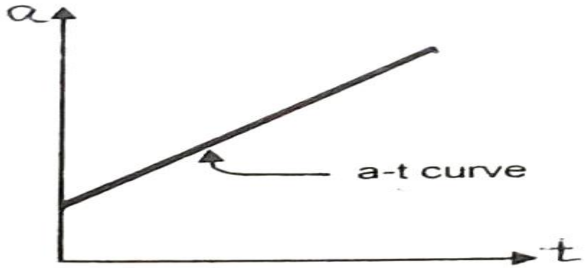


Straight inclined curve

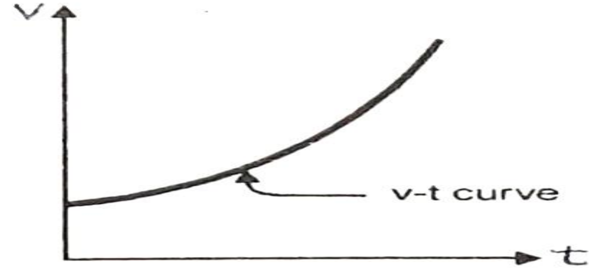


Second degree curve

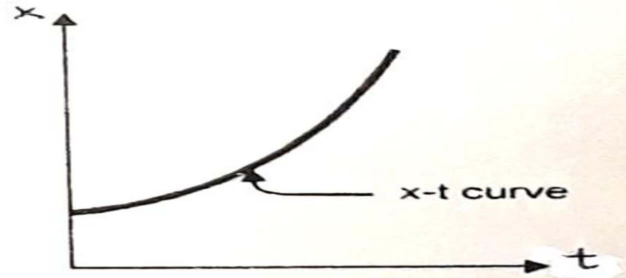
c) Variable Acceleration (Linear Variation) Motion curves



Straight inclined curve

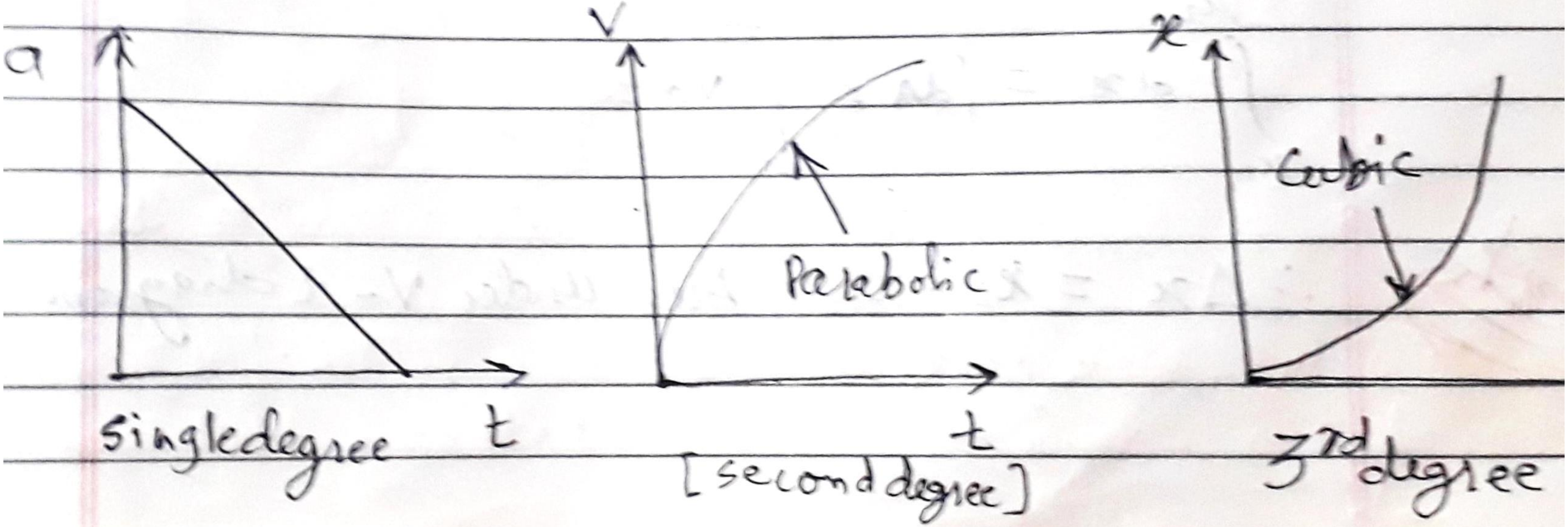


Second degree curve

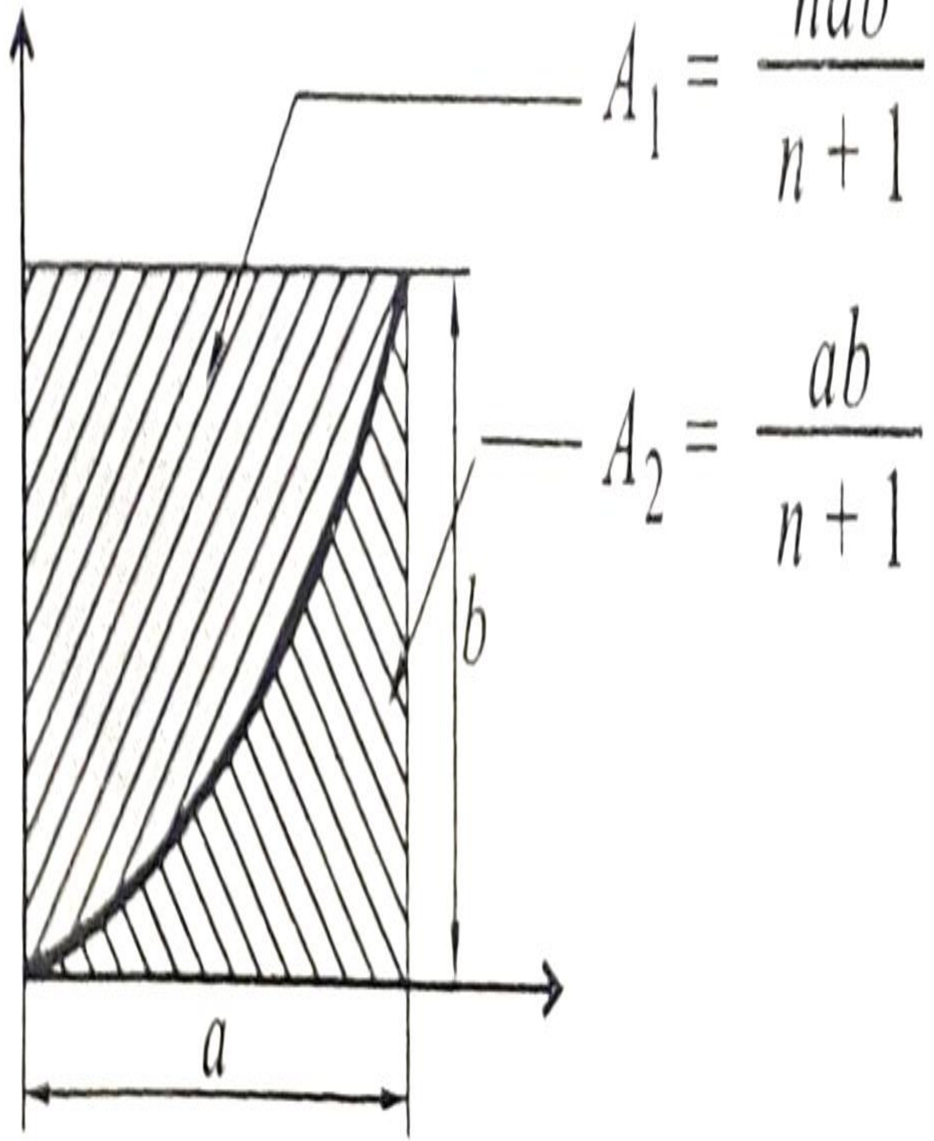


Third degree curve

Acceleration decreases.



Area bounded by curve



$A = A_1 + A_2 = ab$

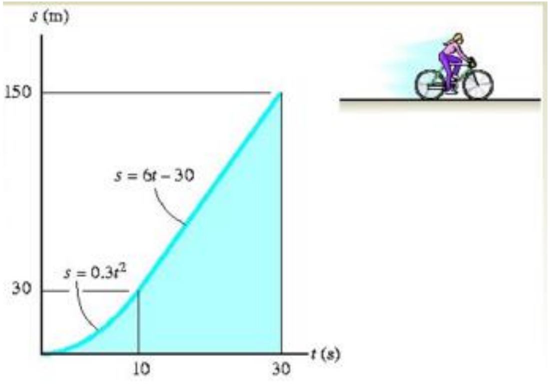
n = Curve of degree of polynomial

Important points to remember

- If a-t curve is horizontal line (zero degree) then v-t curve is inclined line (single degree) and x-t curve is parabolic curve (second degree)
- Slope of motion curve increases from a-t curve towards v-t curve.

Problems

- Q1 A bicycle moves along a straight road such that it position is described by the graph as shown. Construct the *v-t and a-t graphs for 0 ≤ t ≤ 30s.*



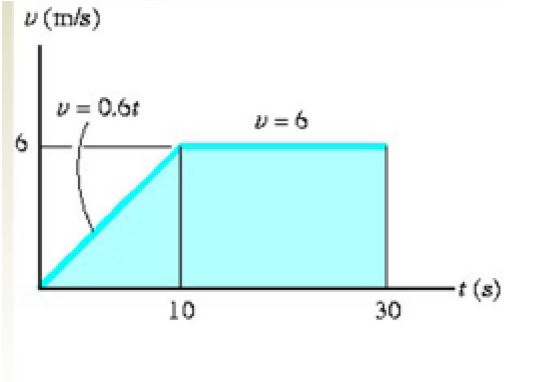
(a)

v-t Graph*. The v-t graph can be determined by* differentiating the eqns. defining the *s-t graph*

$$0 \le t \le 10s; \quad s = 0.3t^2 \quad v = \frac{ds}{dt} = 0.6t$$

$$10s \le t \le 30s; \quad s = 6t - 30 \quad v = \frac{ds}{dt} = 6$$

$$v = \frac{\Delta s}{\Delta t} = \frac{150 - 30}{30 - 10} = 6m/s$$

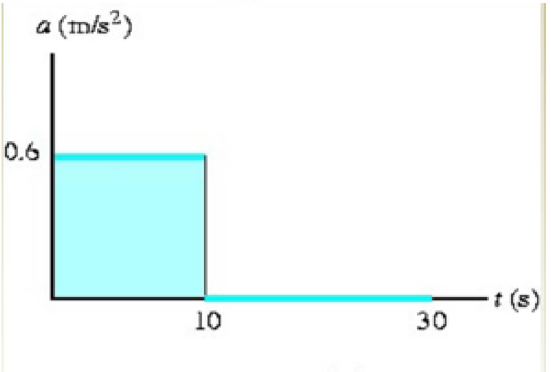


(b)

a-t Graph*. The a-t graph can be determined by* differentiating the eqns. defining the lines of the *v-t graph.*

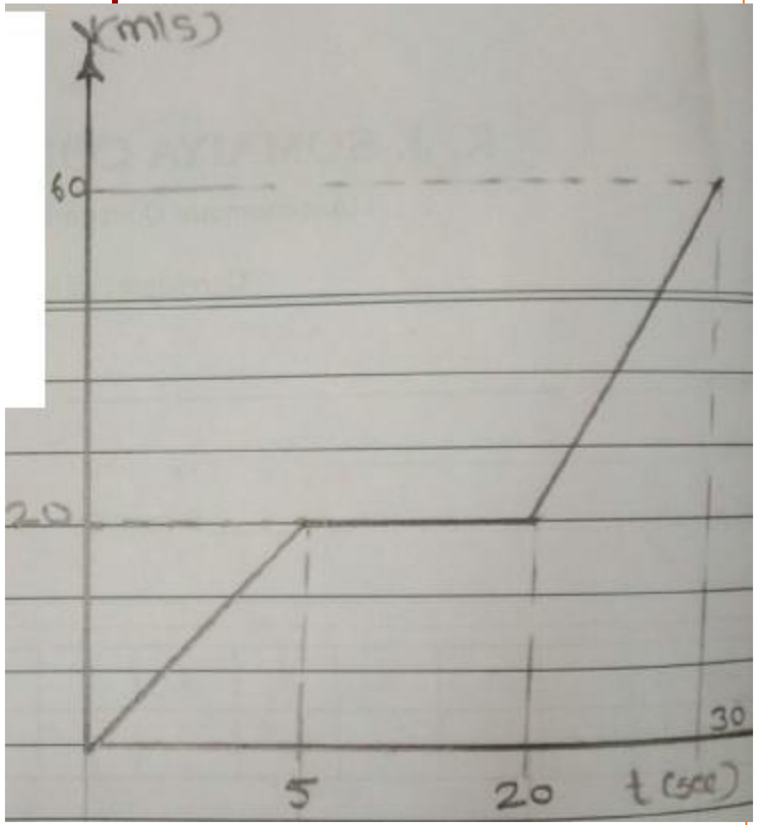
$$0 \le t \le 10s; \quad v = 0.6t \quad a = \frac{dv}{dt} = 0.6$$

$$10 < t \le 30s; \quad v = 6 \quad a = \frac{dv}{dt} = 0$$

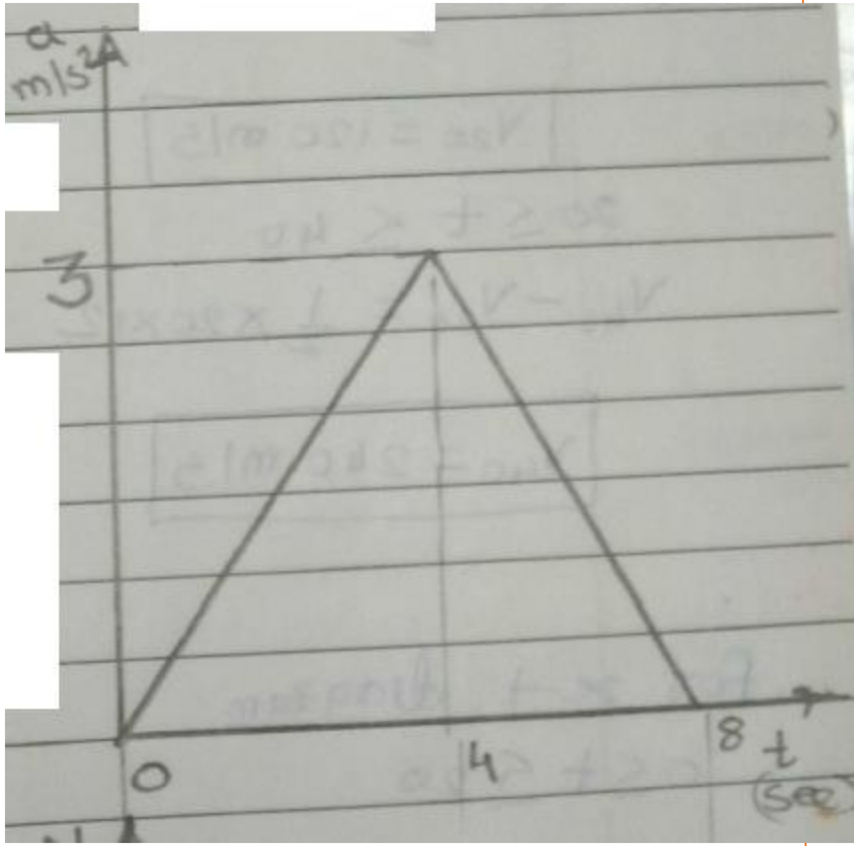


(c)

The motion of a jet plane while travelling along a runway is defined by v-t curve. Construct x-t and v-t graphs for the motion. The plane starts from the rest.



The a-t diagram for a car is shown in the figure. Draw v-t and x-t diagrams. Find the maximum speed attained and maximum distance covered. The car starts from rest from the origin in a straight line.



Q 2 A test car starts from rest and travels along a straight track such that it accelerates at a constant rate for 10 s and then decelerates at a constant rate. Draw the v-t and s-t graphs and determine the time t' needed to stop the car. How far has the car traveled?

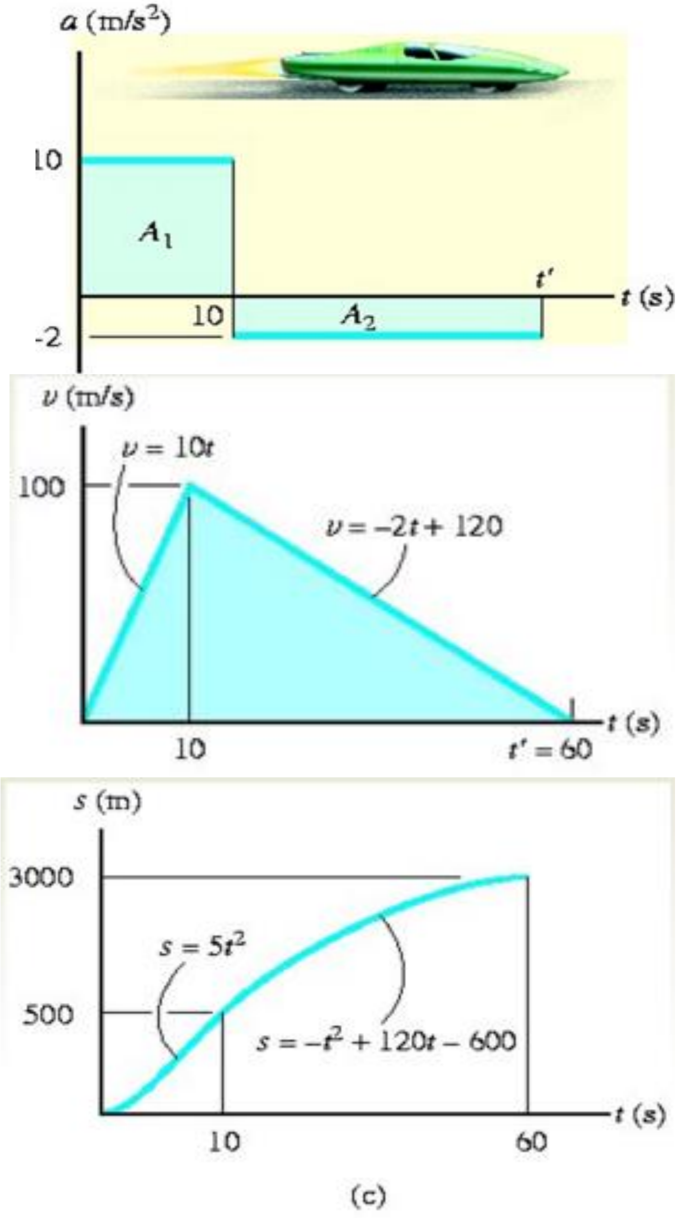
v-t Graph. *The v-t graph can be determined by* integrating the straight-line segments of the *a-t* graph. Using *initial condition v = 0 when t = 0,*

$$0 \le t \le 10s \quad a = 10; \quad \int_0^v dv = \int_0^t 10\,dt, \quad v = 10t$$

When *t = 10s, v = 100m/s, using this as initial* condition for the next time period, we have

$$10s \le t \le t'; \quad a = -2; \quad \int_{100}^v dv = \int_{10}^t -2\,dt, \quad v = -2t + 120$$

When *t = t' we require v = 0. This yield t' = 60 s*



s-t Graph. *Integrating the eqns. of the v-t graph* yields the corresponding eqns. of the *s-t graph*. Using the *initial conditions s = 0 when t = 0,*

$$0 \le t \le 10s; \quad v = 10t; \quad \int_0^s ds = \int_0^t 10t\,dt, \quad s = 5t^2$$

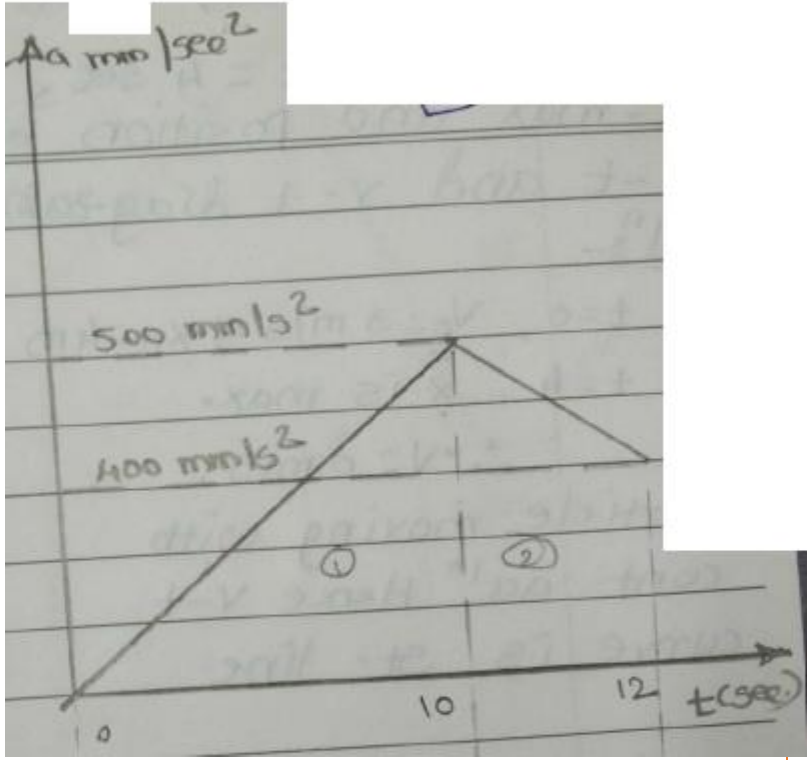
When t = 10s, s = 500m. Using this initial condition,

$$10s \le t \le 60s; \quad v = -2t + 120; \quad \int_{500}^s ds = \int_{10}^t (-2t + 120)dt$$

$$s = -t^2 + 120t - 600$$

When t’ = 60s, the position is s = 3000m

The motion of a particle from rest is given by a-t diagram as shown. Sketch v-t diagram and hence calculate velocity.



A particle moves in a straight line with a velocity-time diagram shown in figure. If s = – 25 m at t = 0, draw displacement-time and acceleration-time diagrams for 0 to 24 seconds.

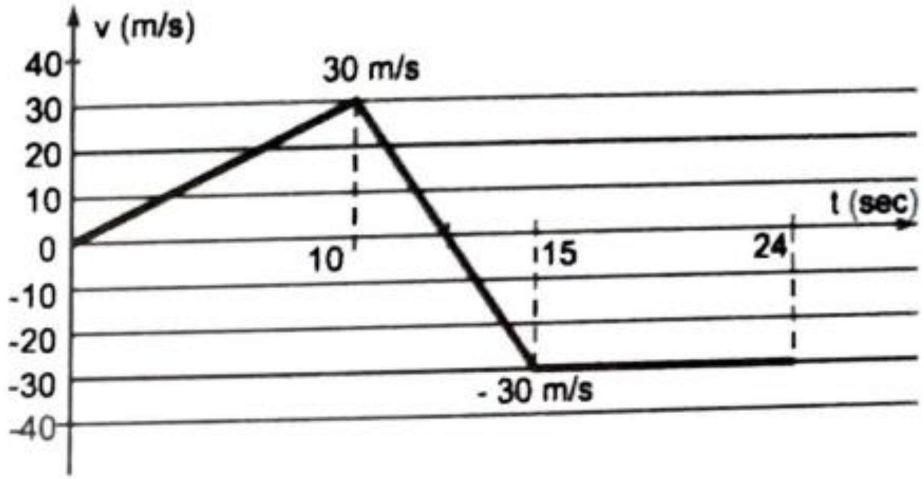
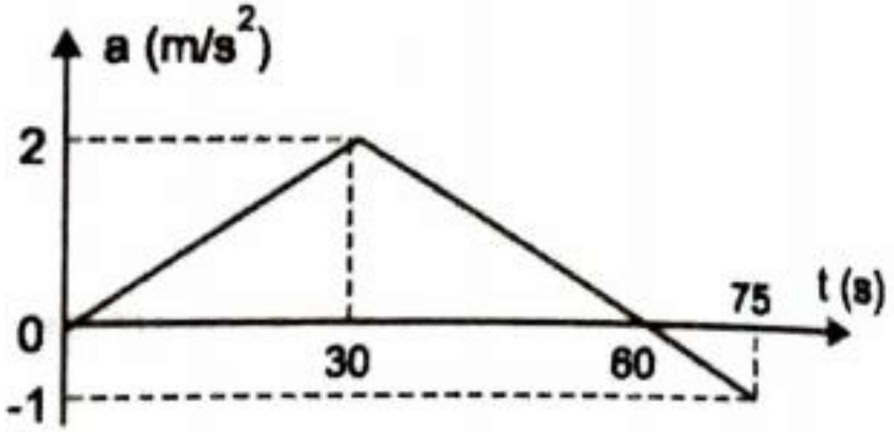


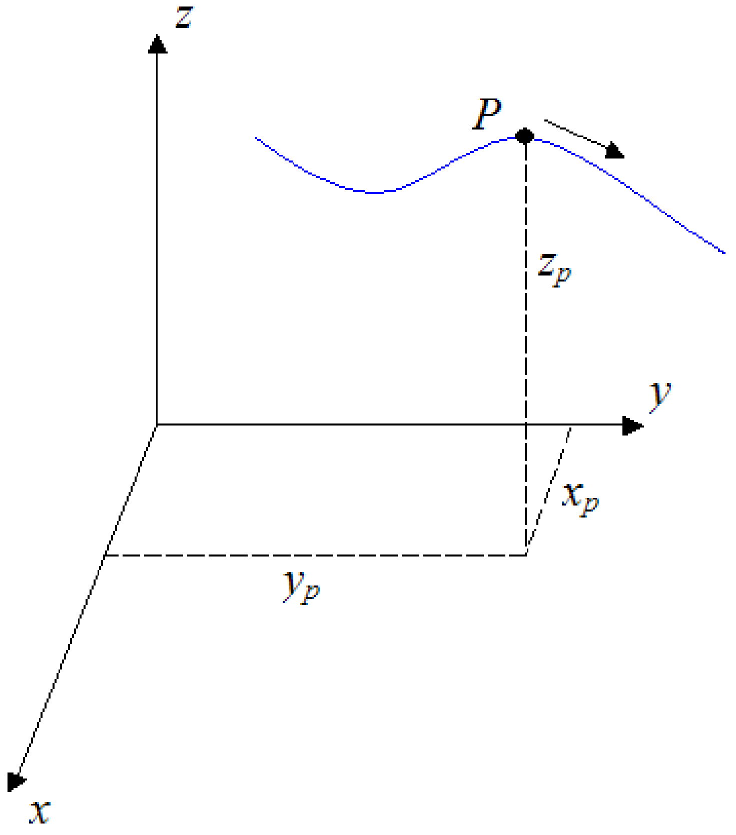
Figure shows (a – t) diagram for particle moving along a straight path for a time interval 0 – 75 sec. Plot (v – t) and (x – t) diagrams and hence find the maximum speed attained by the particle. The particle started from rest from origin.



Curvilinear motion

$$x_p = x_p(t)$$
$$y_p = y_p(t)$$
$$z_p = z_p(t)$$

- Curvilinear motion is defined as motion that occurs when a particle travels along a curved path.
- The curved path can be in two dimensions (in a plane), or in three dimensions.
- To find the velocity and acceleration of a particle experiencing curvilinear motion one only needs to know the position of the particle as a function of time.
- The velocity and acceleration of the particle *P* is given by



$$v_x = \frac{dx_p}{dt} \qquad a_x = \frac{d^2 x_p}{dt^2} \qquad v_p = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$v_y = \frac{dy_p}{dt} \qquad a_y = \frac{d^2 y_p}{dt^2}$$

$$a_p = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$v_z = \frac{dz_p}{dt} \qquad a_z = \frac{d^2 z_p}{dt^2}$$

Curvilinear Motion: Position, Velocity & Acceleration

The softball and the car both undergo curvilinear motion.

- A particle moving along a curve other than a straight line is in *curvilinear motion*.

Analysis of curvilinear motion

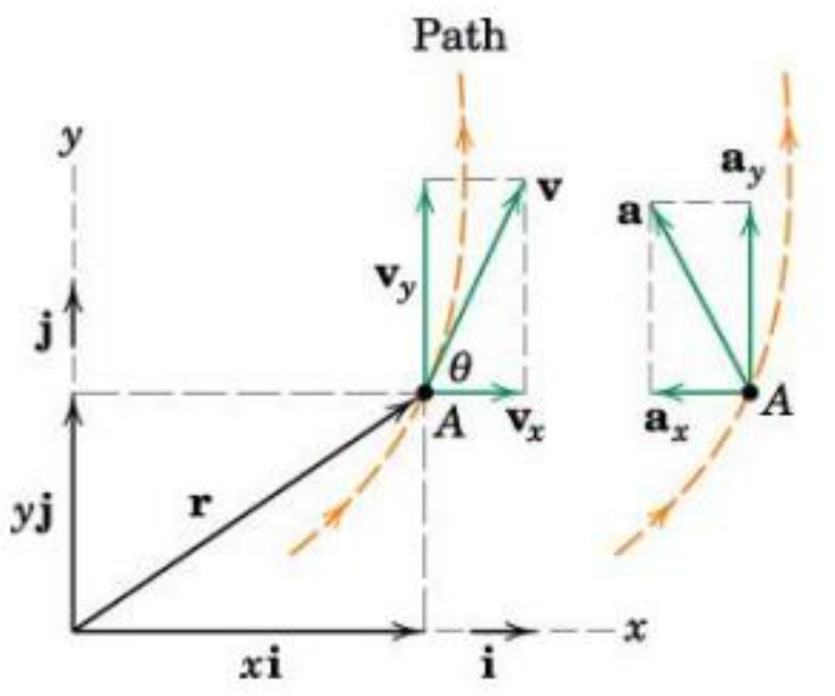
- Rectangular Co-ordinate system
- Normal and Tangential co-ordinate system

Rectangular Coordinates (*x*-*y*)

If all motion components are directly expressible in terms of horizontal and vertical coordinates

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j}$$
$$\mathbf{v} = \dot{\mathbf{r}} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$$
$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$$

$v_x = \dot{x},\ v_y = \dot{y}$ and $a_x = \dot{v}_x = \ddot{x},\ a_y = \dot{v}_y = \ddot{y}$



$$v^2 = {v_x}^2 + {v_y}^2 \qquad v = \sqrt{{v_x}^2 + {v_y}^2} \qquad \tan\theta = \frac{v_y}{v_x}$$
$$a^2 = {a_x}^2 + {a_y}^2 \qquad a = \sqrt{{a_x}^2 + {a_y}^2}$$

Time derivatives of the unit vectors are zero because their magnitude and direction remains constant.

Also, $dy/dx = \tan\theta = v_y/v_x$

Normal and Tangential Coordinates (*n*-*t*)

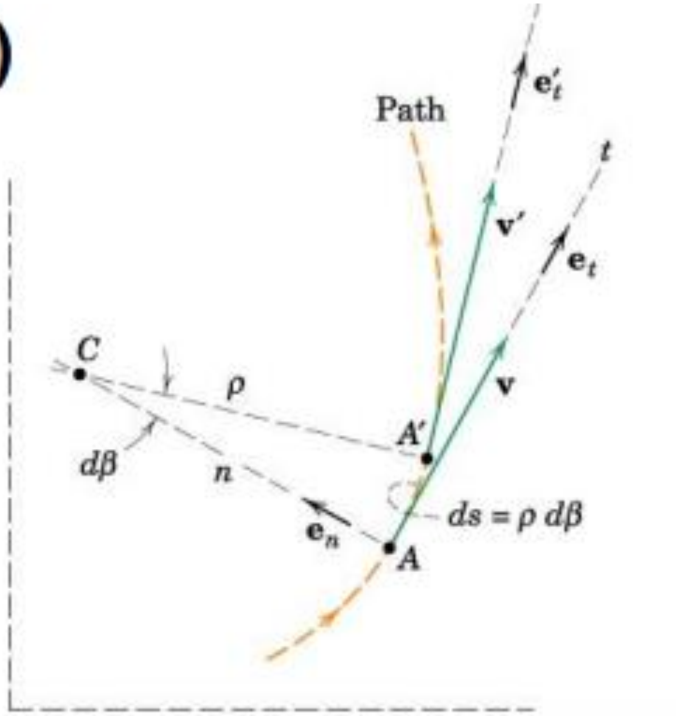
Determination of $\dot{\mathbf{e}}_t$:

→ change in $\mathbf{e}_t$ during motion from *A* to *A'*

→ The unit vector changes to $\mathbf{e}'_t$

The vector difference $d\mathbf{e}_t$ is shown in the bottom figure.

- In the limit $d\mathbf{e}_t$ has magnitude equal to length of the arc $|\mathbf{e}_t|\ d\beta = d\beta$
- Direction of $d\mathbf{e}_t$ is given by $\mathbf{e}_n$

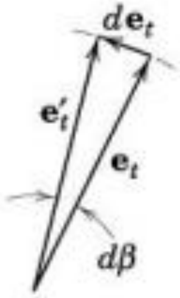


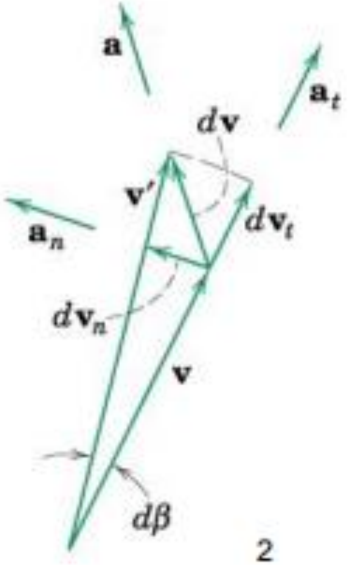
→ We can write: $d\mathbf{e}_t = \mathbf{e}_n\, d\beta$ → $\dfrac{d\mathbf{e}_t}{d\beta} = \mathbf{e}_n$

Dividing by *dt*: $d\mathbf{e}_t / dt = \mathbf{e}_n\,(d\beta/dt)\ \mathbf{e}_n$ → $\dot{\mathbf{e}}_t = \dot{\beta}\mathbf{e}_n$

Substituting this and $v = \rho\, d\beta/dt$ = $v = \rho\dot{\beta}$ in equation for acceleration:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d(v\mathbf{e}_t)}{dt} = v\dot{\mathbf{e}}_t + \dot{v}\mathbf{e}_t \rightarrow \mathbf{a} = \frac{v^2}{\rho}\mathbf{e}_n + \dot{v}\mathbf{e}_t$$





Here:

$$a_n = \frac{v^2}{\rho} = \rho\dot{\beta}^2 = v\dot{\beta}$$

$$a_t = \dot{v} = \ddot{s}$$

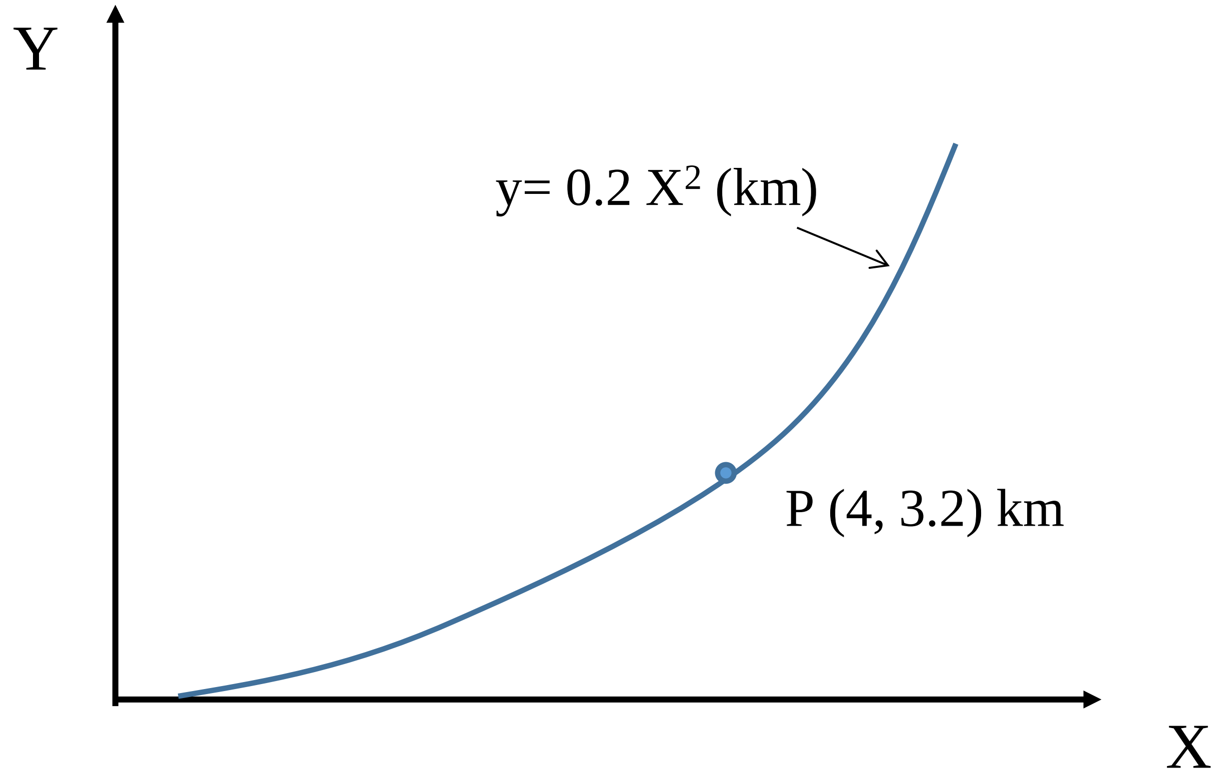
$$a = \sqrt{a_n^2 + a_t^2}$$

Problem

- A particle moves along the path $\bar{r} = (8t^2)i + (t^3 + 5)j$ meters. Where t is in seconds. Determine magnitudes of particles velocity and acceleration when t = 3 seconds. Also determine the equation y = f(x) of the path.

An airplane travels on a curved path. At point ‘P’ it has a speed of 360kmph which is increasing at the rate of 0.5 m/s^2. Determine at ‘P’,

- Magnitude of total acceleration
- Angle made by the acceleration vector with the positive X-axis.



Problem

A particle moves along a hyperbolic path $\frac{x^2}{16} - y^2 = 28$. If the x component of velocity V_x is 4 m/s, and remains constant, determine the magnitude of its velocity and acceleration when it is at point (32m, 6m).

Problem

The position of the charged particle moving in a horizontal plane is measured electronically. This information is fed into a computer, which employees a curve fitting techniques to generate analytical expression for its position given by $\bar{r} = (t^3)i + (t^4)j$, where $\bar{r}$ is in meters and t is in seconds. For t = 1 seconds, determine,

- The acceleration of the particle in rectangular components
- Its normal and tangential acceleration,
- The radius of curvature of the path

Problem

A particle moving in x-y plane and its position is defined by,

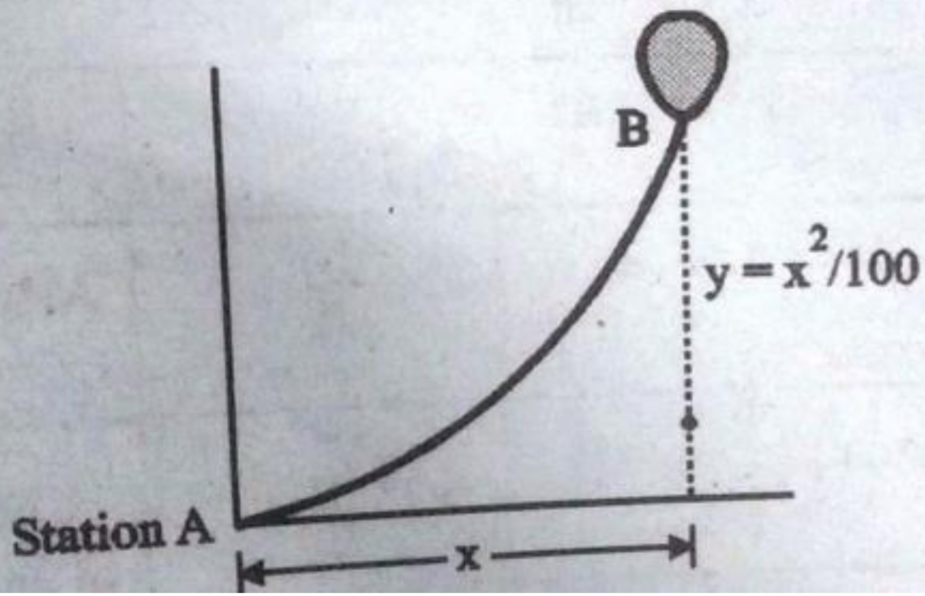
$\bar{r} = \left(\frac{3}{2}t^2\right)i + \left(\frac{2}{3}t^3\right)j$. Find radius of curvature when t = 2 seconds.

At any instant the horizontal position of a balloon is defined by $x = 30t$. If path equation is $y = \frac{x^2}{100}$ determine,

a) The distance of the balloon from the station at A when $t = 2$ sec

b) Magnitude and direction of velocity when $t = 2$sec.

c) The magnitude and direction of acceleration when $t = 2$ sec.



- A car is moving along a curve of radius 300m at a speed 90 kmph. The brakes are suddenly applied, causing speed to decrease at a constant rate of 1.3 m/s^2. Determine the total acceleration,

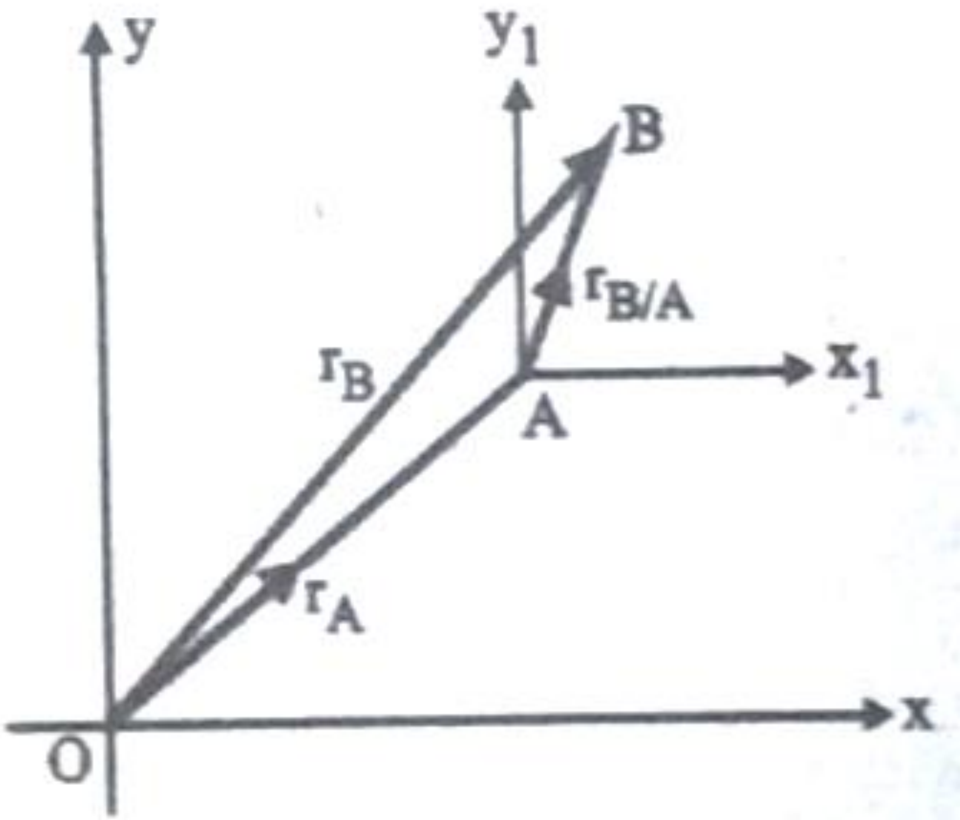
  a. immediately after brakes have been applied.

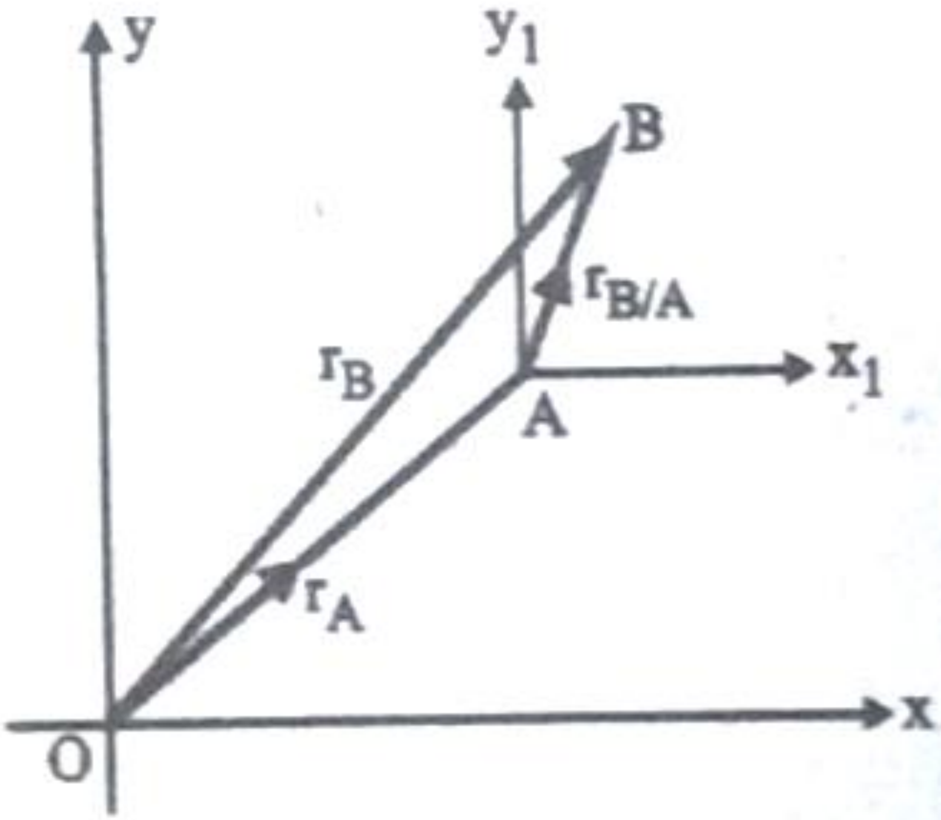
  b. after 5 sec.

- The motion of a particle is defined by $x=4t^2$ and $y=2t^3$ in meters. Determine normal and tangential component acceleration at t=2 sec.
- The velocity of a particle is defined by $v_x = 100 - t^{3/2}$ and $v_y = 100 + 10t - 2t^2$. Determine radius of curvature (1) at the top of its path (2) At t = 12 sec.
- A train enters a curve of radius 800 meters with a speed of 72 kmph. Determine magnitude of total acceleration at the instant the brakes are applied so that train stops by covering a distance of 500 meters along the curve. Also determine the time required by train to come to rest.

Relative Velocity

- The motion of a particle with respect to a fixed frame is called as **absolute motion.**
- The motion of a particle relative to a set of axes which are moving is called as **relative motion.**





Now, by triangle law

$$r_B = r_A + r_{B/A} \qquad \therefore \qquad r_{B/A} = r_B - r_A$$

Similarly

$$v_B = v_A + v_{B/A} \qquad \therefore \qquad v_{B/A} = v_B - v_A$$

and

$$a_B = a_A + a_{B/A} \qquad \therefore \qquad a_{B/A} = a_B - a_A$$

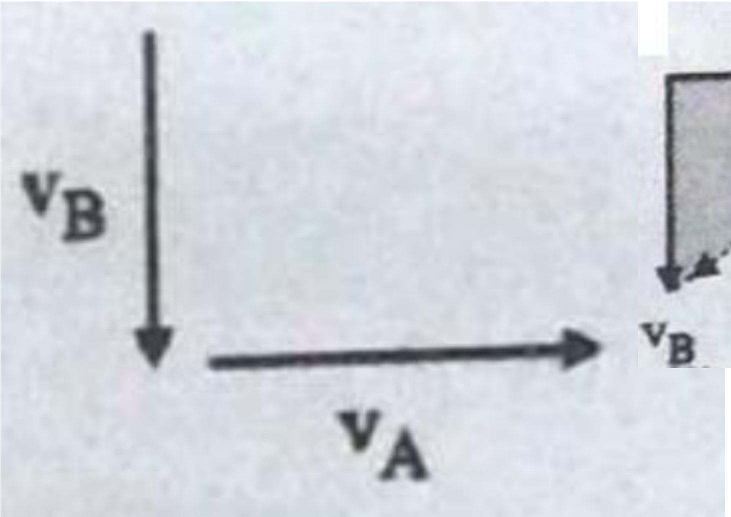
Example:

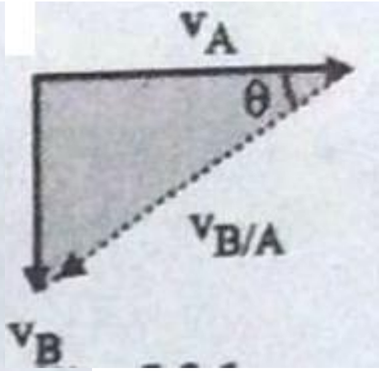
- ✓ Two moving trains
- ✓ Two moving cars.

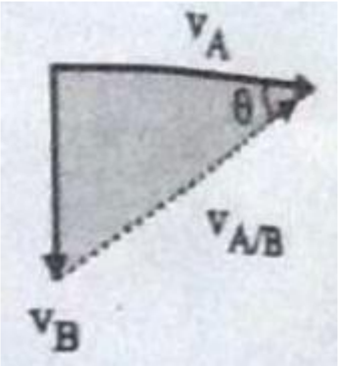
Graphical Approach

- To find relative velocity, place the two velocity vectors with their tails joining at a common points and representing the two sides of a triangle then the closing side of triangle represents relative velocity.

Case 1: If two velocities V_A and V_B are perpendicular





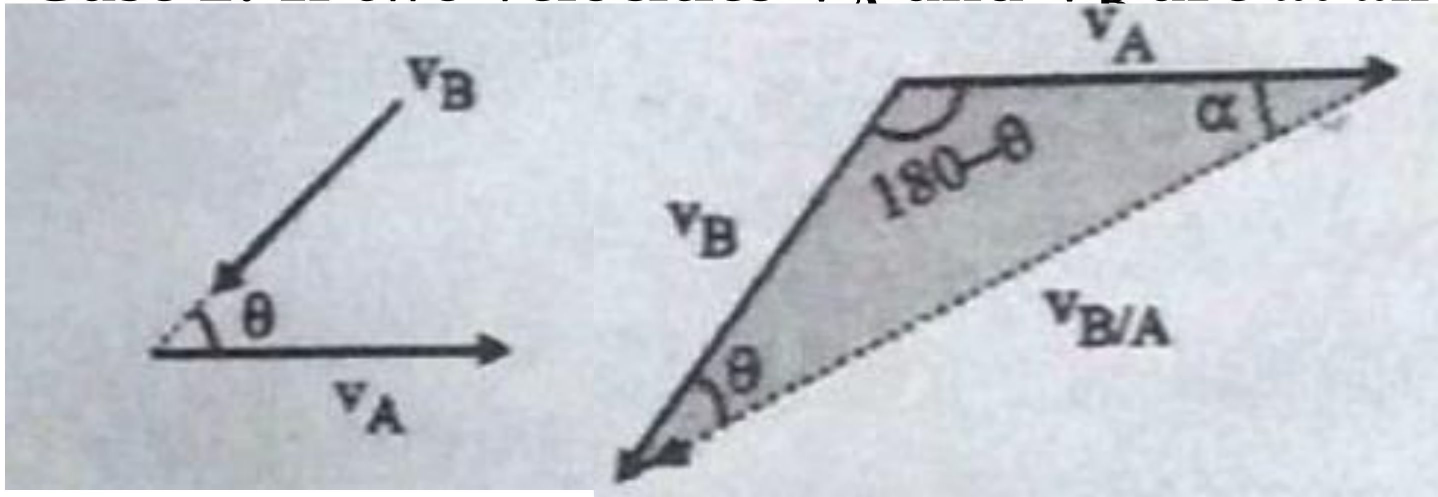


$$v_{B/A} = \sqrt{v_A^2 + v_B^2}$$

$$\tan\theta = \frac{v_B}{v_A}$$

- ❑ The velocity VB/A is directed from A to B and the velocity VA/B is directed from B to A

Case 2: If two velocities V_A and V_B are at an angle θ

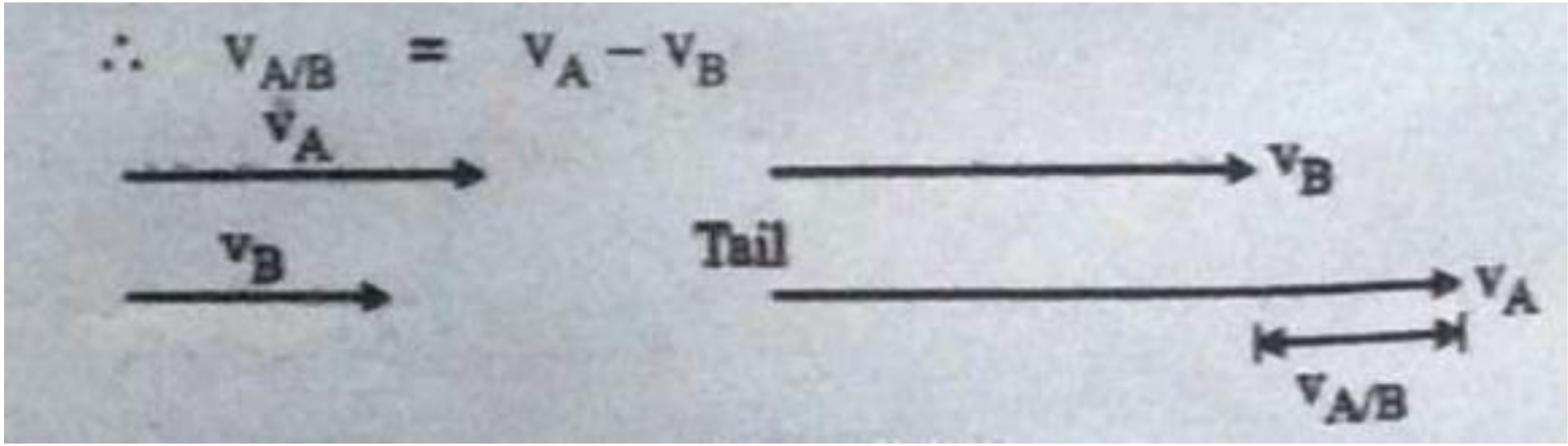


By cosine rule, Magnitude $v_{B/A} = \sqrt{(v_A)^2 + (v_B)^2 - 2v_A \cdot v_B \cos(180-\theta)}$

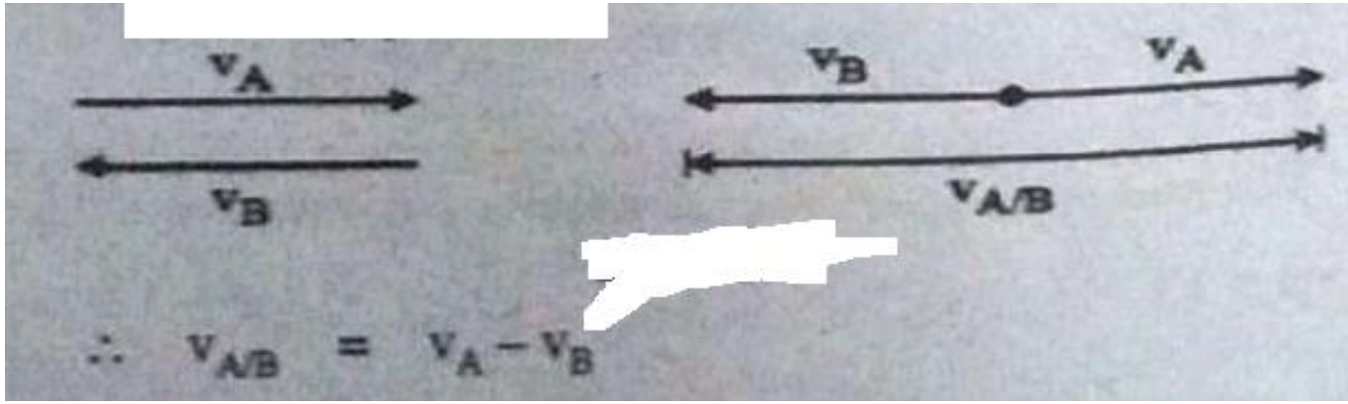
Direction α can be calculated by using sine rule,

$$\frac{v_B}{\sin\alpha} = \frac{v_{B/A}}{\sin(180-\theta)} \quad \therefore \quad \sin\alpha = \frac{v_B \cdot \sin\theta}{v_{B/A}}$$

Case 3: If two velocities V_A and V_B are parallel and V_A not equal to V_B



Case 4: If two velocities V_A and V_B are parallel, Equal and Opposite



1. A train moving at 45 kmph is hit by a stone thrown at right angles to it with a velocity of 22.5 kmph. Find the velocity and direction with which the stone appears to hit a person travelling in the train.

2. A monkey is climbing a tree with a velocity of 10 m/s while a dog running towards the tree chasing the monkey with a velocity of 15 m/s. Find the velocity of dog relative to monkey.

3. Figure shows two cars A and B at a distance of 35 m. Car A is travelling east at a constant speed of 36 kmph. Car B starts from rest and moves south with a constant acceleration of 1.2 m/s^2. Determine,

a. Position
b. Velocity
c. Acceleration of car B relative to car A, 6 seconds after car A crosses the intersection of roads.

4. Two trains leave a station in different direction at the same instant. Train A travels at 360 kmph at 10º west of north. While train B travels at 450 kmph at 60º East of North. Find,

a. Relative velocity of train A with respective to train B
b. Two trains are how much apart 2 minutes later.

5. From point O in figure, a ship A travels in the north making an angle of 45° to the west with velocity of 18 kmph and ship B travels in the east with a velocity of 9 kmph. Find the relative velocity of ship B with respective to ship A.

Figure shows the location of cars A and B at t = 0. Car A starts from rest and travels towards the intersection at a uniform rate of $2\ m/s^2$. Car B travels towards the intersection at a constant speed of 8 m/s. Determine relative position, velocity and acceleration of car B w.r.t car A. at t = 6 sec.