K J SOMAIYA COLLEGE OF ENGINEERING, MUMBAI-77
(CONSTITUENT COLLEGE OF SOMAIYA VIDYAVIHAR UNIVERSITY)
Course Code: 111U06C104
Course Title: Engineering Mechanics

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Module 5

Kinetics of particle		**9**	**CO5**
5.1	Force and acceleration: Introduction to basic concepts, equations of dynamic equilibrium, Newton's second law of motion (only rectilinear motion)		
5.2	Work energy principle		
5.3	Impulse and Momentum: Principle of linear impulse and momentum, law of conservation of momentum, impact and collision, direct central and oblique central impact.		

Kinetics

- Kinetics is a branch of dynamics that deals with the relationship between the change in motion of a body and the forces that cause this change.
- The basis for kinetics is Newton's second law, which states that when an unbalanced force acts on a particle, the particle will accelerate in the direction of the force with a magnitude that is proportional to the force.
- This law can be verified experimentally by applying a known unbalanced force F to a particle, and then measuring the acceleration a.
- Since the force and acceleration are directly proportional, the constant of proportionality, m, may be determined from the ratio $m = F/a$. This positive scalar m is called the mass of the particle.
- Being constant during any acceleration, m provides a quantitative measure of the resistance of the particle to a change in its velocity, that is its inertia.

Newton’s second Law (NSL)

- It can also be stated as if the external unbalanced force acts on a body, the momentum of the body changes. The rate of change of momentum is directly proportional to the force and takes place in the direction of motion.
- Momentum is the quantity of motion possessed by a body. Linear momentum of a body is calculated as a product of mass and velocity of the body

$$\frac{d}{dt}(m\bar{v}) \propto \bar{F}$$

$$\frac{d}{dt}(m\bar{v}) = k\bar{F}$$

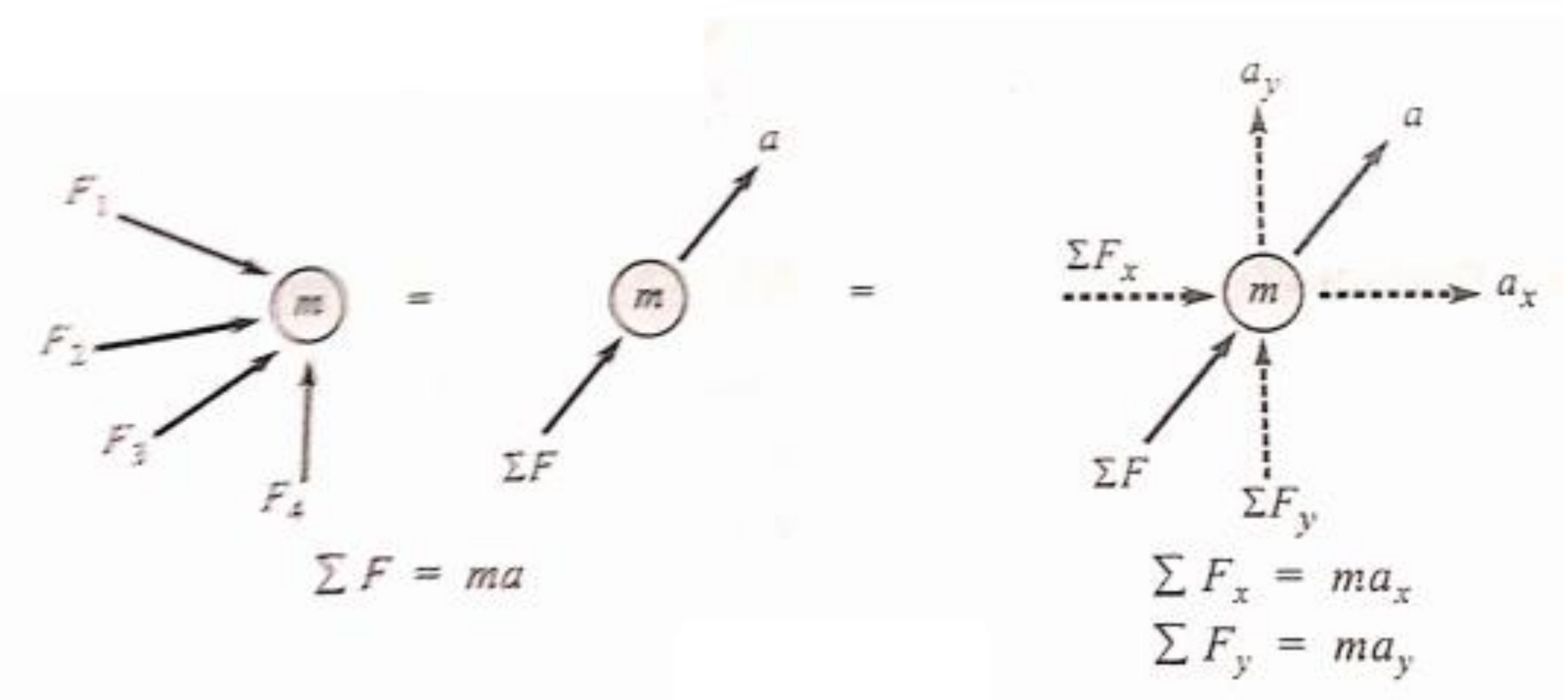
$$m\frac{d\bar{v}}{dt} = k\bar{F}$$

$$m\bar{a} = k\bar{F}$$

when $m = 1$, $a = 1$, $F = 1$ then $k = 1$

$$\therefore \bar{F} = m\bar{a}$$

Rectilinear Motion



D'Alembert's Principle(Dynamic Equilibrium)

- The force system consisting of external forces and inertia force can be considered to keep the particle in equilibrium. Since the resultant force externally acting on the particle is not zero, the particle is said to be in dynamic equilibrium.

- **D'Alemberts' Principle :** *The algebraic sum of external force (ΣF) and inertia force ($-ma$) is equal to zero.*

$$\sum F + (-ma) = 0$$

- **For Rectilinear Motion**

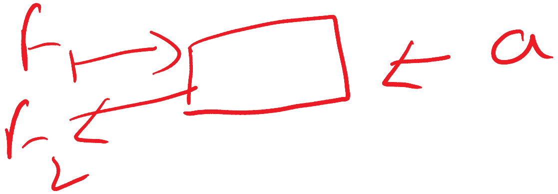
$$\sum F_x + (-ma_x) = 0 \quad \text{and} \quad \sum F_y + (-ma_y) = 0$$

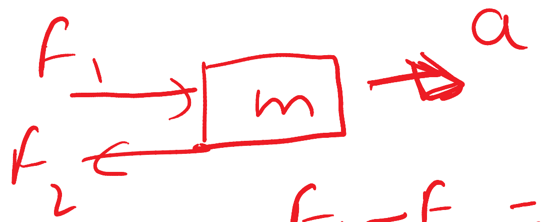
- **For Curvilinear Motion**

$$\sum F_t + (-ma_t) = 0 \quad \text{and} \quad \sum F_n + (-ma_n) = 0$$

Steps for analysis

$-f_1 + f_2 = ma$





NSL → $f_1 - f_2 = ma$

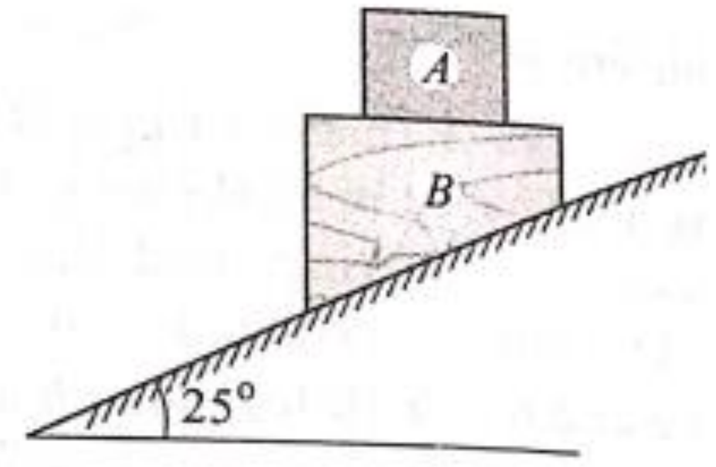
- Draw FBD
- Show the direction of acceleration and consider positive sign along the direction of acceleration.
- Assumption for direction of acceleration:

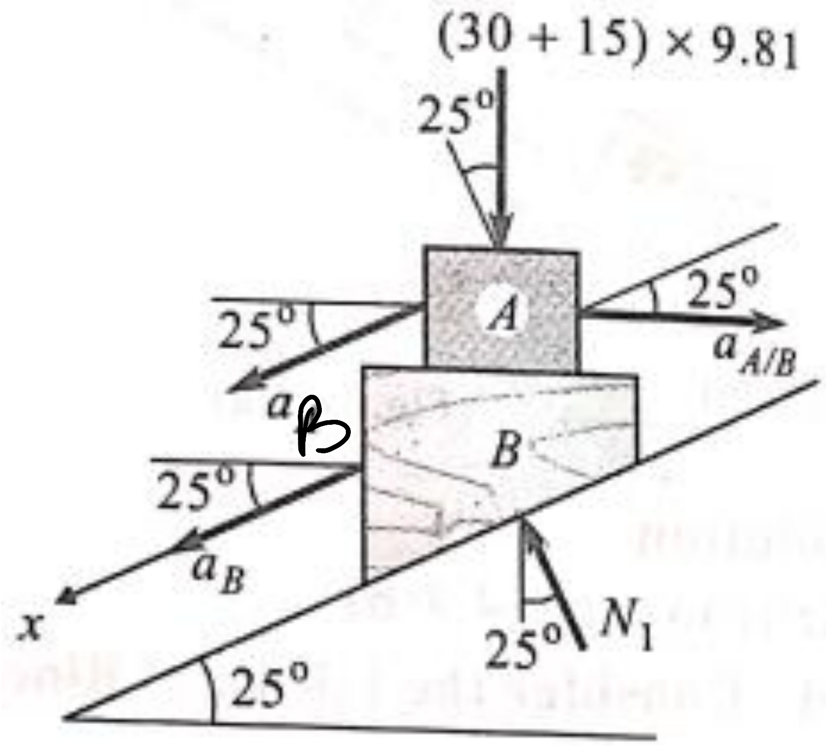
➢ if the friction is not given then assume any direction of acceleration. Positive answer means assumed direction is correct. If -ve, change direc^n of acc^n

➢ If **friction** is given then we must carefully assume the direction of acceleration. Here if we get a **negative answer**, then one should **resolve the problem** by changing the direction.

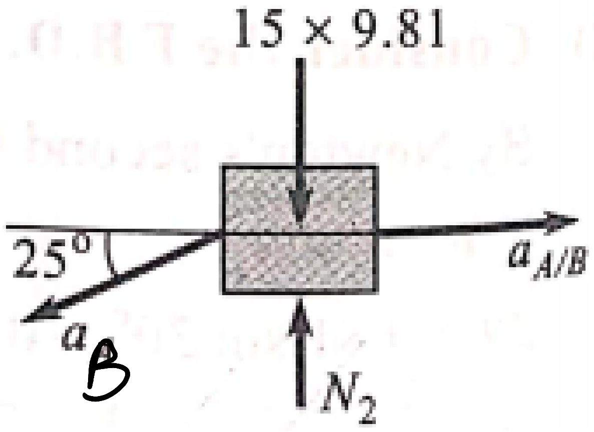
Problem

Two blocks A and B having mass 15 kg and 30 kg respectively are released from rest on an inclined plane as shown. Find the acceleration of each block considering surface to be frictionless.

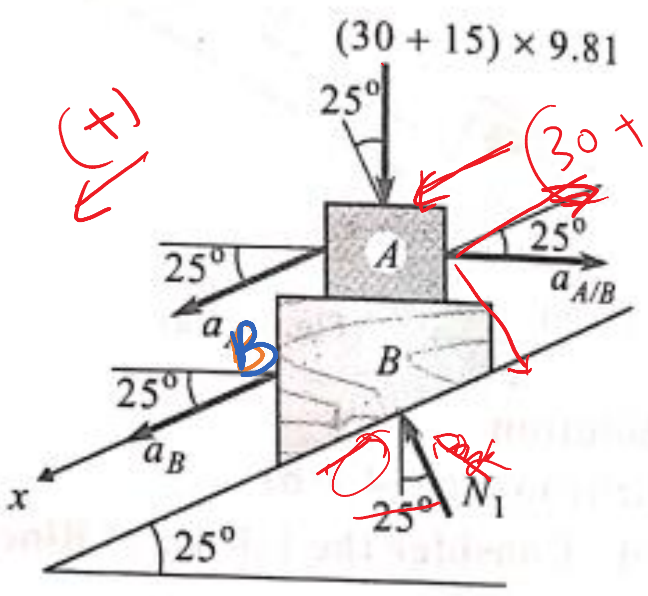




(30 + 15) × 9.81
25°
A
25°
25°
$a_{A/B}$
a_B
B
25°
a_B
x
25°
N_1
25°



15 × 9.81
25°
$a_{A/B}$
a_B
N_2



A has relative motion w.r.t B

$\therefore \bar{a}_{A/B} = \bar{a}_A - \bar{a}_B$

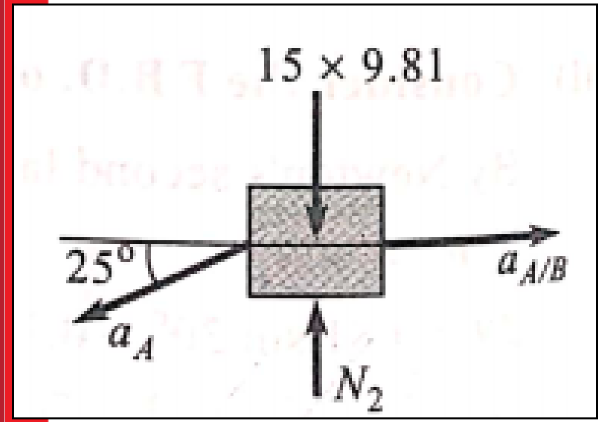
Consider FBD of A & B together

By NSL, $\Sigma F_x = ma_x$

$(30+15)\times 9.81 \sin 25^\circ = 30\, a_B + 15\, a_A$

$= 30\, a_B + 15\, a_B - 15 \cos 25\, a_{A/B}$

$186.57 = 45 a_B - 13.6\, a_{A/B}$



Consider FBD of A

By NSL $\Sigma F_x = ma_x$

$0 = 15 a_{A/B} - 15 a_B \cos 25^\circ \Rightarrow a_{A/B} = 0.9063\, a_B$

$\Rightarrow a_{A/B} = 0.9063\, a_B$ //

$a_B = 5.71\ m/s^2$ 25°

$a_{A/B} = 5.175\ m/s^2 \rightarrow$

$a_A =$

$$\bar{a}_A = \bar{a}_{AB} + \bar{a}_B$$

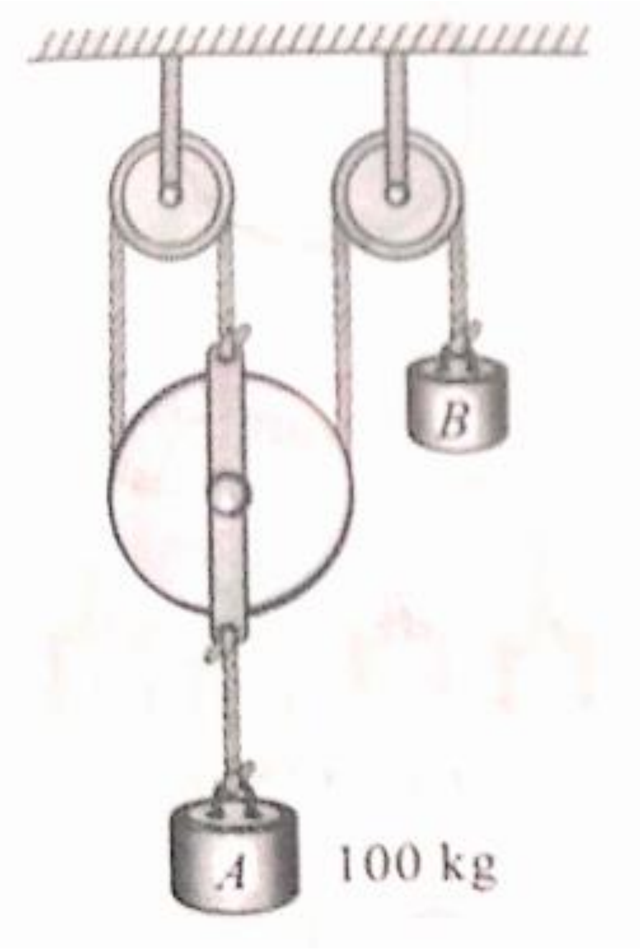
$$\bar{a}_A = -5.175\,i + (5.71\cos 25^\circ\,i - 5.71\sin 25^\circ\,j)$$

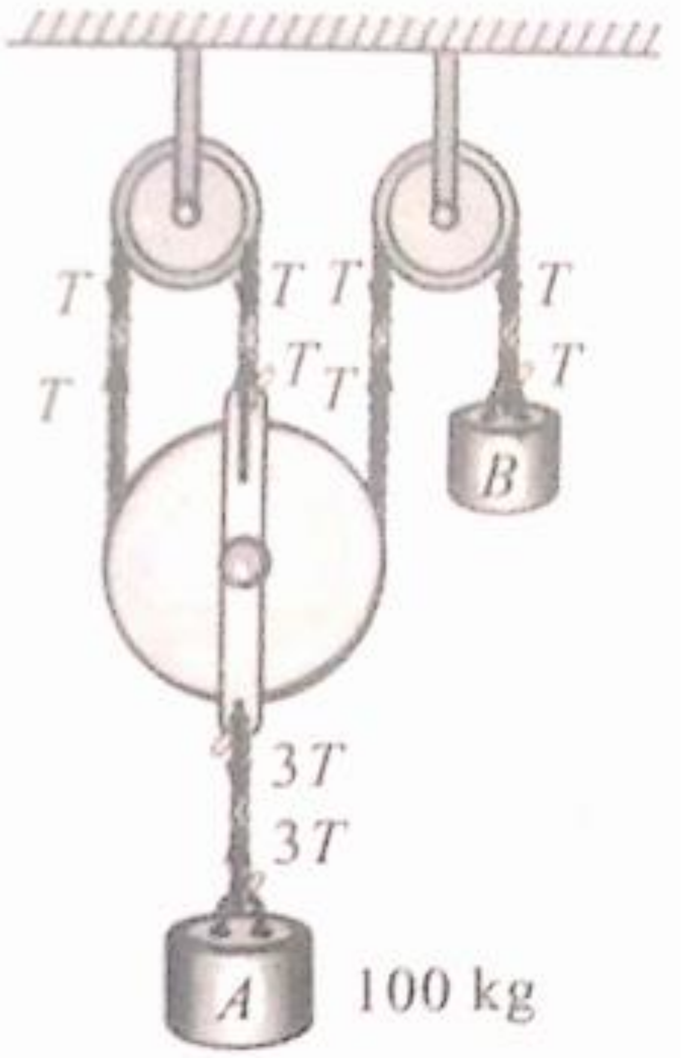
$$= -2.413\,j$$

$$= 2.413\ m/s^2\ (\downarrow)$$

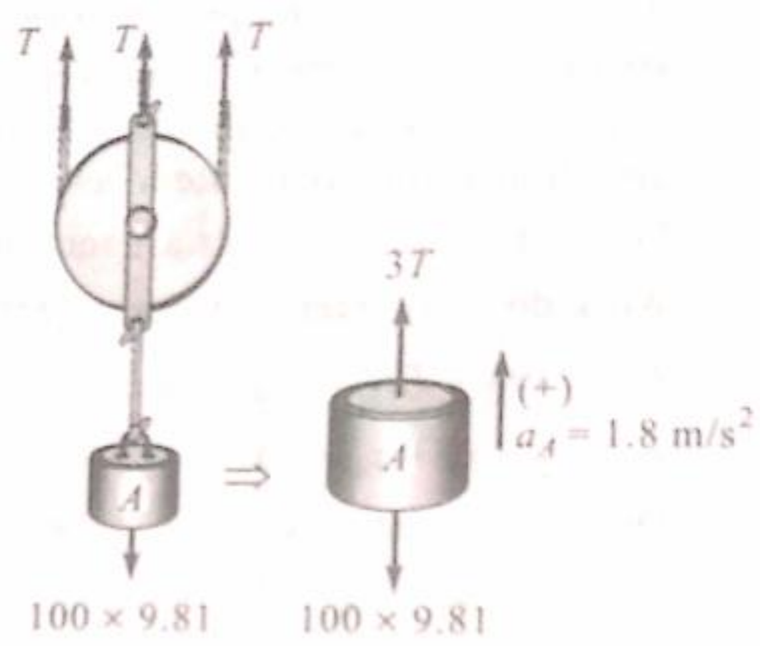
Problem

- Block A of 100 kg moves up with an acceleration of 1.8 m/s^2. Determine the mass of the block B and the corresponding tension in the cable.

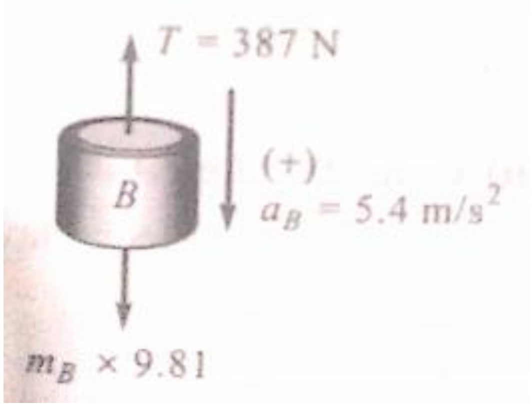




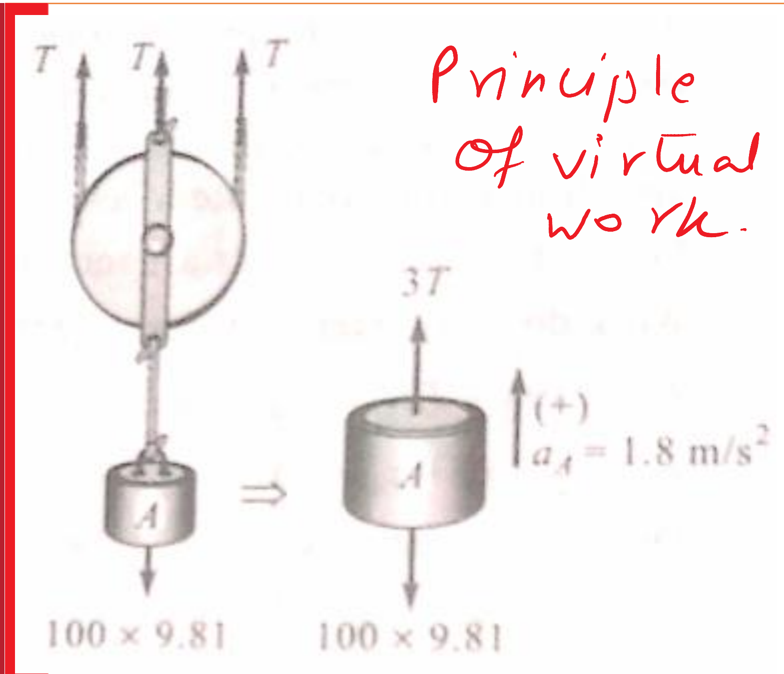
T
T T
T
T
T
T
T
B
3T
3T
A
100 kg



T
T
T
3T
(+)
a_A = 1.8 m/s^2
A
A
100 × 9.81
100 × 9.81



T = 387 N
(+)
a_B = 5.4 m/s^2
B
m_B × 9.81



Principle of virtual work. → Work done by internal forces = 0

$$+3T x_A - T x_B = 0$$

[x_A & 3T direction same ∴ WD +ve

x_B & T direc^n opp ∴ W.D −ve]

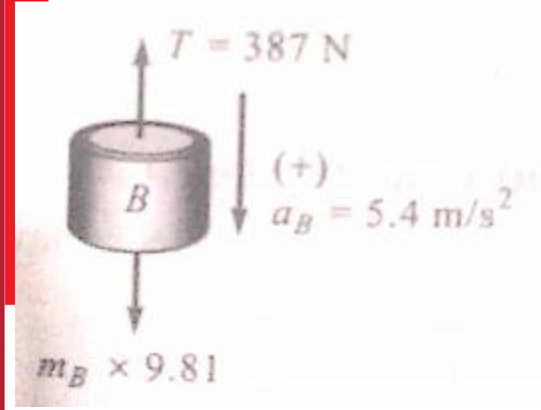
$$3x_A = x_B$$

Diff w.r.t. t we get

$$3V_A = V_B$$

Diff w.r.t. t we get

$$3a_A = a_B = 3 \times 1.8 = 5.4\ m/s^2$$



Consider FBD of A

NSL → $\Sigma F_y = m a_y \Rightarrow 3T - 100 \times 9.81 = 100 \times 1.8$

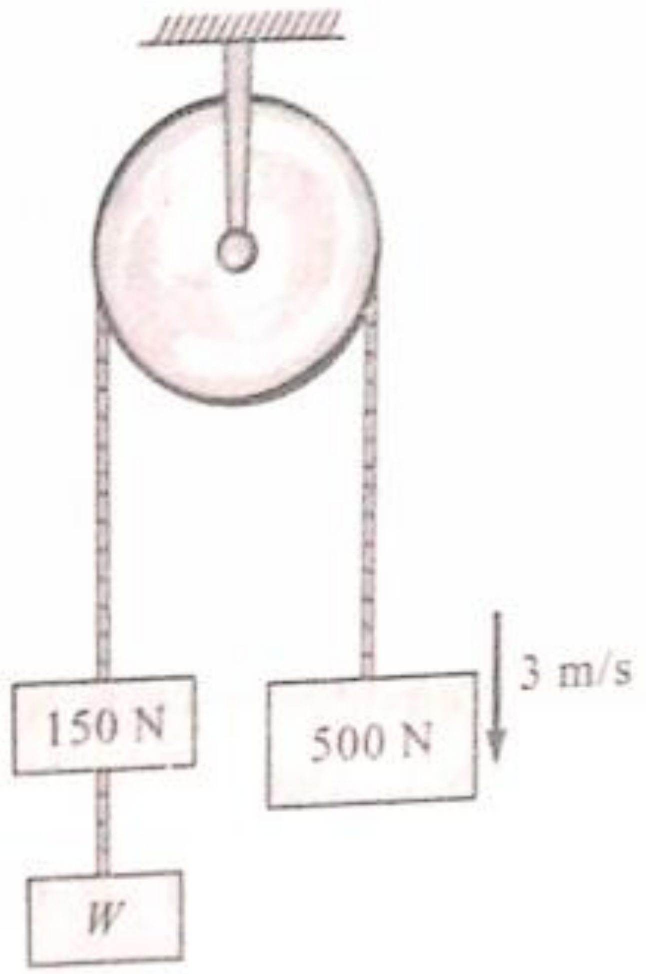
Consider FBD of B

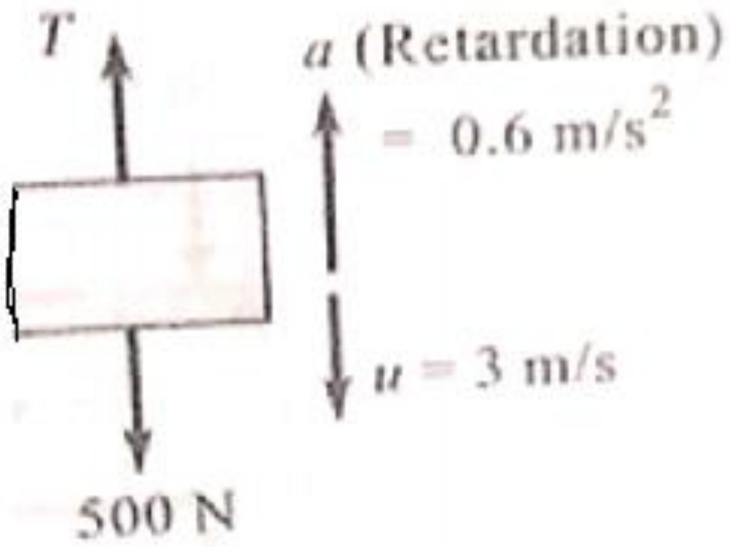
$$m_B \times 9.81 - T = m_B \times a_B$$

$$m_B (9.81 - 5.4) = 387 \Rightarrow m_B = 87.76\ kg$$

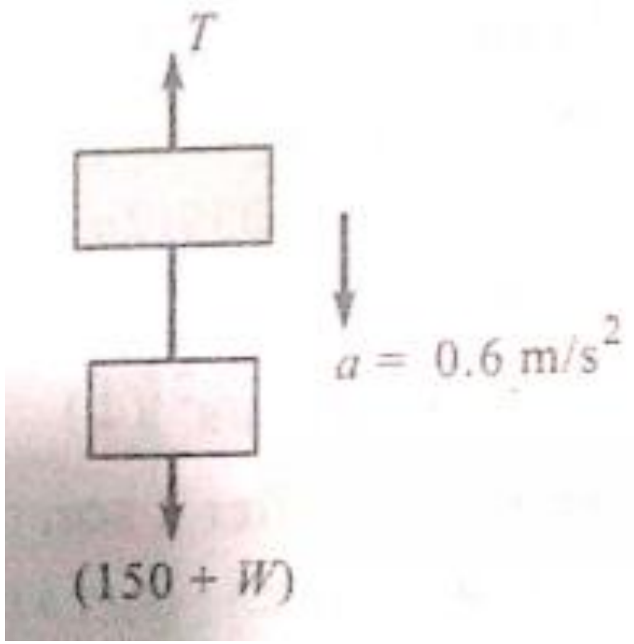
Problem

Determine the weight W required to be attached to 120 N block to bring the system to stop in 5 seconds if at any stage 500 N is moving down at 3 m/s. Assume pulley to be frictionless and massless.

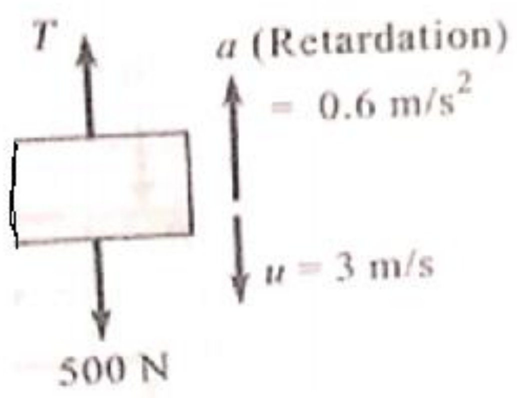


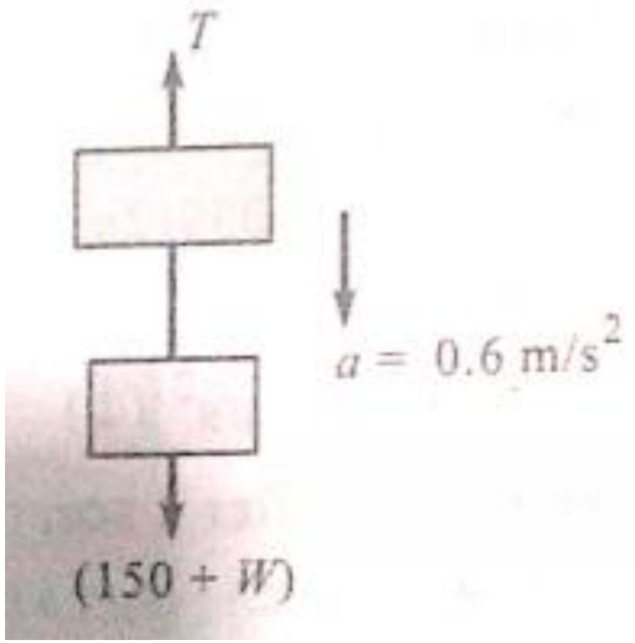


T
a (Retardation)
= 0.6 m/s2
u = 3 m/s
500 N



T
a = 0.6 m/s2
(150 + W)





$v = u + at$

$0 = 3 + a \times 5 \Rightarrow a = -0.6\ m/s^2$

FBD of 500 N Block

NSL → $\Sigma F = ma$

$T - 500 = \frac{500}{9.81} \times 0.6 \Rightarrow T = 530.58\ N$

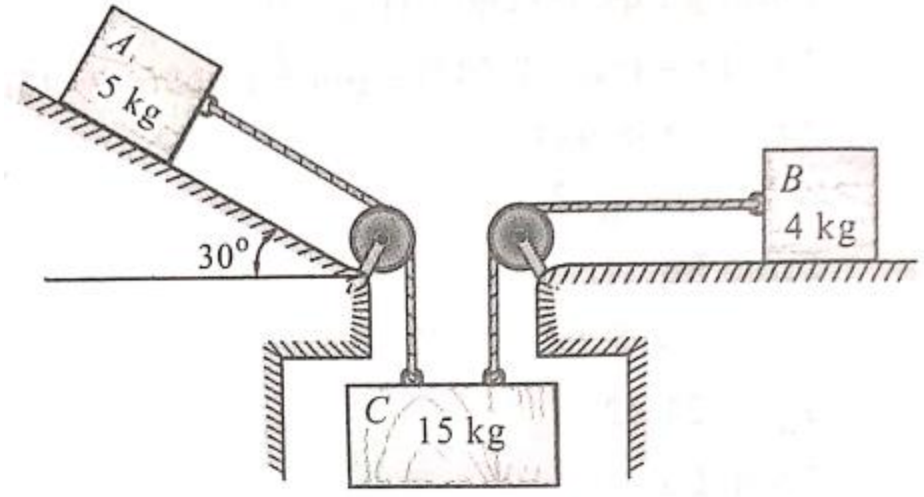
FBD of (150 + W) together

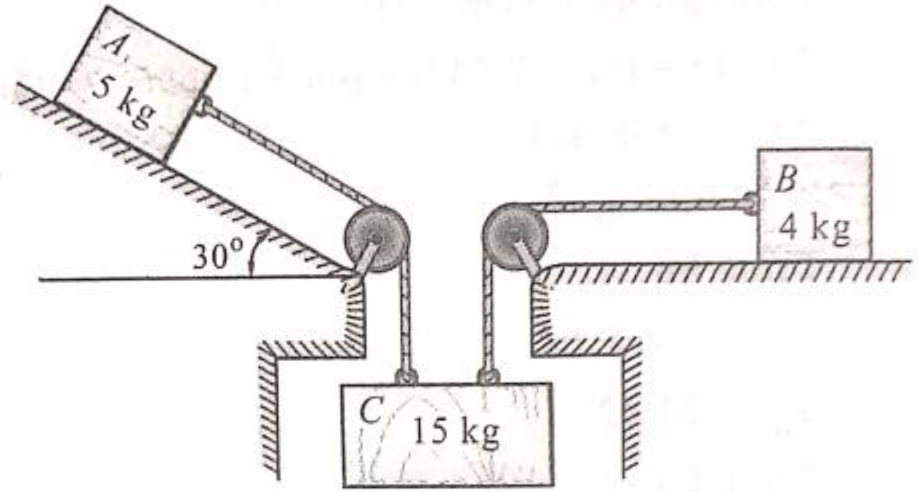
NSL → $\Sigma F = ma$

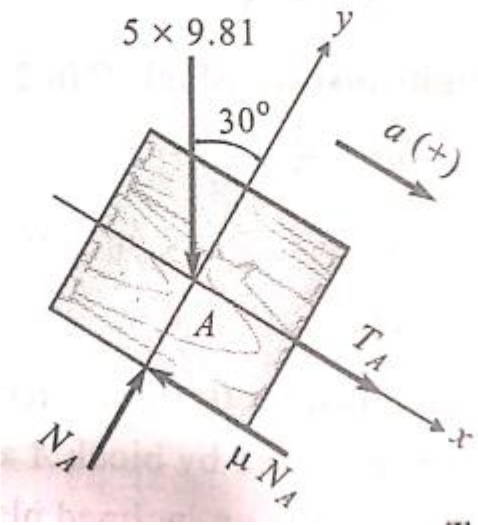
$150 + W - T = \frac{150 + W}{9.81} \times 0.6$

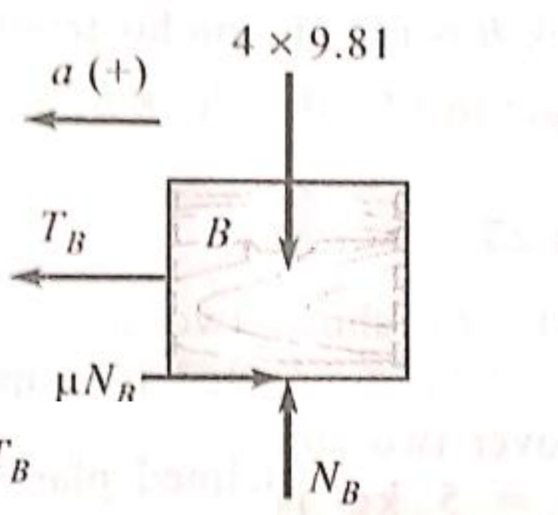
$\Rightarrow W = 415.15\ N$

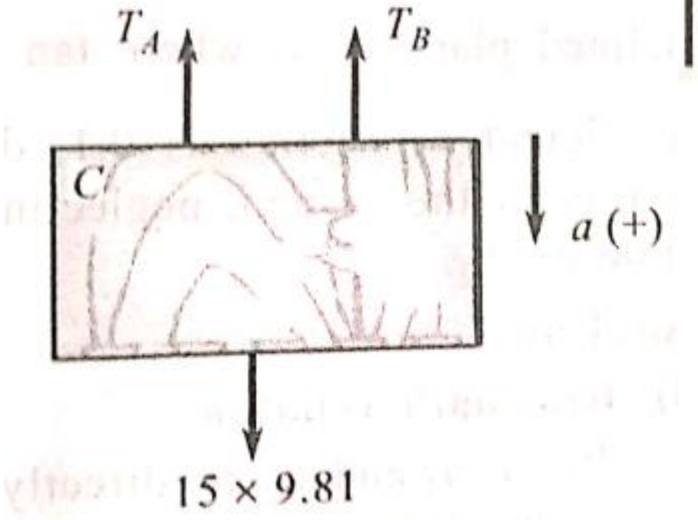
The system is released from rest. What is the height lost by the bodies A, B and C in 2 seconds. Take coeff of kinetic friction at rubbing surfaces as 0.4. also find TA and TB tensions in the wires. Assume pulleys to be weightless and frictionless.

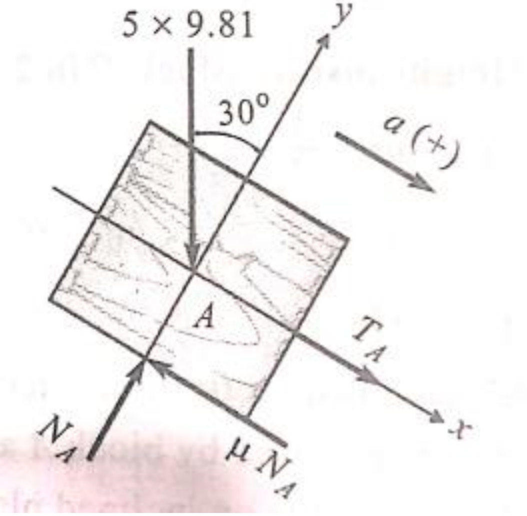


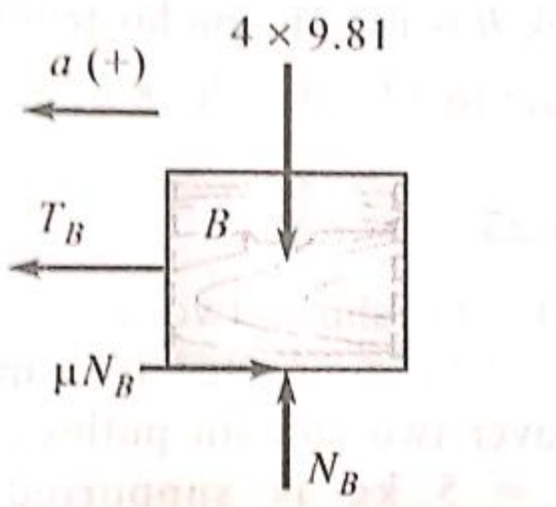

A
5 kg
30°
B
4 kg
C
15 kg

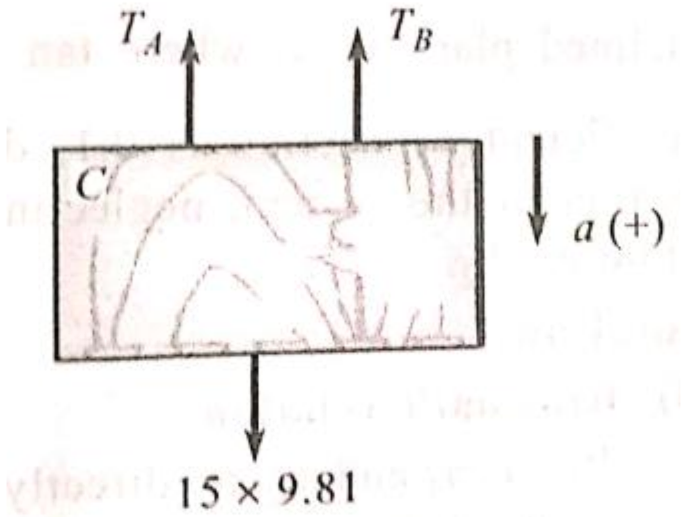

5 × 9.81
y
30°
a (+)
A
T_A
x
N_A
μN_A


a (+)
4 × 9.81
T_B
B
μN_B
N_B


T_A
T_B
C
a (+)
15 × 9.81







∴ All three blocks are connected directly to each other

$a_A = a_B = a_C = a$.

FBD of A

NSL ⇒ $\Sigma F_y = m a_y = 0$

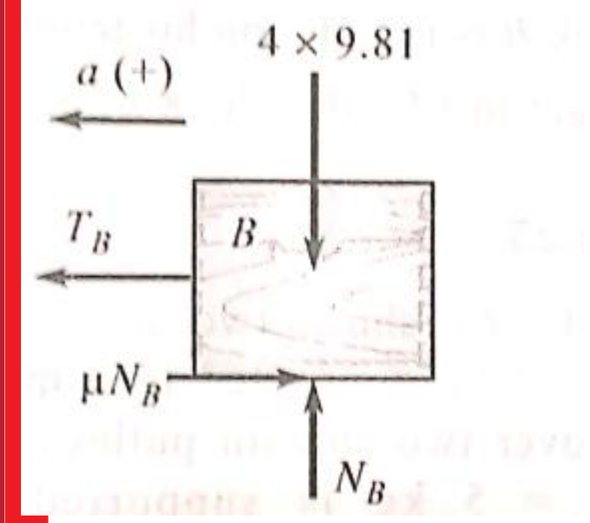
$N_A - 5 \times 9.81 \cos 30 = 0$

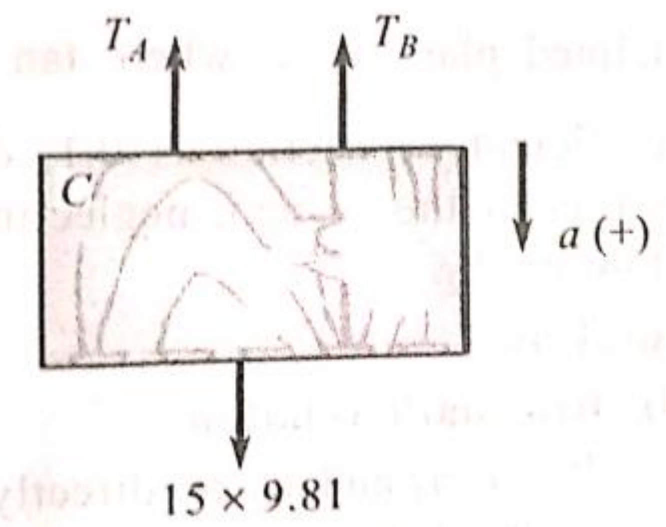
$N_A = 42.48$ N

$\Sigma F_x = m a_x$.

$T_A - \mu N_A + 5 \times 9.81 \sin 30° = 5 \times a$.

$T_A = 5a - 7.533$





FBD of B

NSL → $\Sigma F_y = m a_y$

$N_B - 4 \times 9.81 = 0 \Rightarrow N_B = 39.24\ N.$

$\Sigma F_x = m a_x.$

$T_B - \mu N_B = 4 \times a$

$T_B = 4a + 15.696.$

FBD of C

$\Sigma F_y = m a_y$

$15 \times 9.81 - T_A - T_B = 15 \times a$

$a = 5.79\ m/s^2$

$T_A = 21.42\ N$

$T_B = 38.86\ N$

Ht lost by block C in 2 sec.

$s = ut + \frac{1}{2} a t^2$

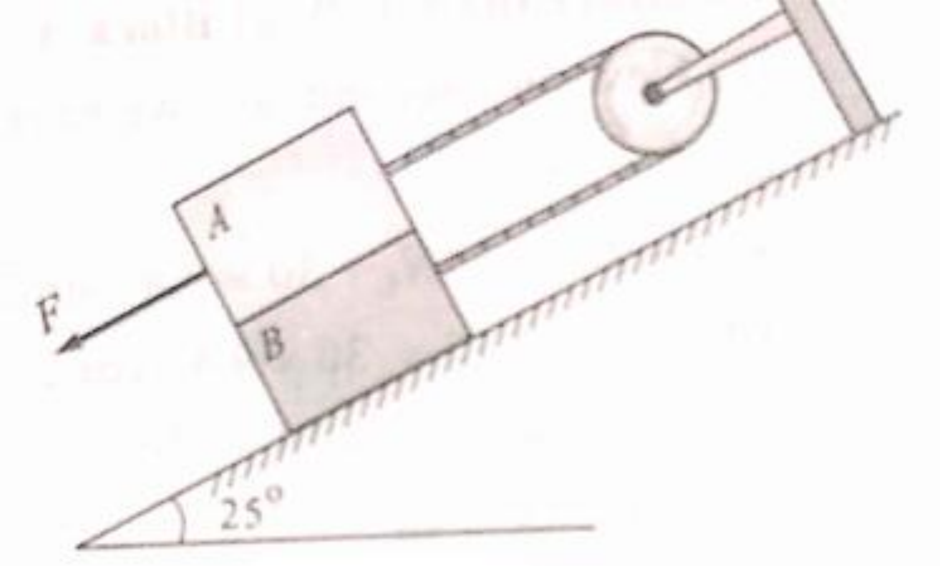
$h_c = 0 + \frac{1}{2} \times 5.79 \times 2^2$

$= 11.57\ m$

$h_A = 11.57 \sin 30$ (inclined plane)

$H_B = 0$ (horizontal)

Problem

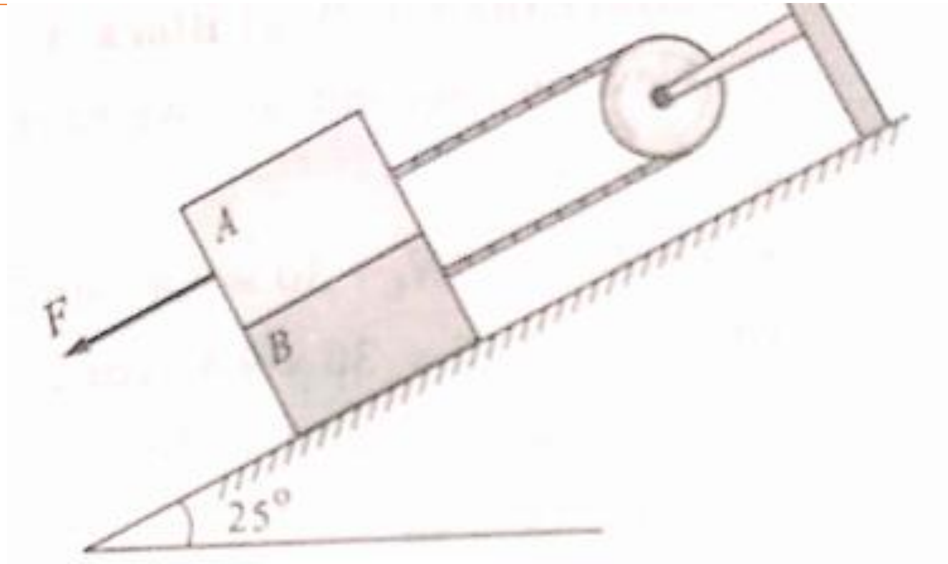


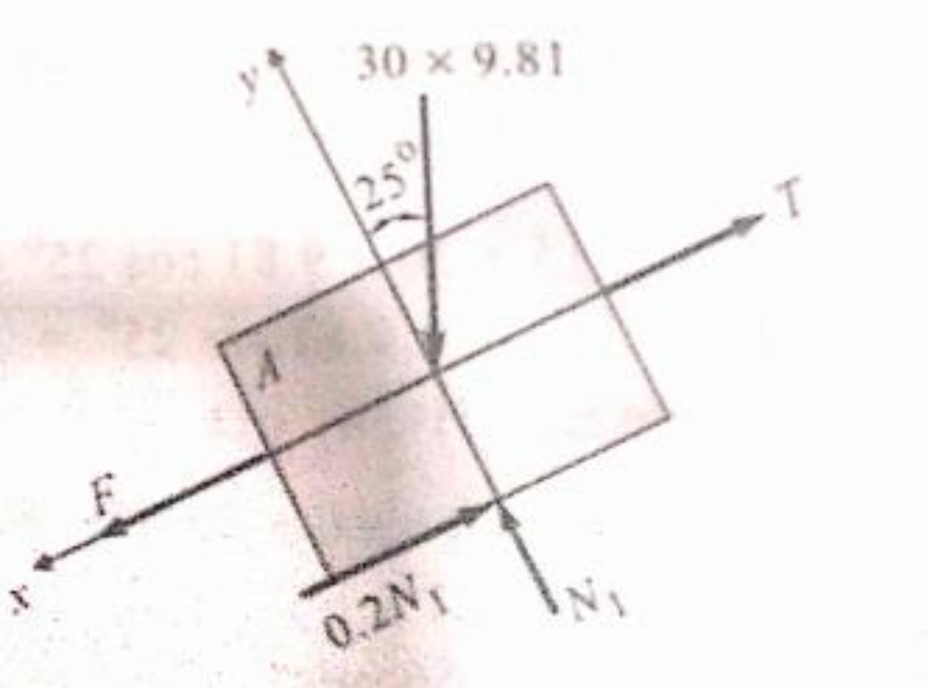
Block A has a mass of 30 kg and B has 20 kg. $\mu s = 0.2$, $\mu k = 0.15$.

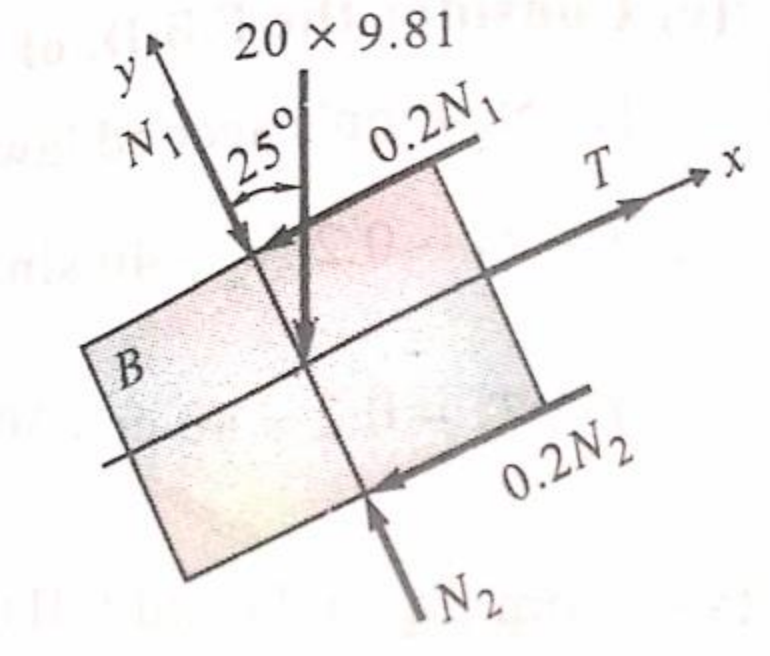
Dertermine:

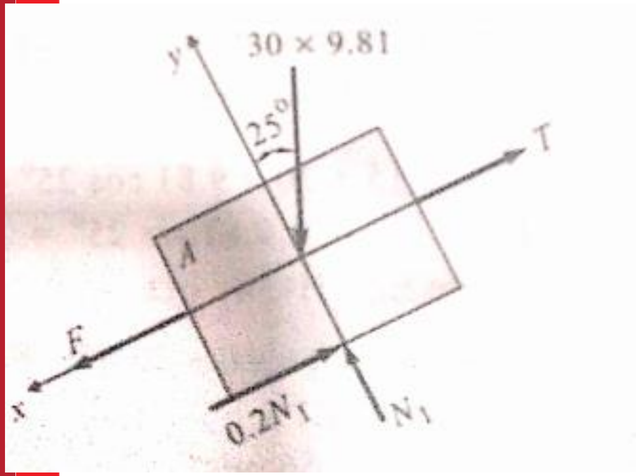
a. The minimum force F to develop impending motion

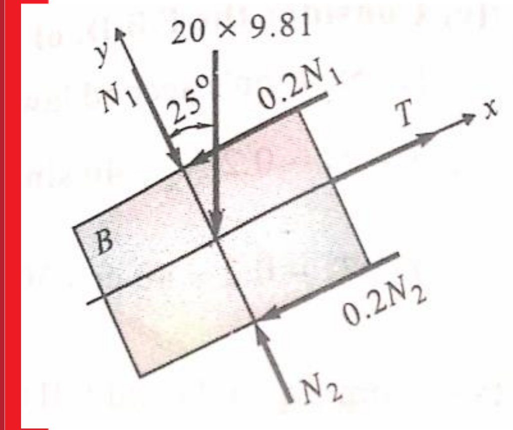
b. Acceleration of A if the applied force F = 400 N.


A
B
F
25°


y
30 × 9.81
25°
T
A
F
x
0.2N1
N1


y
20 × 9.81
N1
25°
0.2N1
T
x
B
0.2N2
N2





Impending motion

$\Sigma F_y = 0 \Rightarrow N_1 = 226.73\ N.$

$\Sigma F_x = 0 \Rightarrow T = F - 71.03$

$\Sigma F_y = 0$

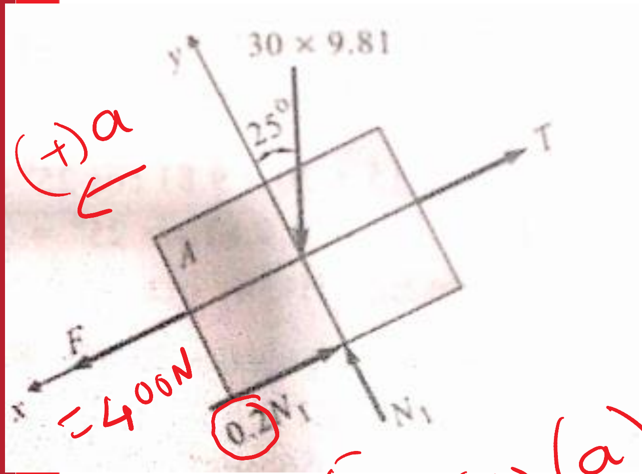
$N_2 = N_1 + 20 * 9.81 \cos 25^\circ$

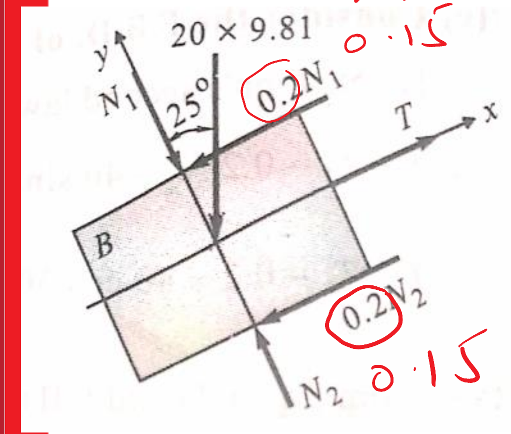
$N_2 = 444.54\ N.$

$\Sigma F_x = 0$

$T = 225.17\ N$

$F = 296.2\ N.$ 25°





Part II

NSL [Block A]

$\Sigma F_x = m a_x$

$400 - T + 30 \times 9.81 \sin 25 = 30 \times a$

$T = 484.38 - 3a$

NSL [Block B]

$T - 0.15(N_1) - 0.15(N_2) - 20 \times 9.81 \sin 25^\circ = 20a$

$N_1 = 30 \times 9.81 \cos 25$

$N_2 = 20 \times 9.81 \cos 25^\circ$

$a = 14.56 \; m/s^2$

$a_A = a_B = 14.56 \; m/s^2$ 25°

Module 5

Kinetics of particle		9	CO5
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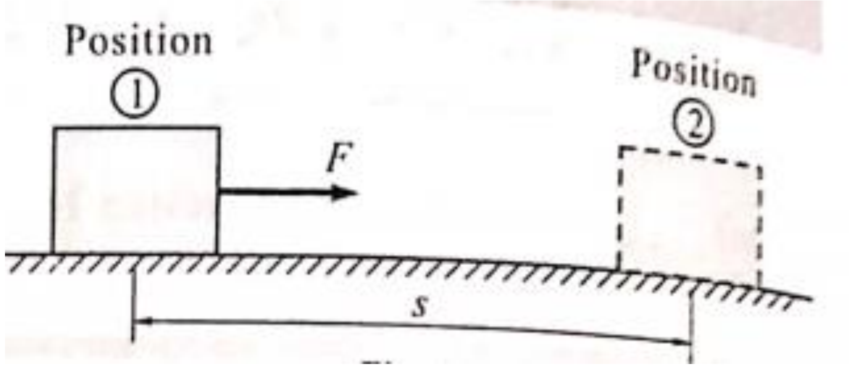
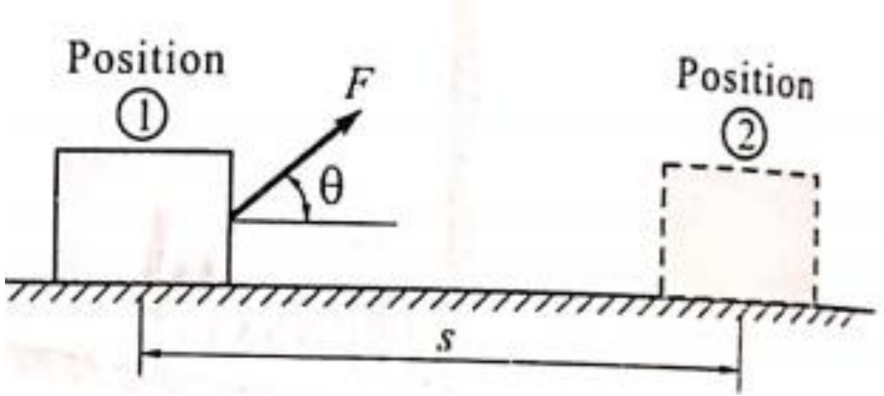
WD by a force

Work done = Force × Displacement

$U = F \times s$





Work done = Component of force in direction of displacement × Displacement

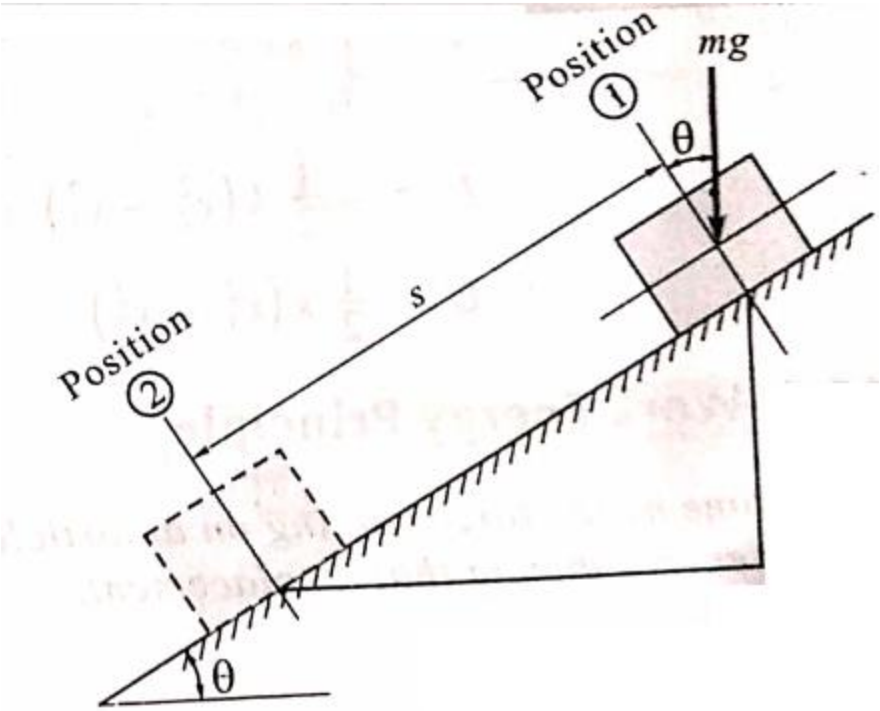
$U = F \cos\theta \times s$

WD by weight

Work done = Component of weight in the direction of displacement × Displacement

$$U = mg \sin\theta \times s$$

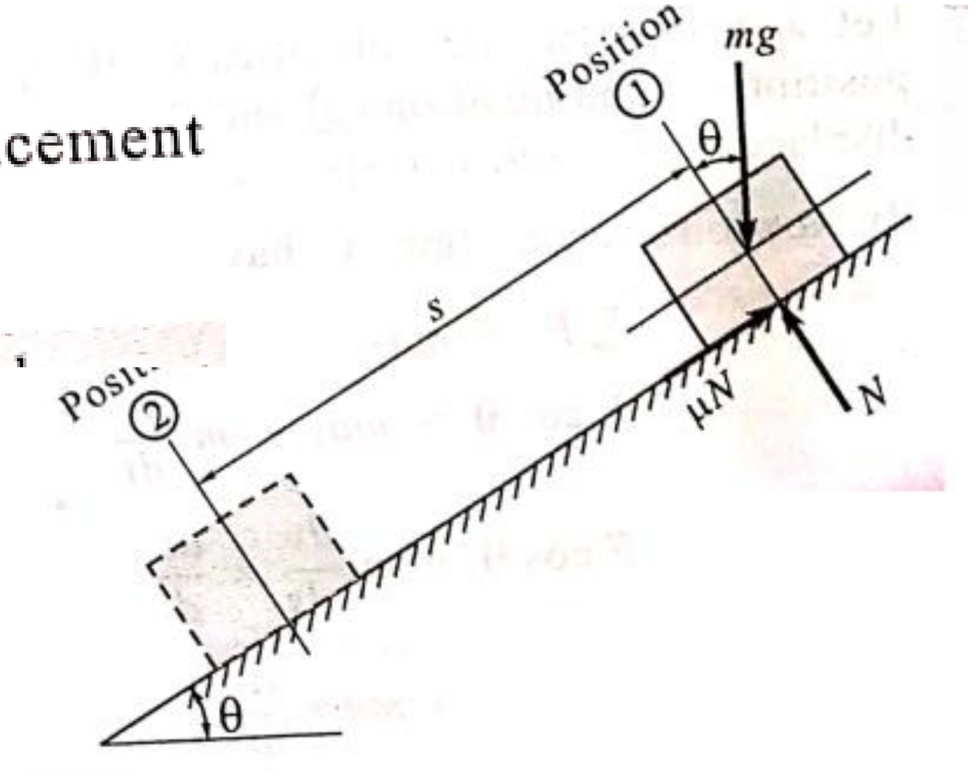
$$U = mg \times s \sin\theta$$



WD by frictional force

Work done = –Frictional force × Displacement

$U = -\mu N \times s$



WD by spring force

Let x_1 be the deformation of spring at position ①.

Let x_2 be the deformation of spring at position ②.

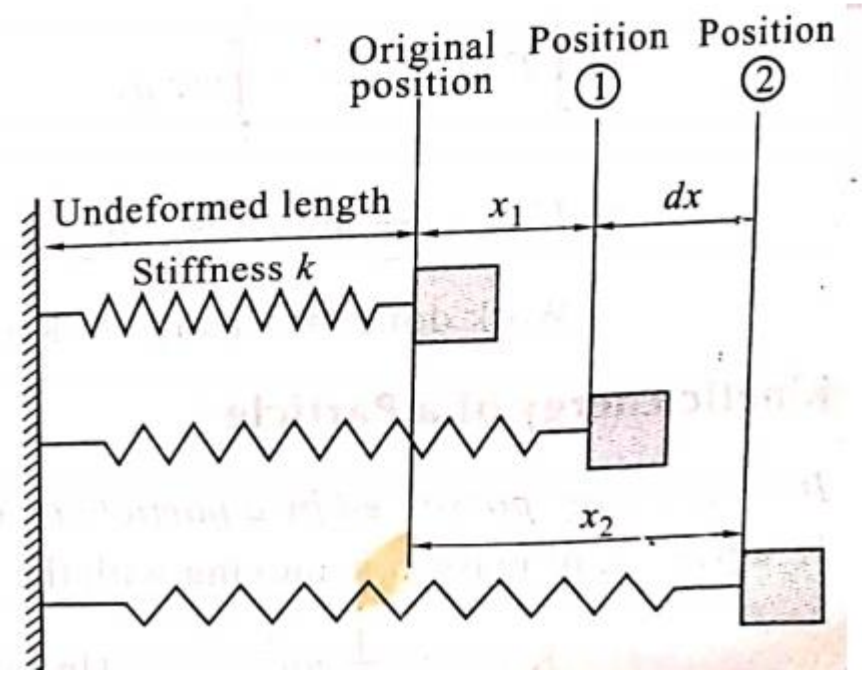
$\therefore$ Spring force $F = -k \times x$

where k is the spring stiffness (N/m)

x is the deformation of spring (m)

–ve sign indicates direction of spring force acts towards original position.

Work done = Spring force × Deformation



$$U = \int_{x_1}^{x_2} -kx\,dx$$

$$\therefore\ U = -\frac{1}{2}k\left(x_2^2 - x_1^2\right)$$

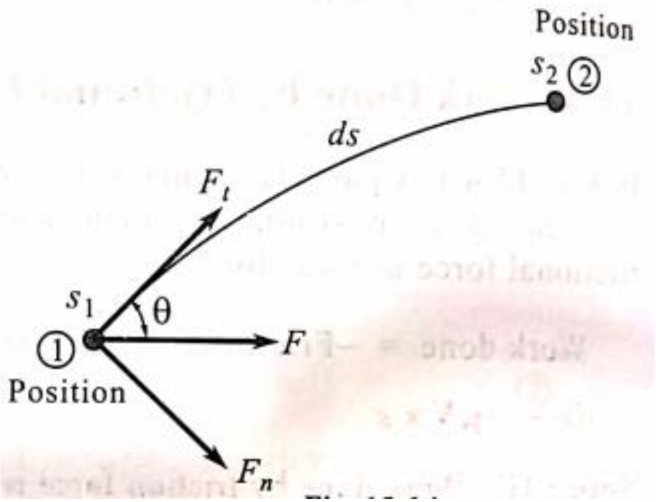
$$\therefore\ U = \frac{1}{2}k\left(x_1^2 - x_2^2\right)$$

Work Energy principle

Work done by forces acting on a particle during some displacement is equal to change in Kinetic energy during that displacement.

Consider the particle having mass m is acted upon by a force F and moving along a path which can be rectilinear or curvilinear

Let v_1 and v_2 be the velocities of the particle at position ① and position ② and the corresponding displacement s_1 and s_2 respectively.



By Newton's second law, we have

$$\Sigma F_t = ma_t$$

$$F\cos\theta = ma_t = m\frac{dv}{dt}$$

$$F\cos\theta = m\frac{dv}{ds}\times\frac{ds}{dt}$$

$$F\cos\theta = mv\times\frac{dv}{ds}$$

$$F\cos\theta\,ds = mv\,dv$$

$$\int_{s_1}^{s_2} F\cos\theta\,ds = \int_{v_1}^{v_2} mv\,dv$$

$$\therefore\ U_{1\text{-}2} = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2$$

Work done = Change in Kinetic Energy

Conservative Forces

If the work of a force is moving the particle from one position to another is independent of the path of the particle and can be expressed as change in potential energy then such forces is called conservative forces

e.g. weight force, spring force, elastic force

Non Conservative Forces

Forces in which work done depends upon the path followed by the particles

e.g. Frictional force, viscous force.

Principle of Conservation of Energy

When the particle is moving from one position to the other under the action of conservative forces (i.e. frictional force does not exist) then by energy conservation principle we can say that the total energy remains constant

$$\text{Total energy} = \text{Kinetic energy} + \text{Potential energy} + \text{Strain energy of spring}$$

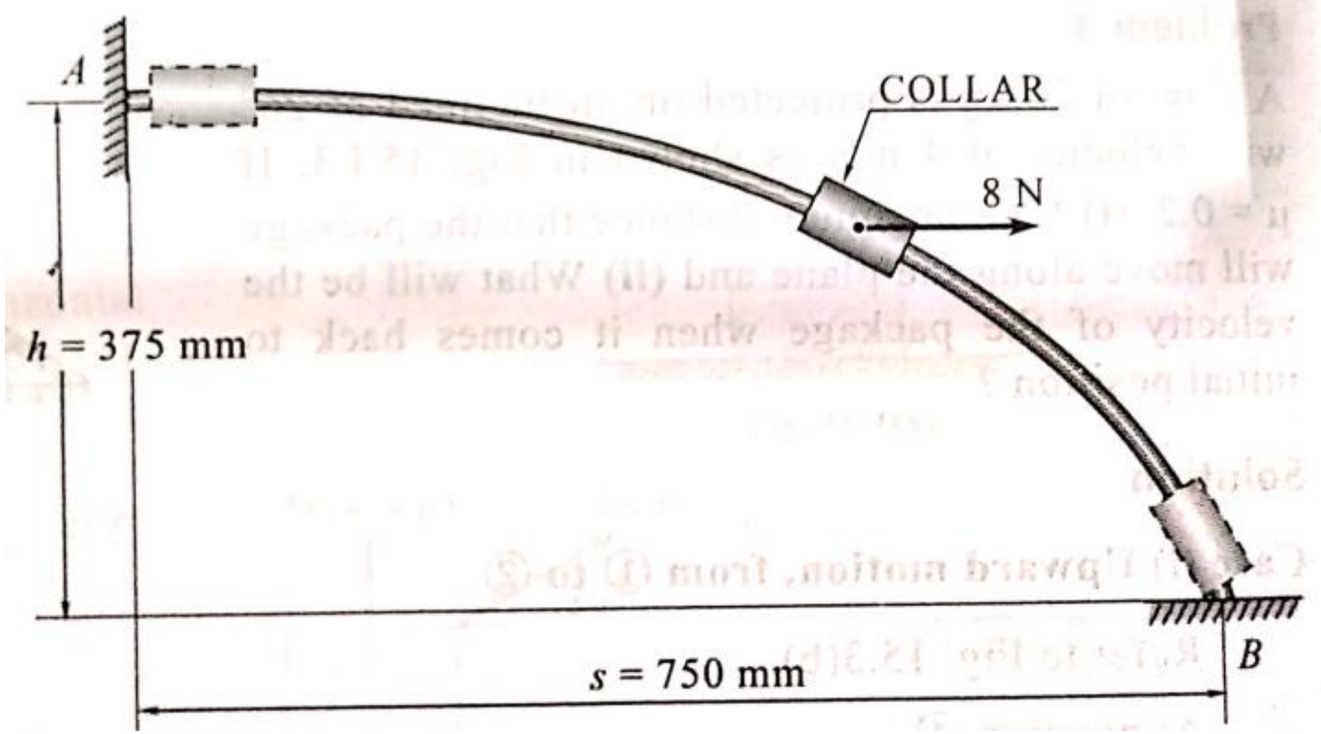
$$\text{Total energy} = \frac{1}{2}mv^2 \pm mgh + \frac{1}{2}kx^2$$

Problem

W.D → wt.

Ext →.

A 0.8 kg collar slides with negligible friction on the fixed rod in the vertical plane as shown. If the collar starts from rest at A under the action of a constant 8N horizontal force, calculate the velocity as it hits the stop at B.



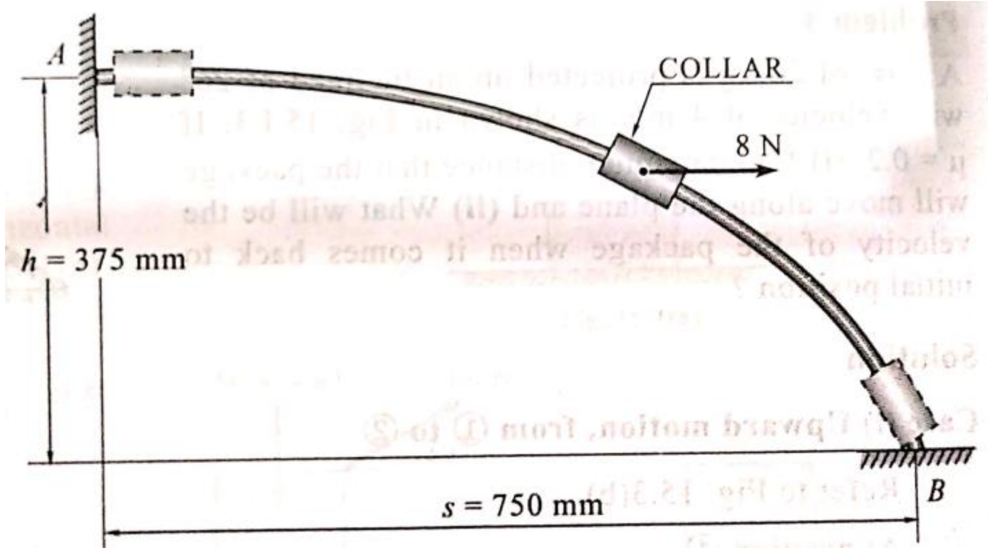
W.E.

WD = change in k.E.

$(P.E) + 8 \times 0.75 = \frac{1}{2} \times 0.8 \times V_B^2 - 0$

$V_A = 0$

$V_B = ?$



By Work Energy principle

Work done = Change in K.E.

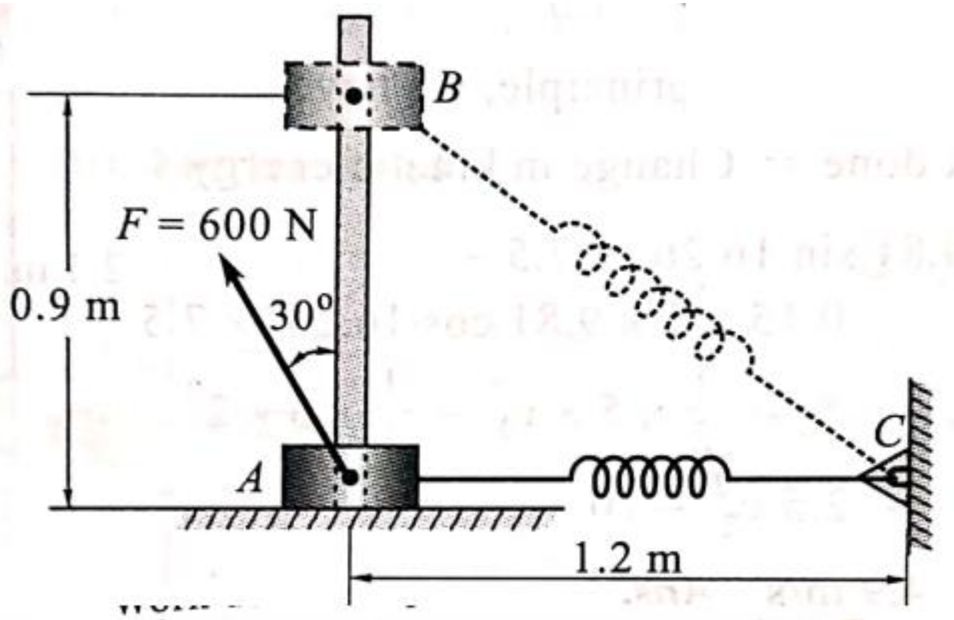
$$mgh + 8 \times s = \frac{1}{2} m V_B^2 - \frac{1}{2} m V_A^2$$

$$0.8 \times 9.81 \times 0.375 + 8 \times 0.75 = \frac{1}{2} \times 0.8 \times V_B^2 - 0$$

$$V_B = 4.728 \text{ m/s}$$

Problem

A collar of mass 15 kg is at rest at A. It can freely slide on a vertical smooth rod AB. The collar is pulled up by a constant force F = 600 N . Unstretched length of the spring is 1 m. calculate the velocity of the collar when it reaches position B. spring constant k = 3 N/mm. AC is horizontal



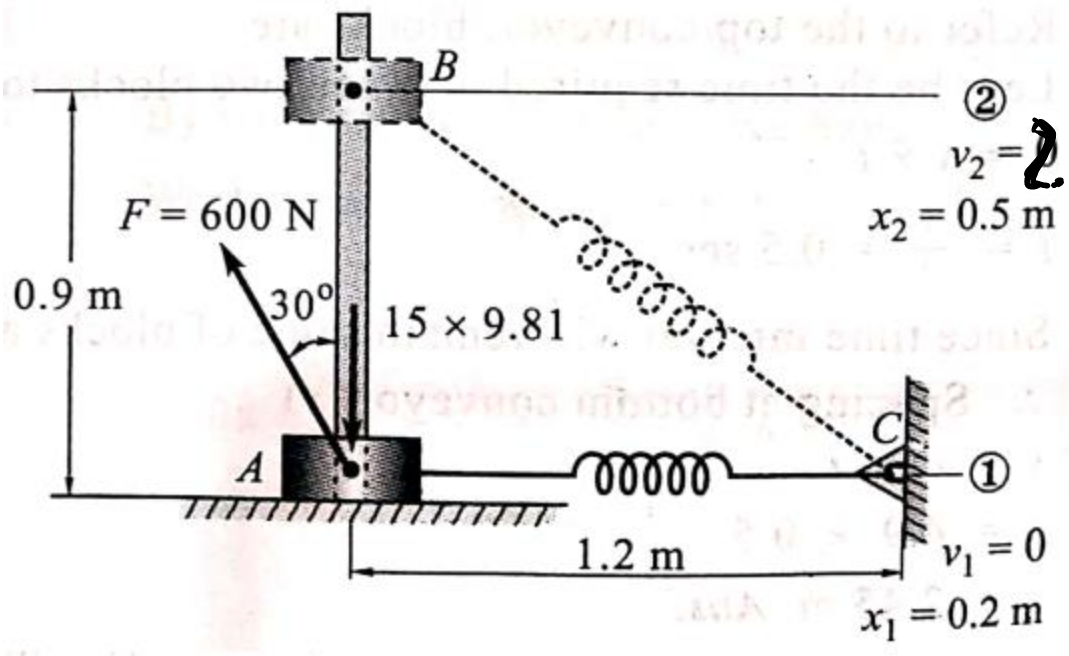
$V_1 = 0$

$x_1 = 1.2 - 1 = 0.2 \text{ m}$

$V_2 = ?$

$x_2 = BC - 1$

$= 1.5 - 1 = 0.5 \text{ m}$



By W.E. principle.

W.D = Change in K.E.

$$600 \cos 30° \times 0.9 - 15 \times 9.81 \times 0.9 + \frac{1}{2} \times 3000 (0.2^2 - 0.5^2)$$

$$= \frac{1}{2} \times 15 \times V_2^2 - 0$$

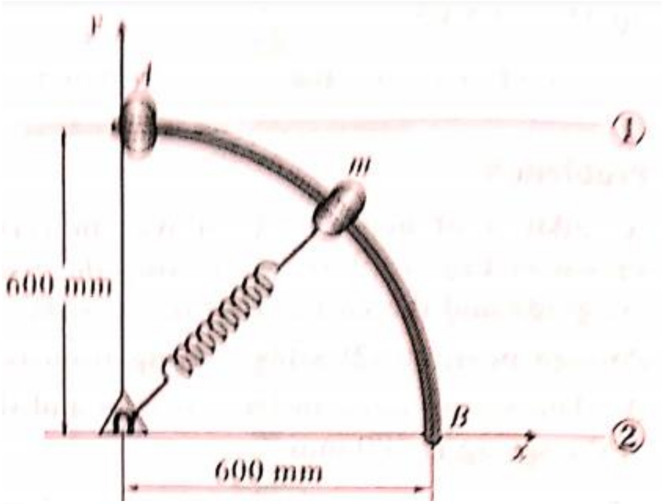
$V_2 = 1.64$ m/s

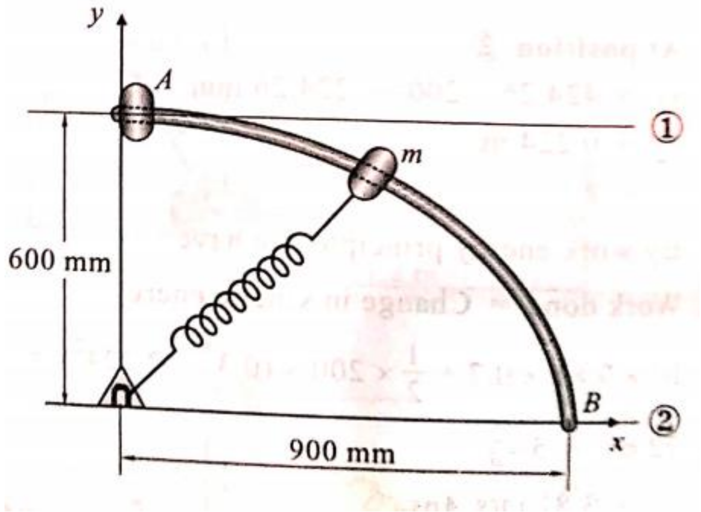
Problem

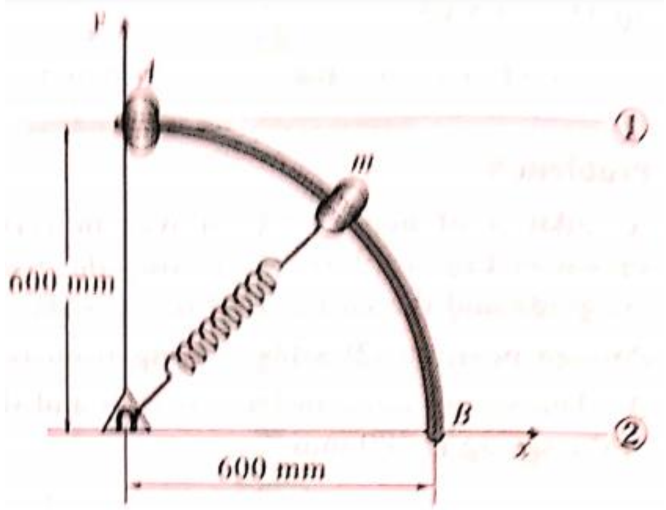
A mass m = 1.8 kg slides from rest at A along the frictionless rod bent into a quarter circle. The spring with modulus k = 16 N/m has an unstretched length of 400 mm.

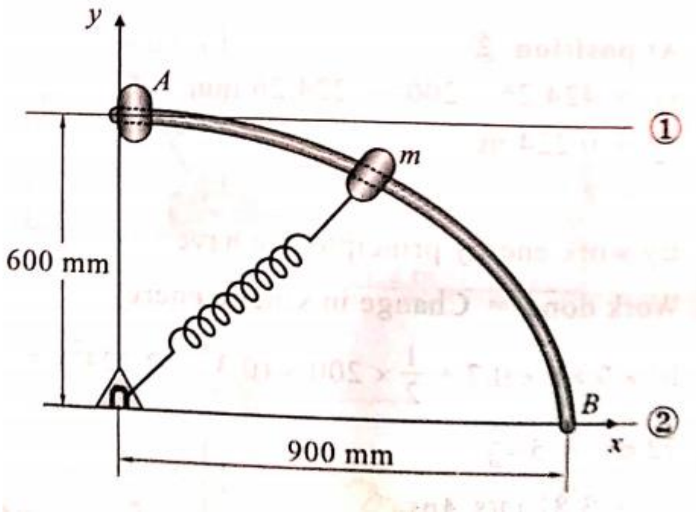
a. Determine the speed of m at B.

b. If the path is elliptical what is the speed at B.









I $V_1 = 0, \quad x_1 = 600 - 400 = 0.2\,m$

$V_2 = ? \quad x_2 = 600 - 400 = 0.2\,m$

W.E principle

$mgh + \frac{1}{2}k(x_1^2 - x_2^2) = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

$1.8 \times 9.81 \times 0.6 + \frac{1}{2} \times 16(0.2^2 - 0.2^2) = \frac{1}{2} \times 1.8 \times v_2^2$

$\Rightarrow V_2 = 3.43\,m/s$

II $V_1 = 0, \; x_1 = 600 - 400 = 0.2\,m$

$V_2 = ?, \; x_2 = 900 - 400 = 0.5\,m$

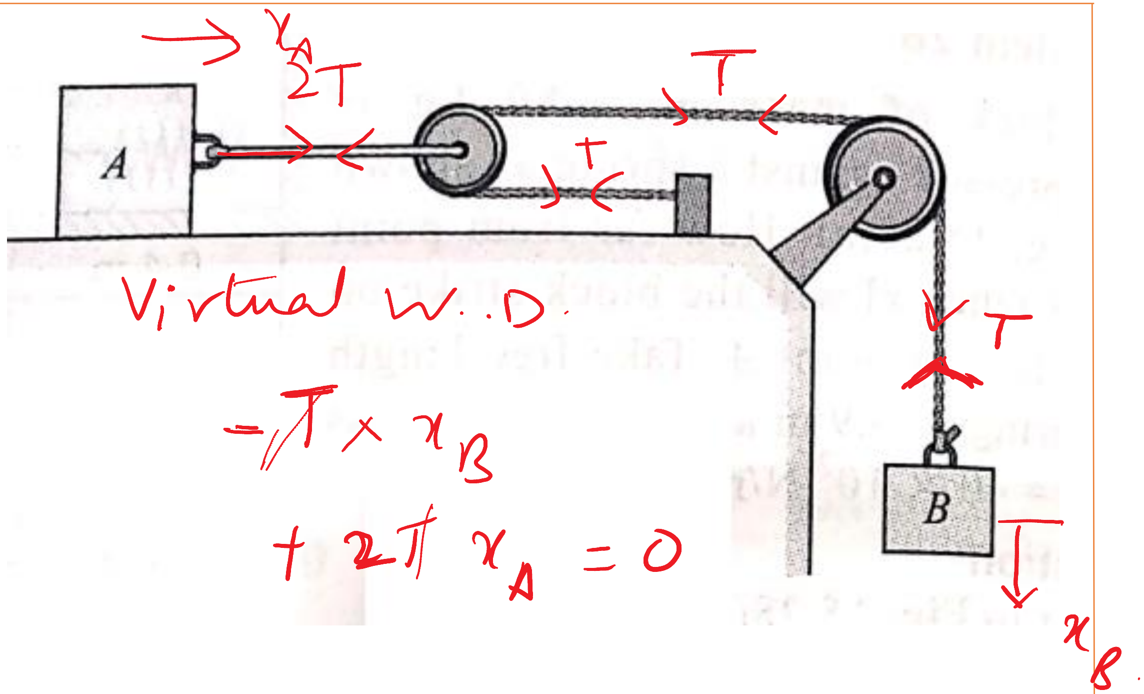
W.E. Principle.

$1.8 \times 9.81 \times 0.6 + \frac{1}{2} 16(0.2^2 - 0.5^2)$

$= \frac{1}{2} \times 1.8 \times v_2^2$

$\Rightarrow V_2 = 3.15\,m/s$

2 blocks A and B having masses 10 kg and 5 kg resp. are connected with cord and pulley system as shown in figure. Determine the velocity of each block when the system is started from rest and block B gets displaced by 2 m. consider $\mu_k = 0.2$ between block A and horizontal surface.



Virtual W.D.

$$-T \times x_B + 2T\, x_A = 0$$

$$2x_A = x_B$$

$$2v_B = v_A$$

Solution

Kinematic relation

$2T x_A - T x_B = 0$

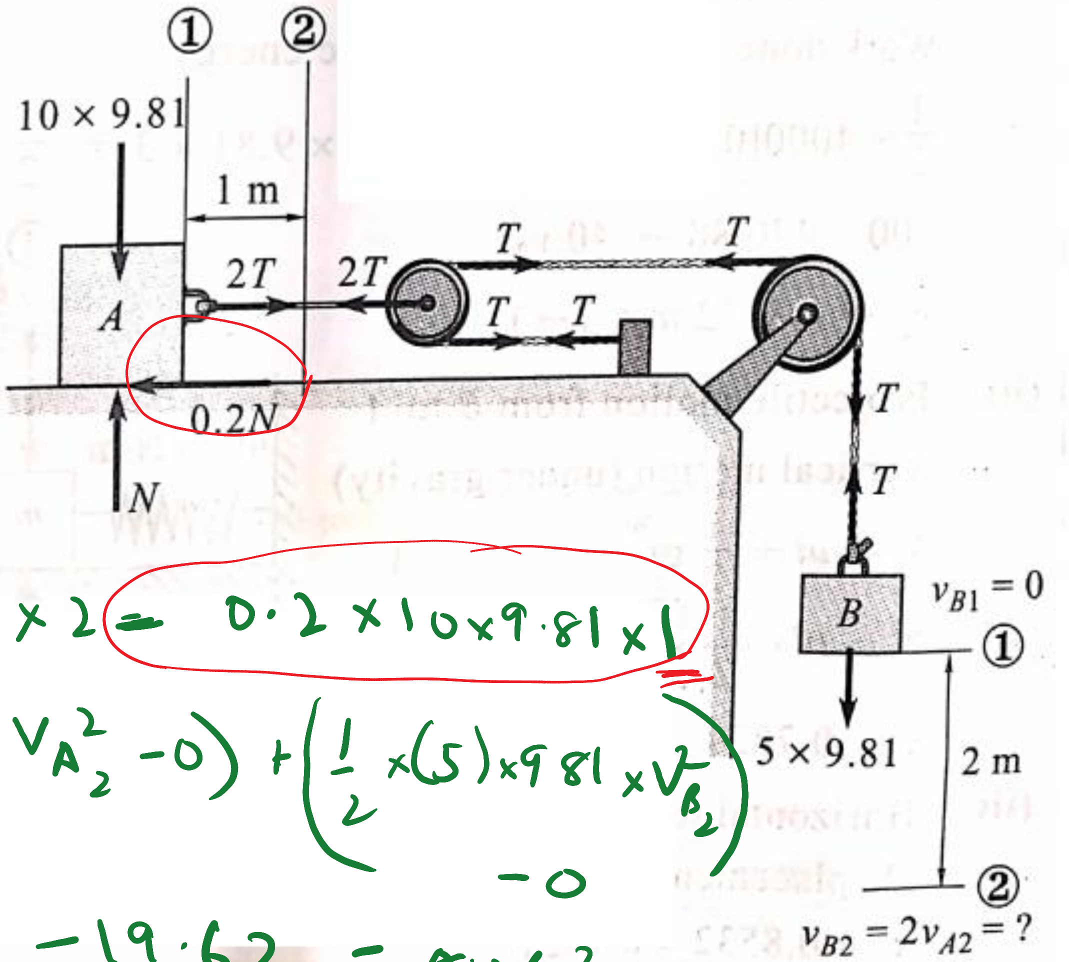
$2x_A = x_B$

$2v_A = v_B$

W.E. Principle

WD = Change in K.E.

$(P.E.)_B$ $(\text{Friction})_A$



$5\times 9.81\times 2 - 0.2\times 10\times 9.81\times 1$

$= \left(\frac{1}{2}\times 10\times v_{A_2}^2 - 0\right) + \left(\frac{1}{2}\times(5)\times 9.81\times v_{B_2}^2 - 0\right)$

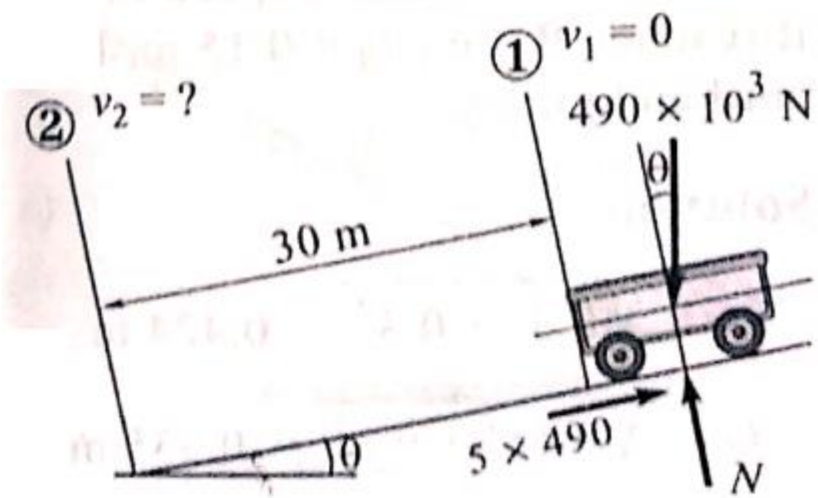
$98.1 - 19.62 = 5 v_{A_2}^2 + 2.5 v_{B_2}^2$

$v_{A_2} = 2.287$ m/s

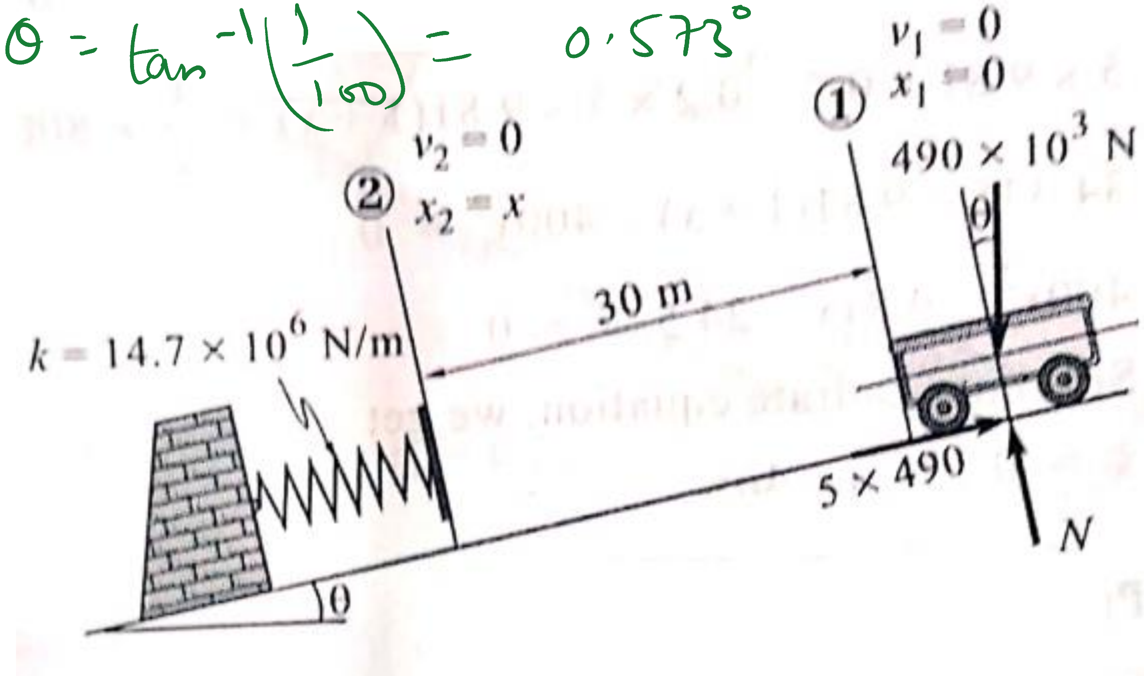
$v_{B_2} = 4.575$ m/s

Problem

A wagon weighing 490 kN starts from rest, runs 30 m down on the inclined surface having slope 1 in 100. and strikes a post as shown in fig. if the rolling resistance of the track is 5 N/kN, find the velocity of wagon when it strikes the post. If the impact is to be cushioned by means of a bumper spring having k = 14.7 kN/mm, determine the maximum compression of the bumper spring.



$\theta = \tan^{-1}\left(\frac{1}{100}\right) = 0.573^\circ$



Velocity of wagon when it strikes post.

W.E $\rightarrow$ WD = change in K.E

$490 \times 10^3 \sin\theta \times 30 - 5 \times 490 \times 30$

$= \frac{1}{2} \times \frac{490}{9.81} \times V_2^2 - 0$

$\Rightarrow V_2 = 1.715$ m/s //

Let x be the max spring deflection

W.E $\rightarrow$ WD = change in K.E.

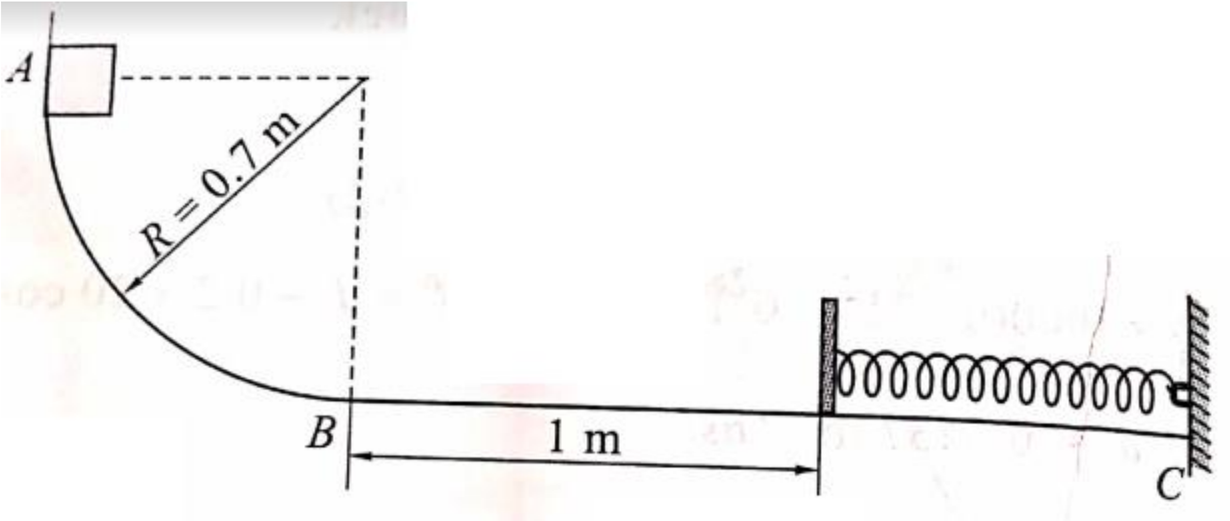
$\frac{1}{2} \times 14.7 \times (0^2 - x^2) \times 10^6$

$+ 490 \times 10^3 \times \sin\theta \times 30 - 5 \times 490 \times 30$

$= 0 - 0$

$\Rightarrow x = 0.1$ m //

Problem



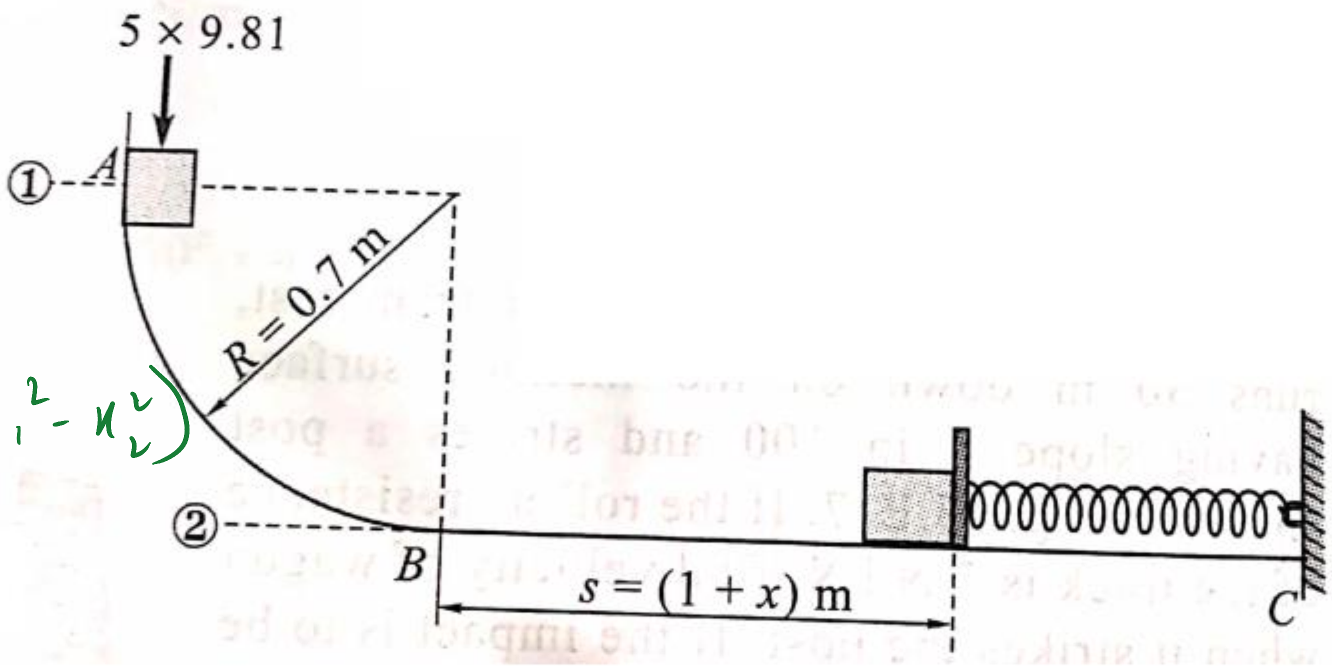
A body of mass M is released from rest at A. AB is a smooth surface. For BC μ= 0.2. k for spring is 0.8 N/m. Determine the maximum compression for spring. AB is a quarter circle of R = 0.7 m.

W.E. Principle

W.D = Change in K.E.

$mgh - \mu N(1+x) + \frac{1}{2}k(x_1^2 - x_2^2)$

$= 0 - 0$



$5 \times 9.81 \times 0.7 - 0.2 \times 5 \times 9.81(1+x) + \frac{1}{2} \times 800(0^2 - x^2) = 0$

$x = 0.236\ m$

Module 5

Kinetics of particle		**9**	**CO5**
5.1	Force and acceleration: Introduction to basic concepts, equations of dynamic equilibrium, Newton's second law of motion (only rectilinear motion)		
5.2	Work energy principle		
5.3	Impulse and Momentum: Principle of linear impulse and momentum, law of conservation of momentum, impact and collision, direct central and oblique central impact.		

Impulse Momentum principle

NSL

$$F = ma$$

$$F = m\frac{dv}{dt} \qquad \left(\because a = \frac{dv}{dt}\right)$$

$$F\,dt = m\,dv$$

Integrating both sides

$$\therefore \int_{t_1}^{t_2} F\,dt = \int_{v_1}^{v_2} m\,dv$$

$$\therefore \int_{t_1}^{t_2} F\,dt = mv_2 - mv_1$$

The term $\int_{t_1}^{t_2} F\,dt$ is called *impulse* and its unit is N.sec.

The term mass × velocity is called *momentum*.

So, we have

Impulse = Final momentum – Initial momentum

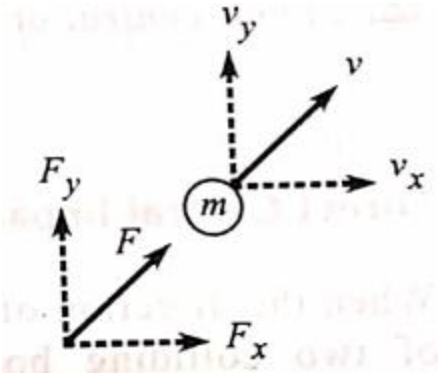
Impulse of a force

- When a large force acts over a small finite period the force is called impulse force.
- When an impulse force acts on a system, non impulsive forces like weight of the body are neglected.

Component form:

$$\int_{t_1}^{t_2} F_x\,dt = mv_{x_2} - mv_{x_1}$$

$$\int_{t_1}^{t_2} F_y\,dt = mv_{y_2} - mv_{y_1}$$



Principle of conservation of momentum

- If resultant force is zero in a particular system, then the impulse momentum equation reduces to initial momentum = final momentum.

$$\int_{t_1}^{t_2} F\,dt = mv_2 - mv_1$$

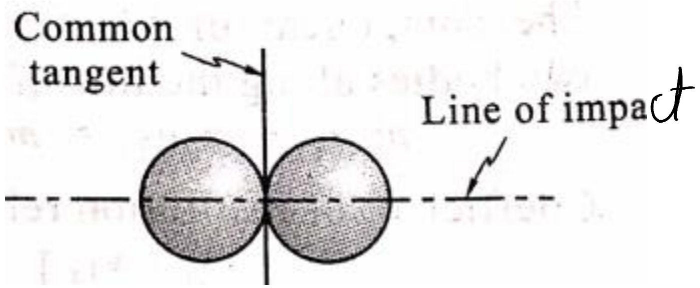
$0 = mv_2 - mv_1 \Rightarrow mv_2 = mv_1$.

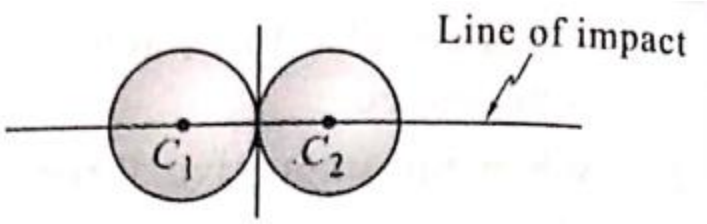
- It happens in many of the force system which comprises of only action and reaction forces. (e.g. gun and shell, man jumping off a boat)

Impact

- Phenomenon of collision of two bodies, which occurs for a very small interval of time and during which two bodies exert very large force on each other, is called an impact.

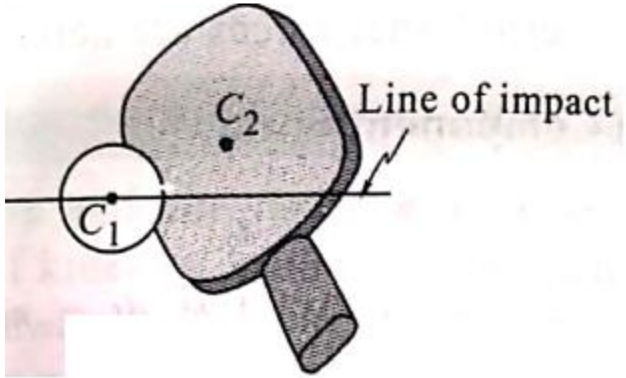
Line of impact: It is the common normal to the surfaces of two bodies in contact during impact.





Central Impact

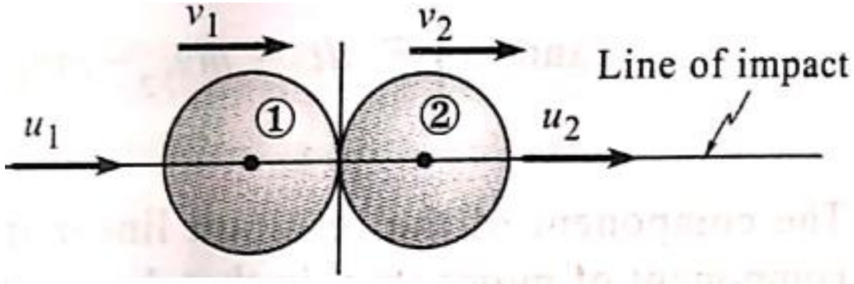
(mass centres lie on line of impact)



Non Central Impact

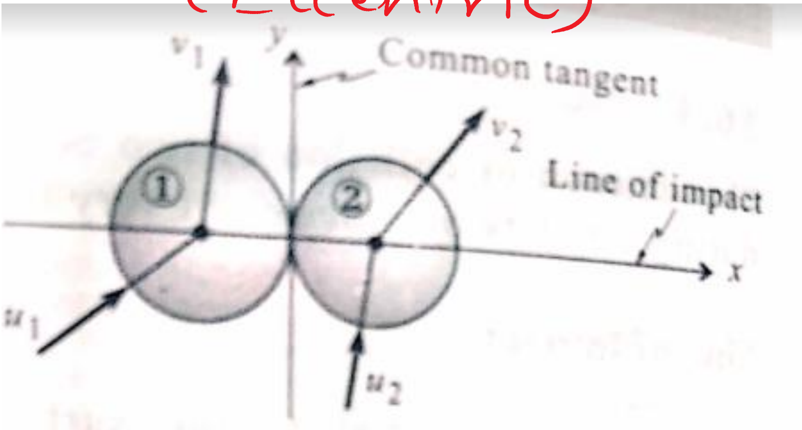
(Eccentric)

(mass centres not on line of impact)



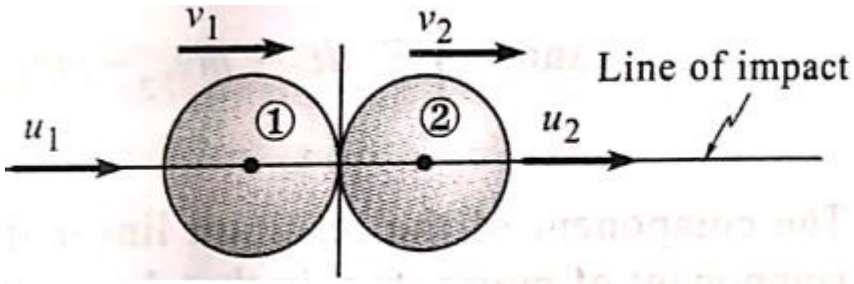
Direct Central Impact

(Direction of motion of mass centres along line of impact)

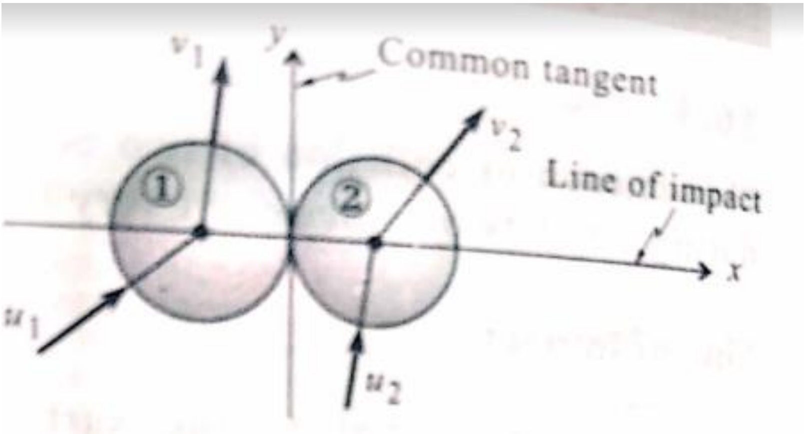


Oblique Central Impact

Direction of motion of mass centres at some angle with line of contact.



Direct Central Impact



Oblique Central Impact

Component of total momentum along line of impact is conserved

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$m_1 u_{1x} + m_2 u_{2x} = m_1 v_{1x} + m_2 v_{2x}$

Coeff of restitution

$e = -\left[\frac{v_2 - v_1}{u_2 - u_1}\right]$

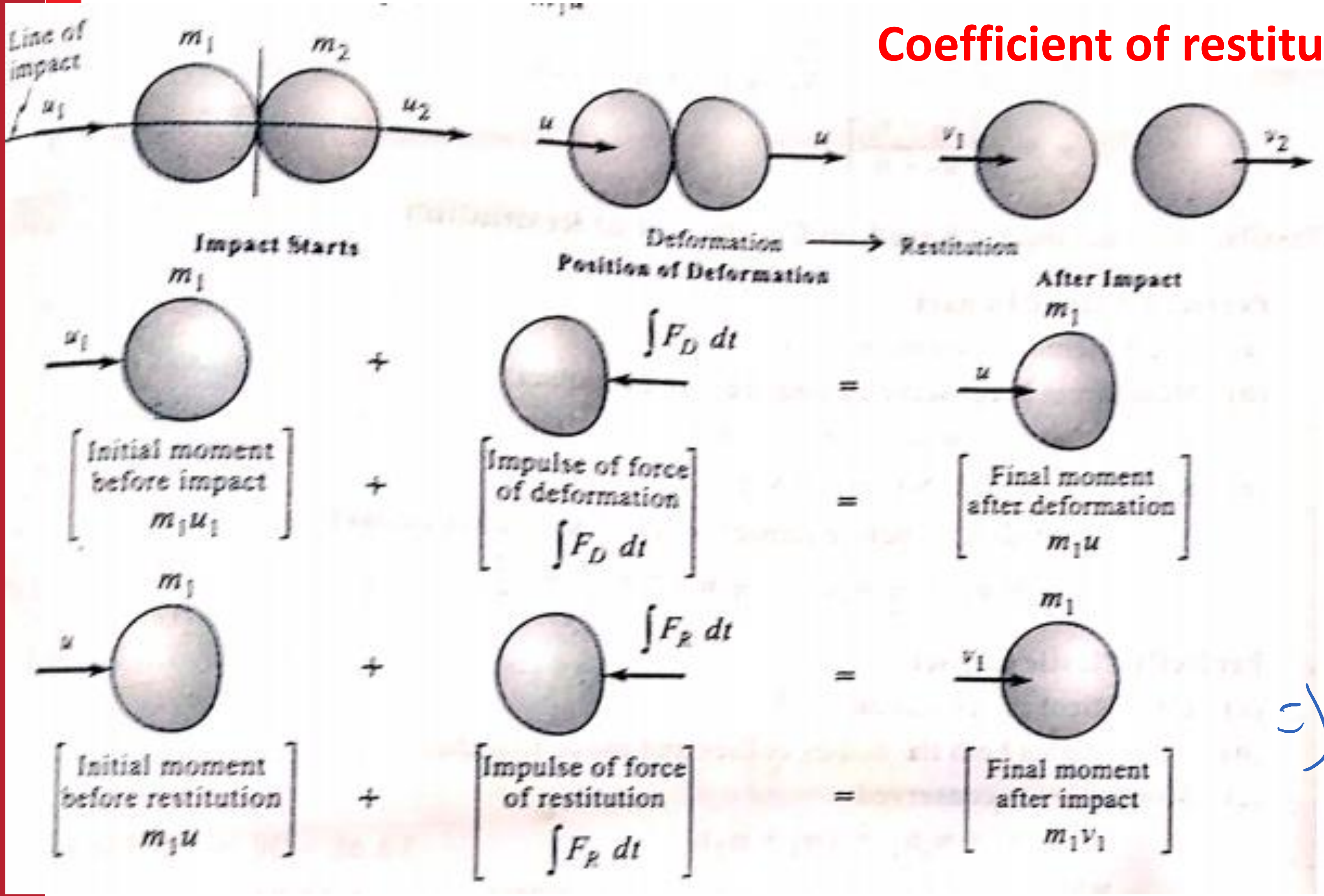
$e = -\left[\frac{v_{2x} - v_{1x}}{u_{2x} - u_{1x}}\right]$

Momentum along tgt is conserved

$m_1 u_{1y} = m_1 v_{1y}$ $\quad u_{1y} = v_{1y}$

$m_2 u_{2y} = m_2 v_{2y}$ $\quad u_{2y} = v_{2y}$

Coefficient of restitution (e)



$$m_1u_1 - \int F_D\,dt = m_1u$$

$$\Rightarrow \int F_D\,dt = m_1u_1 - m_1u$$

$$= m_1u - m_1v_1$$

$$\frac{\int F_R\,dt}{\int F_D\,dt} = \frac{m_1(u-v_1)}{m_1(u_1-u)} = \frac{u-v_1}{u_1-u} = e.$$

||ly for m_2

$$\frac{\int F_R\,dt}{\int F_D\,dt} = \frac{v_2-u}{u-u_2}$$

eliminating u

$$e = \frac{v_2-v_1}{u_1-u_2} = \frac{\text{Velocity of separation}}{\text{Velocity of approach}} = -\left[\frac{v_2-v_1}{u_2-u_1}\right]$$

(coeff of restitution)

Classification of impact based on e

Perfectly Elastic Impact:

- $e = 1$
- Momentum is conserved along line of impact.

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

- KE is conserved.

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

Semi elastic impact

$0<e<1$

Classification of impact based on e

Perfectly Plastic Impact:

- $e = 0$
- After impact both the bodies move together
- Momentum is conserved.

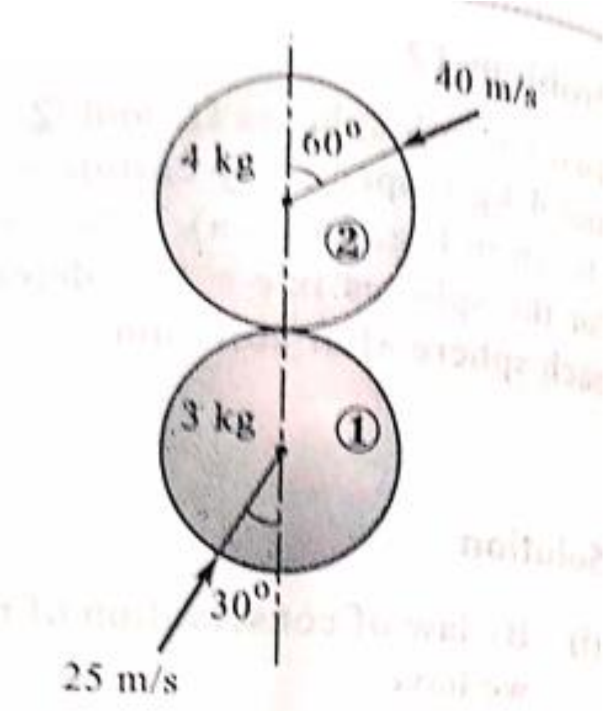
$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) u$$

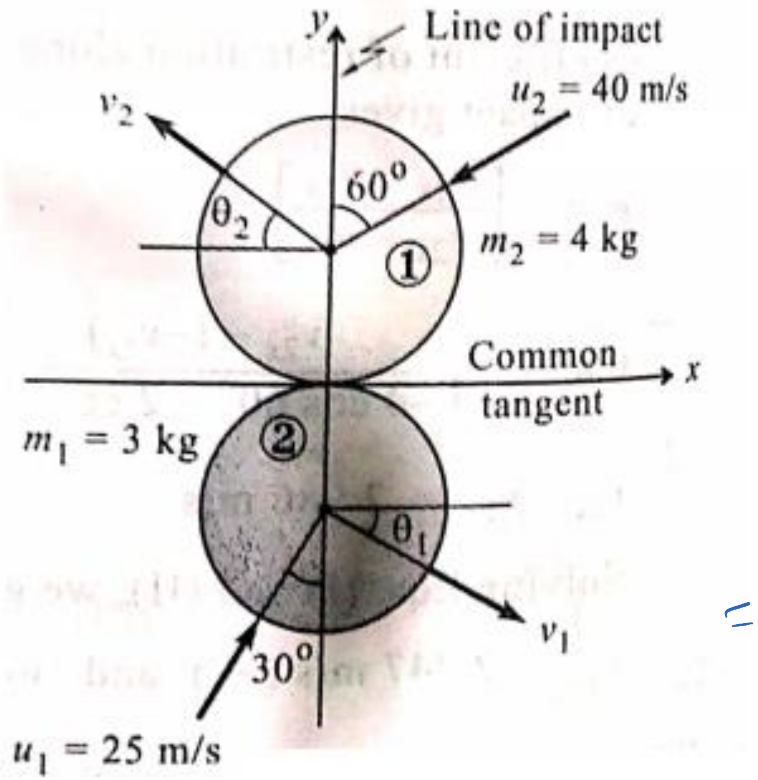
- KE is not conserved. There is loss of KE during impact.

$$\text{Loss of K.E} = \left(\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2\right) - \left(\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2\right)$$

Problem

2 smooth balls of mass 3 kg and 4 kg are moving with velocities 25 m/s and 40 m/s resp at an angle of 30o and 60o with vertical as shown. If the coefficient of restitution between them is 0.8, find the magnitude and direction of velocities of these balls after impact.





By Law of conservation of momentum along line of impact

$$m_1 u_{1y} + m_2 u_{2y} = m_1 v_{1y} + m_2 v_{2y}$$

$$3 \times 25 \cos 30 + 4 \times (-4 \cos 60^\circ) = 3(-v_{1y}) + 4(v_{2y})$$

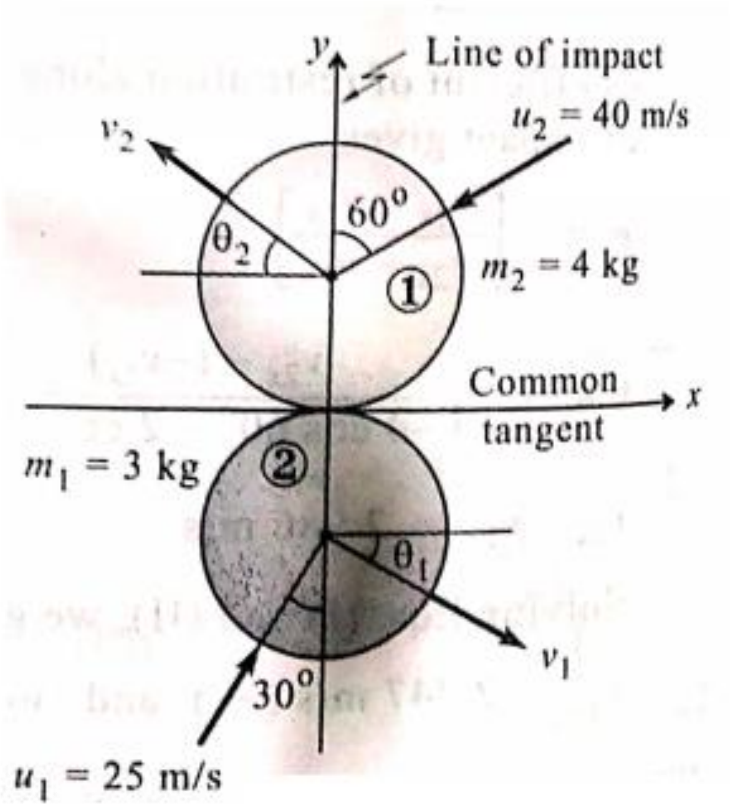
$$-3v_{1y} + 4v_{2y} = -15.05$$

$$e = -\left[\frac{v_{2y} - v_{1y}}{u_{2y} - u_{1y}}\right] \therefore 0.8 = -\left[\frac{v_{2y} - (-v_{1y})}{-40 \cos 60 - 25 \cos 30}\right]$$

$$\Rightarrow v_{1y} + v_{2y} = 33.33 \text{ m/s}$$

$$v_{1y} = 21.19 \text{ m/s } (\downarrow)$$

$$v_{2y} = 12.13 \text{ m/s } (\uparrow)$$



Component of velocity before & after impact is conserved along common tangent.

$V_{1x} = u_{1x} = 25 \sin 30^\circ = 12.5$ m/s (→)

$V_{2x} = u_{2x} = 40 \sin 60^\circ = 34.64$ m/s (←)

$$V_1 = \sqrt{V_{1x}^2 + V_{1y}^2} = 24.60 \text{ m/s}$$

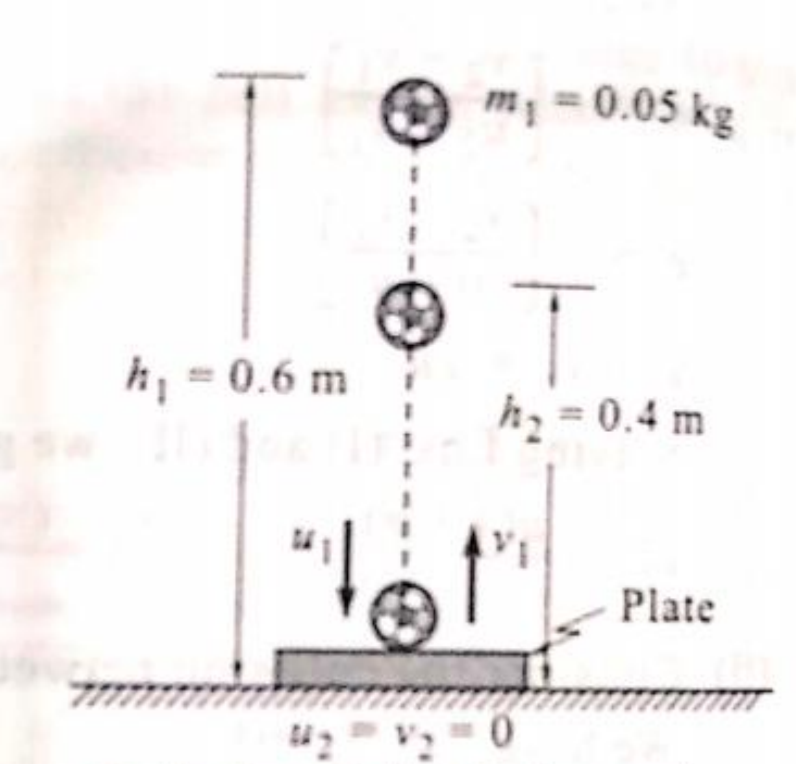
$$\theta_1 = \tan^{-1}\left(\frac{V_{1y}}{V_{1x}}\right) = 59.46^\circ$$

$$V_2 = \sqrt{V_{2x}^2 + V_{2y}^2} = 36.7 \text{ m/s}$$

$$\theta_2 = \tan^{-1}\left(\frac{V_{2y}}{V_{2x}}\right) = 19.30^\circ$$

Problem

A 50 gm ball is dropped from a height of 600 mm on a small plate as shown in figure. It rebound to a height of 400 mm when the plate directly rests on the ground and to a height of 250 mm when a foam rubber mat is placed between the plate and the ground. Determine the coefficient of restitution between the plate and the ball and mass of the plate.



(velocity of plate before and after impact will be zero) vel. of plate

(1) When plate is on ground, $\underbrace{u_2 = v_2}_{}= 0$

② → Plate
① → ball

$u_1 = \sqrt{2gh_1}\ (\downarrow)$

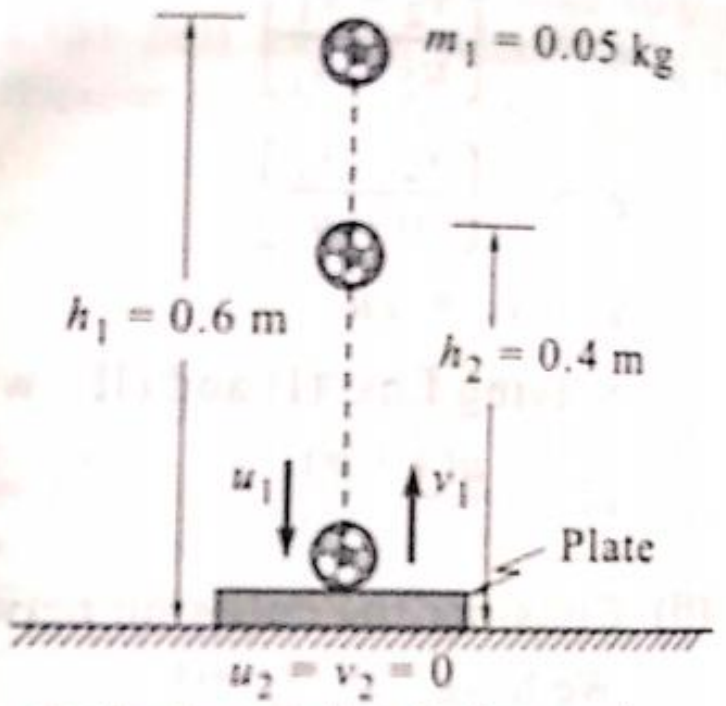
$= \sqrt{2 \times 9.81 \times 0.6} = 3.43\ m/s\ (\downarrow)$

$v_1 = \sqrt{2gh_2}\ (\uparrow)$

$= \sqrt{2 \times 9.81 \times 0.4} = 2.8\ m/s\ (\uparrow)$

$e = -\left[\dfrac{v_2 - v_1}{u_2 - u_1}\right] = -\left[\dfrac{0 - 2.8}{0 - (-3.43)}\right]$

$= 0.816$ //



(velocity of plate before and after impact will be zero)

(2) Plate is on rubber foam mat.

$u_1 = \sqrt{2gh_1} = 3.43\ m/s\ (\downarrow)$

$v_1 = \sqrt{2gh_2} = \sqrt{2g \times 0.25} = 2.215\ m/s\ (\uparrow)$

$e = -\left[\frac{v_2 - v_1}{u_2 - u_1}\right]$

$0.816 = -\left[\frac{-v_2 - 2.215}{0 - (-3.43)}\right]$

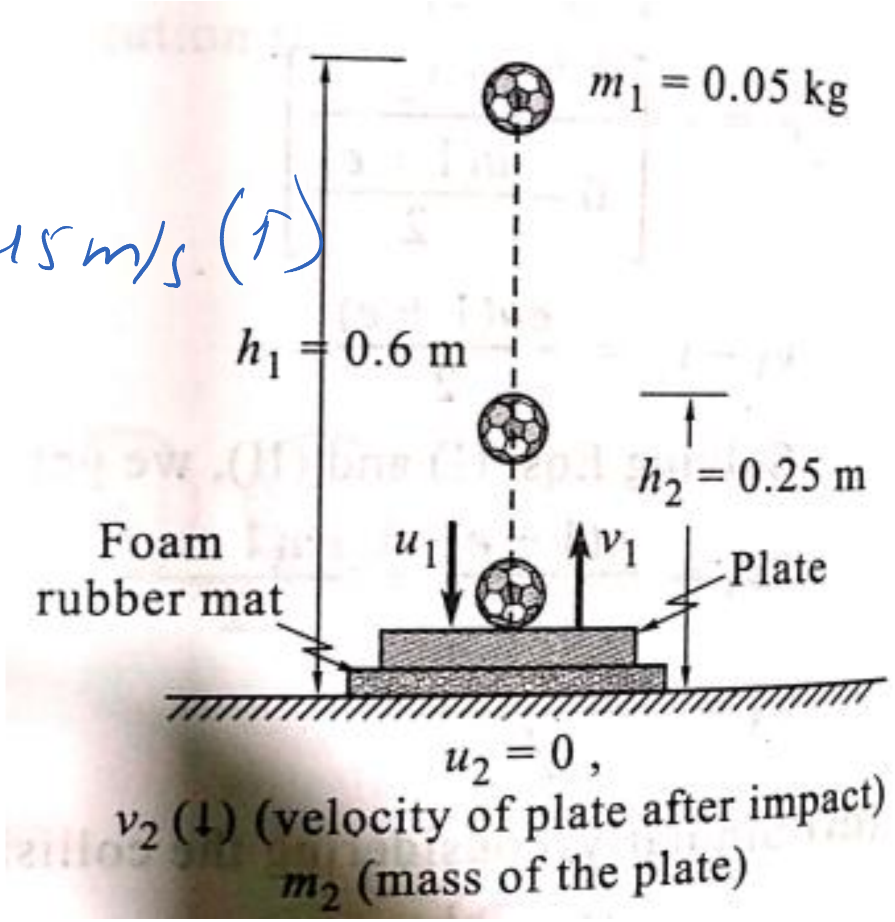
$v_2 = 0.584\ m/s\ (\downarrow)$

By law of cons. of momentum.

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

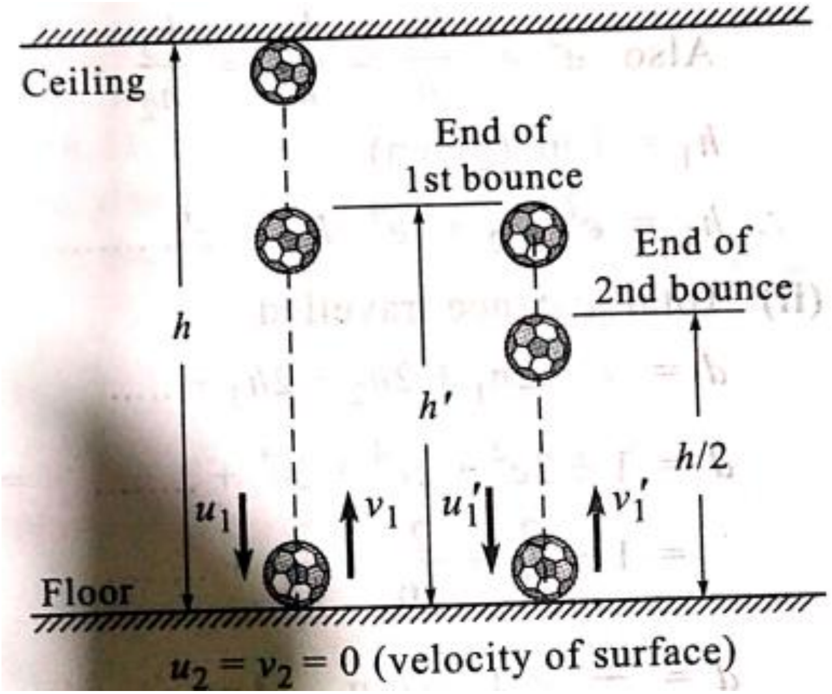
$0.05 \times (-3.43) + m_2(0) = 0.05 \times 2.215 + m_2(-0.584)$

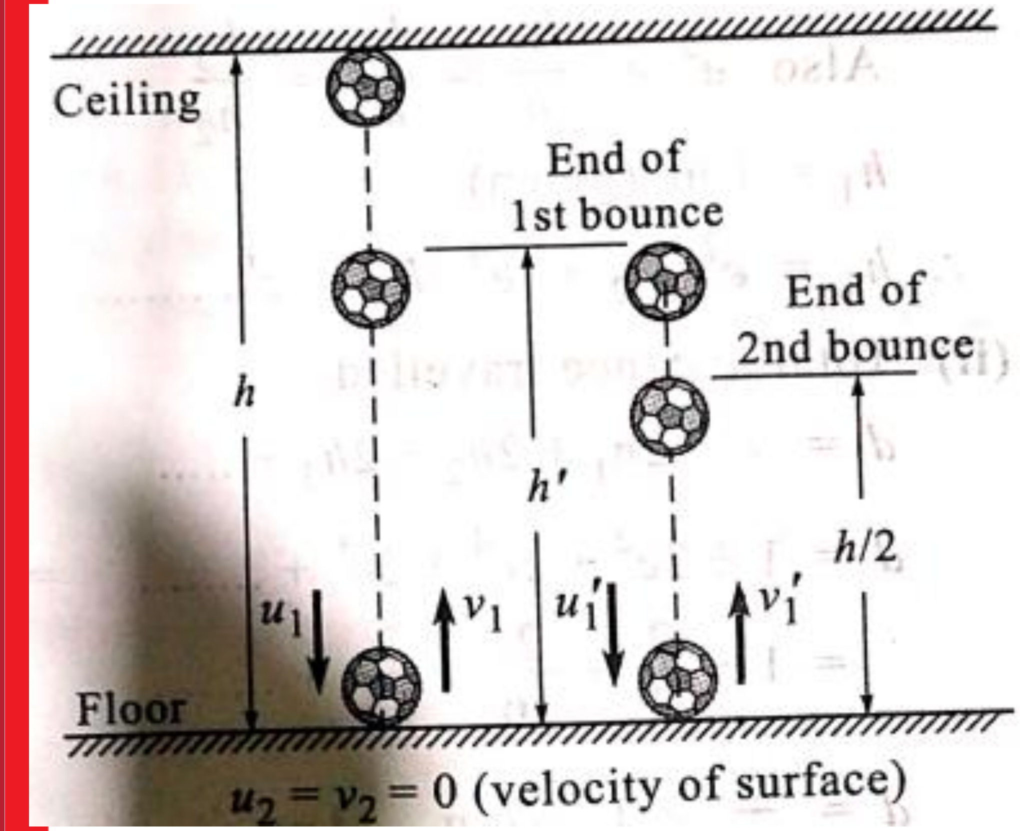
$m_2 = 0.483\ kg$.



Problem

A heavy elastic ball drops from the ceiling of a room and after rebounding twice from the floor reaches a height equal to one half of the height of the ceiling. Find the coefficient of restitution.





1st bound.

$$e = -\left[\frac{v_2 - v_1}{u_2 - u_1}\right]$$

$$= -\left[\frac{0 - \sqrt{2gh_1'}}{0 - (-\sqrt{2gh})}\right]$$

$$e^2 = h'/h \qquad h' = he^2.$$

2nd bounce.

$$e = -\left[\frac{v_2 - v_1'}{u_2 - u_1'}\right]$$

$$= -\left[\frac{0 - \sqrt{2gh/2}}{0 - \sqrt{2gh'}}\right] = \sqrt{\frac{1}{2e^2}}$$

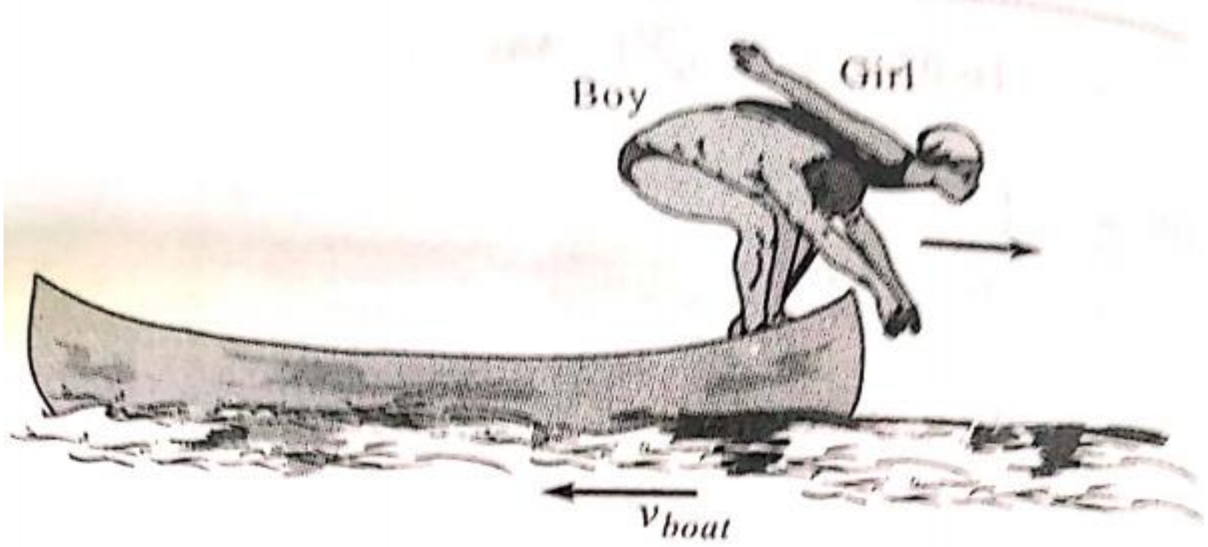
$$e^2 = \frac{1}{2e^2} \qquad e^4 = \frac{1}{2} \quad e = 0.841$$

Problem

A boy of mass 60 kg and girl of mass 50 kg dive off the end of a boat of mass 160 kg with a horizontal velocity of 2 m/s relative to the boat as shown in the figure. Considering the boat to be initially at rest, find its velocity just after

a. Both the boy and girl dive off simultaneously

b. The boy dives first followed by girl.



When both dive off simultaneously

Vel. of boy & girl relative to boat is 2 m/s

$$V_{boy/boat} = V_{boy} - V_{boat}$$

$$2 = V_{boy} - (-V_{boat})$$

$$V_{boy} = 2 - V_{boat}$$

$$V_{girl} = 2 - V_{boat}$$

By Conservation of momentum principle

Initial mom = final mom.

$$0 = 60(2 - V_{boat}) + 50(2 - V_{boat}) + 160 \times (-V_{boat})$$

$$V_{boat} = 1.227 \text{ m/s } (\leftarrow)$$

Boy dives first followed by girl.

→ When boy dives & girl is on boat

By cons. of mom.

Initial mom = final momentum

$$0 = 60(2 - V_{boat}) + 160(-V_{boat})$$

$$V_{boat} = 0.5455 \text{ m/s } (\leftarrow)$$

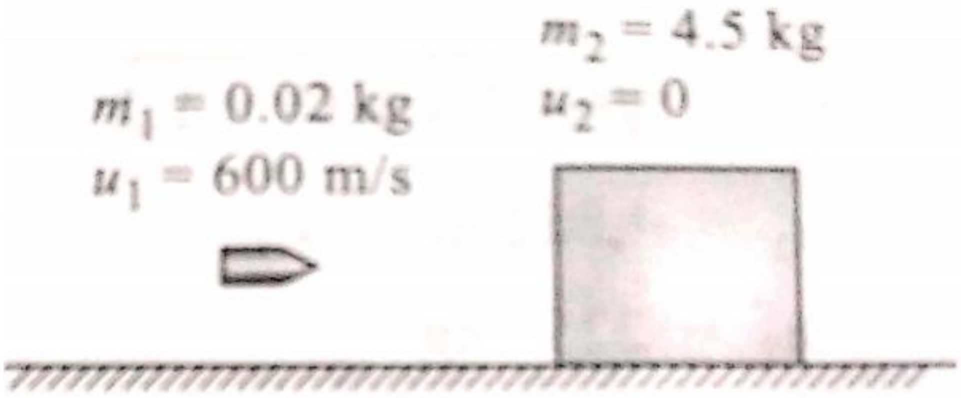
→ Later when girl jumps

By cons. of mom

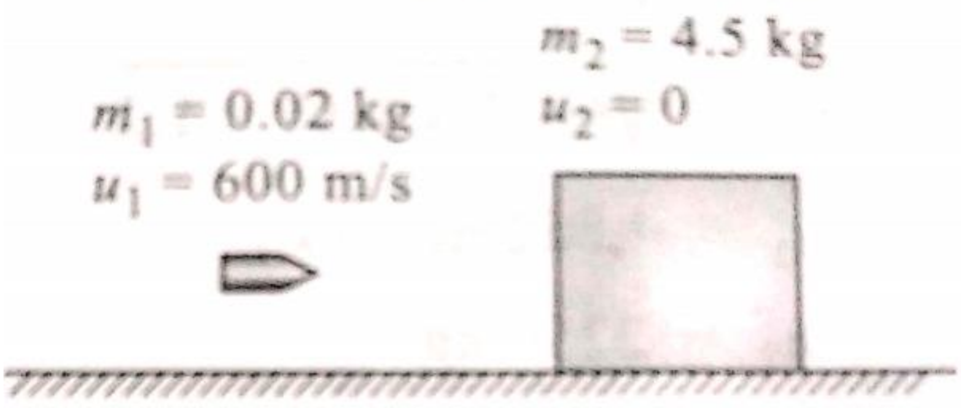
$$160(-0.5455) = 50(2 - V_{boat}) + 160(-V_{boat})$$

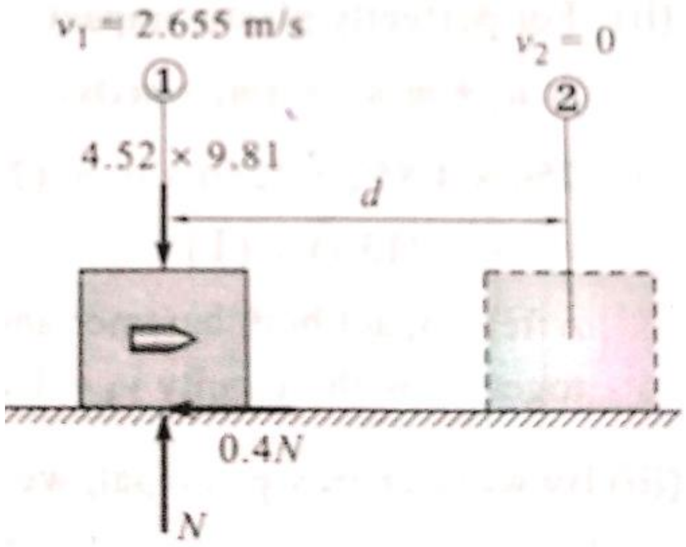
$$V_{boat} = 0.8918 \text{ m/s } (\leftarrow)$$

Problem



A 20 gm bullet is fired with a velocity of magnitude 600 m/s into a 4.5 kg block of wood which is stationary as shown in figure. Knowing that the coefficient of kinetic friction between the block and the floor is 0.4, determine a. how far the block will move and b. percentage of initial energy lost in friction between the block and the floor.





Law of conservation of momentum

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) V_1$$

$$0.02 \times 600 + 0 = (4.52) V_1$$

$$V_1 = 2.655 \text{ m/s } (\rightarrow)$$

W.E. principle

W.D = Ch. in K.E.

$$-0.4 \times 4.52 \times 9.81 \times d = 0 - \frac{1}{2} \times 4.52 \times 2.655^2$$

$$d = 0.9 \text{ m}$$

$$\text{Initial K.E.} = \frac{1}{2} \times 0.02 \times 600^2$$

$$= 3600 \text{ J}$$

Energy lost in friction

$$= 0.4 \times 4.52 \times 9.81 \times 0.9$$

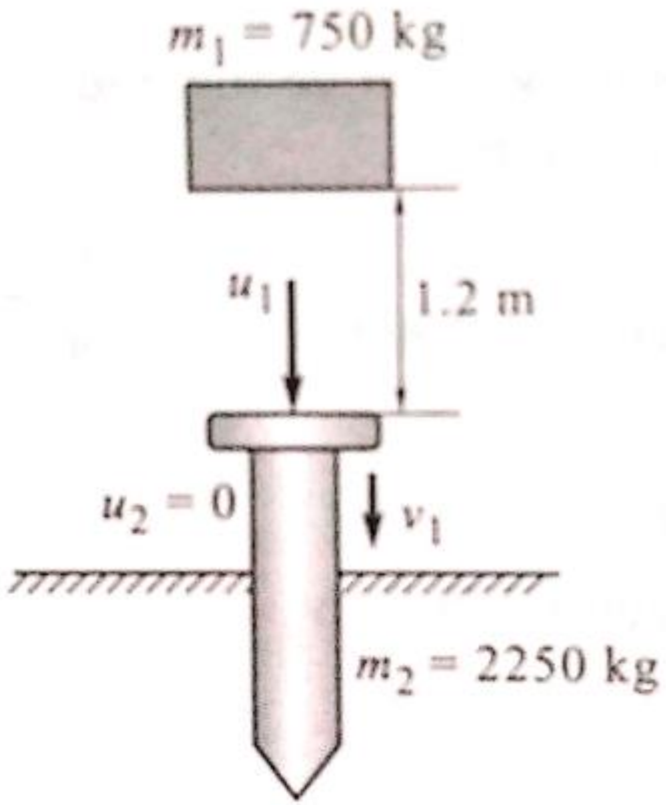
$$= 15.93 \text{ J}$$

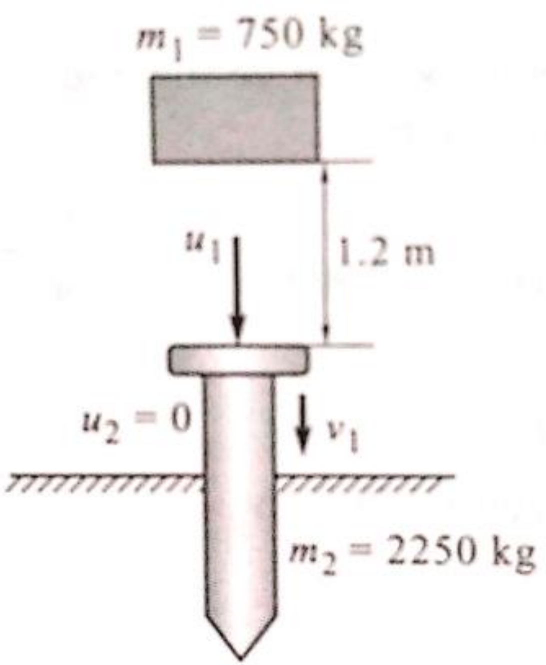
$$\% \text{ loss} = 15.93/3600$$

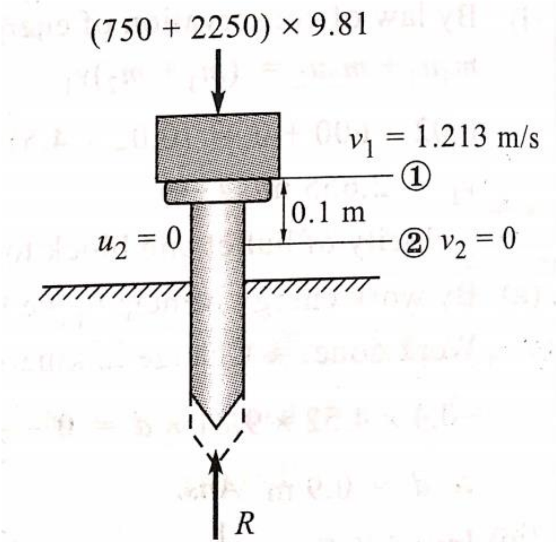
$$= 0.44\%$$

Problem

A 750 kg hammer of a drop hammer pile driver falls from a height of 1.2 m onto the top of a pile as shown. The pile is driven 100 mm into the ground. Assume perfectly plastic impact, determine the average resistance of the ground to penetration. Assume mass of the pile as 2250 kg.







W.E principle.

W.D = Change in K.E.

$(750+2250)\times 9.81\times 0.1 - R\times 0.1 = 0 - \frac{1}{2}\times(750+2250)\times 1.213^2$

$R = 51482.3\ N(\uparrow)$

$u_1 = \sqrt{2gh_1}$

$= \sqrt{2g\times 1.2}$

$= 4.852\ m/s\ \downarrow$

Cons. of momentum

$m_1u_1 + m_2u_2 = (m_1+m_2)v_1$

$750\times 4.852 + 0 = (750+2250)v_1$

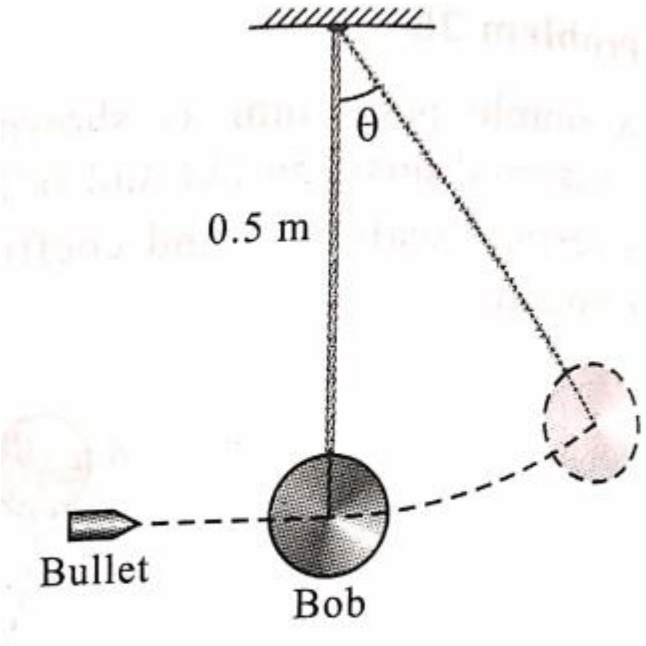
$v_1 = 1.213\ m/s\ (\downarrow)$

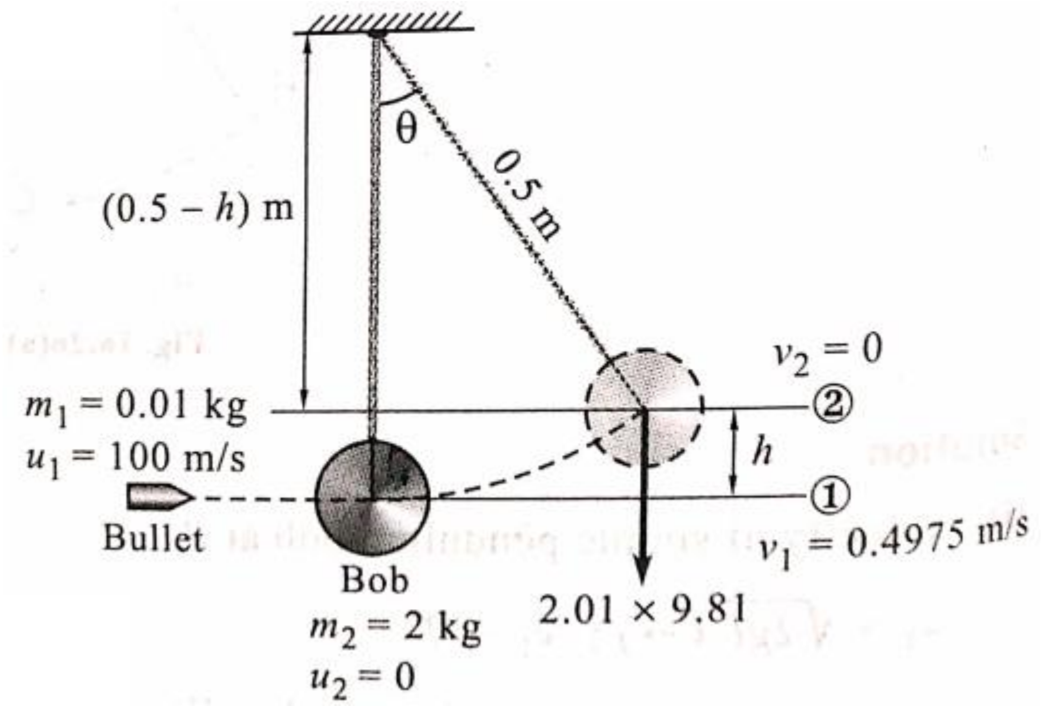
Problem

A bullet of mass 10 gm is moving with a velocity of 100 m/s and hits a 2 kg bob of a simple pendulum horizontally as shown. Determine the maximum angle through which the pendulum string 0.5 m long may swing if

a. the bullet gets embedded in the bob

b. The bullet escapes from the other end of the bob with a velocity of 10 m/s





CASE I: Bullet embedded in bob

Law of cons. of momentum.

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) V_1$$

$$0.01 \times 100 + 2 \times 0 = (0.01 + 2) V_1$$

$$V_1 = 0.4975 \text{ m/s}$$

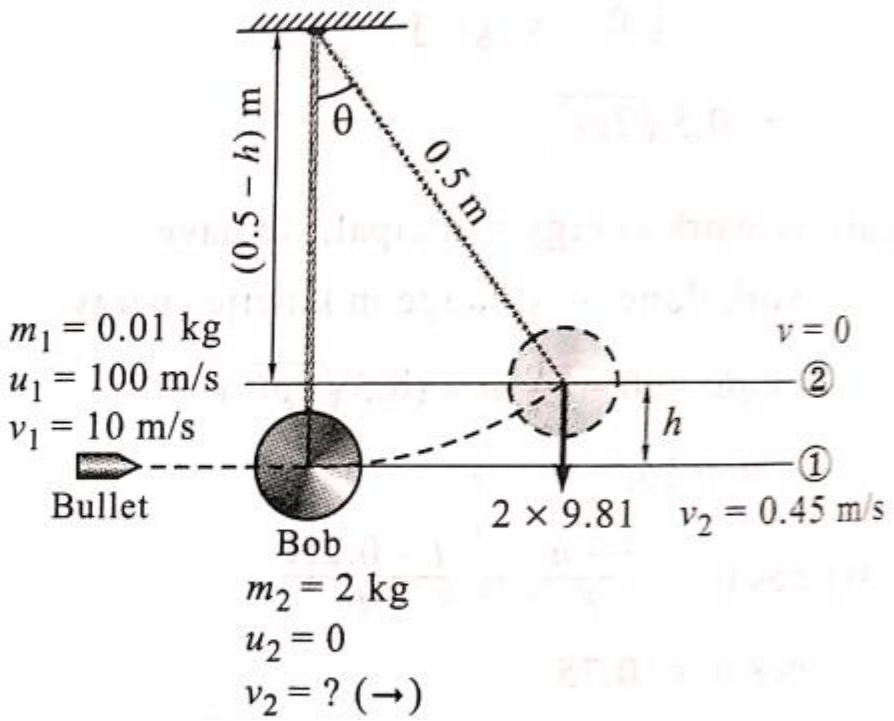
W.E. Principle.

$$-2.01 \times 9.81 \times h = 0 - \frac{1}{2} \times 2.01 \times 0.4975^2$$

$$h = 0.0127 \text{ m}$$

$$\cos\theta = \frac{0.5 - h}{0.5}$$

$$\theta = 12.94^\circ$$



WE principle:

$$-2 \times 9.81 \times h = 0 - \frac{1}{2} \times 2 \times 0.45^2$$

$$h = 0.01032 \text{ m}$$

$$\theta = \cos^{-1}\left(\frac{0.5 - 0.01032}{0.5}\right)$$

$$= 11.66^\circ$$

Case 2: Bullet escapes at 10 m/s.

Law of cons. of momentum ⇒

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 V_2$$

$$0.01 \times 100 + 2 \times 0 = 0.01 \times 10 + 2 \times V_2$$

$$V_2 = 0.45 \text{ m/s}$$

References for preparing this ppt:

Engineering Mechanics - Hibbeler, H. C.and Gupta

2. *Engineering Mechanics – N.H. Dubey*

3. *Web sources*