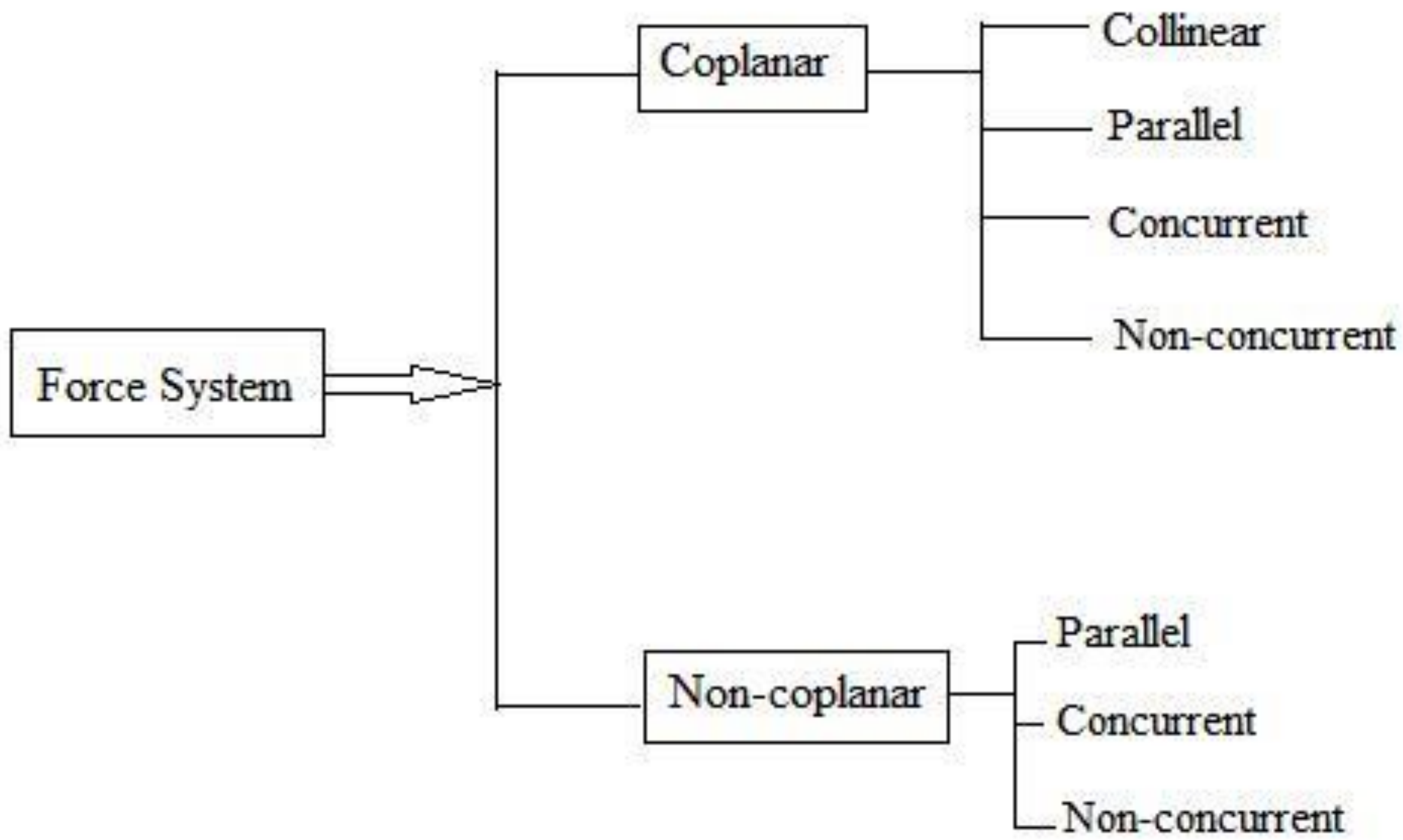
# K J SOMAIYA COLLEGE OF ENGINEERING, MUMBAI-77

## (CONSTITUENT COLLEGE OF SOMAIYA VIDYAVIHAR UNIVERSITY)

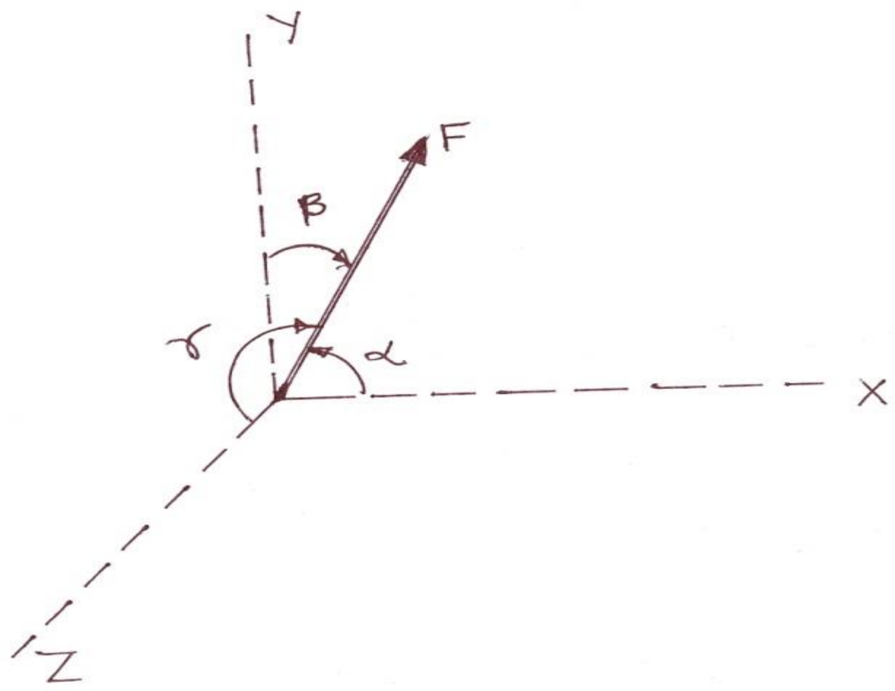
**Presented by:**
**Prof. R. B. Pansare**


# Classification of system of forces



**A Force in space**: The force system which is acting in different planes is called as non-coplanar force system or space forces.

A Force is said to be in space if its line of action makes an angle α, β and γ with respect to rectangular co-ordinate axes X, Y and Z respectively as shown the Fig.

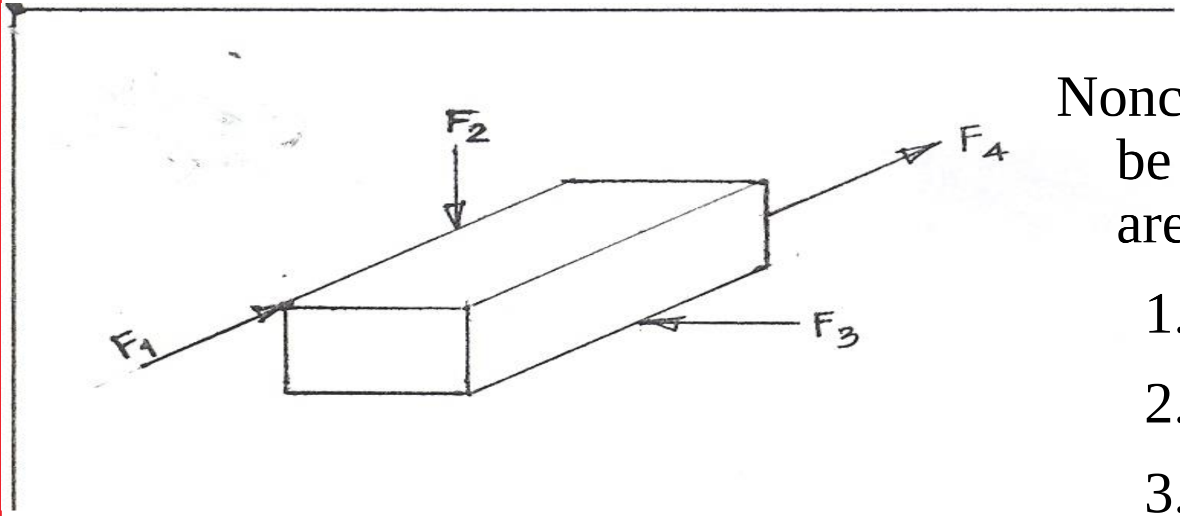


# Forces in space

## Noncoplanar system of forces (Forces in Space) and Their Classifications

System of forces which do not lie in a single plane is called non-coplanar system of forces(Forces in space ).

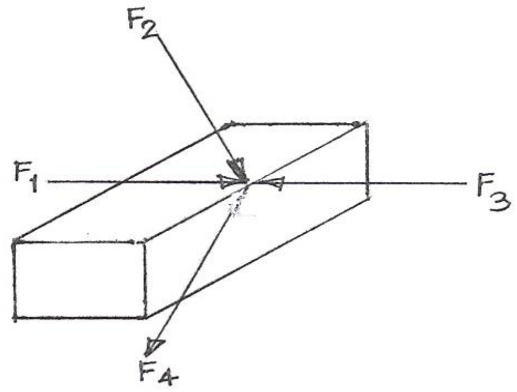
A typical noncoplanar system of forces (forces in space) is shown in the Fig. below

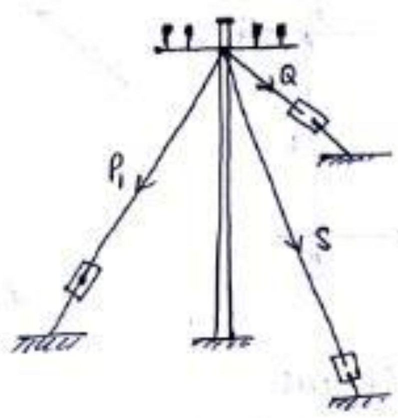


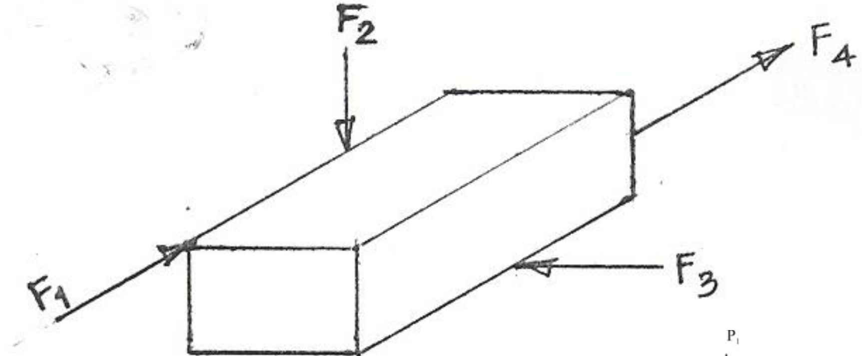

Noncoplanar system of forces(Forces in space) can be broadly classified into three categories. They are

1. Concurrent noncoplanar system of forces
2. Non-concurrent noncoplanar system of forces
3. Noncoplanar parallel system of forces

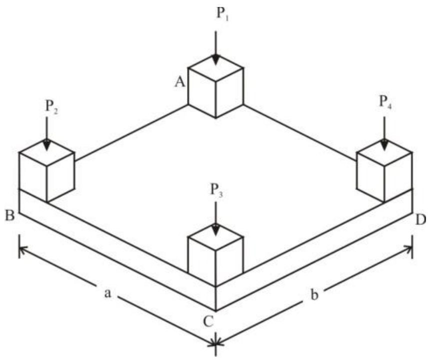
1. **<u>Concurrent noncoplanar system of forces</u>**: Forces which meet at a point with their lines of action do not lie in a plane are called “Concurrent noncoplanar system of forces”. A typical system of Concurrent noncoplanar system of forces is shown in the Fig.



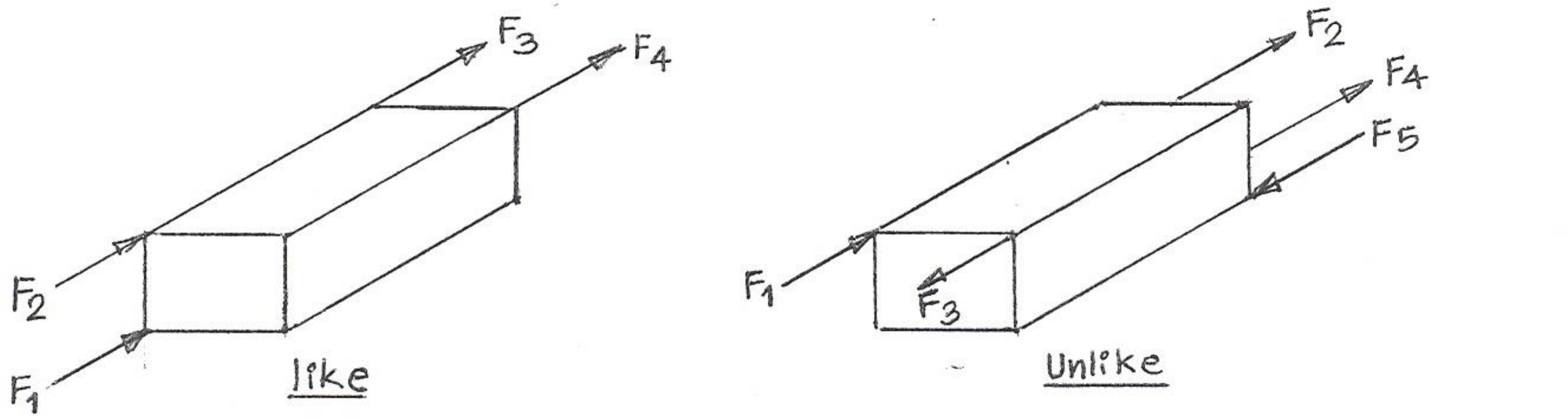




2. **N<u>on-concurrent noncoplanar system of forces</u>**: Forces which do not meet at a point and their lines of action do not lie in a plane, such forces are called “Non-concurrent noncoplanar system of forces”. A typical system of non-concurrent noncoplanar system of forces is shown in the Fig.

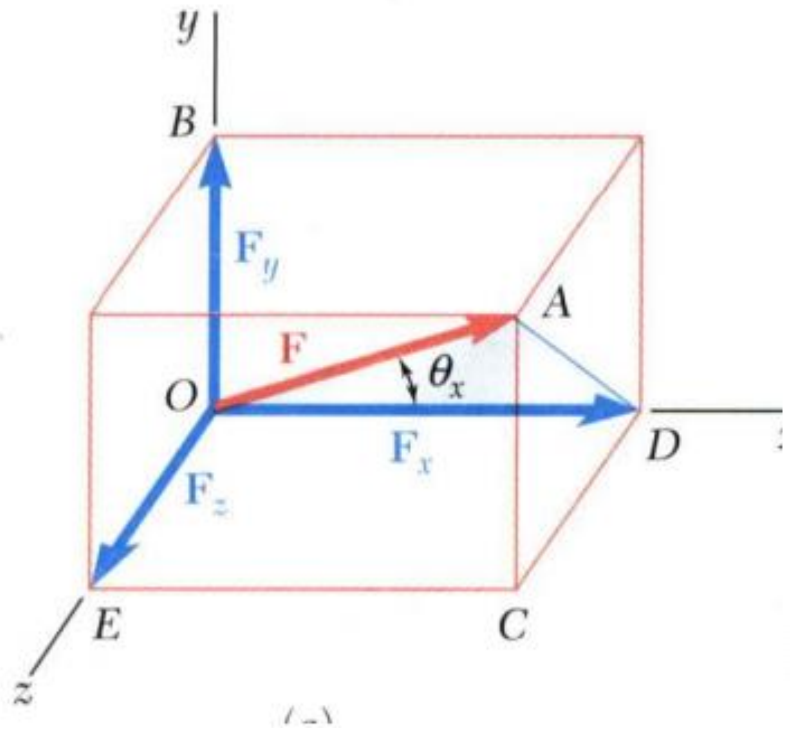



Non- Coplanar and non-Concurrent force system

3. **Noncoplanar parallel system of forces:** If lines of action of all the forces in a system are parallel and they do not lie in a plane such a system is called Non-coplanar parallel system of forces. If all the forces are pointing in one direction then they are called Like parallel forces otherwise they are called unlike parallel forces as shown in the Fig.



# Rectangular components of a force in space




- With the angles between **F** and the axes,

$$F_x = F\cos\theta_x \quad F_y = F\cos\theta_y \quad F_z = F\cos\theta_z$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$= F\left(\cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}\right)$$

$$= F\vec{\lambda}$$

$$\vec{\lambda} = \cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}$$

- $\vec{\lambda}$ is a **unit vector** along the line of action of **F**; $\cos\theta_x$, $\cos\theta_y$, and $\cos\theta_z$ are the **direction cosines**

Now applying Pythagorean theorem to the triangles OAB and OCD

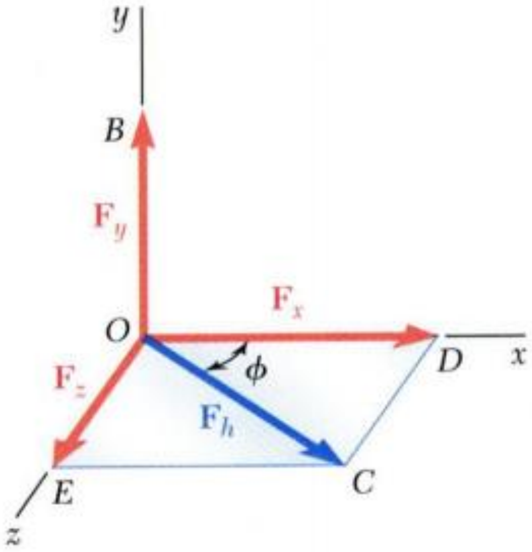
$F^2 = (OA)^2 = OB^2 + BA^2 = F_y^2 + F_h^2$ ----------------(1)

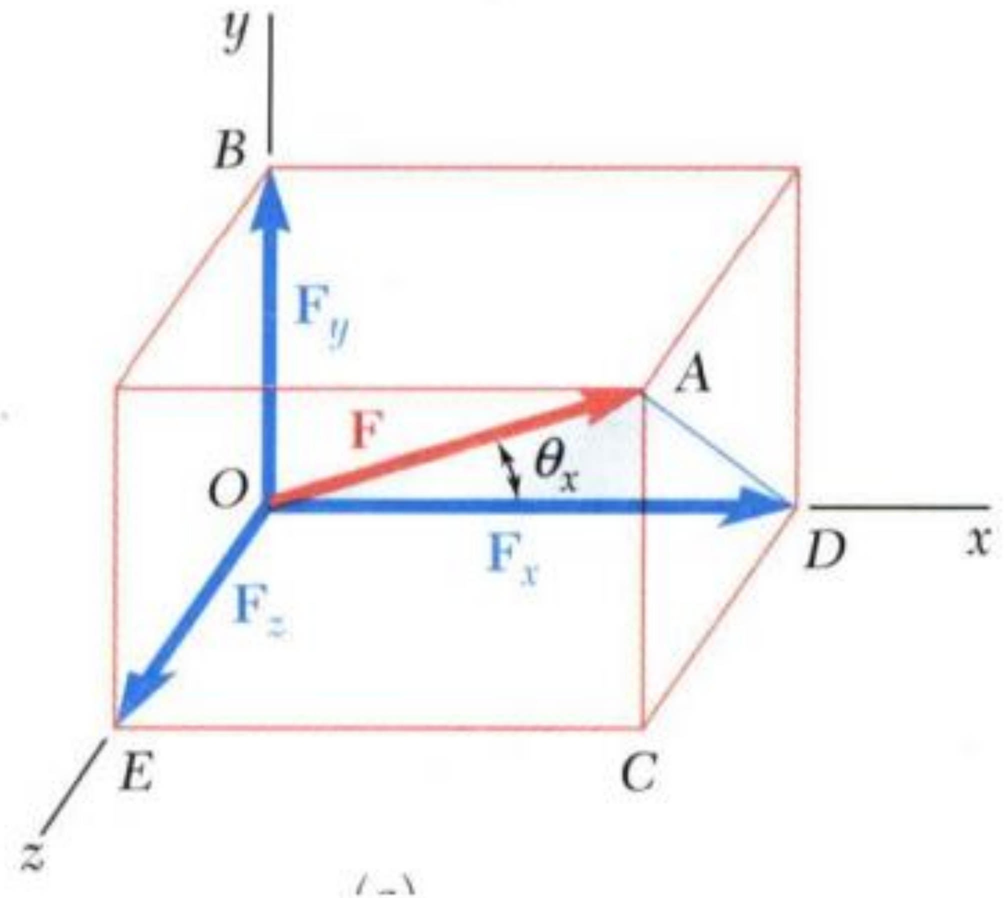
$F_h^2 = OC^2 = OD^2 + DC^2 = F_x^2 + F_z^2$ ----------------(2)

Substituting equation (2) into the equation (1), we get

$F^2 = F_x^2 + F_y^2 + F_z^2$

$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$ -----(3)



From the above Figure (Fig.)

$F_x = F \cos \theta_x$, $F_y = F \cos\theta_y$, $F_z = F \cos\theta_z$ ------------(4)

Where $\theta_x$, $\theta_y$, $\theta_z$ are the angles formed by the force F with X, Y, Z axes respectively.
$F_x$, $F_y$, $F_z$ are the rectangular components of the force F in the directions of X, Y, Z axes respectively.

$\cos \theta_x = F_x/F$; $\cos\theta_y = F_y/F$; $\cos\theta_z = F_z/F$

Substituting equation (4) into the equation (3), we get

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

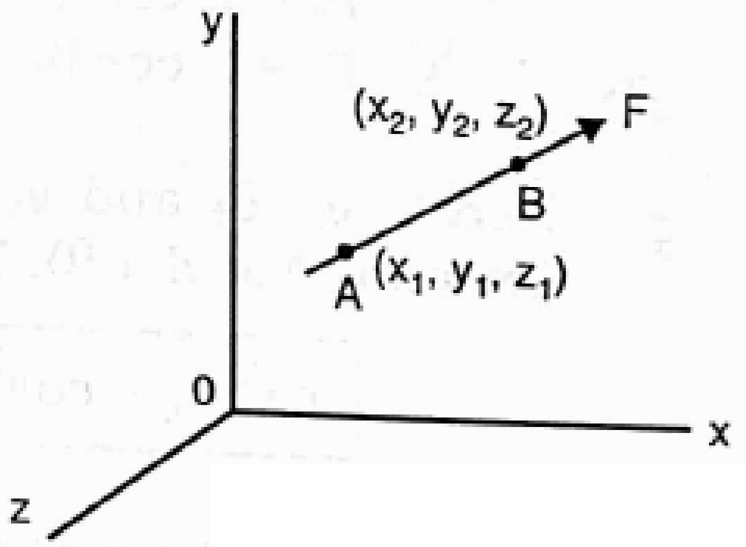
$$F = \sqrt{F^2\cos^2\theta_x + F^2\cos^2\theta_y + F^2\cos^2\theta_z}$$

$$F^2 = F^2 ( \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z )$$

$$\mathbf{1 = \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z} \quad \text{-------------(5)}$$

# Force in vector form

Fig. shows a force of magnitude F in space passing through A $(x_1, y_1, z_1)$ and B $(x_2, y_2, z_2)$. The force in vector form is




$$\bar{F} = F.\hat{e}_{AB}$$

$$\bar{F} = F\left(\frac{(x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}\right)$$

$$\bar{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j} + F_z\,\mathbf{k}$$

... Force in vector form

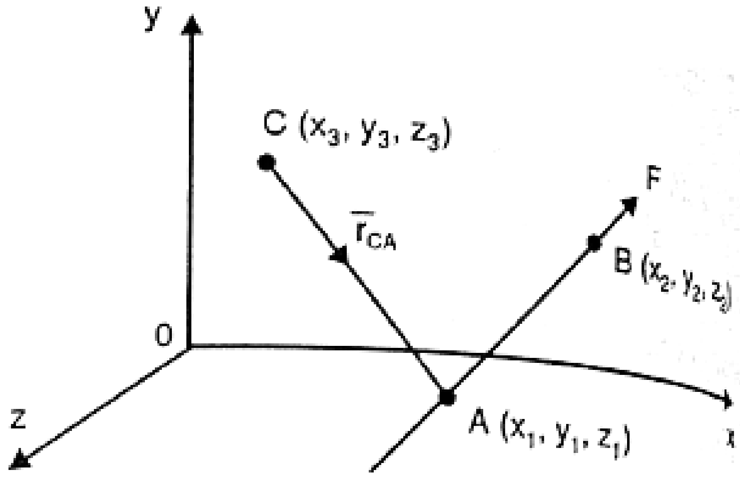
Note: **i**, **j** *and* **k** *printed in bold type denote unit vectors along the x, y and z axis respectively.*

# Moment of a force

Step 1: Put the force in vector form i.e.

$$\overline{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j} + F_z\,\mathbf{k}$$

Step 2: Find the position vector extending from the moment centre to any point on the force i.e. $\overline{r} = r_x\,\mathbf{i} + r_y\,\mathbf{j} + r_z\,\mathbf{k}$




Step 3: Perform the cross product of the position vector and the force vector to get the moment vector
i.e.

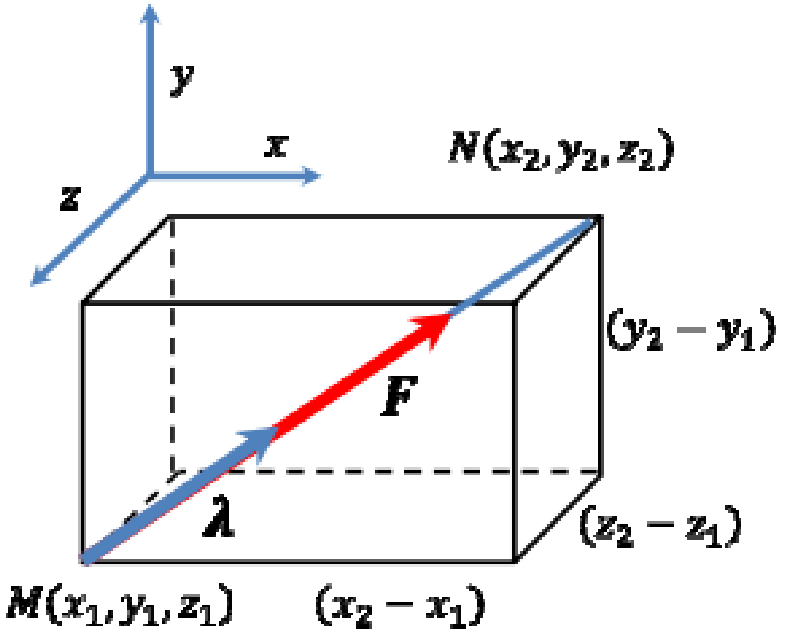
$$\overline{M}^{F}_{point} = \overline{r} \times \overline{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

# Rectangular Components in Space

Direction of the force is defined by the location of two points

$M(x_1, y_1, z_1)$ and $N(x_2, y_2, z_2)$




$\boldsymbol{d}$ is the vector joining $M$ and $N$

$$\boldsymbol{d} = d_x\boldsymbol{i} + d_y\boldsymbol{j} + d_z\boldsymbol{k}$$

$$d_x = (x_2 - x_1) \qquad d_y = (y_2 - y_1)$$

$$d_z = (z_2 - z_1)$$

$$\boldsymbol{F} = F\boldsymbol{\lambda}$$

$$= F\left(\frac{d_x\boldsymbol{i} + d_y\boldsymbol{j} + d_z\boldsymbol{k}}{d}\right)$$

$$F_x = F\frac{d_x}{d} \qquad F_y = F\frac{d_y}{d} \qquad F_z = F\frac{d_z}{d}$$

**Resultant of concurrent forces in Space**:-

Resolve all the forces into their rectangular components in X, Y and Z axes directions. Adding algebraically all the horizontal components in the x direction gives

$$R_x = \sum F_x,$$

Similarly adding algebraically all the components in y and z directions yield the following relations

$$R_y = \sum F_y,$$

$$R_z = \sum F_z$$

Thus magnitude of resultant

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

Angles $\theta_x$, $\theta_y$, $\theta_z$ resultant forms with the axes of coordinates are obtained by

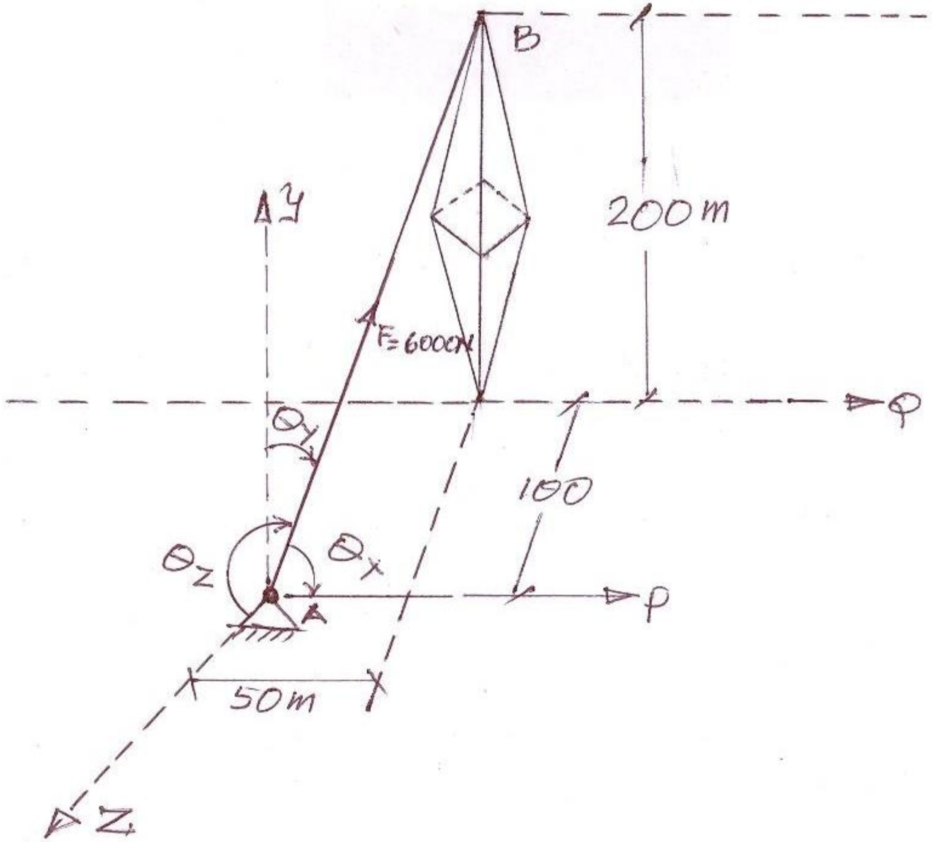
$$Cos\theta_x = \frac{R_x}{R}; Cos\theta_y = \frac{R_y}{R}; Cos\theta_z = \frac{R_z}{R}$$

**_Problems:_**

(1) A tower guy wire is anchored by means of a bolt at A is shown in the following Figure. The tension in the wire is 6000N. Determine

(a) The components $F_x$, $F_y$, $F_z$ of the forces acting on the bolt.

(b) The angles $\theta_x$, $\theta_y$, $\theta_z$ defining the direction of the force.



**Solution:** (a) Here $d_x = 50m$, $d_y = 200m$, $d_z = -100m$

Total distance A to B

$$d = \sqrt{d_x^2 + d_y^2 + d_z^2}$$

$$= \sqrt{(50)^2 + (200)^2 + (-100)^2}$$

$$= 229.13 \text{ m}$$

Using the equation, $F_x/d_x = F_y/d_y = F_z/d_z = F/d$

$\therefore F_x = d_x \,.\, (F/d) = (50 \times 6000)/ 229.13 = 1309.3 \text{ N}$

$F_y = d_y \,.\, (F/d) = (200 \times 6000)/ 229.13 = 5237.20 \text{ N}$

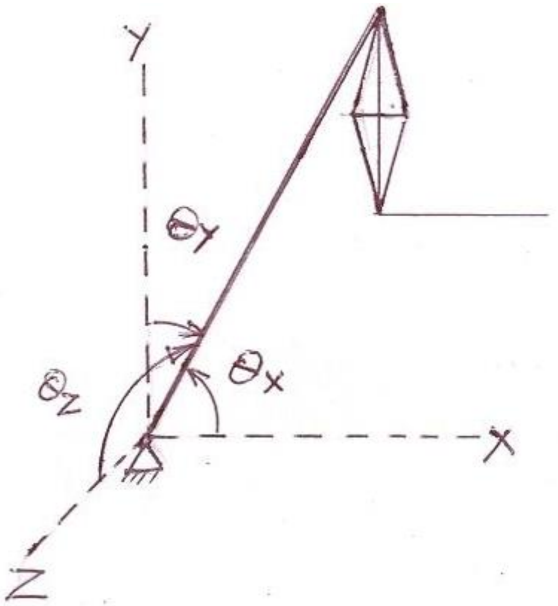
$F_z = d_z \,.\, (F/d) = (-100 \times 6000)/ 229.13 = -2618.6 \text{ N}$

(b) Directions of the force:

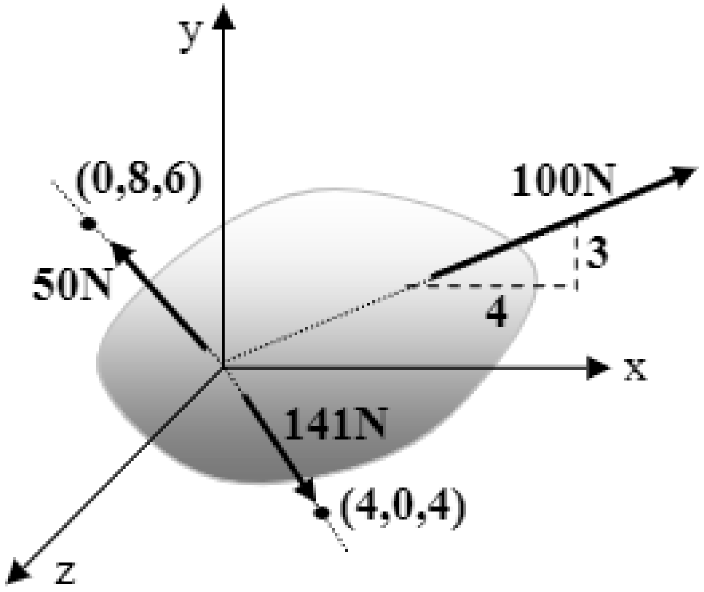
$\cos\theta_x = d_x/d$ , $\theta_x = \cos^{-1}(50/229.13) = 77.4°$

$\theta_y = \cos^{-1}(d_y/d) = \cos^{-1}(200/229.13) = 29.2°$

$\theta_z = \cos^{-1}(d_z/d) = \cos^{-1}(-100/229.13) = 115.88°$



**Example:** Find the resultant of the three concurrent forces (passing through origin) shown in the figure. The 100N force lies in the X-Y plane.



Let us first represent these forces as vectors in rectangular Cartesian coordinate system as follows:

$$\bar{F}_1 = 100\left(\left(\frac{4}{\sqrt{4^2+3^2}}\right)\hat{i}+\left(\frac{3}{\sqrt{4^2+3^2}}\right)\hat{j}+0\hat{k}\right)=\left(80\hat{i}+60\hat{j}+0\hat{k}\right)\text{N}$$

$$\bar{F}_2 = 50\left(\left(\frac{0}{\sqrt{0^2+8^2+6^2}}\right)\hat{i}+\left(\frac{8}{\sqrt{0^2+8^2+6^2}}\right)\hat{j}+\left(\frac{6}{\sqrt{0^2+8^2+6^2}}\right)\hat{k}\right)=\left(0\hat{i}+40\hat{j}+30\hat{k}\right)\text{N}$$

$$\bar{F}_3 = 141\left(\left(\frac{4}{\sqrt{4^2+0^2+4^2}}\right)\hat{i}+\left(\frac{0}{\sqrt{4^2+0^2+4^2}}\right)\hat{j}+\left(\frac{4}{\sqrt{4^2+0^2+4^2}}\right)\hat{k}\right)=\left(100\hat{i}+0\hat{j}+100\hat{k}\right)\text{N}$$

$$\bar{R}=\bar{F}_1+\bar{F}_2+\bar{F}_3=180\hat{i}+100\hat{j}+130\hat{k}\ \text{N}$$

$$=\sqrt{180^2+100^2+130^2}\left(\left(\frac{180}{\sqrt{180^2+100^2+130^2}}\right)\hat{i}+\left(\frac{100}{\sqrt{180^2+100^2+130^2}}\right)\hat{j}+\left(\frac{130}{\sqrt{180^2+100^2+130^2}}\right)\hat{k}\right)\text{N}$$

$$=244\left(0.74\hat{i}+0.41\hat{j}+0.53\hat{k}\right)\text{N}$$

- A force of magnitude 50 KN is acting at point A (2,3,4) m towards point B (6, -2, -3) m. Find the moment of the given force about a point D (-1, 1, 2) m

**Solution:** The force in vector form is

$$\bar{F} = F.\hat{e}_{AB}$$

$$= 50\left(\frac{4\mathbf{i} - 5\mathbf{j} - 7\mathbf{k}}{\sqrt{4^2 + 5^2 + 7^2}}\right)$$

$$= 21.08\,\mathbf{i} - 26.35\,\mathbf{j} - 36.89\,\mathbf{k} \quad \text{kN}$$

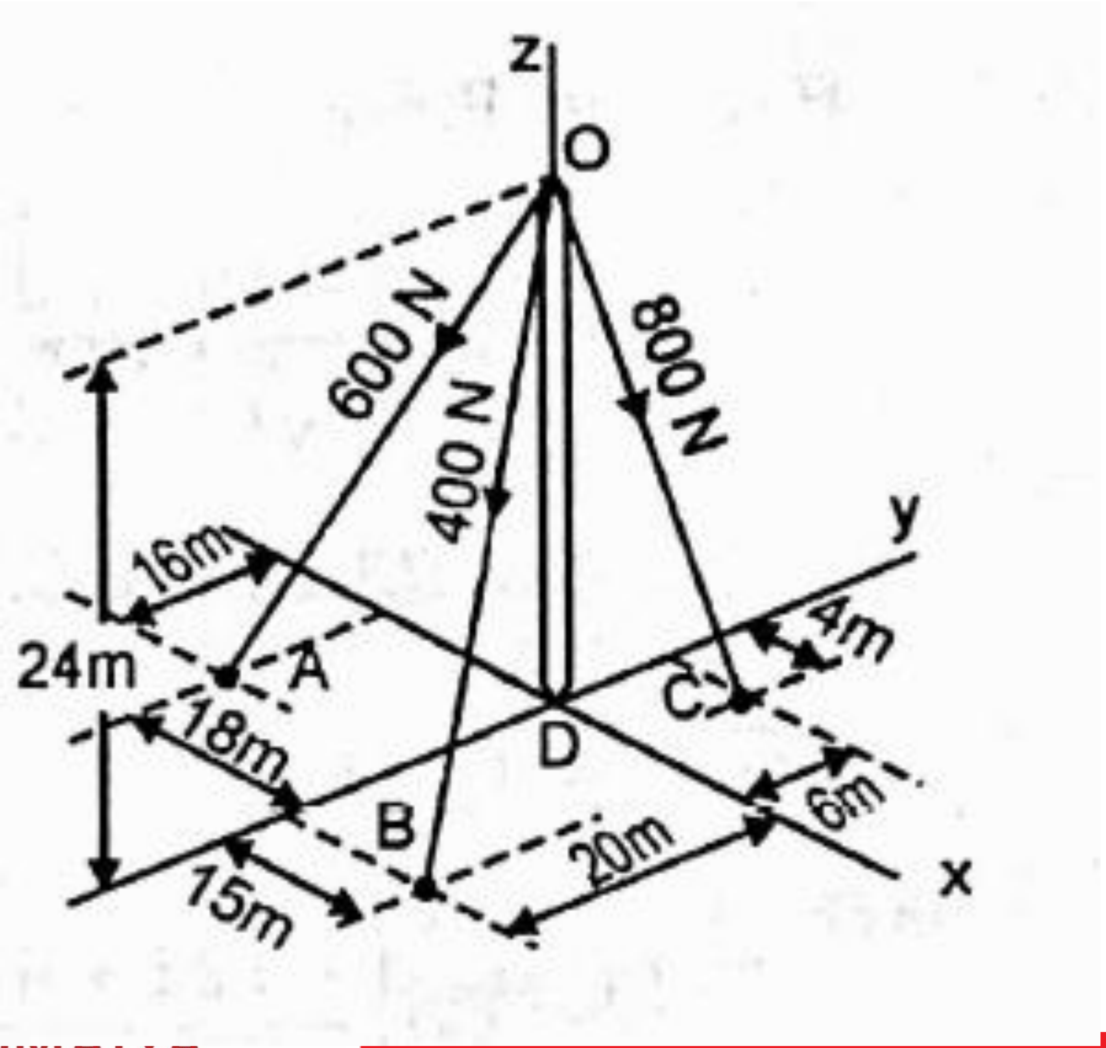
$$\bar{M}_D^F = \bar{r}_{DA} \times \bar{F}$$

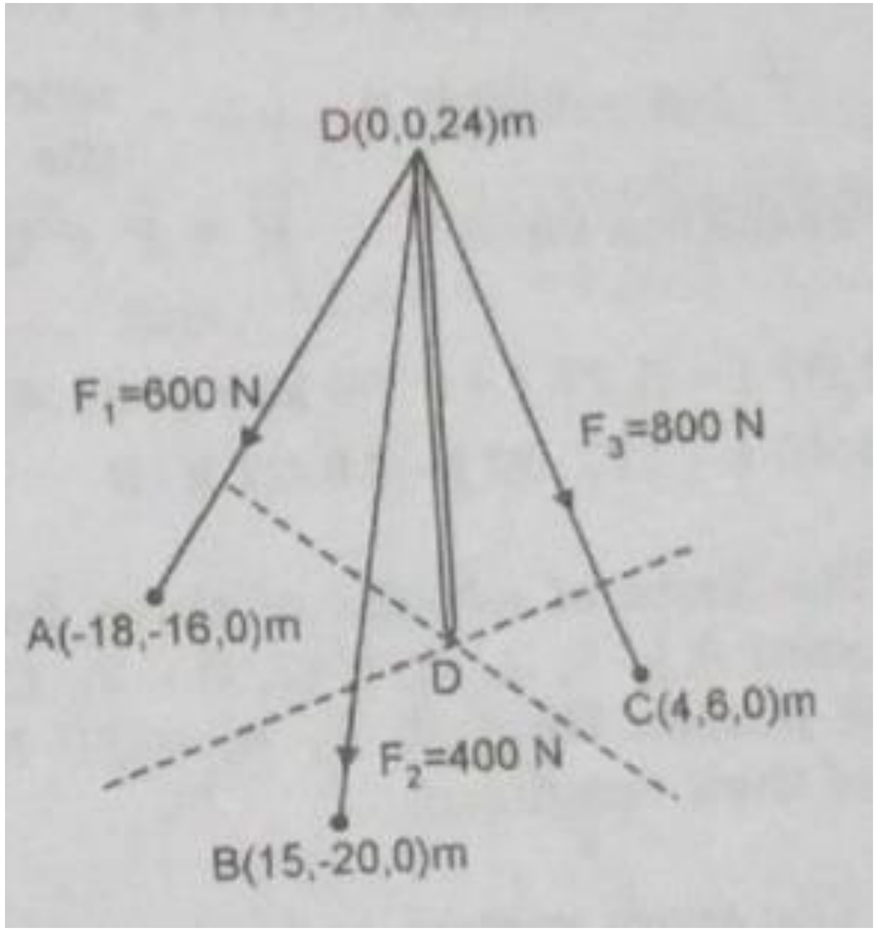
Here $\bar{r}_{DA} = 3\,\mathbf{i} + 2\,\mathbf{j} + 2\,\mathbf{k}$ m

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & 2 \\ 21.08 & -26.35 & -36.89 \end{vmatrix}$$

$$\therefore \quad \bar{M}_D^F = -21.08\,\mathbf{i} + 152.8\,\mathbf{j} - 121.2\,\mathbf{k} \quad \text{kNm}$$

The tower is held in place by three cables. If the force of each cable acting on the tower is as shown in figure, determine the resultant.



Let $\bar{F}_1$ be the force in cable OA

$$\bar{F}_1 = F_1 . \hat{e}_{OA}$$

$$\therefore \quad = 600\left[\frac{-18\mathbf{i} - 16\mathbf{j} - 24\mathbf{k}}{\sqrt{18^2 + 16^2 + 24^2}}\right]$$

$$= -317.6\mathbf{i} - 282.4\mathbf{j} - 423.5\mathbf{k} \text{ N}$$

Let $\bar{F}_2$ be the force in cable OB

$$\bar{F}_2 = F_2 . \hat{e}_{OB}$$

$$\therefore \quad = 400\left[\frac{15\mathbf{i} - 20\mathbf{j} - 24\mathbf{k}}{\sqrt{15^2 + 20^2 + 24^2}}\right]$$

$$= +173.1\mathbf{i} - 230.8\mathbf{j} - 277\mathbf{k} \text{ N}$$

Let $\bar{F}_3$ be the force in cable OC

$$\bar{F}_3 = F_3 . \hat{e}_{OC}$$

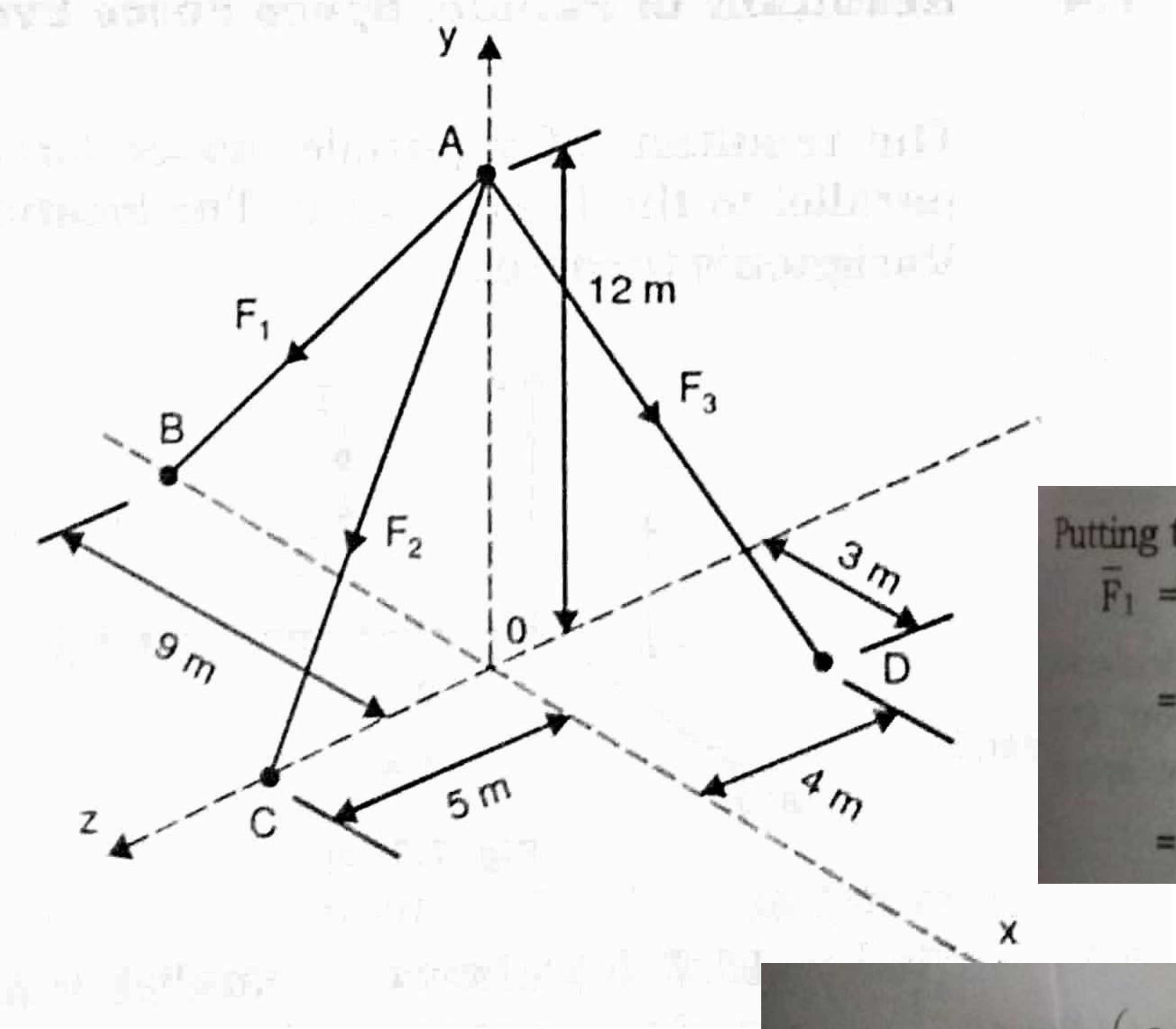
$$\therefore \quad = 800\left[\frac{4\mathbf{i} + 6\mathbf{j} - 24\mathbf{k}}{\sqrt{4^2 + 6^2 + 24^2}}\right]$$

$$= 127.7\mathbf{i} + 191.5\mathbf{j} - 766.2\mathbf{k} \text{ N}$$

Resultant force $\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3$

$$\therefore \quad \bar{R} = -16.8\mathbf{i} - 321.7\mathbf{j} - 1466.7\mathbf{k} \text{ N}$$

The resultant of the three concurrent space forces at A is $\bar{R} = -788j$ N. Find the magnitude of F1, F2 and F3 force.




**Solution:** This is a concurrent space force system of three forces. To put the forces in vector form, we need the coordinates of the points through which the forces pass.

From the figure the coordinates are, A (0, 12, 0) m, B (– 9, 0, 0) m, C (0, 0, 5) m and D (3, 0, – 4) m.

Putting the forces in vector form.

$$\bar{F}_1 = F_1.\hat{e}_{AB}$$

$$= F_1\left(\frac{-9\,\mathbf{i} - 12\,\mathbf{j}}{\sqrt{9^2 + 12^2}}\right)$$

$$= F_1(-0.6\,\mathbf{i} - 0.8\,\mathbf{j})\ \text{N}$$

$$\bar{F}_2 = F_2.\hat{e}_{AC}$$

$$= F_2\left(\frac{-12\,\mathbf{j} + 5\,\mathbf{k}}{\sqrt{12^2 + 5^2}}\right)$$

$$= F_2(-0.923\,\mathbf{j} + 0.385\,\mathbf{k})\ \text{N}$$

$$\bar{F}_3 = F_3.\hat{e}_{AD} = F_3\left(\frac{3\,\mathbf{i} - 12\,\mathbf{j} - 4\,\mathbf{k}}{\sqrt{3^2 + 12^2 + 4^2}}\right) \quad \therefore \quad \bar{F}_3 = F_3(0.231\,\mathbf{i} - 0.923\,\mathbf{j} - 0.308\,\mathbf{k})\ \text{N}$$

The resultant of the forces at A is $\overline{R} = -788\,\mathbf{j}$ N

$$\overline{R} = \overline{F}_1 + \overline{F}_2 + \overline{F}_3$$

$$0\,\mathbf{i} - 788\,\mathbf{j} + 0\,\mathbf{k} = F_1(-0.6\,\mathbf{i} - 0.8\,\mathbf{j}) + F_2(-0.923\,\mathbf{j} + 0.385\,\mathbf{k}) + F_3(0.231\,\mathbf{i} - 0.923\,\mathbf{j} - 0.308\,\mathbf{k})$$

$$0\,\mathbf{i} - 788\,\mathbf{j} + 0\,\mathbf{k} = (-0.6\,F_1 + 0.231\,F_3)\,\mathbf{i} + (-0.8\,F_1 - 0.923\,F_2 - 0.923\,F_3)\,\mathbf{j} + (0.385\,F_2 - 0.308\,F_3)\,\mathbf{k}$$

Equating the coefficients

$$-0.6\,F_1 + 0.231\,F_3 = 0 \quad \ldots\ldots\ldots\ldots (1)$$

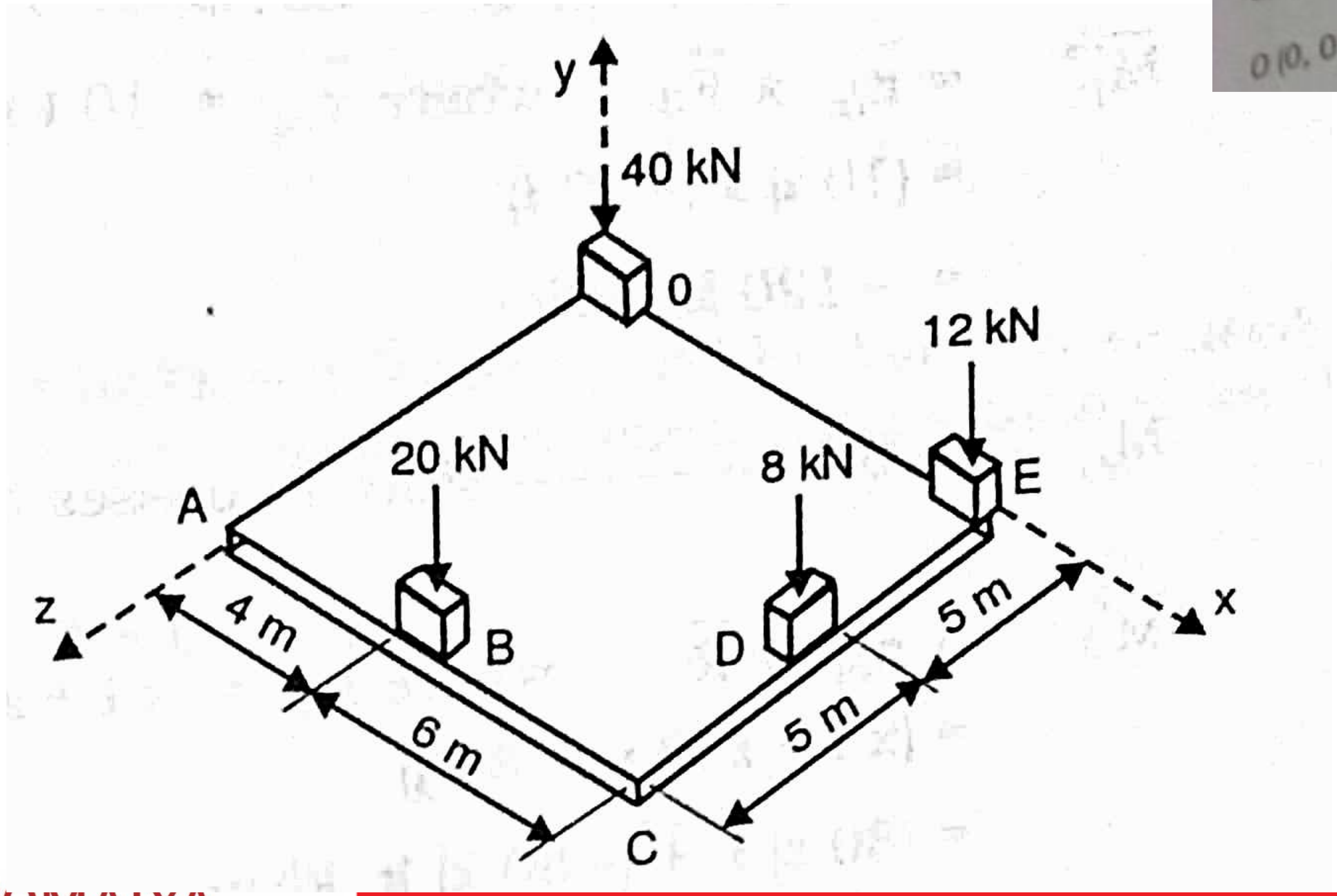
$$-0.8\,F_1 - 0.923\,F_2 - 0.923\,F_3 = -788 \quad \ldots\ldots\ldots\ldots (2)$$

$$0.385\,F_2 - 0.308\,F_3 = 0 \quad \ldots\ldots\ldots\ldots (3)$$

Solving equations (1), (2) and (3) we get,

$F_1 = 154$ N, $F_2 = 320$ N, $F_3 = 400$ N

A square foundation mat supports the four columns as shown in figure. Determine the magnitude and point of application of the resultant of the four loads.




O (0, 0, 0), B (4, 0, 10), D (10, 0, 5), E (10, 0, 0)

Putting the forces in vector form

Let $F_1 = 20$ kN

∴ $\bar{F}_1 = -20\,\mathbf{j}$ kN sinc

Similarly

Let $F_2 = 8$ kN

∴ $\bar{F}_2 = -8\,\mathbf{j}$ kN

Let $F_3 = 12$ kN

∴ $\bar{F}_3 = -12\,\mathbf{j}$ kN

Let $F_4 = 40$ kN

∴ $\bar{F}_4 = -40\,\mathbf{j}$ kN

The resultant $\overline{R} = \overline{F}_1 + \overline{F}_2 + \overline{F}_3 + \overline{F}_4$

$\overline{R} = (-20\,\mathbf{j}) + (-8\,\mathbf{j}) + (-12\,\mathbf{j}) + (-40\,\mathbf{j})$

$\therefore \quad \overline{R} = -80\,\mathbf{j}$ kN

Point of application of the resultant:

Let the resultant act at a point P (x, 0, z) in the plane of the foundation mat. To use Varignon's theorem, we need to find the moments of all the forces and also of the resultant about point O.

$\overline{M}_O^{F_1} = \overline{r}_{OB} \times \overline{F}_1$ where $\overline{r}_{OB} = 4\,\mathbf{i} + 10\,\mathbf{k}$ m

$= (4\,\mathbf{i} + 10\,\mathbf{k}) \times (-20\,\mathbf{j})$

$= 200\,\mathbf{i} - 80\,\mathbf{k}$ kNm

$\overline{M}_O^{F_2} = \overline{r}_{OD} \times \overline{F}_2$ where $\overline{r}_{OD} = 10\,\mathbf{i} + 5\,\mathbf{k}$

$= (10\,\mathbf{i} + 5\,\mathbf{k}) \times (-8\,\mathbf{j})$

$= 40\,\mathbf{i} - 80\,\mathbf{k}$ kNm

$\overline{M}_O^{F_3} = \overline{r}_{OE} \times \overline{F}_3$ where $\overline{r}_{OE} = 10\,\mathbf{i}$ m

$= (10\,\mathbf{i}) \times (-12\,\mathbf{j})$

$= -120\,\mathbf{k}$ kNm

$\overline{M}_O^{F_4} = 0$ ---------- since $F_4$ passes through O

$\overline{M}_O^{R} = \overline{r}_{OP} \times \overline{R}$ where $\overline{r}_{OP} = x\,\mathbf{i} + z\,\mathbf{k}$

$= (x\,\mathbf{i} + z\,\mathbf{k}) \times (-80\,\mathbf{j})$

$= (80\,z)\,\mathbf{i} + (-80\,x)\,\mathbf{k}$ kNm

Using Varignon's theorem

$$\Sigma\,\overline{M}_O^{F} = \Sigma\,\overline{M}_O^{R}$$

$$\overline{M}_O^{F_1} + \overline{M}_O^{F_2} + \overline{M}_O^{F_3} + \overline{M}_O^{F_4} = \overline{M}_O^{R}$$

$$(200\,\mathbf{i} - 80\,\mathbf{k}) + (40\,\mathbf{i} - 80\,\mathbf{k}) + (-120\,\mathbf{k}) = (80\,z)\,\mathbf{i} + (-80x)\,\mathbf{k}$$

$$\therefore \quad 240\,\mathbf{i} - 280\,\mathbf{k} = (80\,z)\,\mathbf{i} + (-80\,x)\,\mathbf{k}$$

equating the coefficients

$240 = 80\,z$

$z = 3$ m

$-280 = -80\,x$

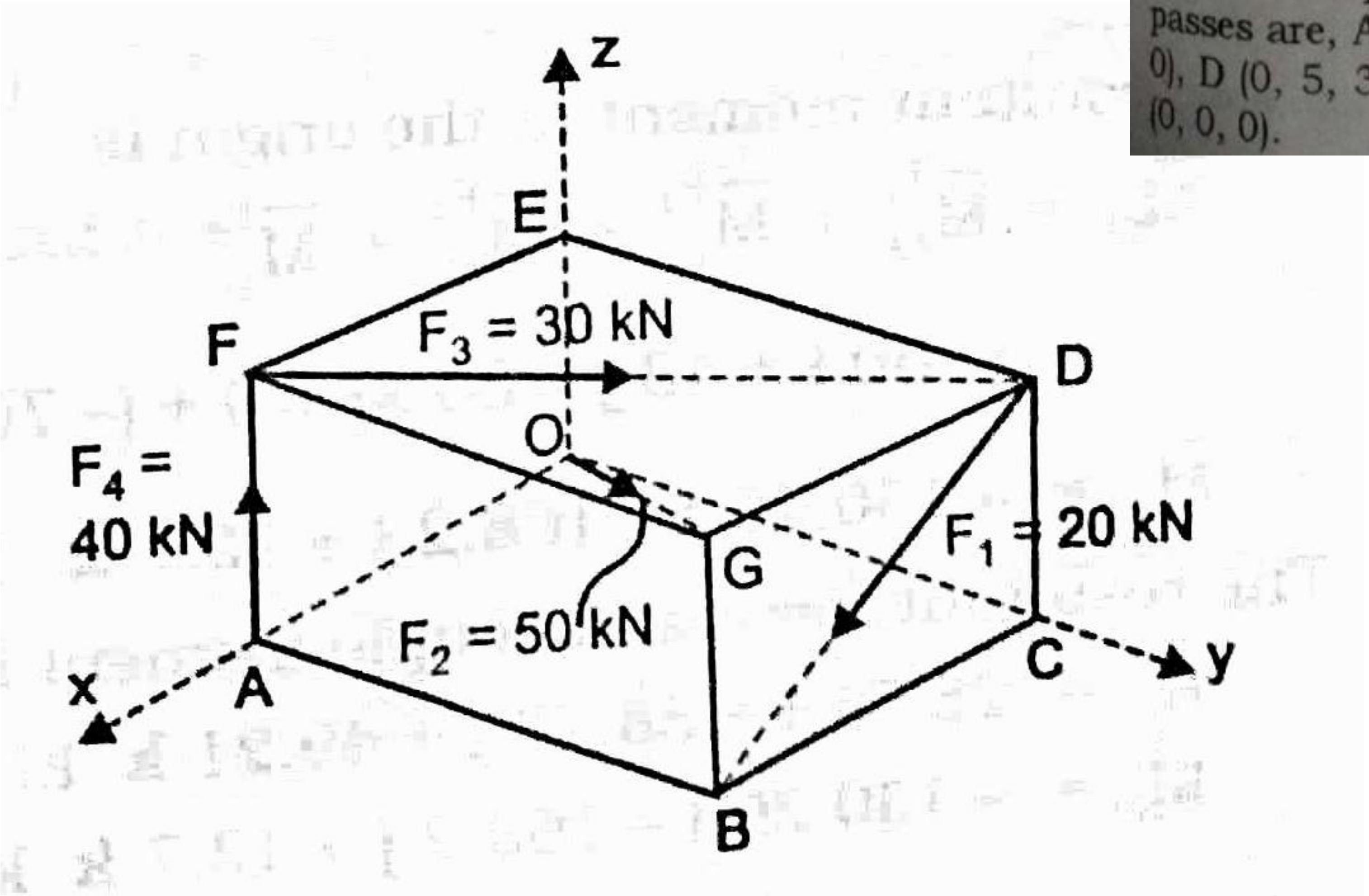
$x = 3.5$ m

$\therefore$ The resultant $\overline{R} = -80\,\mathbf{j}$ kN passes through point P (3.5, 0, 3) m

Determine the resultant force and couple moment about the origin of the force system shown in figure. L(OA) = 4 m, L(OC) = 5 m, L(OE) = 3 m

various points through

passes are, A (4, 0, 0), B (4, 5, 0), C (0, 5, 0), D (0, 5, 3), F (4, 0, 3), G (4, 5, 3) and O (0, 0, 0).



Putting the forces in vector form

$$\bar{F}_1 = F_1.\hat{e}_{DB}$$

$$= 20\left(\frac{4\mathbf{i} - 3\mathbf{k}}{\sqrt{4^2 + 3^2}}\right)$$

$$= 16\,\mathbf{i} - 12\,\mathbf{k}\ \text{kN}$$

$$\bar{F}_2 = F_2.\hat{e}_{OG}$$

$$= 50\left(\frac{4\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}}{\sqrt{4^2 + 5^2 + 3^2}}\right)$$

$$= 28.28\,\mathbf{i} + 35.35\,\mathbf{j} + 21.21\,\mathbf{k}\ \text{kN}$$

$$\bar{F}_3 = F_3.\hat{e}_{FD}$$

$$= 30\left(\frac{-4\mathbf{i} + 5\mathbf{j}}{\sqrt{4^2 + 5^2}}\right) \quad \therefore \quad \bar{F}_3 = -18.74\,\mathbf{i} + 23.42\,\mathbf{j}\ \ \text{kN}$$

$$\bar{F}_4 = 40\,\mathbf{k}\ \ \text{kN}$$

The resultant force $\bar{R} = \bar{F}_1 + \bar{F}_2 + F_3 + F_4$

$$\bar{R} = (16\,\mathbf{i} - 12\,\mathbf{k}) + (28.28\,\mathbf{i} + 35.35\,\mathbf{j} + 21.21\,\mathbf{k}) + (-18.74\,\mathbf{i} + 23.42\,\mathbf{j}) + (40\,\mathbf{k})$$

$$\therefore\ \bar{R} = 25.54\,\mathbf{i} + 58.77\,\mathbf{j} + 49.21\,\mathbf{k}\ \ \text{kN}$$

Taking moment of all forces about the specified point, which is the origin.

$$\bar{M}_O^{F_1} = \bar{r}_{OD} \times \bar{F}_1 \qquad \text{where } \bar{r}_{OD} = 5\,\mathbf{j} + 3\,\mathbf{k}\ \text{m}$$

$$= (5\,\mathbf{j} + 3\,\mathbf{k}) \times (16\,\mathbf{i} - 12\,\mathbf{k})$$

$$= -60\,\mathbf{i} + 48\,\mathbf{j} - 80\,\mathbf{k}\ \ \text{kNm}$$

$$\bar{M}_O^{F_2} = 0 \qquad \text{since } F_2 \text{ passes through O}$$

$$\bar{M}_O^{F_3} = \bar{r}_{OD} \times \bar{F}_3 \qquad \text{where } \bar{r}_{OD} = 5\,\mathbf{j} + 3\,\mathbf{k}\ \text{m}$$

$$= (5\,\mathbf{j} + 3\,\mathbf{k}) \times (-18.74\,\mathbf{i} + 23.42\,\mathbf{j})$$

$$= -70.26\,\mathbf{i} - 56.22\,\mathbf{j} + 93.7\,\mathbf{k}\ \ \text{kNm}$$

$$\bar{M}_O^{F_4} = \bar{r}_{OA} \times \bar{F}_4 \qquad \text{where } \bar{r}_{OA} = 4\,\mathbf{i}\ \text{m}$$

$$= (4\,\mathbf{i}) \times (40\,\mathbf{k})$$

$$= -160\,\mathbf{j}\ \text{kNm}$$

The resultant moment at the origin is

$$\overline{M}_O = \overline{M}_O^{F_1} + \overline{M}_O^{F_2} + \overline{M}_O^{F_3} + \overline{M}_O^{F_4}$$

$$= (-60\,\mathbf{i} + 48\,\mathbf{j} - 80\,\mathbf{k}) + 0 + (-70.26\,\mathbf{i} - 56.22\,\mathbf{j} + 93.7\,\mathbf{k}) + (-160\,\mathbf{j})$$

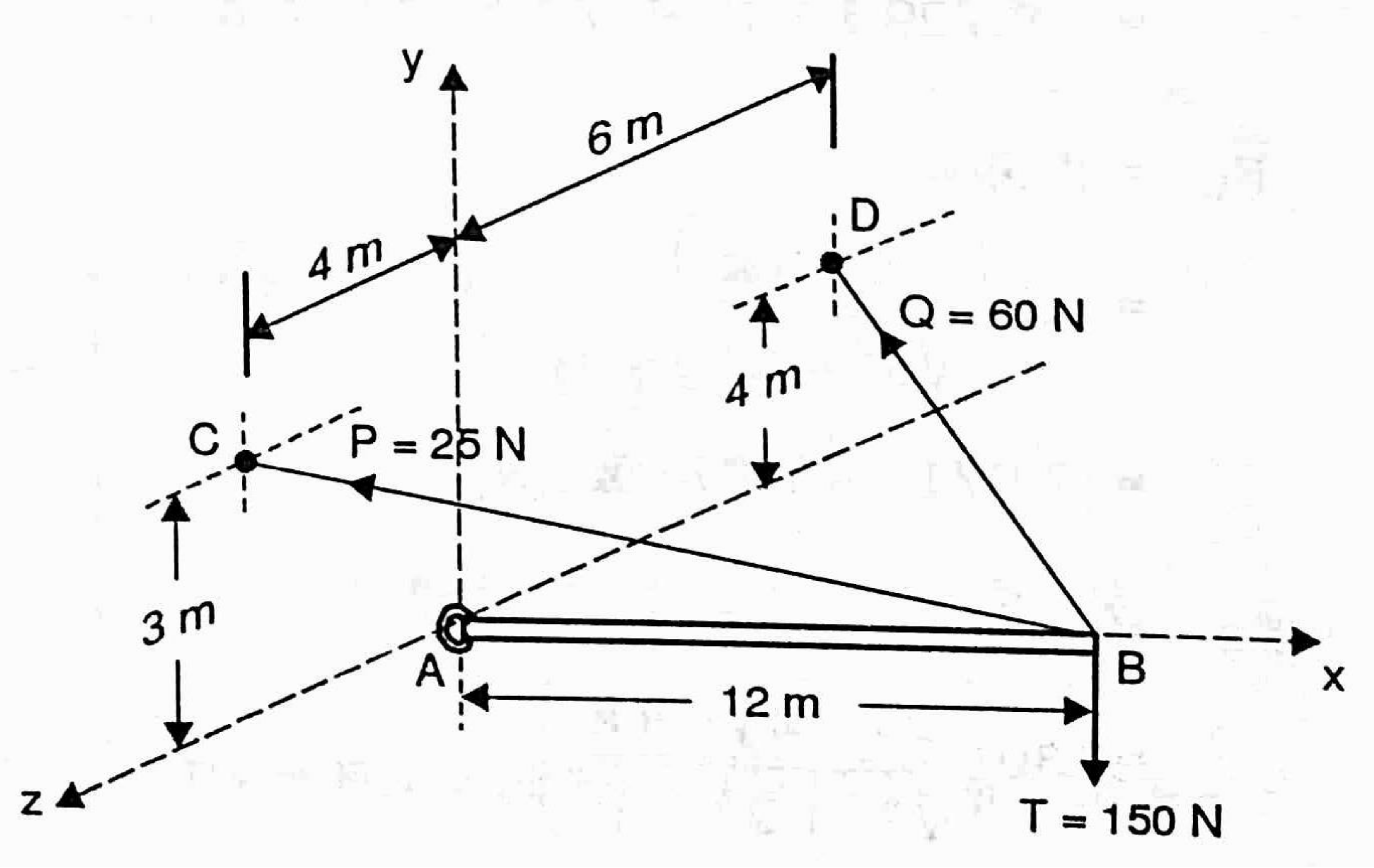
$$\therefore\ \overline{M}_O = -130.26\,\mathbf{i} - 168.2\,\mathbf{j} + 13.7\,\mathbf{k}\ \text{kNm}$$

The resultant force and couple moment at the origin is

$$\overline{R} = 25.54\,\mathbf{i} + 58.77\,\mathbf{j} + 49.21\,\mathbf{k}\ \text{kN}$$

$$\overline{M}_O = -130.26\,\mathbf{i} - 168.2\,\mathbf{j} + 13.7\,\mathbf{k}\ \text{kNm}$$

Three forces P, Q and T acts at point B. Find the resultant of these forces.



The lines of actions of three forces concurrent at origin O pass respectively through point A(-1,2,4), B(3,0,-3), C(2,-2,4). Force F1 = 40 N passes through A, F2 = 10 N passes through B, F3 = 30 N passes through C. Find the magnitude and direction of their resultant.