Engineering Mechanics

Module 1.1 – Resultant of forces

Rajesh Pansare

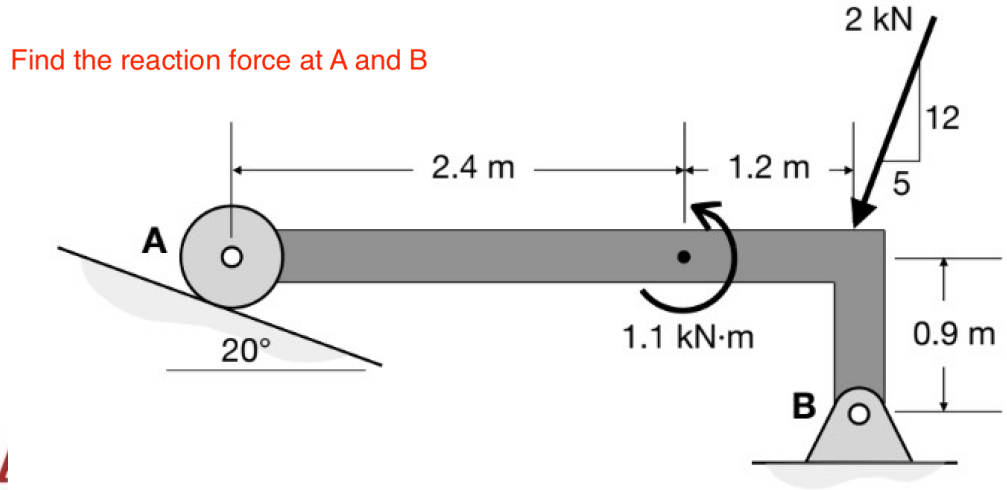
Contents

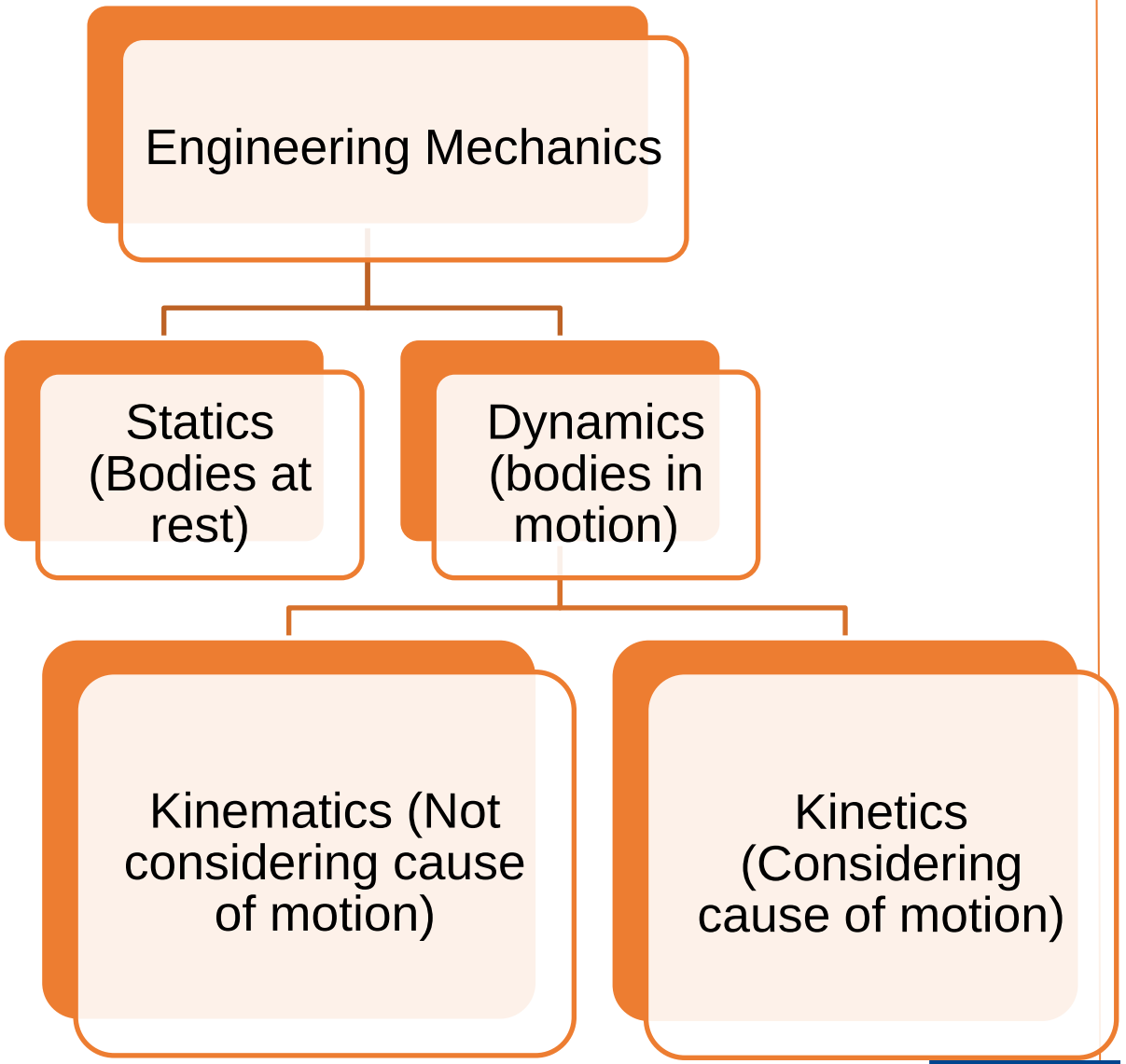
System of forces	
1.1	System of coplanar forces: Resultant of concurrent forces, parallel forces, non-concurrent non parallel system of forces, moment of force about a point, couples, Varignon's theorem, Principle of transmissibility of forces

Engineering Mechanics

What is mechanics?

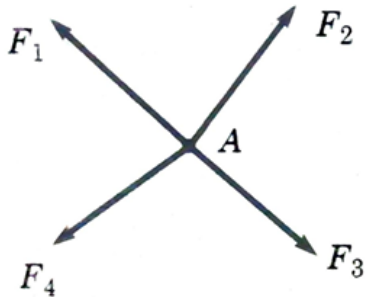
It is the branch of engineering science which deals with effect and analysis of forces acting on the body, which may be at rest or in the motion

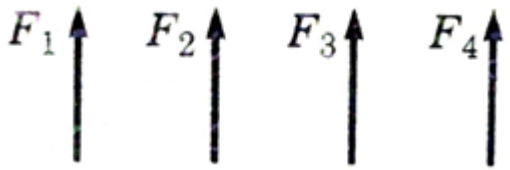


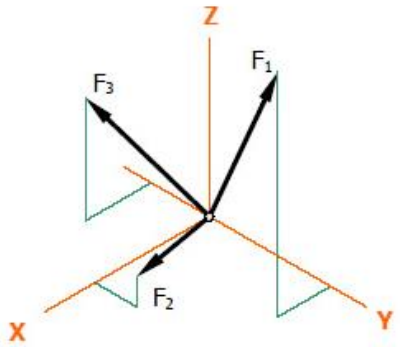


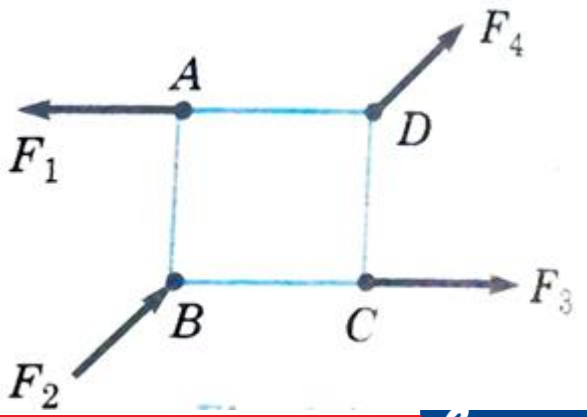
Types of different force system (F.S.) acting on a body

- Force systems
  - Co-planer force system
    - Concurrent F.S.
    - Non-concurrent F.S.
  - Non-co-planer force system
    - Concurrent F.S.
    - Non-concurrent F.S.









Resultant of a force system and its importance

Basic terminologies: Force and moment

Force: It is an external agency acting on the body which will cause the motion of the body from one location to another

Moment: The turning tendency of the body due to application of a force on it about a particular point is called moment. Moment magnitude is given by force * perpendicular distance

What is resultant?

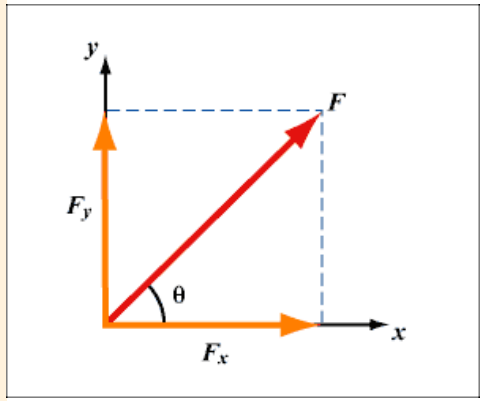
It is a single equivalent force acting on a body producing the same effect as that of multiple forces producing on it.

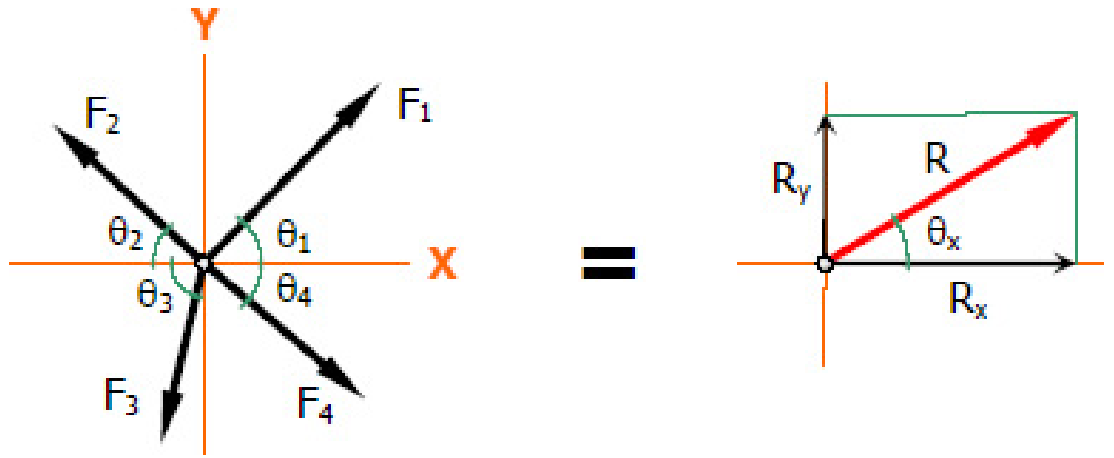
$$R_x = \Sigma F_x$$

$$R_y = \Sigma F_y$$

$$R = \sqrt{{R_x}^2 + {R_y}^2}$$

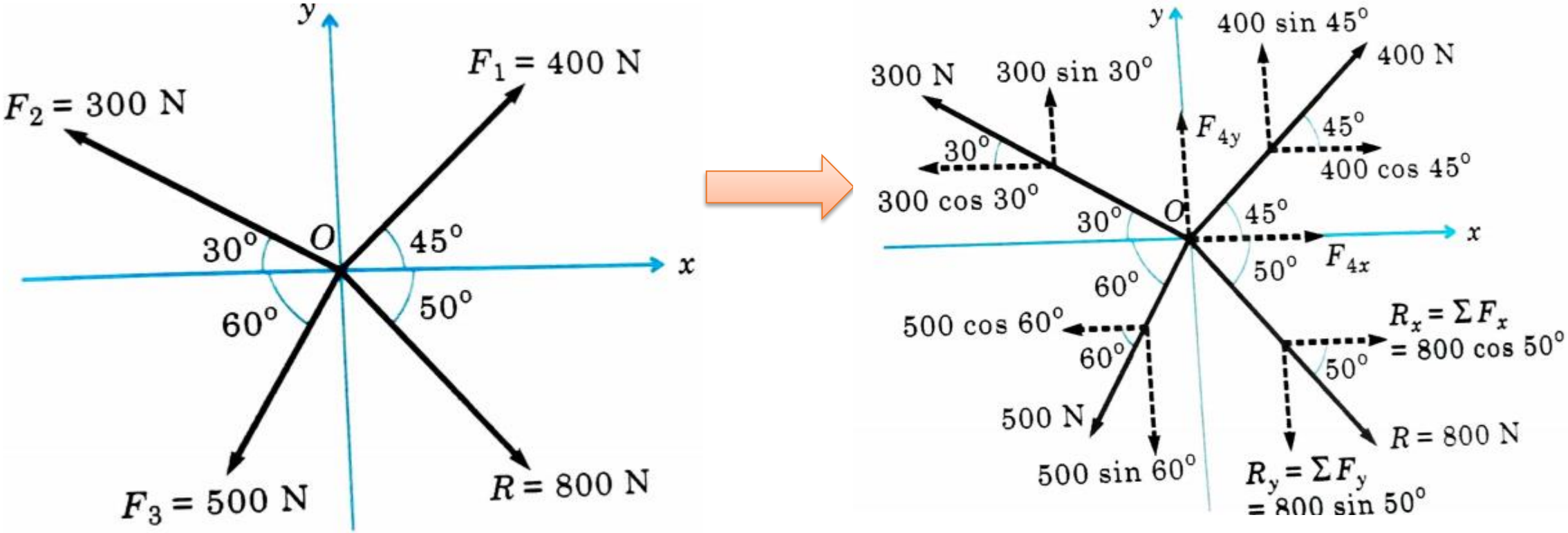
$$\tan\theta_x = \frac{R_y}{R_x}$$



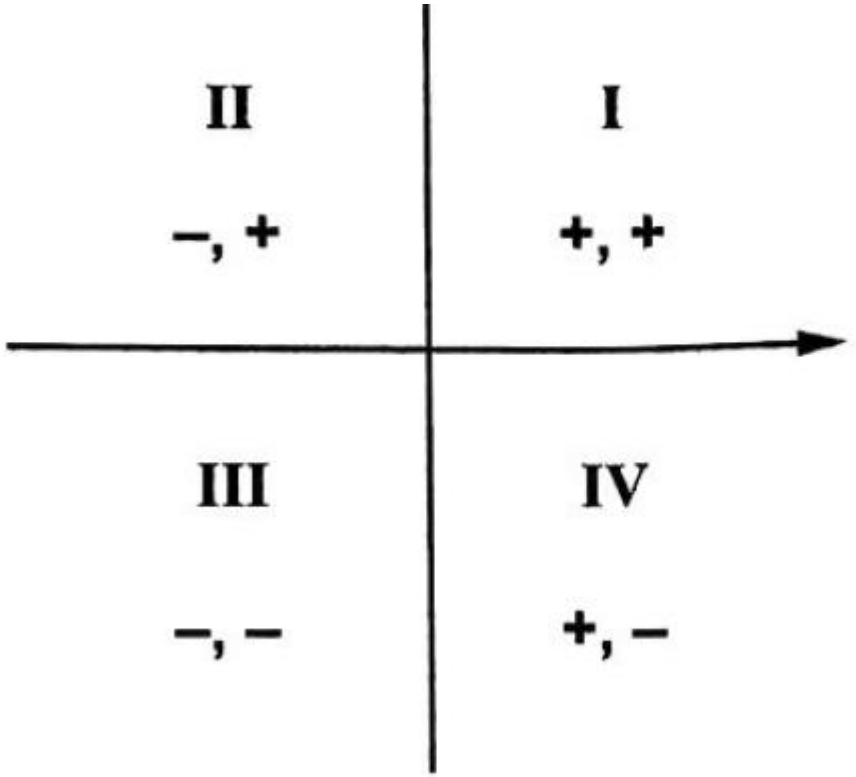


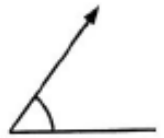
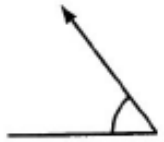
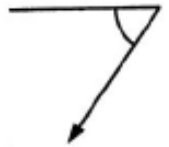
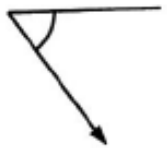
Resolution of force

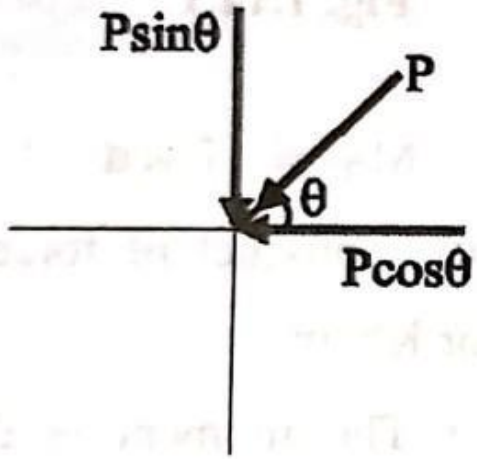
Resolution of any particular force is nothing but representing a single force into its equivalent rectangular components



Sign convention and symbolic representation of Resultant

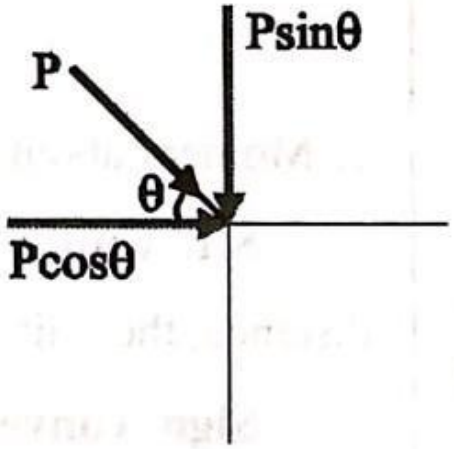


Quadrant	Symbol
I	
II	
III	
IV	



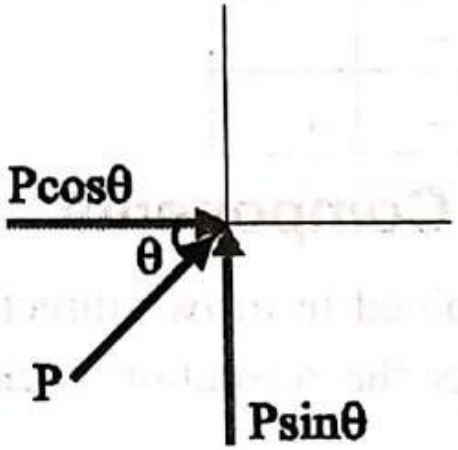
$P_x = -P\cos\theta$

$P_y = -P\sin\theta$



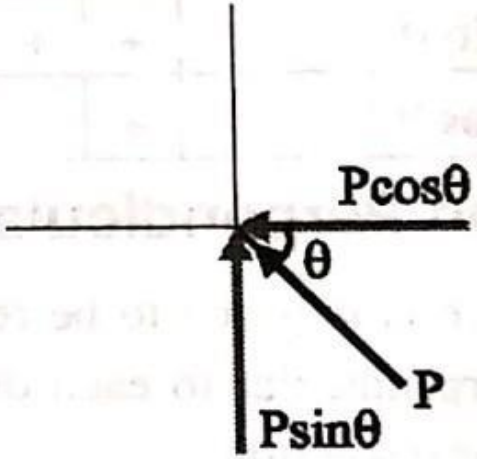
$P_x = P\cos\theta$

$P_y = -P\sin\theta$



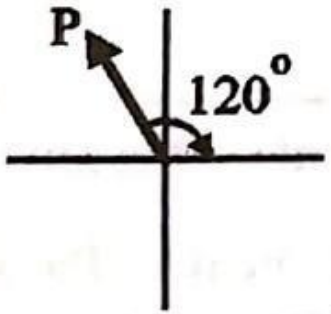
$P_x = P\cos\theta$

$P_y = P\sin\theta$



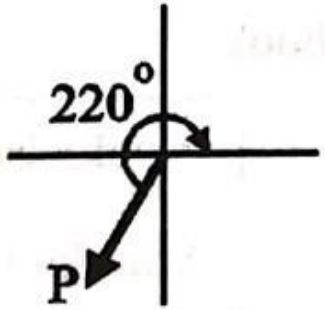
$P_x = -P\cos\theta$

$P_y = +P\sin\theta$



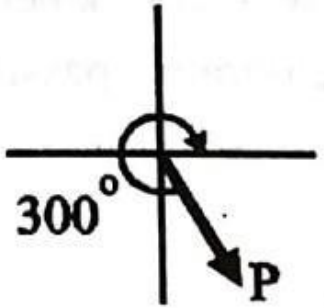
$P_x = +P\cos 120^\circ$

$P_y = +P\sin 120^\circ$



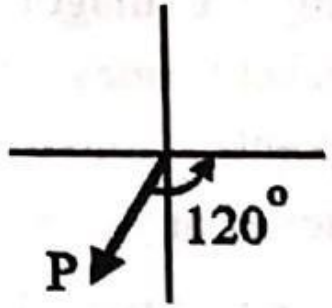
$P_x = +P\cos 220^\circ$

$P_y = +P\sin 220^\circ$



$P_x = +P\cos 300^\circ$

$P_y = +P\sin 300^\circ$

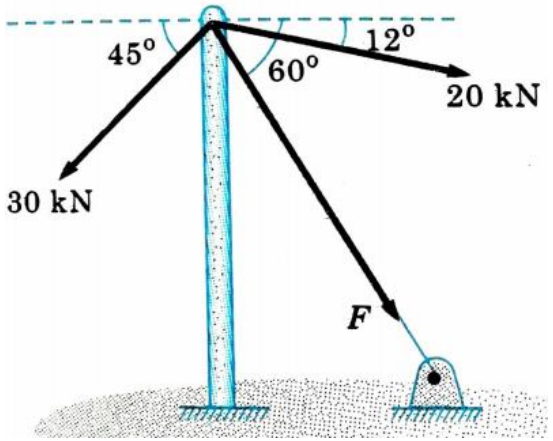


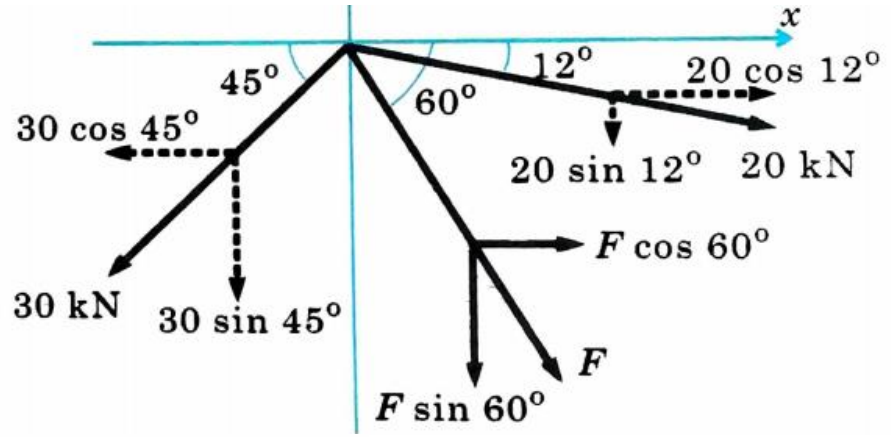
$P_x = +P\cos(-120^\circ)$

$P_y = +P\sin(-120^\circ)$

Resultant of coplanar concurrent force system

Problem 1 : For the force system shown in fig. determine the value of force F so that the resultant of the system is vertical





$$\Sigma F_x = 20 \cos 12^\circ + F \cos 60^\circ - 30 \cos 45^\circ = 0$$

$$\therefore F = 3.3 \text{ kN}$$

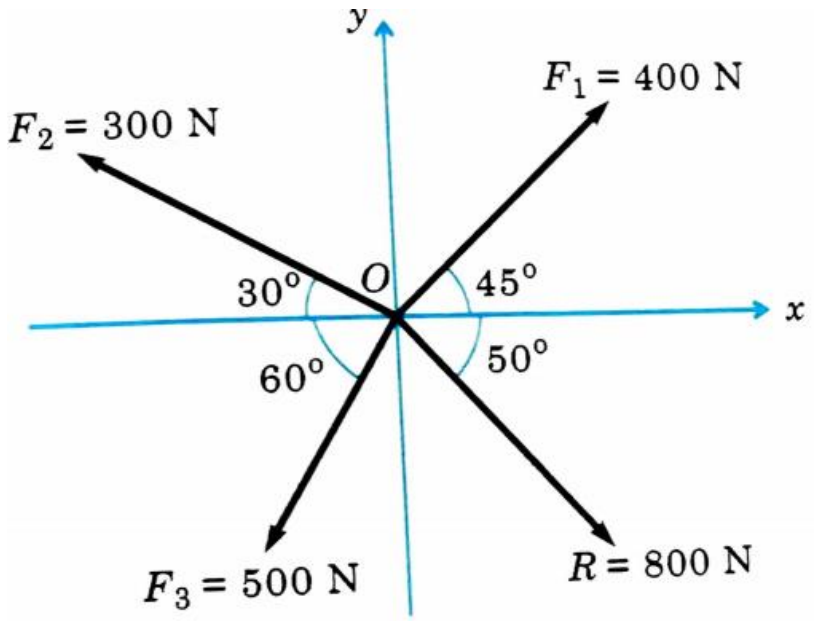
$$R = \Sigma F_y = -30 \sin 45^\circ - F \sin 60^\circ - 20 \sin 12^\circ$$

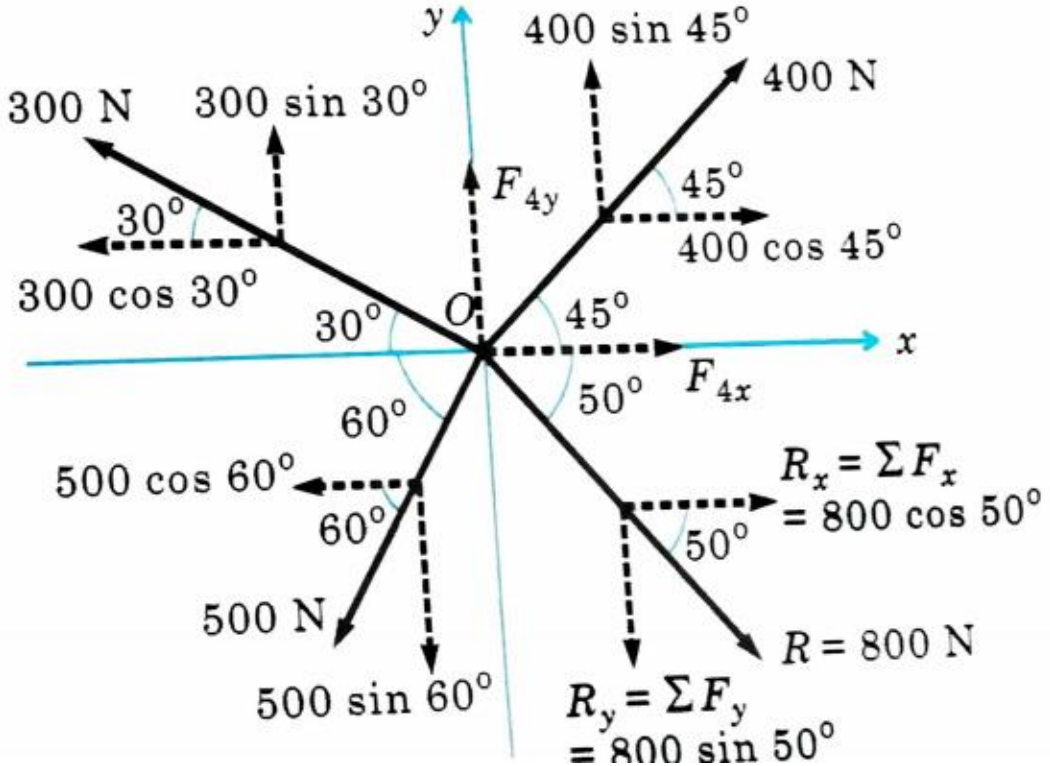
$$= -30 \sin 45^\circ - 3.3 \sin 60^\circ - 20 \sin 12^\circ$$

$$= -28.23 \text{ kN} = 28.23 \text{ kN} (\downarrow)$$

Resultant of coplanar concurrent force system

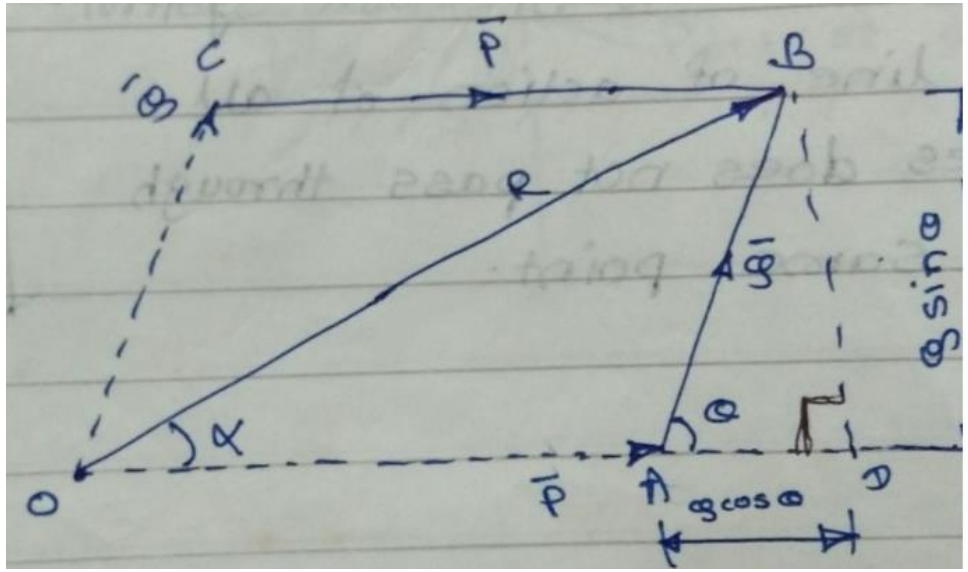
Practise problem : Find the force F4 completely so that the resultant of the force system is as shown in fig.





LAW OF PARALLELOGRAM OF FORCES

- If two forces, acting at a point, are represented in magnitude and direction by the two sides of a parallelogram drawn from one of its angular points, their resultant is represented both in magnitude and direction by the diagonal of the parallelogram passing through that angular point.



Let OA and OC represent the forces P and Q acting at a point O and inclined to each other at an angle α then the resultant R and direction ‘α’ (shown in figure) will be given by

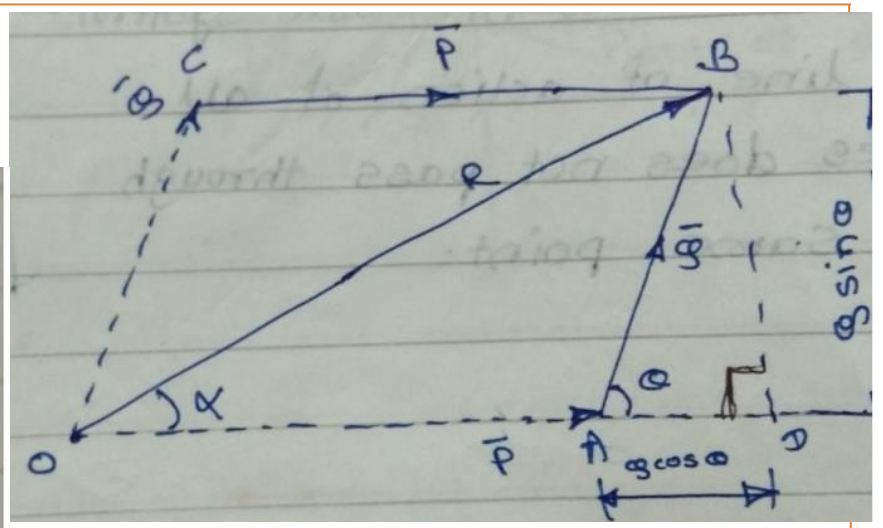
$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

and $$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

Case (i): If P = Q, then $\tan\theta = \tan(\alpha/2) \Rightarrow \theta = \alpha/2$

Case (ii): If the forces act at right angles, so that $\alpha = 90°$, we have $R = \sqrt{P^2 + Q^2}$ and $\tan\theta = \frac{Q}{P}$

Proof



Let, $OC = AB = Q$; $OA = P$; $OB = R$.

In ΔODB,

$$OB^2 = OD^2 + BD^2$$

$$OB^2 = (OA + AD)^2 + BD^2$$

$$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$$

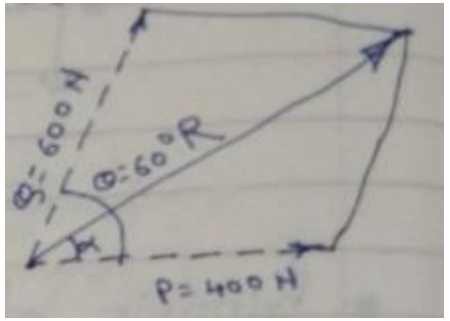
$$= P^2 + 2PQ\cos\theta + Q^2\cos^2\theta + Q^2\sin^2\theta$$

$$\boxed{R^2 = P^2 + Q^2 + 2PQ\cos\theta}$$

Also, $\tan\alpha = \frac{BD}{OD} = \frac{BD}{OA + AD}$

$$\boxed{\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}}$$

- Two forces of 400N and 600N act at an angle 60º to each other. Determine the resultant in magnitude and direction if the forces have same sense.



Method (I) Parallellogram law

Let both the forces are Pull type.

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$R = \sqrt{(400)^2 + (600)^2 + 2\times400\times600\times\cos60}$$

$$\boxed{R = 871.78\ N}$$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta} = \frac{600\sin60}{400 + 600\cos60}$$

$$\therefore \tan\alpha = 0.742$$

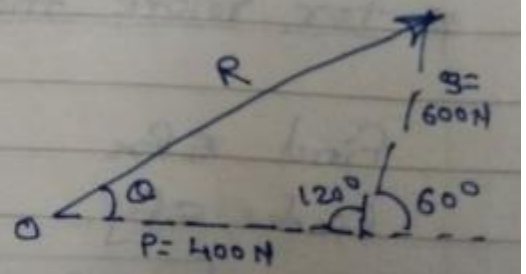
$$\boxed{\alpha = 36.59^\circ}$$

Method (II) Triangle law:

By cosine rule,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

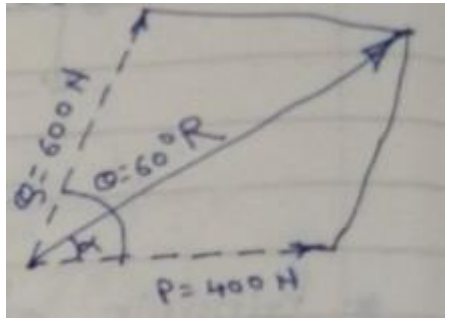
$$\boxed{R = 871.78\ N}$$



By sine rule,

$$\frac{600}{\sin\theta} = \frac{871.78}{\sin 120}$$

$$\boxed{\therefore \theta = 36.59^\circ}$$



Method (III) Resolution of force

$$\Sigma F_x = 400 + 600\cos 60 = 700\ N\ (\rightarrow)$$

$$\Sigma F_y = 0 + 600\sin 60 = 519.62\ N\ (\uparrow)$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{700^2 + 519.62} = \underline{871.78\ N}$$

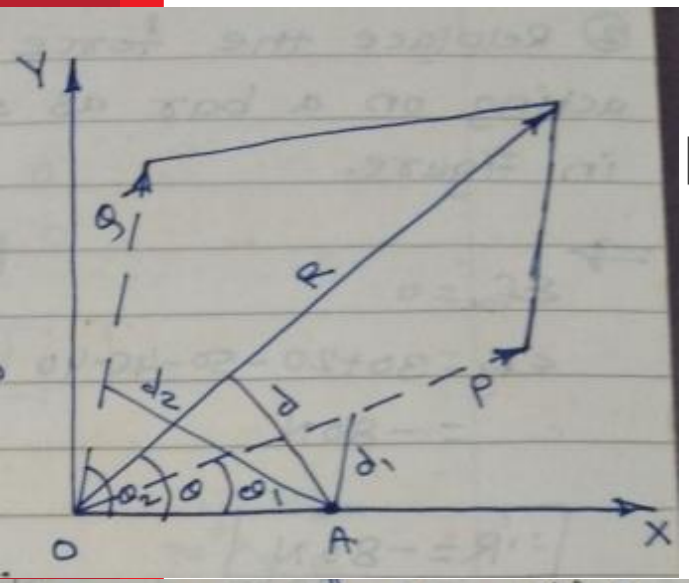
$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x} = \frac{519.62}{700}$$

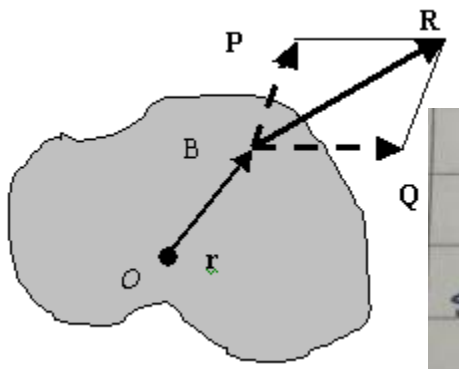
$$\boxed{\therefore \theta = 36.59^\circ}$$

Varignon's theorem

Statement: Moment of a resultant about any point is equal to the sum of the moments of the all forces about the same point.

Proof





$M_0 = r \times R = r \times P + r \times Q$

Let R is force acting at point O.
It is resolved in two components P & Q
let θ_1, θ_2 & θ are angles of P, Q & R resp.

∴ Moment of R about any point A is R×d.
similarly, Sum of moments of P & Q about same point A, is $P \times d_1 + Q \times d_2$
sum of y components of P & Q is $P_y + Q_y$

$\therefore R_y = P_y + Q_y$

also, $R_y = R\sin\theta$, $P_y = P\sin\theta_1$, $Q_y = Q\sin\theta_2$

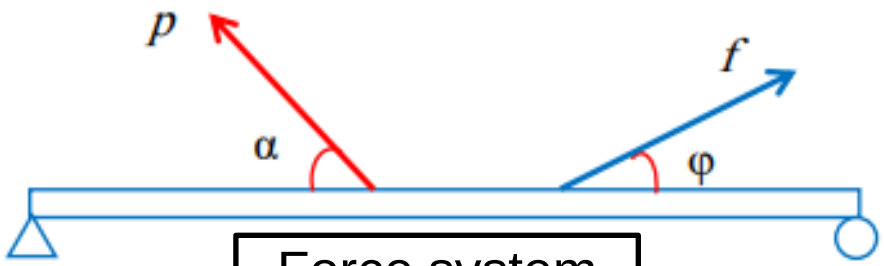
$\therefore R\sin\theta = P\sin\theta_1 + Q\sin\theta_2$

multiply both sidy by length OA,

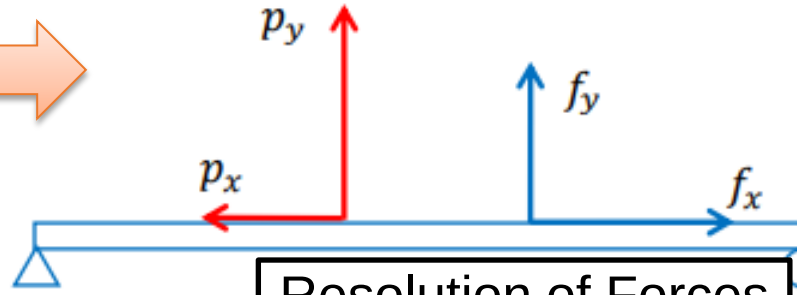
$\therefore R \times OA\sin\theta = P \times OA\sin\theta_1 + Q \times OA\sin\theta_2$

$$\therefore R \times d = P \times d_1 + Q \times d_2$$

Resultant of Non- concurrent coplaner force system



Force system



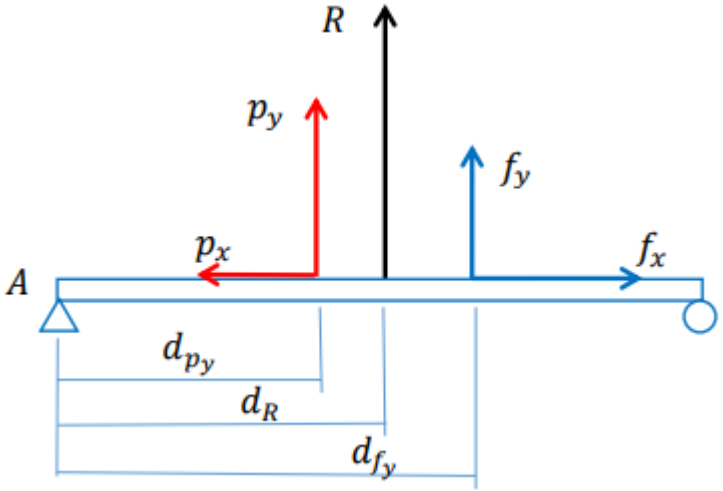
Resolution of Forces

$$f_x = f \times \cos\varphi$$

$$f_y = f \times \sin\varphi$$

$$p_x = p \times \cos\alpha$$

$$p_y = p \times \sin\alpha$$



$$\sum M \text{ about } A =$$

$$R \times d_R = \sum (F_y \times d_f) = p_y \times d_{p_y} + f_y \times d_{f_y}$$

Location of the resultant

$$R_x = \Sigma F_x$$

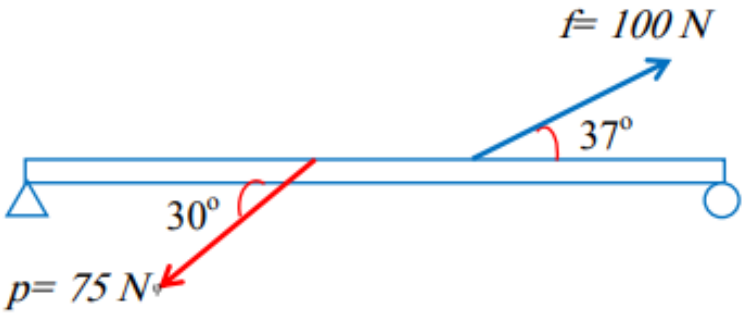
$$R_y = \Sigma F_y$$

$$R = \sqrt{{R_x}^2 + {R_y}^2}$$

$$\tan\theta_x = \frac{R_y}{R_x}$$

Calculation of resultant

Problem- Determine the resultant of the force system and also locate the same



$$f_x = 100 \times \cos 37 = 79.86\ N \rightarrow$$

$$f_y = 100 \times \sin 37 = 60.18\ N \uparrow$$

$$p_x = 75 \times \cos 30 = -64.95\ N = 64.95\ N \leftarrow$$

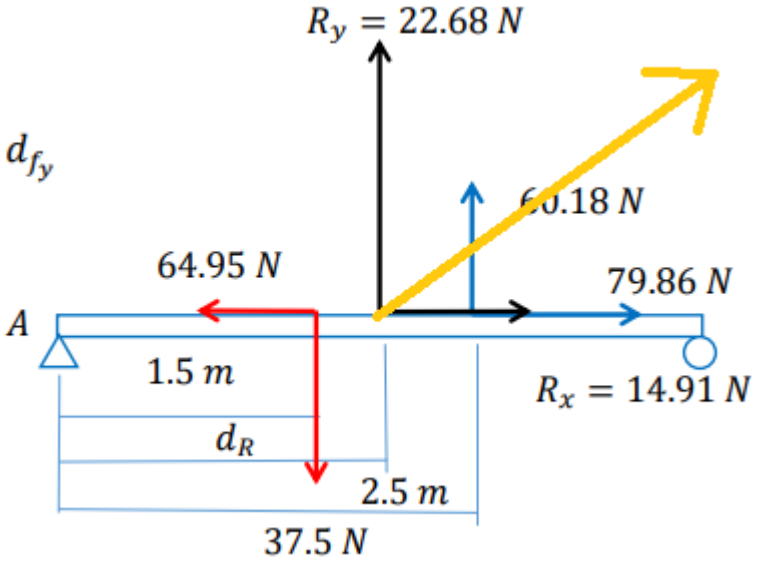
$$p_y = 75 \times \sin 30 = -37.5\ N = 37.5N \downarrow$$

$$\sum M \text{ about } A =$$

$$R_y \times d_R = \sum (F_y \times d_f) = -p_y \times d_{p_y} + f_y \times d_{f_y}$$

$$22.68 \times d_R = -37.5 \times 1.5 + 60.18 \times 2.5$$

$$d_R = 4.15 \text{ m}$$



Location of resultant

Resultant calculation

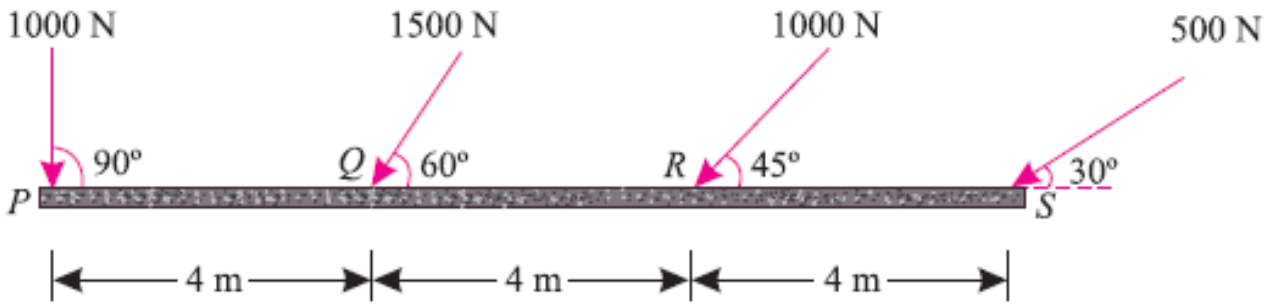
$$R_x = \sum F_x = f_x + p_x = 79.86 - 64.95 = 14.91\ N \rightarrow$$

$$R_y = \sum F_y = f_y + p_y = 60.18 - 37.5 = 22.68\ N \uparrow$$

$$R = \sqrt{14.91^2 + 22.68^2} = 27.14\ N$$

$$Slope = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{22.68}{14.91}\right) = 56.69°$$

Problem – Determine Resultant and x-intercept of the F. S.



$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(1890)^2 + (3256)^2} = 3765 \text{ N}$$

$$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{3256}{1890} = 1.722 \quad \text{or} \quad \theta = 59.8°$$

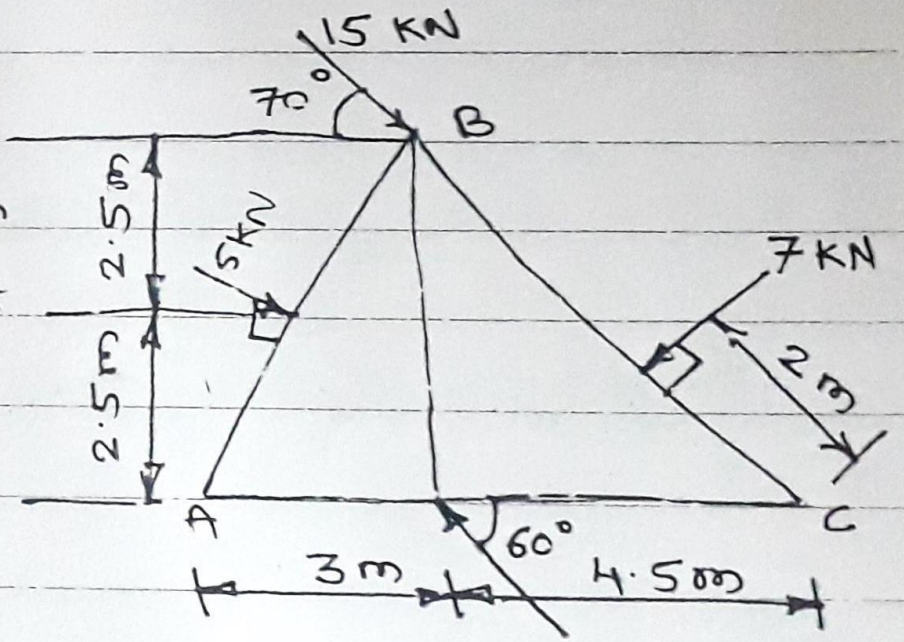
To determine x-intercept, the moments of all the forces are taken about point P

$$3256\,x = (1000 \times 0) + (1500 \times 0.866)\,4 + (1000 \times 0.707)8 + (500 \times 0.5)12$$
$$= 13\,852$$

$$x = \frac{13\,852}{3256} = 4.25 \text{ m}$$

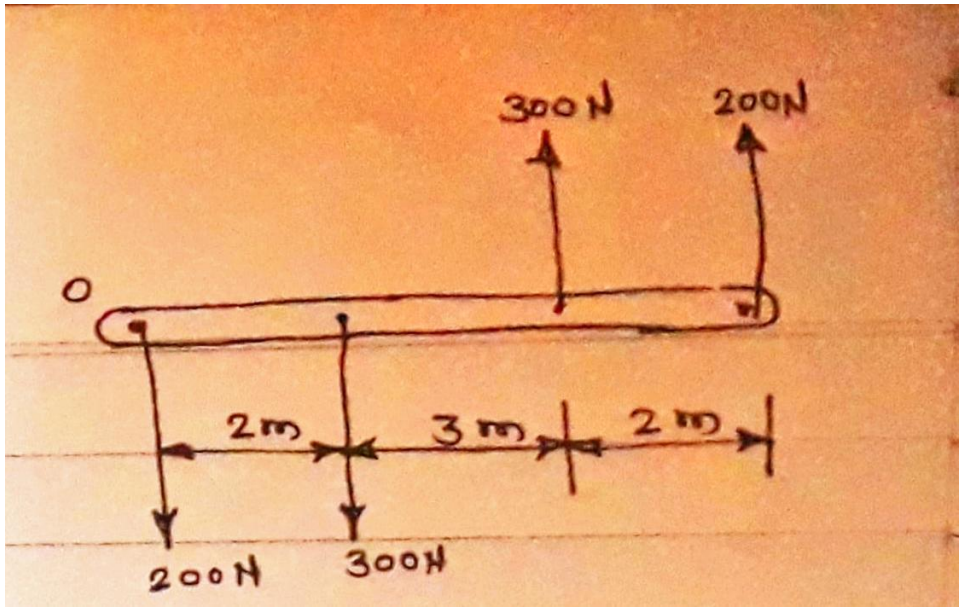
Practice problems

(4) A triangular plate ABC is subjected to four coplanar forces as shown in figure. Find the resultant completely and locate its position with respect to point A.

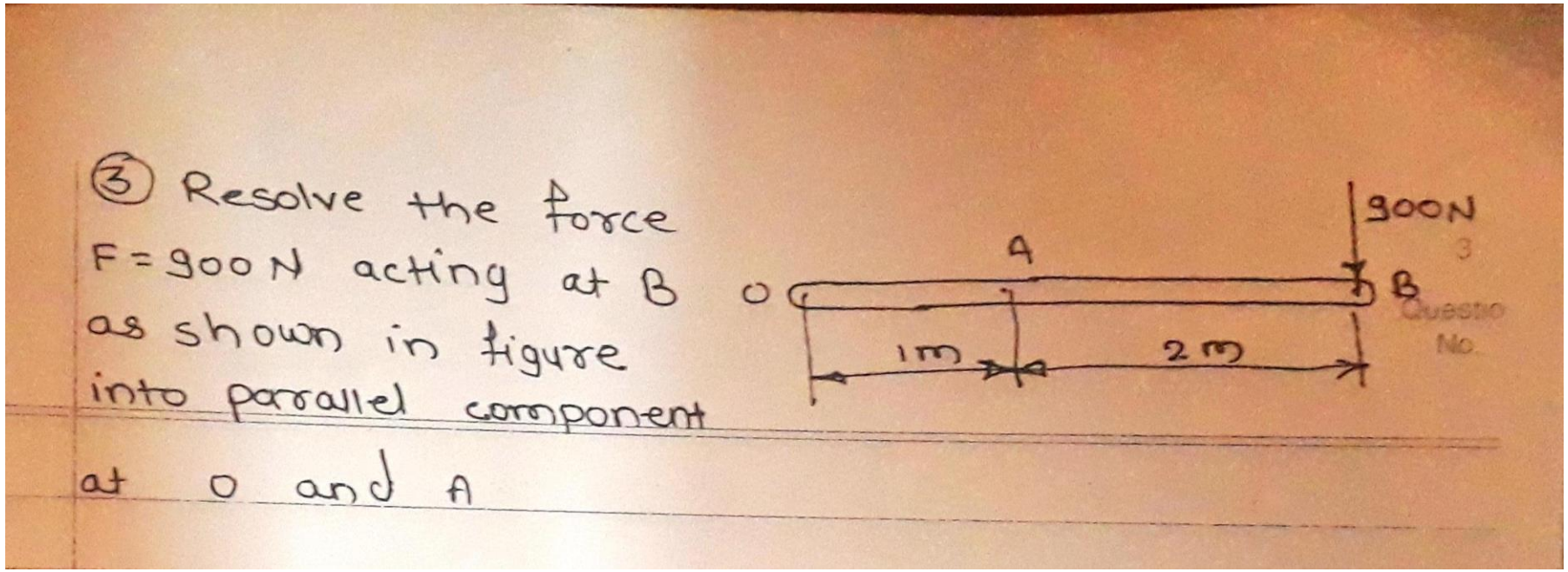


Practice problem

- Find the resultant of the force system



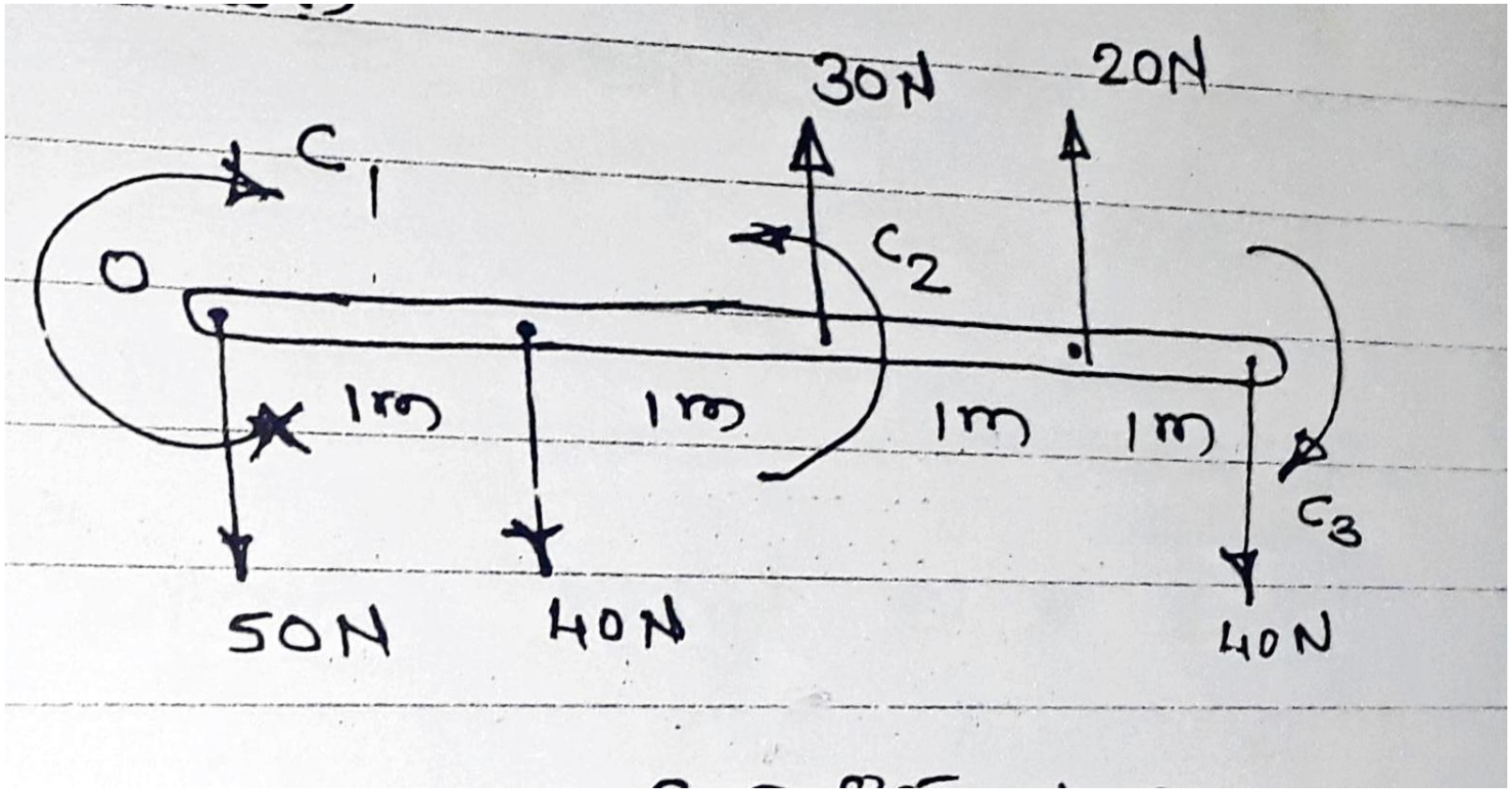
Practice problem



Practice problem

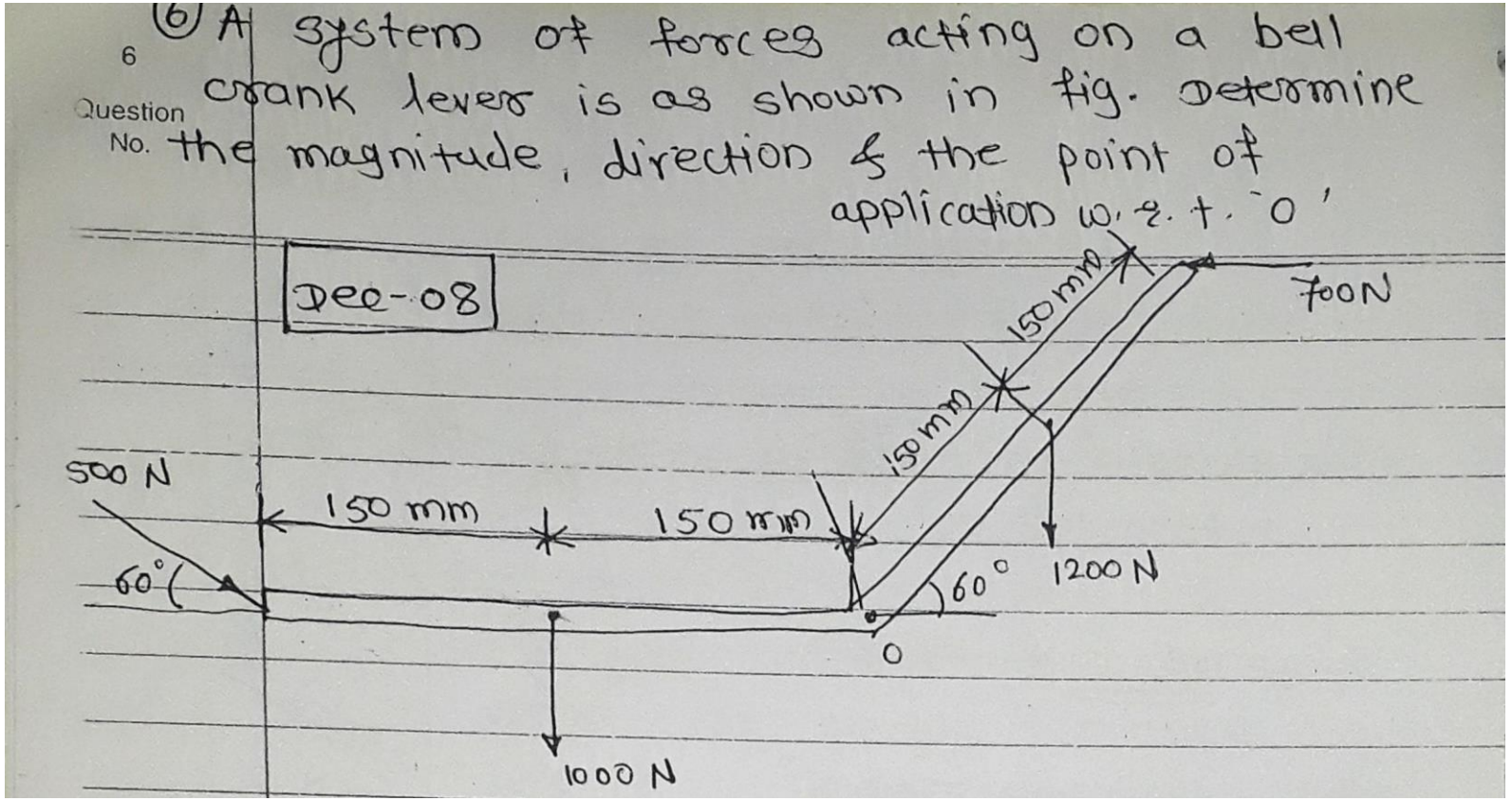
- Replace the force system acting on a bar as shown in figure.

C_1 = 85 N-m
C_2 = 65 N-m
C_3 = N-m

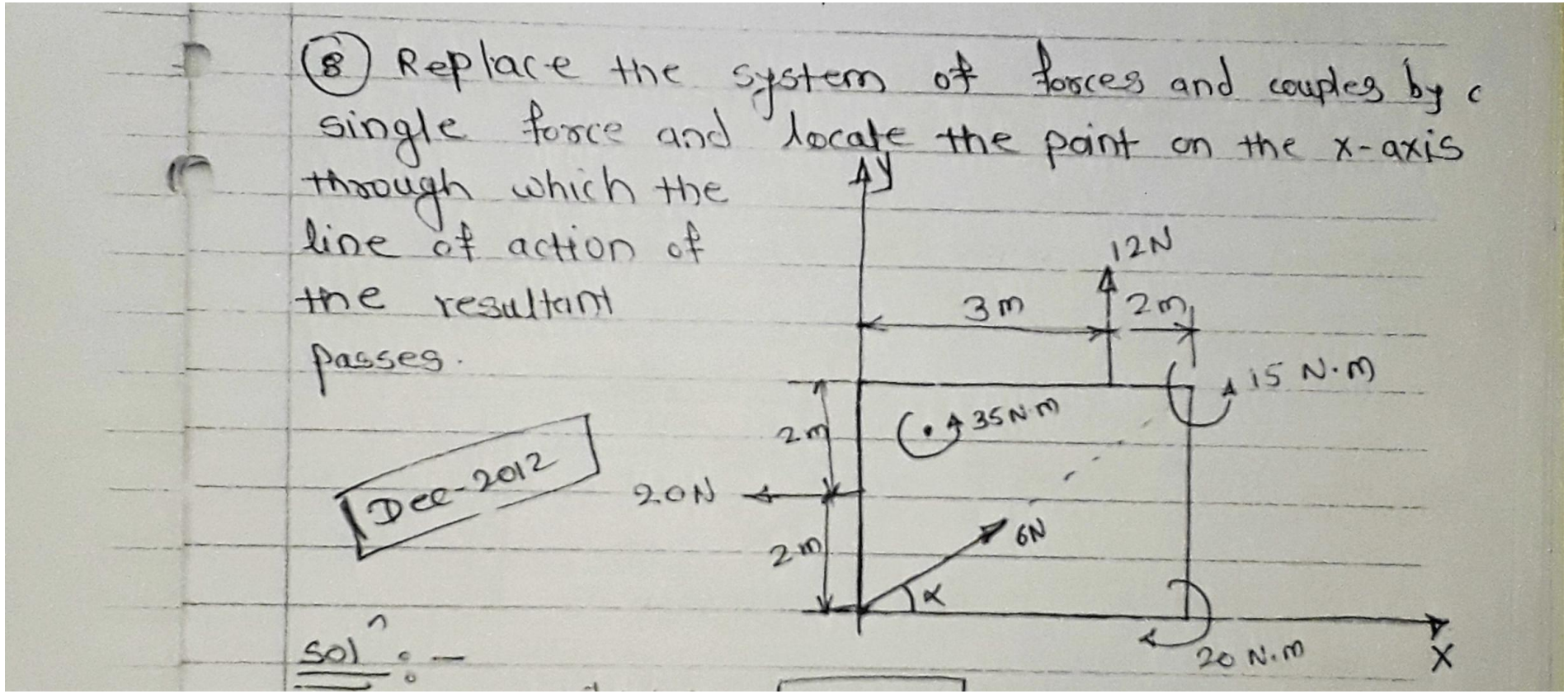


Practice problem

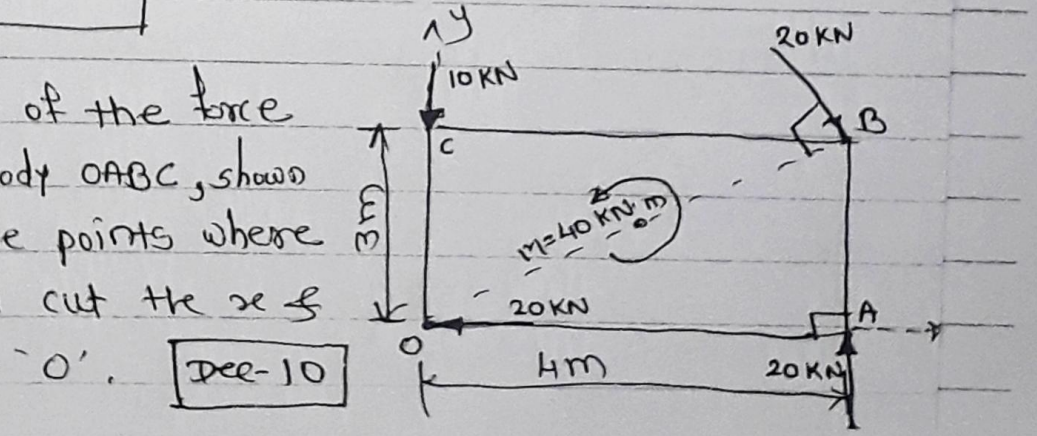
- Solve the given problem



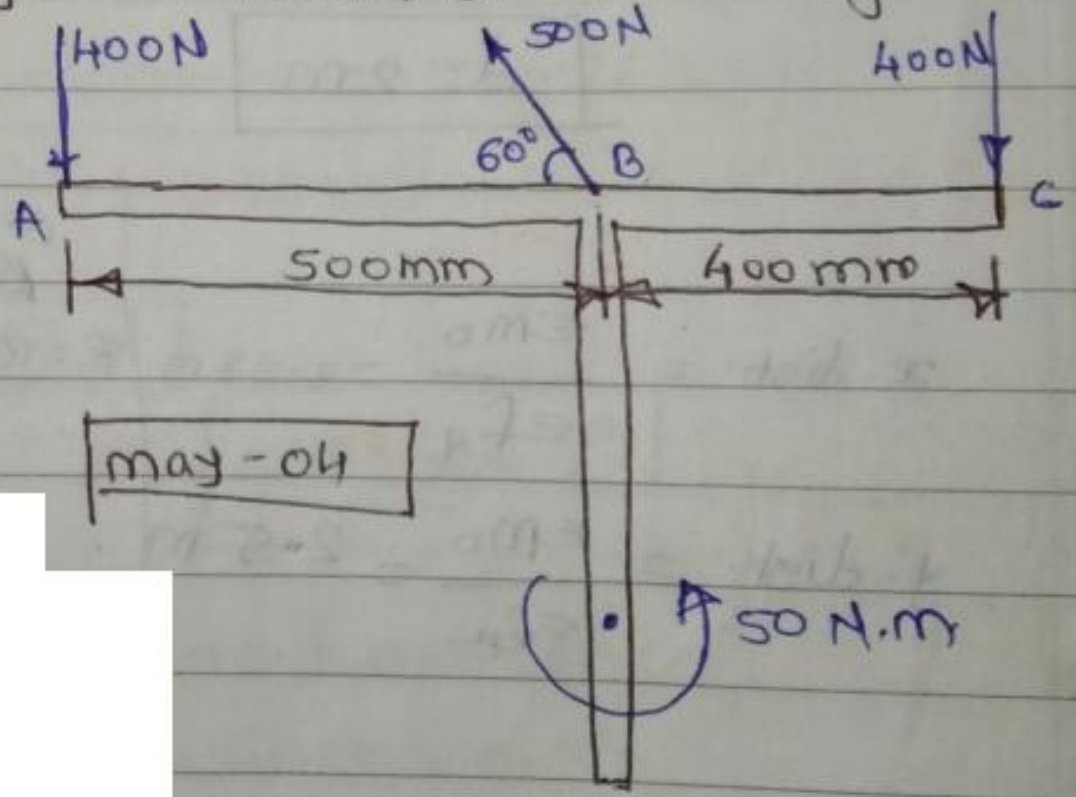
Practice problem



(7) Find the resultant of the force system acting on a body OABC, shown in fig. Also find the points where the resultant will cut the x & y axis from point 'O'. Dec-10



A bracket is subjected to a coplanar force system as shown in fig. Determine the magnitude & line of action from A of the single resultant of the system. If the resultant is to pass through the point B, what should be the magnitude & direction of couple?



may - 04