Equilibrium of Force System

Problems

Problem:

Block $P = 5$ kg and block Q of mass m kg is suspended through the chord is in the equilibrium position as shown in Fig. 3.2(a). Determine the mass of block Q.

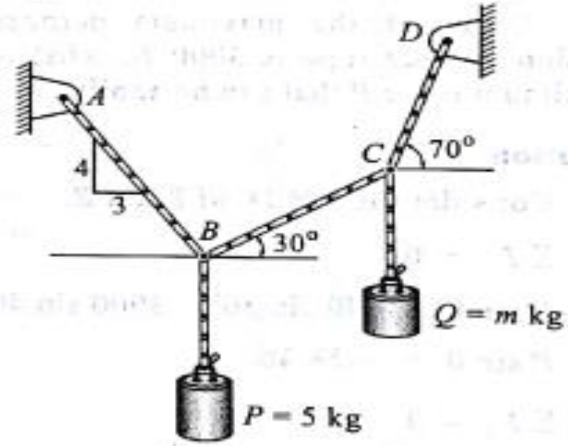


Fig. 3.2(a)

Solution

(i) Consider the F.B.D. of Point B.

(ii) By Lami's theorem,

$$\frac{5 \times 9.81}{\sin 96.87^\circ} = \frac{T_{AB}}{\sin 120^\circ} = \frac{T_{BC}}{\sin 143.13^\circ}$$

$$\therefore T_{AB} = 42.79 \text{ N}$$

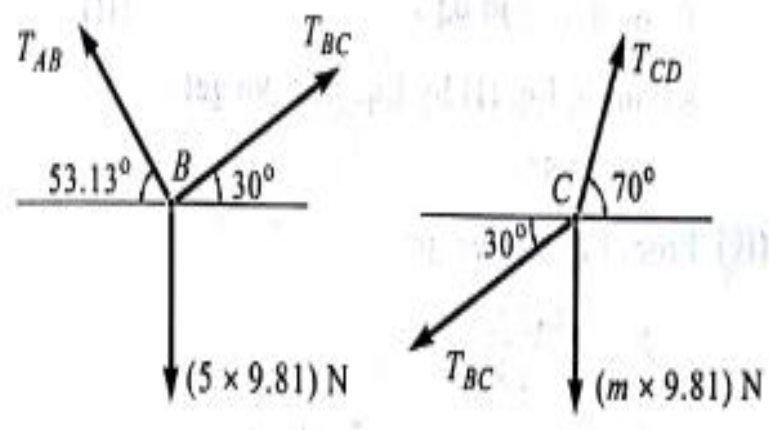
$$T_{BC} = 29.64 \text{ N}$$

(iii) Consider the F.B.D. of Point C.

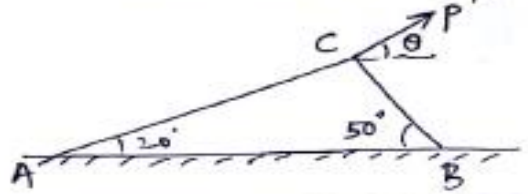
(iv) By Lami's theorem,

$$\frac{m \times 9.81}{\sin 140^\circ} = \frac{29.64}{\sin 160^\circ}$$

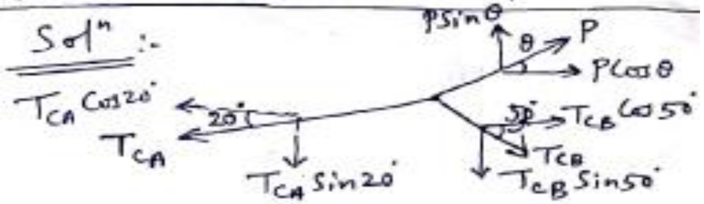
$\therefore m = 5.678$ kg ***Ans.***



Q. Two pipes are tied together at C. If max. permissible tension in each rope is 3.5 KN. What is the max. force (P) that can be applied & in what direction?



Sol^n :-



$\Sigma F_x = 0 \quad \therefore \quad P\cos\theta + T_{CB}\cos 50° - T_{CA}\cos 20° = 0$

$T_{CA} = T_{CB} = 3.5$ KN $\quad \therefore \quad P\cos\theta = 1.039$ — ①

$\Sigma F_y = 0 \quad \therefore \quad P\sin\theta - T_{CB}\sin 50° - T_{CA}\sin 20° = 0$

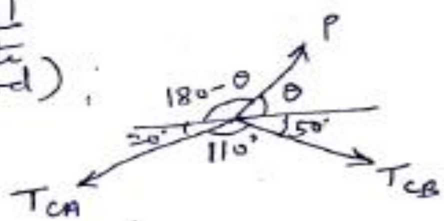
$\therefore \quad P\sin\theta = 3.878$ — ②

$T_{CA} = T_{CB} = 3.5$ KN

$\therefore \quad \frac{P\sin\theta}{P\cos\theta} = \frac{3.878}{1.039} \quad \therefore \quad \tan\theta = 3.732 \quad \therefore \quad \boxed{\theta = 75°}$

$\therefore \quad P\cos 75° = 1.039 \quad \therefore \quad \boxed{P = 4 \text{ KN}}$

By Lami's theorum; (Alternate Method).



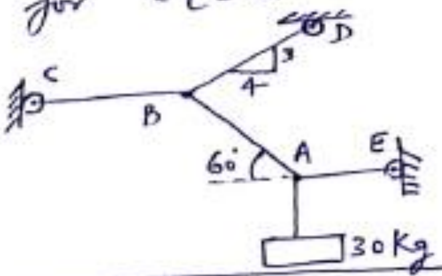
$$\frac{P}{\sin 110°} = \frac{3.5}{\sin(200-\theta)} = \frac{3.5}{\sin(50+\theta)}$$

$\therefore \quad \sin(200-\theta) = \sin(50+\theta)$

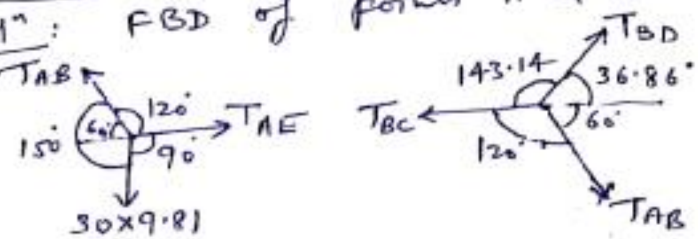
$\therefore \quad 200 - \theta = 50 + \theta \quad \therefore \quad \boxed{\theta = 75°}$

$\therefore \quad P = 3.5 \times \frac{\sin 110°}{\sin(200-75°)} = \boxed{4 \text{ KN}}$ Ans.

Q. A 30 Kg pipe is supported at A by a system of five chords. Determine the force in each chord for Eq≡m.



Sol^n: FBD of points A & B



For point A:
$$\frac{30\times 9.81}{\sin 120^\circ} = \frac{T_{AB}}{\sin 90^\circ} = \frac{T_{AE}}{\sin 150^\circ}$$

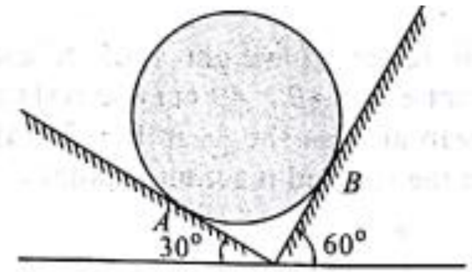
$\therefore\ T_{AB} = 339.82\,N \qquad T_{AE} = 169.91\,N$

For Point B:- $T_{BD} = 490.6\,N$ & $T_{BC} = 562.44\,N$

$$\frac{T_{AB}}{\sin 143.14} = \frac{T_{BD}}{\sin 120^\circ} = \frac{T_{BC}}{\sin 96.86^\circ}$$

Problem:

A cylinder of mass 50 kg is resting on a smooth surface which are inclined at 30° and 60° to horizontal as shown in Fig. 3.8(a). Determine the reaction at contact A and B.



Solution

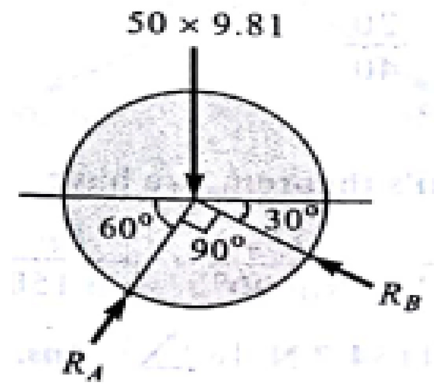
(i) Consider the F.B.D. of the cylinder.

(ii) By Lami's theorem, we have

$$\frac{50 \times 9.81}{\sin 90^\circ} = \frac{R_A}{\sin 120^\circ} = \frac{R_B}{\sin 150^\circ}$$

$$R_A = 424.79 \text{ N}$$

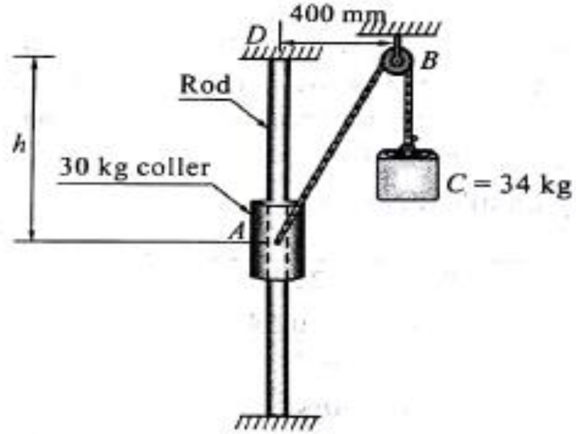
$$R_B = 245.25 \text{ N} \qquad \textbf{\textit{Ans.}}$$



F.B.D. of Cylinder

Problem:

The 30 kg collar may slide on frictionless vertical rod and is connected to a 34 kg counter weight. Find the value of h for which the system is in equilibrium.



Solution

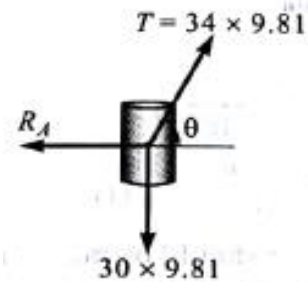
Consider the F.B.D. of the collar

By Lami's theorem,

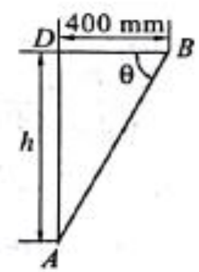
$$\frac{T}{\sin 90^\circ} = \frac{30 \times 9.81}{\sin (180^\circ - \theta)}$$

$$\sin \theta = \frac{30 \times 9.81}{34 \times 9.81} \times \sin 90^\circ$$

$$\therefore \theta = 61.93^\circ$$



F.B.D. of Collar



$$\tan 61.93^\circ = \frac{h}{400}$$

$h = 750$ mm ***Ans.***

Problem

Two identical rollers each of mass 50 kg are supported by an inclined plane and a vertical wall as shown in Fig. Assuming smooth surfaces, find the reactions induced at the point of support A, B and C.

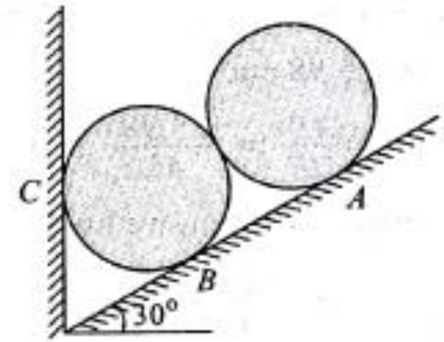


Fig.

Solution

(i) Consider F.B.D. of both rollers together and let R be the radius of rollers.

(ii) $\Sigma M_O = 0$

$R_A \times 2R - 50 \times 9.81 \cos 30^\circ \times 2R = 0$

$R_A = 424.79 \text{ N } (60^\circ)$ ***Ans.***

(iii) $\Sigma F_y = 0$

$R_B \cos 30^\circ + R_A \cos 30^\circ - 50 \times 9.81 - 50 \times 9.81 = 0$

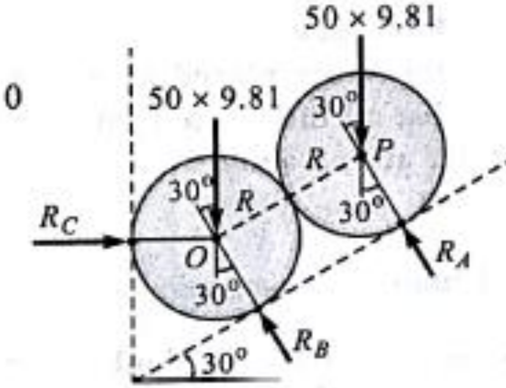
$R_B = 707.97 \text{ N } (60^\circ)$ ***Ans.***

(iv) $\Sigma F_x = 0$

$R_C - R_A \sin 30^\circ - R_B \sin 30^\circ = 0$

$R_C - 424.79 \sin 30^\circ + 707.97 \sin 30^\circ$

$R_C = 566.38 \text{ N } (\rightarrow)$ ***Ans.***



Problem

Two spheres A and B are resting in a smooth through as shown in Fig. 3.15(a). Draw the free body diagrams of A and B showing all the forces acting on them, both in magnitude and direction. Radius of spheres A and B are 250 mm and 200 mm, respectively.

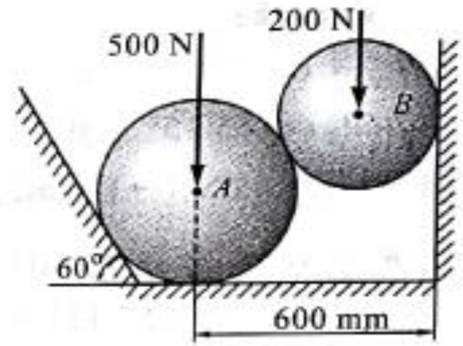


Fig. 3.15(a)

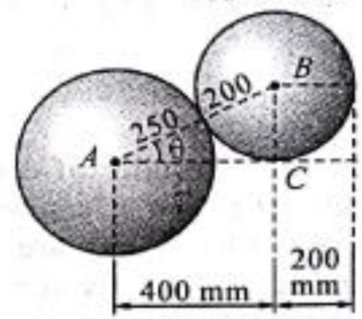


Fig. 3.15(b)

Solution

(i) From Fig. 3.15(b). $AB = 450$ mm and $AC = 400$ mm

$$\cos\theta = \frac{AC}{AB} = \frac{400}{450} \qquad \therefore\ \theta = 27.27^\circ$$

(ii) Consider the F.B.D. of Sphere *B* [Fig. 3.15(c)]

By Lami's theorem,

$$\frac{200}{\sin 152.73^\circ} = \frac{R_1}{\sin 117.27^\circ} = \frac{R_2}{\sin 90^\circ}$$

$\therefore R_1 = 388$ N $(\leftarrow)$ and

$R_2 = 436.51$ N $\left(\angle 27.27^\circ\right)$ ***Ans.***

(iii) Consider the F.B.D. of Sphere A [Fig. 3.15(d)]

$\Sigma F_x = 0$

$R_4 \cos 30^\circ - 436.51 \cos 27.27^\circ = 0$

$R_4 = 448\text{ N}$ ($\measuredangle 30^\circ$) ***Ans.***

$\Sigma F_y = 0$

$-500 + R_3 - 436.51 \sin 27.27^\circ + 448 \sin 30^\circ = 0$

$R_3 = 476\text{ N}$ ($\uparrow$) ***Ans.***

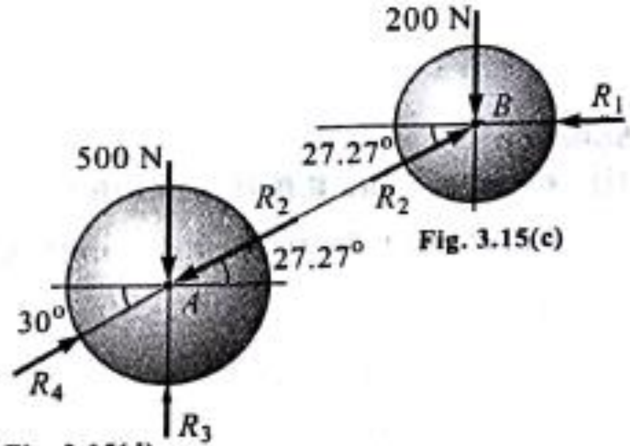


Fig. 3.15(c)

Fig. 3.15(d)

Ques:- Two smooth spheres of weight 100N and of radius 250 mm each are in equilibrium in a horizontal channel of width 900 mm as shown. Find the reaction at the surface of contact A, B, C, D assuming all the surfaces to be smooth.

Solution :- figure (b) shows F.B.D. of combined spheres. Reactions R_A, R_B and R_C are perpendicular to their respective surfaces.

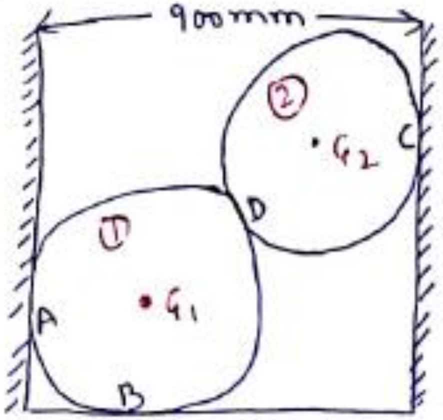


fig (A)

Applying condition of equilibrium,

$(+\uparrow)\ \Sigma F_y = 0$; $R_B - 100 - 100 = 0$

$\boxed{R_B = 200N\,(\uparrow)}$ Ans

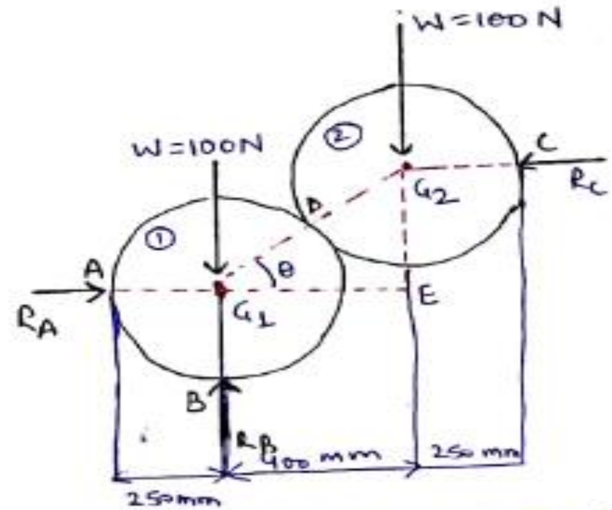


fig (B) F.B.D of both spheres

$(+\rightarrow)\ \Sigma F_x = 0;\quad R_A - R_C = 0$

$R_A = R_C$ ------ (I)

From the geometry of the figure, we have

length $G_1G_2 = 2r = 2\times250 = 500$ mm

and length $G_1E = 400$ mm

$G_2E = \sqrt{(G_1G_2)^2 - (G_1E)^2} = \sqrt{(500)^2 - (400)^2}$

$= 300$ mm

$(+\circlearrowleft)\ \Sigma M_{G_1} = 0$

$-100\times400 + R_C\times300 = 0$

$$\therefore \boxed{R_C = 133.333\ N\ (\leftarrow)}\ \text{Ans}$$

Substituting the value of R_C in equation (1), we get

$$\boxed{R_A = 133.333\ N\ (\rightarrow)}\ \text{Ans}$$

To find the reaction at D, draw F.B.D of sphere 1 separately as shown in figure Ex 52(c).

Applying conditions of equilibrium to sphere (1)

$(+\rightarrow)\ \Sigma F_x = 0;\ R_A - R_D \cos\theta = 0$

$$\cos\theta = \frac{G_1E}{G_1G_2} = \frac{400}{500} = 0.8$$

$$\therefore\ 133.333 - R_D \times 0.8 = 0$$

$$\boxed{R_D = 166.667\ N}\ \text{Ans}$$

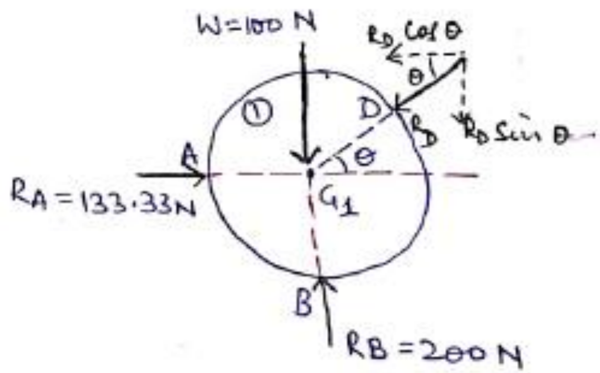
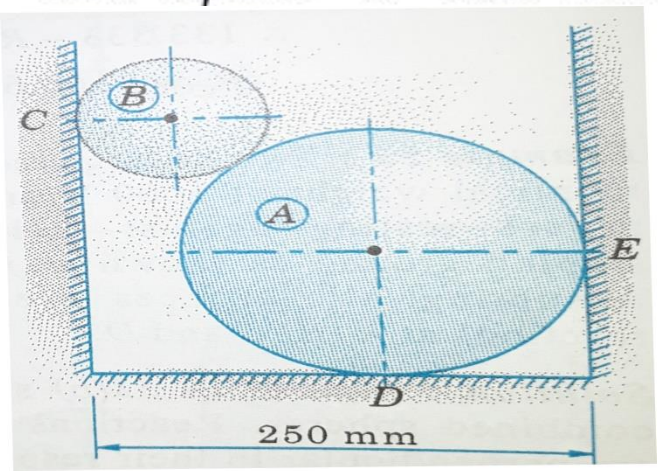
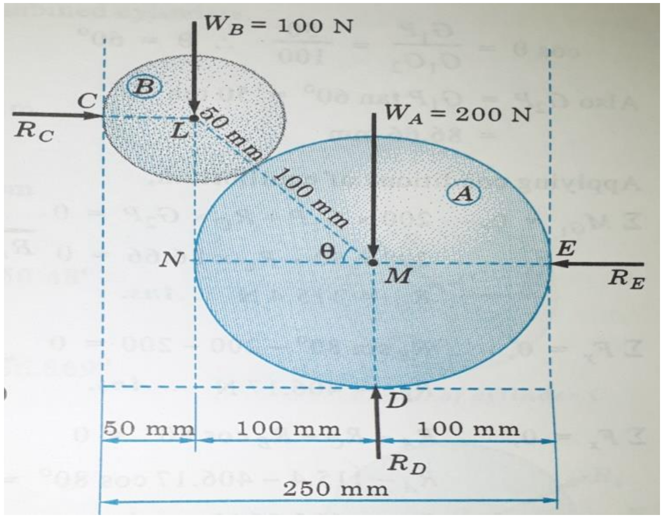


fig (c) : F.B.D of sphere (1)

Q. Two smooth spheres A & B of weight 200 N & 100N resp. are resting ~~on~~ against ~~the~~ two smooth vertical walls & smooth horizontal floor as shown. The radius of sphere A is 100 mm & radius of sphere B is 50 mm. Find the reaction from vertical wall & horizontal floor. Also find reaction exerted by each sphere on the other?



Solution:



FBD

Solⁿ :-

$NM = 250 - r_B - r_A$

$= 250 - 50 - 100$

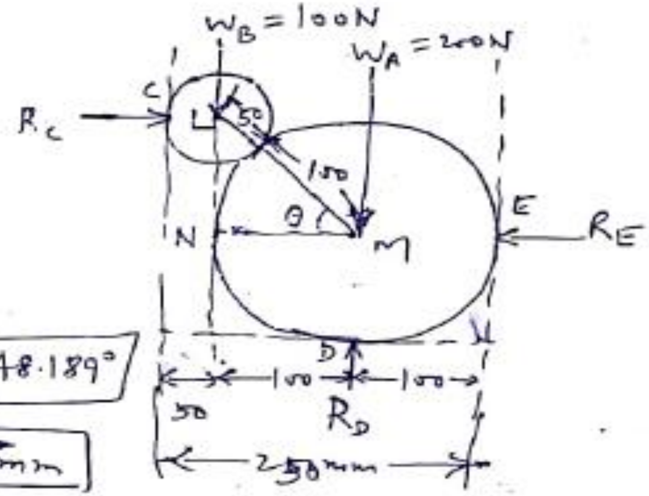
$\therefore NM = 100\ mm$

$ML = r_A + r_B = 100 + 50$

$= 150\ mm$

$\cos\theta = \frac{NM}{LM} = \frac{100}{150} \quad \therefore \boxed{\theta = 48.189^\circ}$

$\sin\theta = \frac{LN}{LM} \Rightarrow \boxed{LN = 111.8\ mm}$



COE :- $\Sigma f_x = 0 \quad \therefore R_c - R_E = 0 \quad \therefore R_c = R_E$ — ① eqⁿ

$\Sigma f_y = 0 \quad \therefore R_D - 100 - 200 = 0 \quad \therefore \boxed{R_D = 300\ N \uparrow}$

$\Sigma M = 0 \quad \therefore 100 \times MN - R_c \times LN = 0$

$100 \times 100 - R_c \times 111.8 = 0 \quad \therefore R_c = 89.45\ N\ (\rightarrow)$

R_c in ① eqⁿ $\quad \therefore R_E = 89.45\ N\ (\leftarrow)$

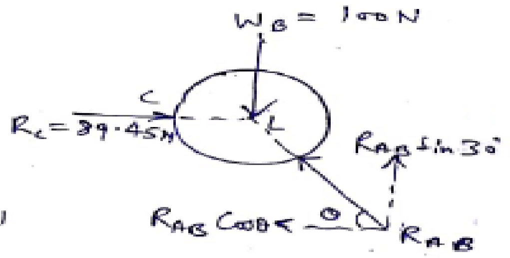
To find Reaction exerted by each sphere :-

COE :-

$\Sigma F_y = 0 \quad \therefore \quad R_{AB} \sin\theta - W_B = 0$

Here $\theta = 48.189^\circ$ & $W_B = 100\,N$

$\therefore \boxed{R_{AB} = 134.165\,N}$



Ques: Two spheres A and B of weight 1000 N and 750 N respectively are kept as shown in the figure Ex (a). Determine the reaction at all contact point 1, 2, 3 and 4. Radius of A = 400 mm and Radius of B = 300 mm.

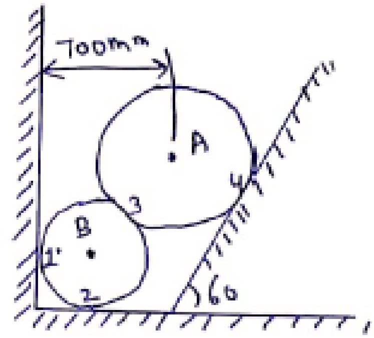


Fig (A)

Solu^n :- Draw F.B.D of spheres A and B as shown in figure (B) and apply conditions of equilibrium

From F.B.D of sphere A

$\Sigma F_y = 0.$

$R_3 \sin 55.15^\circ + R_4 \sin 30^\circ = 1000$

$\Sigma F_x = 0,$

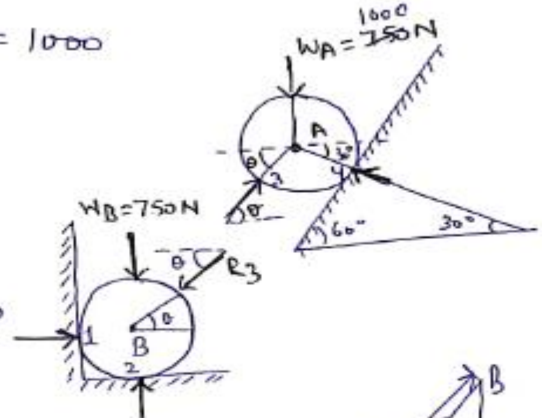
$R_3 \cos 55.15^\circ = R_4 \cos 30^\circ$

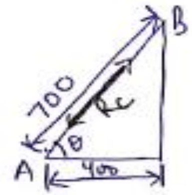
$\therefore R_3 = 1.516\ R_4$

From (I)

$1.516\ R_4 \sin 55.15^\circ + R_4 \sin 30^\circ = 1000$

$\therefore R_3 = 869.4\,N$ and $R_4 = 573.48\,N$





$\cos\theta = \frac{400}{700}$

$\theta = 55.15^\circ$

fig B : F.B.D of spheres A and B.

From FBD of sphere B

$\Sigma F_x = 0,$

$R_1 = R_3 \cos 55.15^\circ = 869.4 \cos 30^\circ$

$\therefore R_1 = 496.8\ N$

$\Sigma F_y = 0.$

$R_2 = 750 + R_3 \sin 55.15^\circ$

$\therefore R_2 = 1463.47\ N$

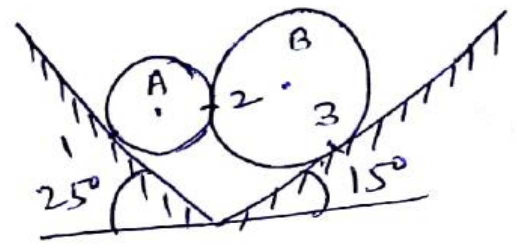
Thus, we have

$R_1 = 496.8$, $R_2 = 1463.47\ N$,

$R_3 = 869.4N \quad R_4 = 573.48\ N$ Ans

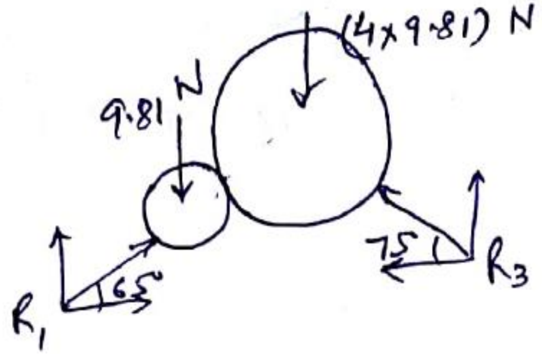
Question:

Determine the reactions at points of contact 1, 2 & 3.
Assume smooth surfaces.
Take $m_A = 1$ kg, $m_B = 4$ kg



Solution:

$R_1 = ?$, $R_2 = ?$, $R_3 = ?$ $r_A = 1$ cm
$r_B = 4$ cm
$m_A = 1$ kg, $m_B = 4$ kg



COE :- $\Sigma F_x = 0$; $\xrightarrow{+ive}$

$R_1 \cos 65 - R_3 \cos 75 = 0$ — ①

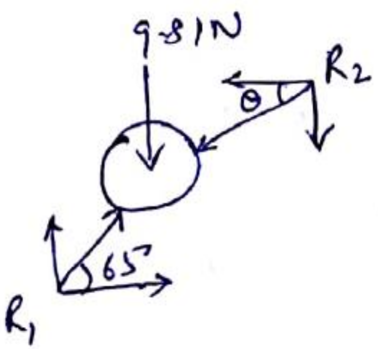
$\Sigma F_y = 0$; $\uparrow$ +ive

$R_1 \sin 65 + R_3 \sin 75 - 9.81 - 39.24 = 0$

$R_1 \sin 65 - R_3 \sin 75 = 49.05$ — ②

Solve eq ① & ②, we get → $\boxed{R_1 = 19.75\ N,\ \theta = 65°}$

$\boxed{R_3 = 32.25\ N,\ \theta = 75°}$



COE :- $\Sigma F_x = 0$; $\xrightarrow{+ive}$

$19.75 \cos 65 - R_2 \cos\theta = 0$

$\therefore R_2 \cos\theta = 8.347$ — ③

$\Sigma F_y = 0$; $\uparrow$ +ive

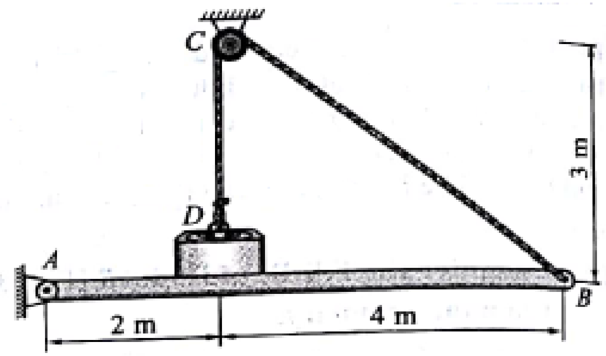
$19.75 \sin 65 - 9.81 - R_2 \sin\theta = 0$

$\therefore R_2 \sin\theta = 8.0895$ — ④

$\therefore$ Solve eq ③ & ④, we get $\boxed{R_2 = 11.62\ N,\ \theta = 44.1°}$

Problem

A uniform beam *AB* hinged at *A* is kept horizontal by supporting and setting a 50 kN weight with the help of a string tied at *B* and passing over a smooth peg at *C*, as shown in Fig. . The beam weight is 25 kN. Find the reaction at *A* and *C*.



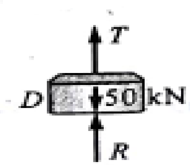
Solution

(i) Consider the F.B.D. of Block *D*

$$\Sigma F_y = 0$$

$$R + T - 50 = 0$$

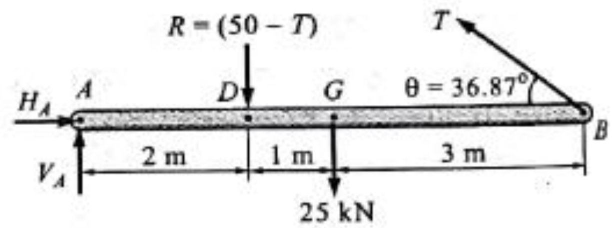
$$\therefore R = 50 - T$$



FBD of Block D

(ii) Consider the F.B.D. of Beam AB

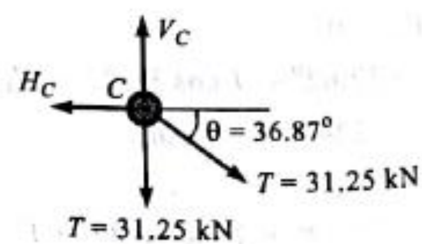
$$\Sigma M_A = 0$$
$$-(50 - T)2 - 25 \times 3 + T \sin 36.87^\circ \times 6 = 0$$
$$\therefore T = 31.25 \text{ kN}$$
$$\Sigma F_x = 0$$
$$H_A - 31.25 \cos 36.87^\circ = 0$$
$$\therefore H_A = 25 \text{ kN } (\rightarrow) \quad \textit{Ans.}$$
$$\Sigma F_y = 0$$
$$V_A - (50 - T) - 25 + T \sin 36.87^\circ = 0$$
$$\therefore V_A = 25 \text{ kN } (\uparrow) \quad \textit{Ans.}$$



FBD of Beam AB

(iii) Consider the F.B.D. of Peg C

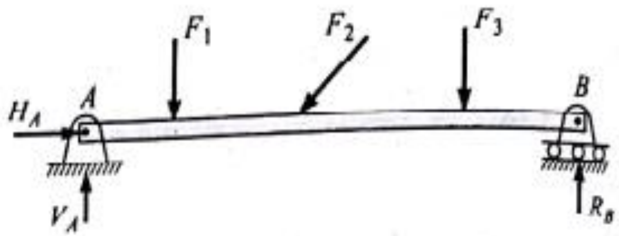
$$\Sigma F_x = 0$$
$$31.25 \cos 36.87^\circ - H_C = 0$$
$$\therefore H_C = 25 \text{ kN } (\leftarrow) \quad \textit{Ans.}$$
$$\Sigma F_y = 0$$
$$V_C - 31.25 - 31.25 \sin 36.87^\circ = 0$$
$$\therefore V_C = 50 \text{ kN } (\uparrow) \quad \textit{Ans.}$$



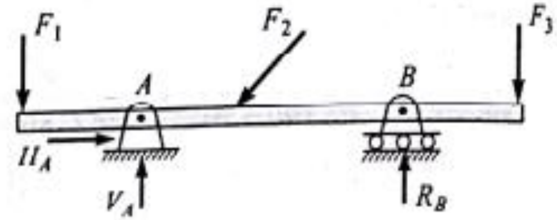
FBD of Peg C

Classification of Beam :

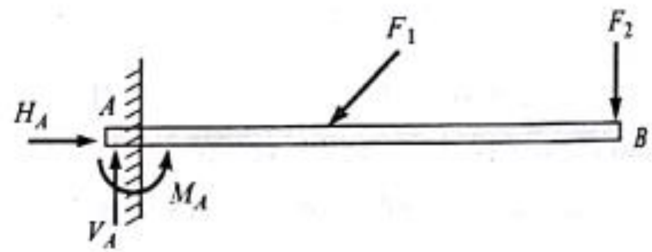
1. **Simply Supported Beam :** As the name indicates, it is the simplest of all beams which is supported by a hinge at one end and a roller at the other end.



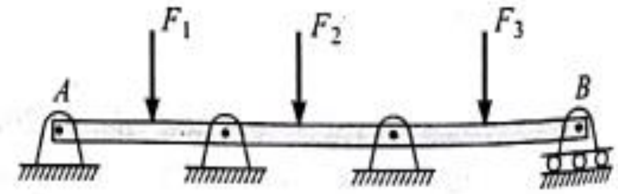
2. **Simply Supported Beam with Overhang:** Here, one end or both the ends of simply supported beam is projected beyond the supports which means that the portion of beam extends beyond the hinge and roller supports.



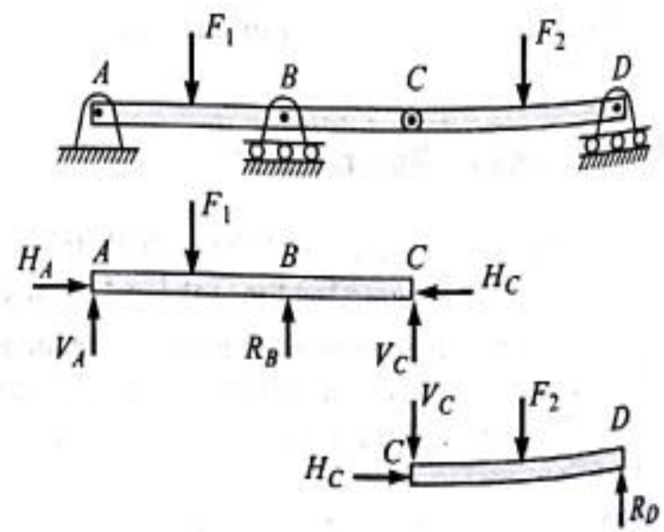
3. **Cantilever Beam :** A beam which is fixed at one end and free at the other end is called a *cantilever beam*. The fixed end is also known as built-in support. The common example is wall bracket, projected balconies, etc. One end of the beam is cast in concrete and is nailed, bolted, riveted or welded. The fixed end does not allow horizontal linear movement, vertical linear movement or rotational movement.



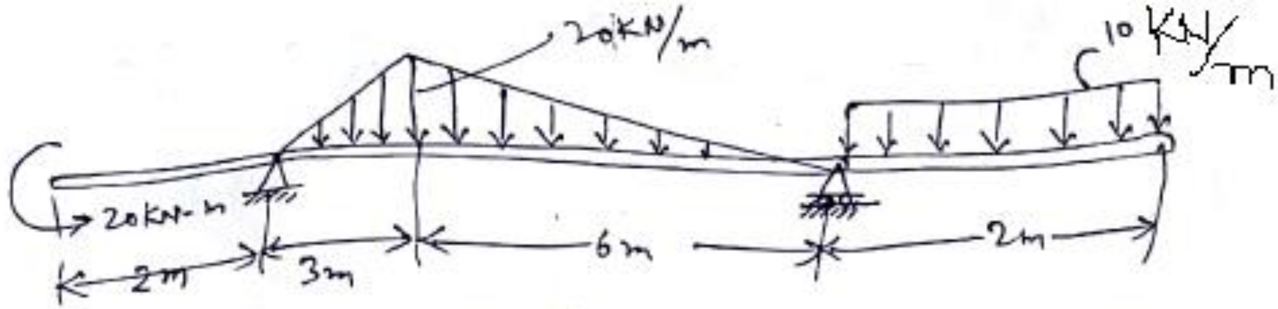
4. **Continuous Beam :** A beam which has more than two support is said to be a *continuous beam*. The extreme left and right supports are the end supports of the beam. Two intermediate supports are shown. Such beams are also called *statically indeterminate beams* because the reactions cannot be obtained by the equation of equilibrium.



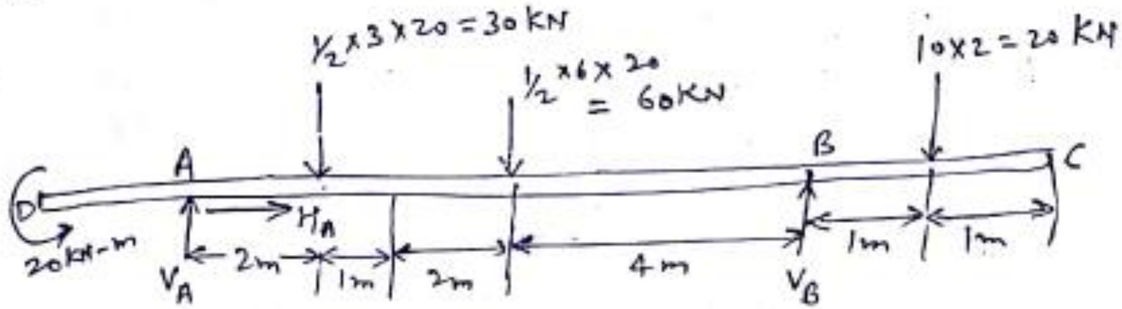
5. **Beams Linked with Internal Hinges :** Here two or more beams are connected to each other by pin joint and continuous beam is formed. Such a joint are called *internal hinges*. Internal hinges allow us to draw F.B.D. of beam at its joint, if required.



Problem :



Sol^n



$\Sigma F_x = 0 \quad \therefore \quad H_A = 0$

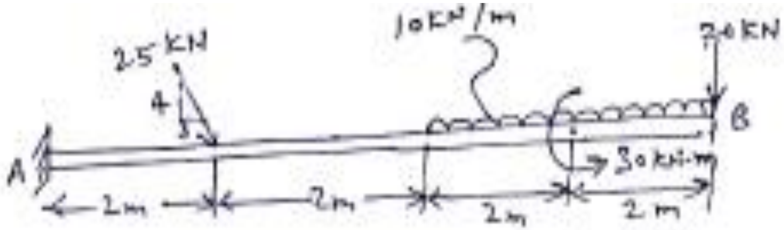
$\Sigma M_A = 0 \quad \therefore \; 20 - 30 \times 2 - 60 \times 5 + V_B \times 9 - 20 \times 10 = 0$

$\therefore \boxed{V_B = 60\,kN \uparrow}$

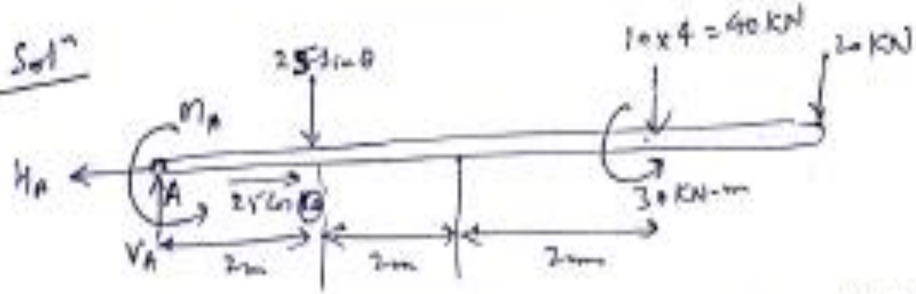
$\Sigma F_y = 0 \quad \therefore \quad V_A + V_B - 30 - 60 - 20 = 0 \qquad \therefore V_A = 50\,kN (\uparrow)$

Problem:

Q.



Soln



COE :- $\Sigma F_x = 0$ $\quad -H_A + 25\cos\theta = 0$ $\quad \therefore H_A = 25 \times 0.6 = 15$ KN

$\Sigma F_y = 0$ $\quad \therefore V_A - 25\sin\theta - 40 - 20 = 0$ $\quad \therefore \boxed{V_A = 80\text{ KN}}$

$\Sigma M_A = 0$ $\quad \therefore M_A - 25\sin\theta \times 2 - 40 \times 6 - 20 \times 8 + 30 = 0$

$\therefore M_A = 410$ KN.m

Problem

Calculate the support reactions for the beam shown in Fig.

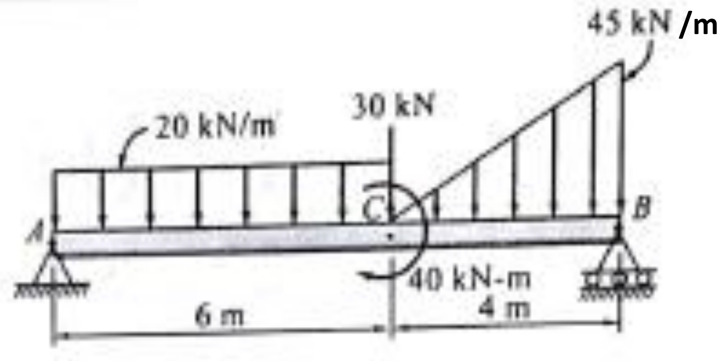
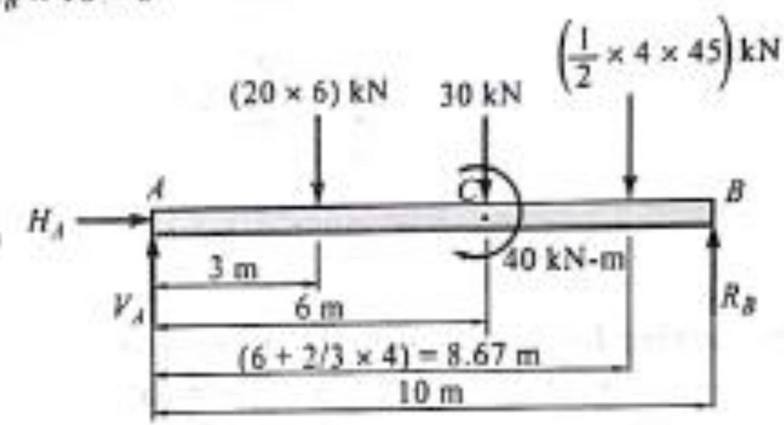


Fig.

Solution

(i) Consider the F.B.D. of Beam AB



FBD of Beam AB

(ii) $\Sigma M_A = 0$

$-120 \times 3 - 30 \times 6 - 40 - 90 \times 8.67 + R_B \times 10 = 0$

$R_B = 136.03$ kN (↑)

(iii) $\Sigma F_x = 0$

$H_A = 0$

(∵ there is no horizontal force acting)

(iv) $\Sigma F_y = 0$

$V_A - 120 - 30 - 90 + 136.03 = 0$

$V_A = 103.97$ kN (↑) *Ans.*

Problem

Find the support reactions at A and B for the beam loaded as shown in Fig.

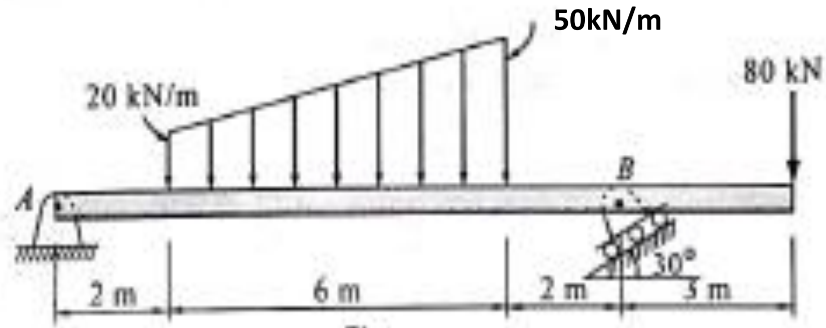


Fig.

Solution

(i) Consider the F.B.D. of Beam AB

(ii) $\Sigma M_A = 0$

$R_B \sin 60^\circ \times 10 - 120 \times 5 - 90 \times 6 - 80 \times 13 = 0$

$R_B = 251.73$ kN $(60^\circ \measuredangle)$

(iii) $\Sigma F_x = 0$

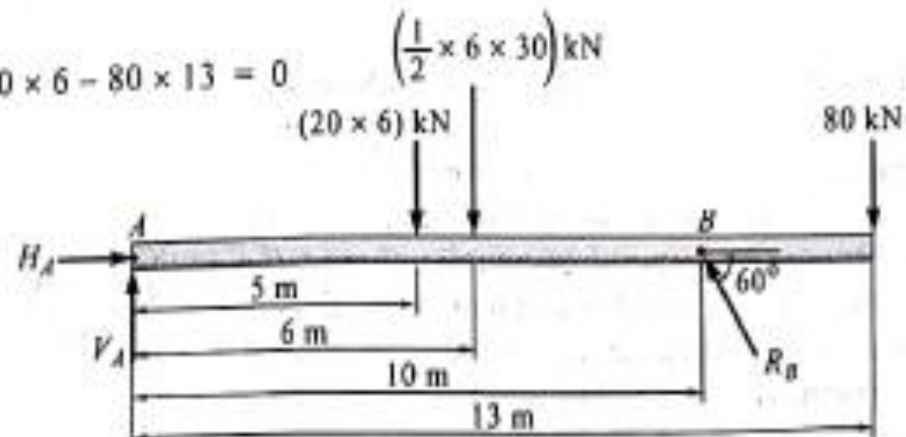
$H_A - 251.73 \cos 60^\circ = 0$

$H_A = 125.87$ kN $(\rightarrow)$

(iv) $\Sigma F_y = 0$

$V_A - 120 - 90 + 251.73 \sin 60^\circ - 80 = 0$

$V_A = 72$ kN $(\uparrow)$ *Ans.*



FBD of Beam AB

Problem

Find analytically the support reaction at B and the load P, for the beam shown in Fig. , if the reaction of support A is zero.

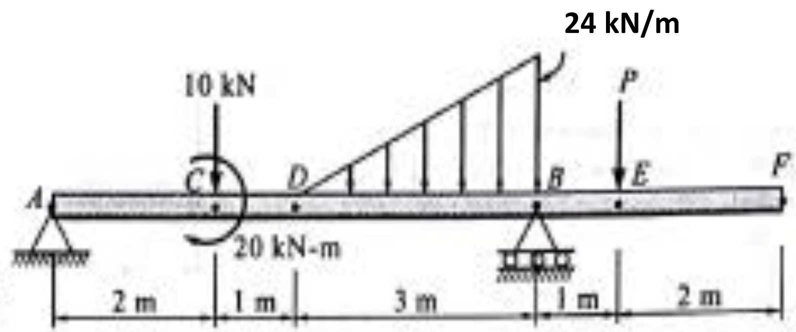


Fig.

Solution

(i) Consider the F.B.D. of Beam AF

(ii) $\Sigma F_y = 0$

$V_A + R_B - 10 - 36 - P = 0$ ($V_A = 0$ given)

$R_B - P = 46$ ...(I)

(iii) $\Sigma M_A = 0$

$R_B \times 6 - 10 \times 2 - 20 - 36 \times 5 - P \times 7 = 0$

$6 R_B - 7 P = 220$ ...(II)

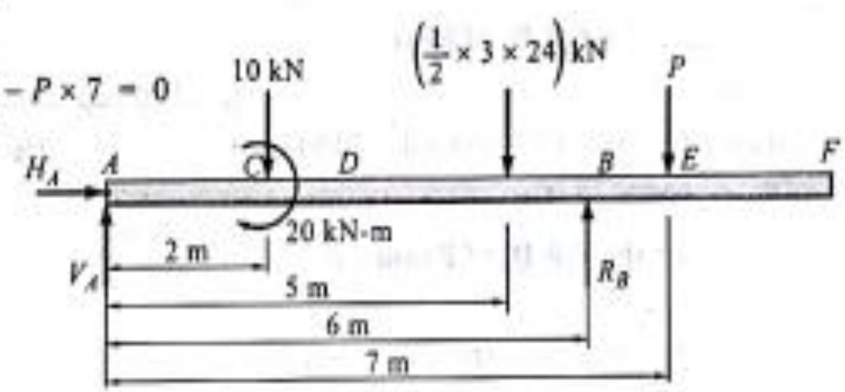
(iv) Solving Eqs. (I) and (II)

$R_B = 102$ kN ($\uparrow$) *Ans.*

(v) From Eq. (I)

$P = 102 - 46$

$P = 56$ kN ($\downarrow$) *Ans.*



FBD of Beam AF

Problem

Find the support reactions at A and F for the given Fig.

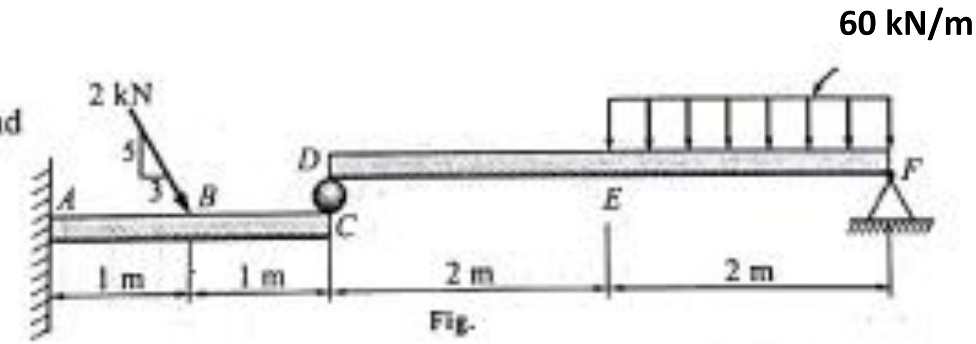


Fig.

Solution

(i) Consider the F.B.D. of Beam *DF*

$\Sigma M_F = 0$

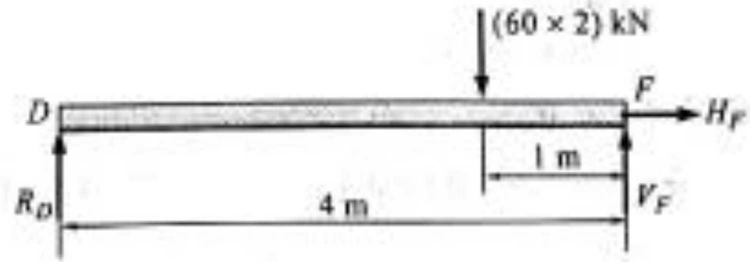
$120 \times 1 - R_D \times 4 = 0 \quad \therefore R_D = 30 \text{ kN}$

$\Sigma F_x = 0 \qquad \therefore H_F = 0$

$\Sigma F_y = 0$

$R_D + V_F - 120 = 0$

$V_F = 90 \text{ kN } (\uparrow)$ ***Ans.***



FBD of Beam DF

(ii) Consider the F.B.D. of Beam *AC*

$\Sigma M_A = 0$

$M_A - 2 \sin 59.04° \times 1 - 30 \times 2 = 0$

$M_A = 61.72 \text{ kN-m } (\circlearrowleft)$

$\Sigma F_x = 0$

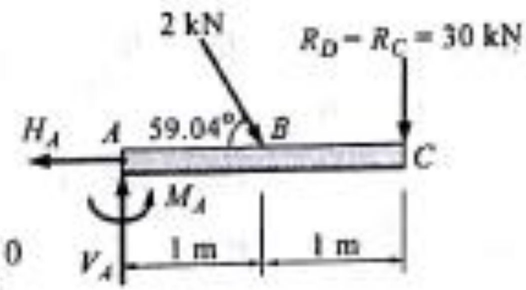
$2 \cos 59.04° - H_A = 0$

$H_A = 1.03 \text{ kN } (\leftarrow)$

$\Sigma F_y = 0$

$V_A - 2 \sin 59.04° - 30 = 0$

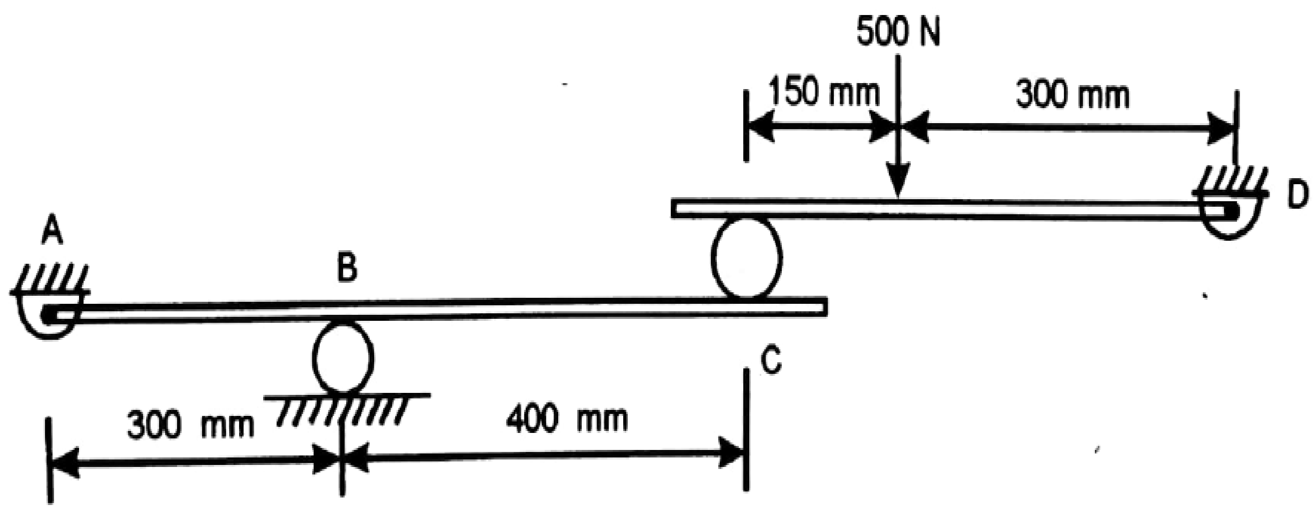
$V_A = 31.72 \text{ kN } (\uparrow)$ ***Ans.***



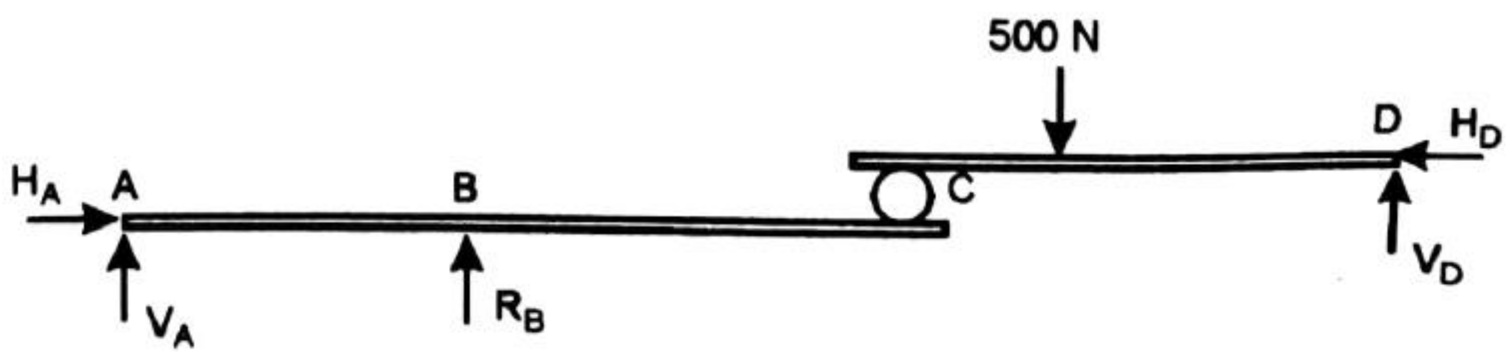
FBD of Beam AC

Problem:

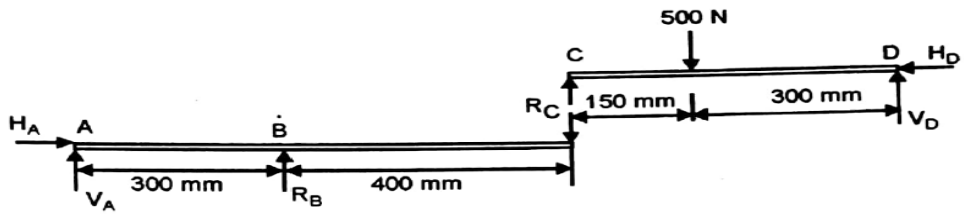
For a lever system shown, find the support reactions.



Solution: FBD of the system of two connected bodies is shown as -



Let us therefore isolate the two bodies and apply COE to each of them.



Applying COE to body CD.

$M_D = 0$ ↺ + ve

$+ (500 \times 300) - (R_C \times 450) = 0$

$R_C = 333.33$ N

$\therefore$ $R_C = 333.33$ N ↑ on body CD.

$\Sigma F_Y = 0$ ↑ + ve

$333.33 - 500 + V_D = 0$

$V_D = 166.67$ N

$\therefore$ $V_D = 166.67$ N ↑

$\Sigma F_X = 0$ → + ve

$H_D = 0$

$\therefore$ The total reaction at D is $R_D = 166.67$ N ↑ .... **Ans.**

Applying COE to body AC

using $R_C = 333.33$ N ↓ on body AC

$\Sigma M_A = 0$ ↺ + ve

$- (333.33 \times 700) + (R_B \times 300) = 0$

$R_B = 777.7$ N

$\therefore$ $R_B = 777.7$ N ↑ .... **Ans.**

$\Sigma F_Y = 0$ ↑ +ve

$V_A + 777.7 - 333.3 = 0$

$V_A = -444.4$

$\therefore$ $V_A = 444.4$ N ↓

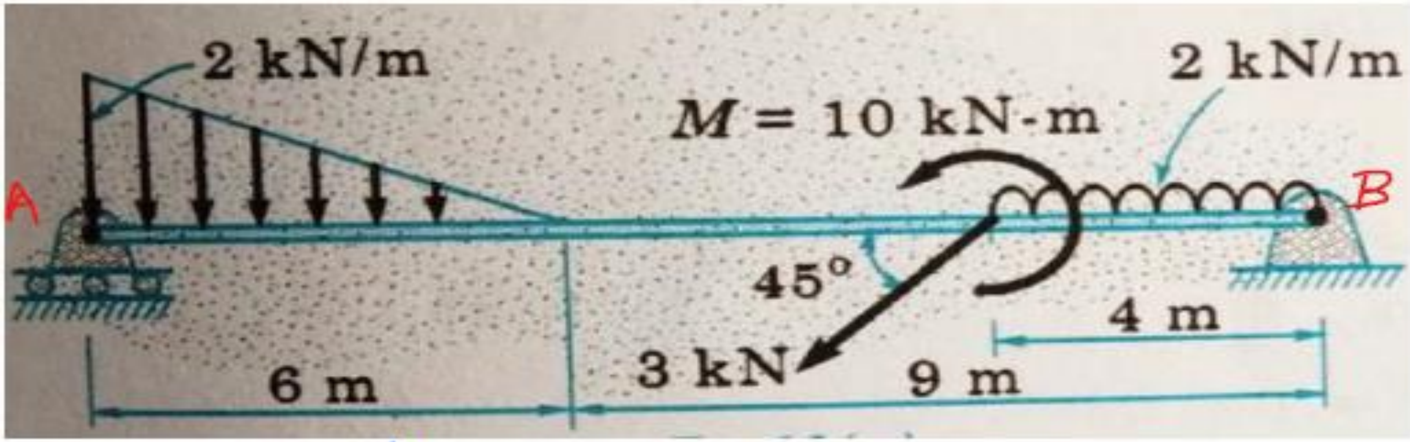
$\Sigma F_X = 0$ → + ve

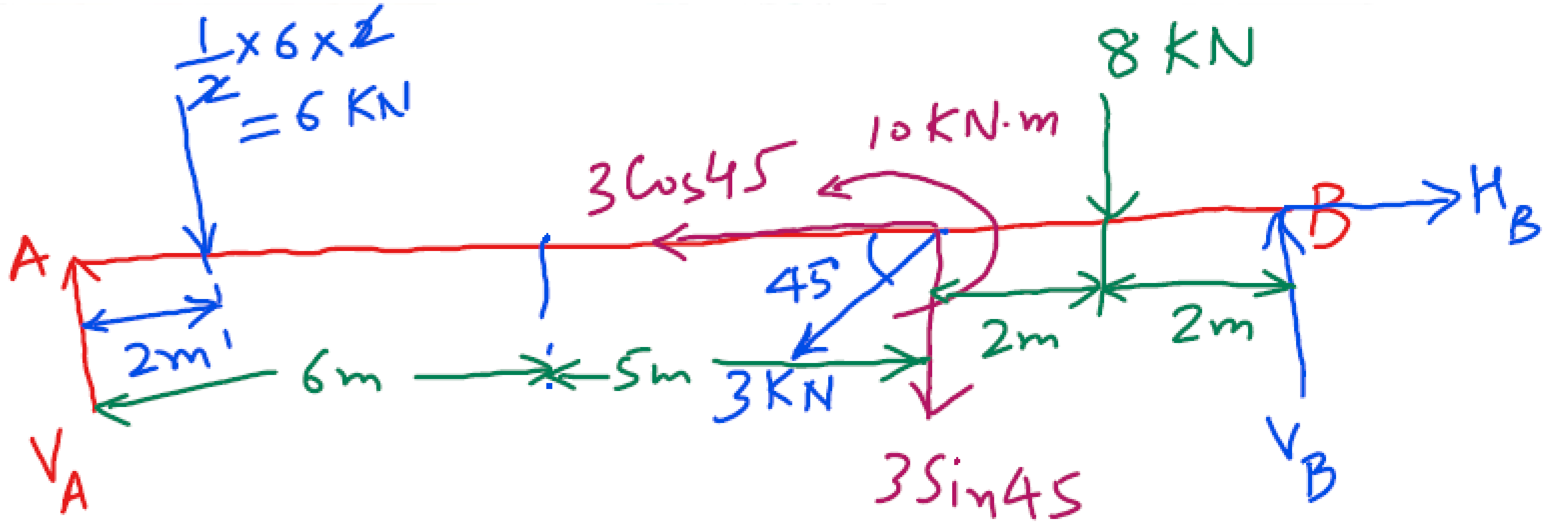
$H_A = 0$

$\therefore$ The total reaction at A is $R_A = 444.4$ N ↓ .... **Ans.**

Problem:

Determine the reactions at all supports of the beam AB as shown in figure.





COE :- $\sum F_x = 0$; $\xrightarrow{+}$

$H_B - 3\cos 45 = 0$ $\therefore \boxed{H_B = 2.121 \text{ KN} (\rightarrow)}$

$\sum F_y = 0$; $\uparrow +$ $\Rightarrow V_A - 6 - 3\sin 45 - 8 + V_B = 0$

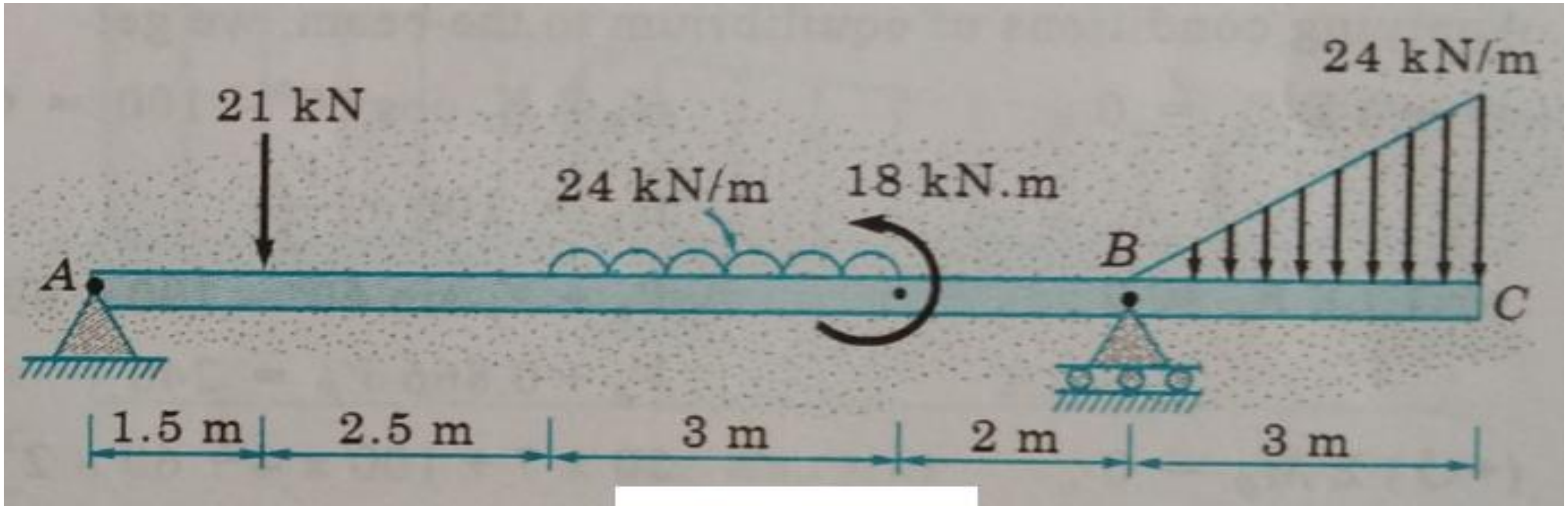
$\sum M_B = 0$; $+\circlearrowleft$ $\Rightarrow -(V_A \times 15) + (6 \times 13) + \{(3\sin 45) \times 4\}$ $+10 + (8 \times 2) = 0$

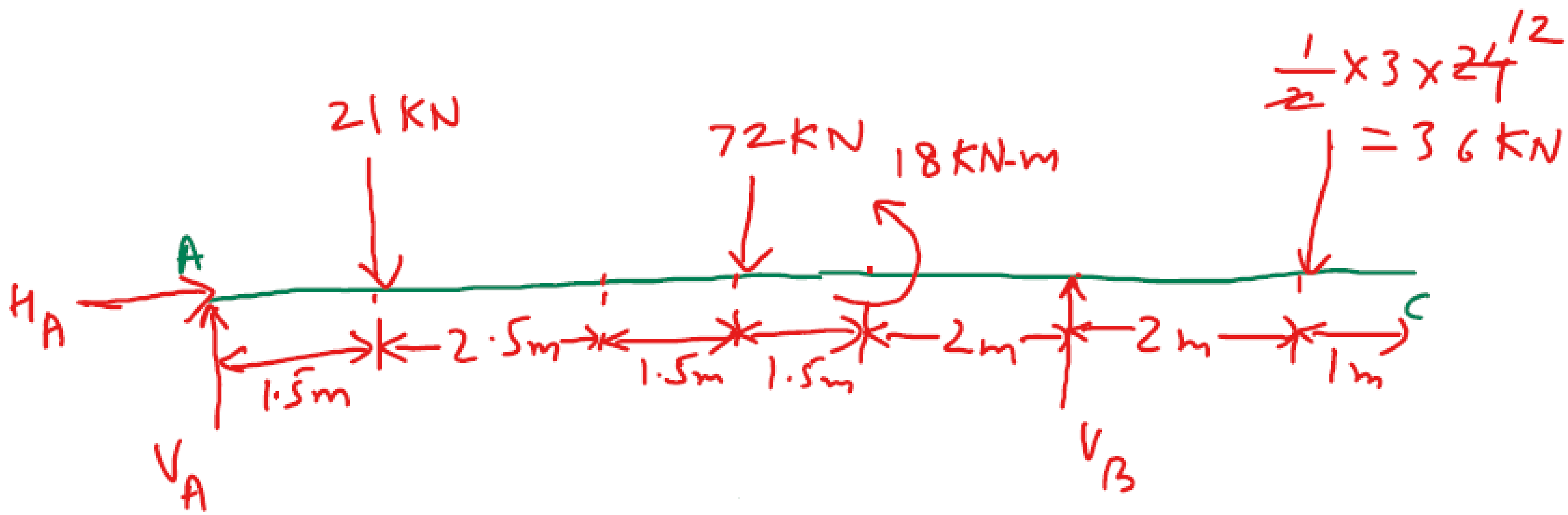
$\boxed{V_A = 7.5 \text{ KN} (\uparrow)}$

$\boxed{V_B = 8.621 \text{ KN} (\uparrow)}$

Problem:

Determine the reactions at all supports of the beam AB as shown in figure.





COE :- $\Sigma F_x = 0$; $\xrightarrow{+}$

$\boxed{H_A = 0}$

$\Sigma F_y = 0$; $\uparrow +$

$V_A - 21 - 72 + V_B - 36 = 0 \Rightarrow V_A = 39.5\ kN (\uparrow)$

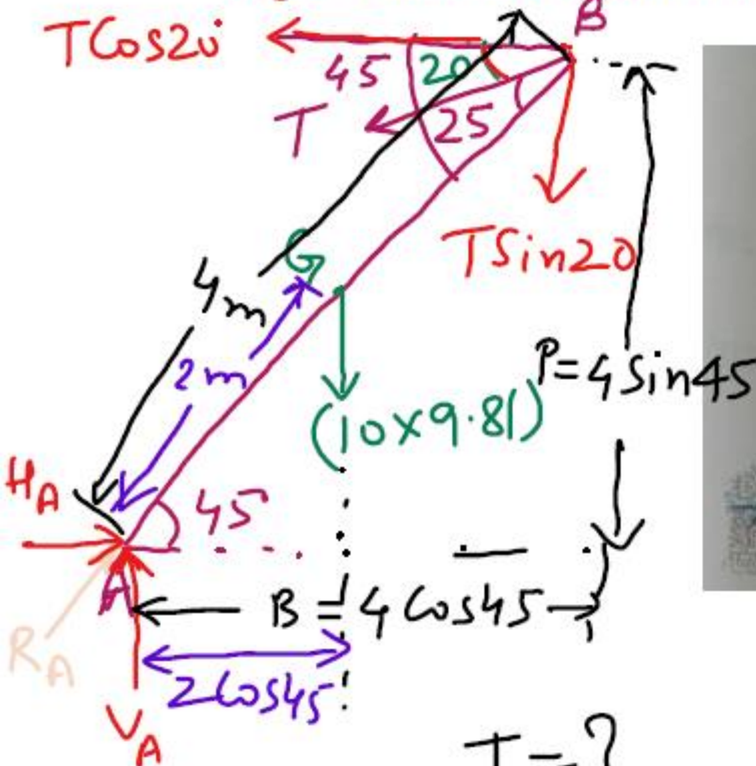
$\Sigma M_A = 0$; $+\circlearrowleft$

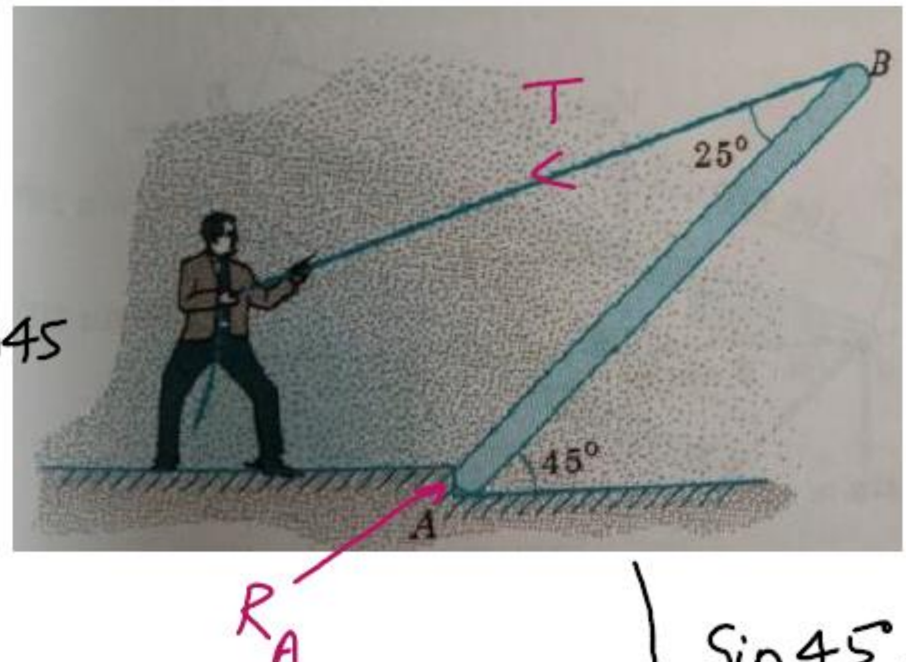
$-(21 \times 1.5) - (72 \times 5.5) + (18) + (V_B \times 9) - (36 \times 11) = 0$

$V_B = 89.5\ kN\ (\uparrow)$

Problem:

A man raises a 10 Kg joist of length 4 m by pulling on a rope. Find the tension T in the rope and the reaction at A.





$T = ?$, $R_A = ?$

$\sin 45° = \frac{P}{4}$

$P = 4 \sin 45°$

COE; $\Sigma M_A = 0$; $+\circlearrowleft$

$$\{(T\cos 20) \times (4\sin 45)\} - \{(T\sin 20) \times 4\cos 45\}$$
$$- \{(10 \times 9.81) \times 2\cos 45\} = 0$$

$$\boxed{T = 82.098\,N}$$

$\cos 45 = \frac{B}{4}$

$B = 4\cos 45$

$\cos 45 = \frac{b}{2}$

$b = 2\cos 45$

$\Sigma F_x = 0$; $\overset{+}{\rightarrow}$

$H_A - T\cos 20 = 0$ $\therefore H_A - 82.098\cos 20 = 0$

$H_A = 77.146\,N\ (\rightarrow)$

$\Sigma F_y = 0$; $\uparrow +$

$V_A - 82.098\sin 20 - (10 \times 9.81) = 0$

$V_A = 126.179\,N\ (\uparrow)$

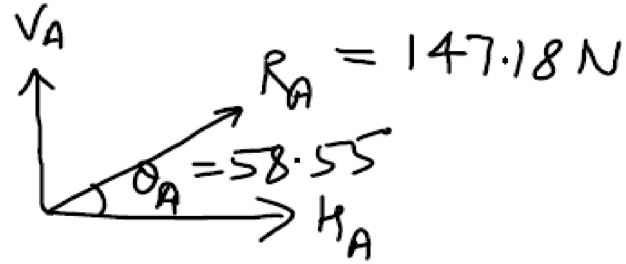
$$R_A = \sqrt{(H_A)^2 + (V_A)^2}$$

$$= \sqrt{(77.146)^2 + (126.179)^2}$$

$$R_A = 147.89\,N$$

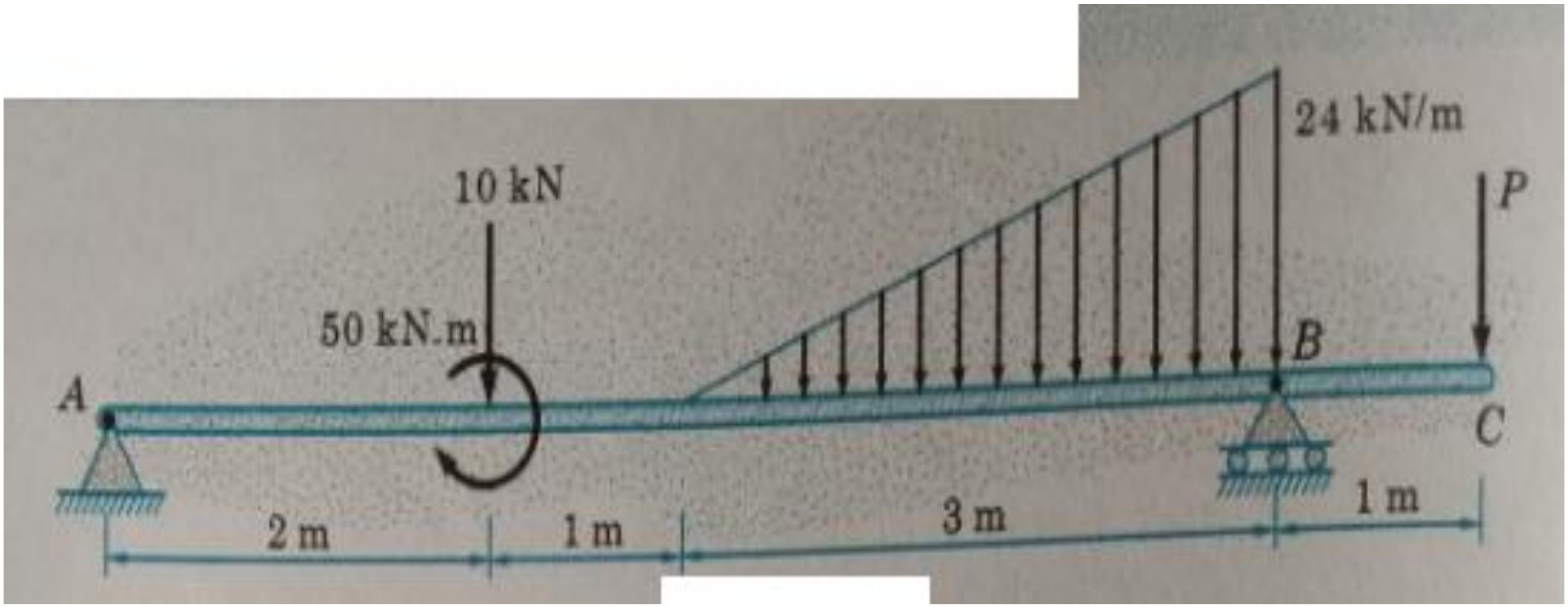
$$\theta_A = \tan^{-1}\left(\frac{V_A}{H_A}\right) = \tan^{-1}\left(\frac{126.179}{77.146}\right)$$

$$\theta_A = 58.55^\circ$$



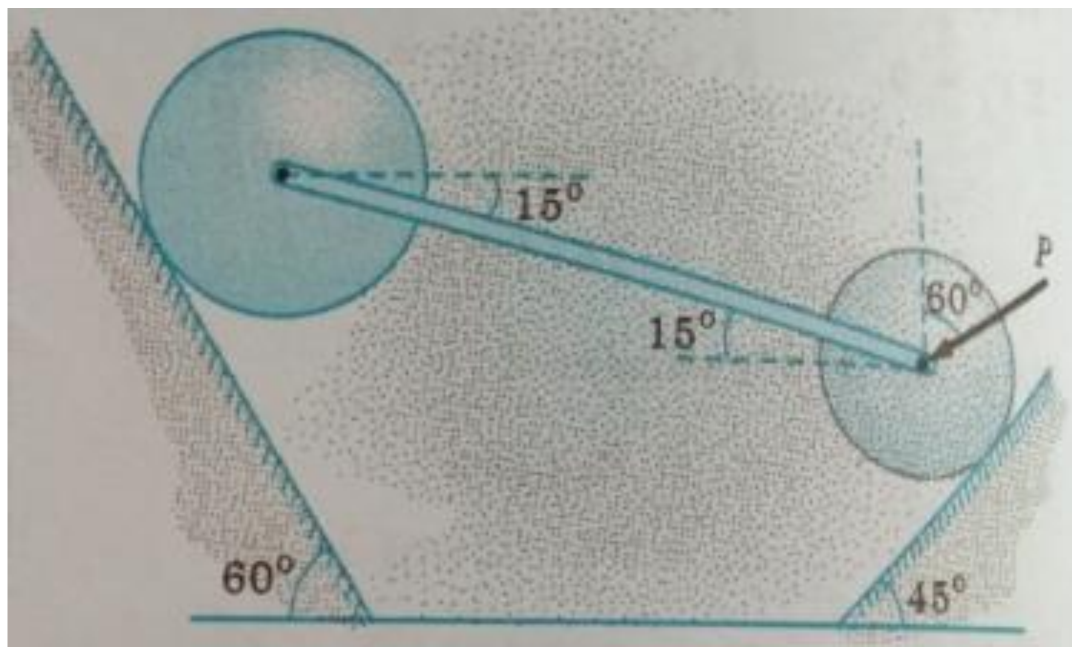
Problem for practice:

Find analytically the support reaction at B and load P for the beam shown in figure if reaction at support A is zero.



Problem for practice:

: Two cylinders, A of weight 4000 N and B of weight 2000 N rest on smooth inclines as shown in *figure* They are connected by a bar of negligible weight hinged to each cylinder at its, geometric center by smooth pins. Find the force P to be applied such that it will hold the system in the given position.



Problem for practice:

Three smooth spheres rest against two inclined smooth planes as shown. Determine

a) The reaction force at contact points when $\theta = 30^\circ$

b) The minimum angle θ for which the spheres remain in equilibrium.

Take for sphere 1 weight = 500 N and radius = 0.2 m

for spheres 2 and 3 weight = 1000 N and radius = 0.4 m