# Co-ordinate system in Curvilinear Motion

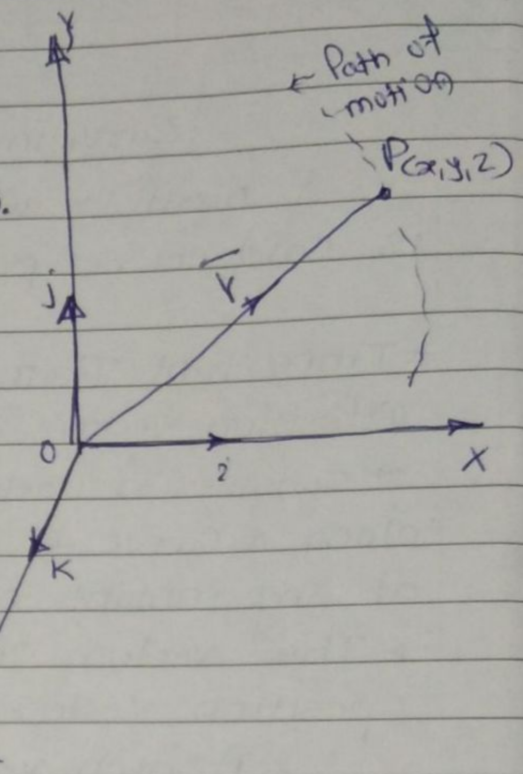
1. Rectangular co-ordinate system
2. Normal & Tangential co-ordinate system
3. Radial & Transverse C.O.S.

## 1. Rectangular co-ordinate system

→ consider a particle moving along a curve
→ At any instant, particle is positioned at $P(x, y, z)$.
→ The position vector will be,

$\bar{r} = xi + yj + zk$ ...for space curve

$\bar{r} = xi + yj$ ...for plane curve.




* Velocity and Acceleration components:

let, $\overline{OP} = \bar{r} = xi + yj$

$$\therefore \bar{v} = \frac{d\bar{r}}{dt} = \left(\frac{dx}{dt}\right) i + \left(\frac{dy}{dt}\right) j$$

$$\boxed{\therefore \bar{V} = V_x i + V_y j}$$

where,

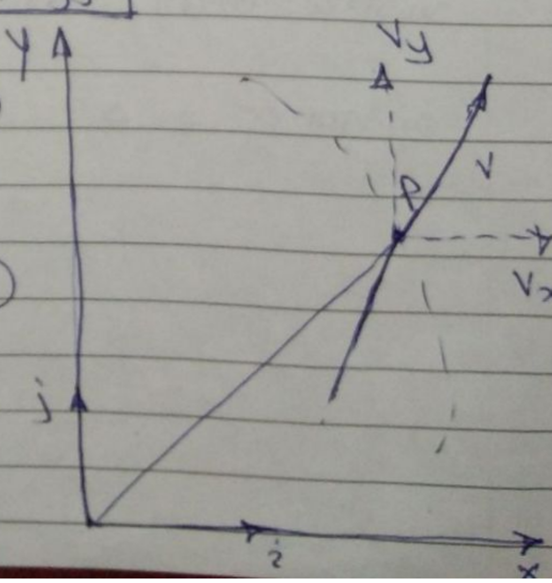
$\therefore V_x = \frac{dx}{dt} = \dot{x}$ (x - component)

$\therefore V_y = \frac{dy}{dt} = \dot{y}$ (y - component)

Also,

$V = \sqrt{V_x^2 + V_y^2}$

$\tan\alpha = V_y / V_x$



Now,

$$\bar{V} = V_x \hat{i} + V_y \hat{j}$$

$$\bar{a} = \frac{d\bar{v}}{dt} = \left(\frac{dv_x}{dt}\right)\hat{i} + \left(\frac{dv_y}{dt}\right)\hat{j}$$

$$\boxed{\therefore\ \bar{a} = a_x \hat{i} + a_y \hat{j}}$$

Where,

$$a_x = \frac{dv_x}{dt} = \dot{v}_x \text{ (x - component)}$$

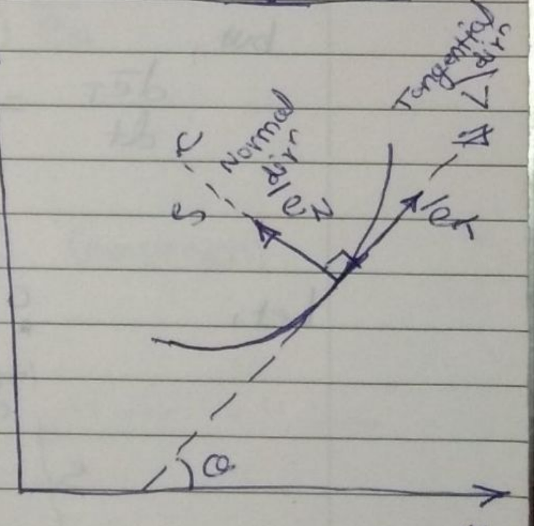
$$a_y = \frac{dv_y}{dt} = \dot{v}_y \text{ (y - component)}$$

$$\therefore\ a = \sqrt{a_x^2 + a_y^2}$$

$$\therefore\ \tan\beta = a_y / a_x.$$

## (2) Normal & Tangential co-ordinate system

→ consider a particle moving along a curve is located at point 'P' at any instant of time 't'.



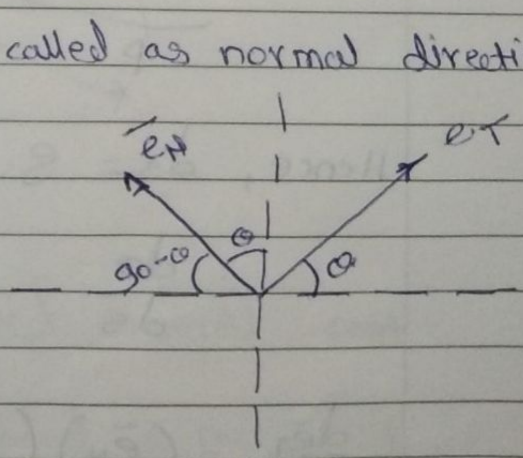

→ The dir^n along tangent is called tangential dir^n and velocity is always tangential to the path.

→ Dir^n $\perp^{ar}$ to tangential is called as normal direction.

Let,

$\bar{e}_T$ ⇒ unit vector along tangential dir^n

$\bar{e}_N$ → unit vector along normal direction.




$\therefore\ \bar{e}_T = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$ and $\bar{e}_N = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$

Diff. w.r.t. '$\theta$'

$\therefore \frac{d\bar{e}_T}{d\theta} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$ & $\frac{d\bar{e}_N}{d\theta} = -\cos\theta\,\hat{i} - \sin\theta\,\hat{j}$

$\therefore \frac{d\bar{e}_T}{d\theta} = \bar{e}_N$ —① $\frac{d\bar{e}_N}{d\theta} = -\bar{e}_T$ —②

Now,

$V_T = \bar{V}$ ... Velocity is always tangential

$V_N = 0$ ... ~~Tangential~~ Normal Component is zero.

We know,

$$\bar{a} = \frac{d\bar{v}}{dt} = \frac{d(\bar{v})}{dt}$$

$$= \frac{d(v \cdot \bar{e}_T)}{dt}$$

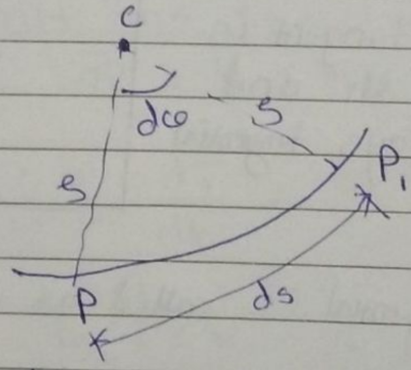
$$= \frac{dv}{dt} \cdot \bar{e}_T + \frac{d\bar{e}_T}{dt} \cdot v \quad \text{—③}$$

but,

$$\frac{d\bar{e}_T}{dt} = \frac{d\bar{e}_T}{d\theta} \cdot \frac{d\theta}{ds} \cdot \frac{ds}{dt} \quad \text{...④}$$

... by double chain rule

Let,




Hence, $ds = \rho \cdot d\theta$

$\therefore \frac{d\theta}{ds} = \frac{1}{\rho}$ ... sub. in eq$^n$ ④

$\frac{d\bar{e}_T}{dt} = (\bar{e}_N)\left(\frac{1}{\rho}\right)(v)$ ... sub. in eq$^n$ ③

$$\therefore \bar{a} = \left(\frac{dv}{dt}\right) \bar{e}_T + \bar{e}_N \cdot \frac{1}{\rho} \cdot v^2$$

$$\therefore \bar{a} = \left(\frac{dv}{dt}\right) \bar{e}_T + \left(\frac{v^2}{\rho}\right) \bar{e}_N .$$

compairing with,

$$\bar{a} = (a_T)\,\bar{e}_T + (a_N)\,\bar{e}_N$$

We get,

- $a_T = \dfrac{dv}{dt}$ $\Rightarrow$ Tangential Component of accl$^n$

$a_N = \dfrac{v^2}{\rho}$ $\Rightarrow$ Normal —"— —"—

Magnitude, $a = \sqrt{a_T^2 + a_N^2}$

dir$^n$ $\tan\alpha = \dfrac{a_T}{a_N}$

→ $a_N$ is always directed towards centre

→ $a_T$ is positive if speed is increasing

→ $a_T$ is negative if speed is decreasing

Radius of curvature

$$|S| = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2} \quad \text{--- (1)}$$