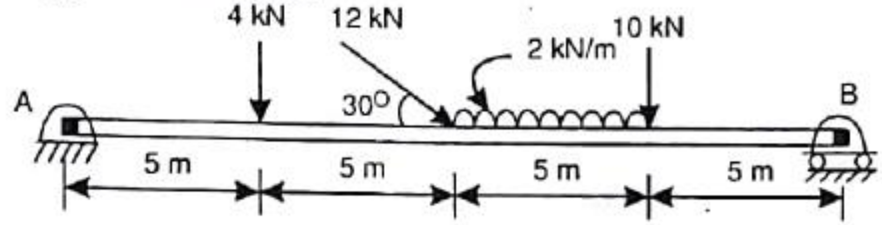
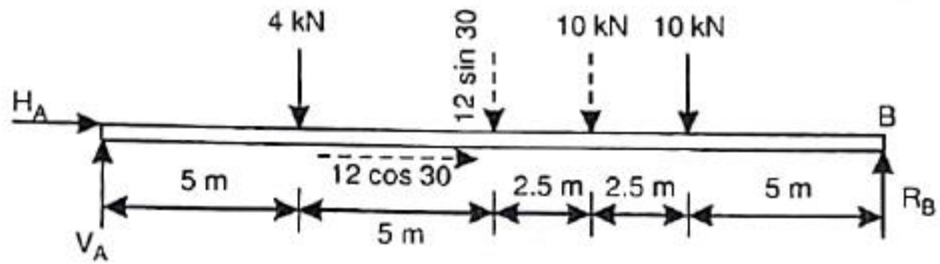
Equilibrium of Force System

Ex. 3.1 A beam AB is hinged at end A and roller supported at end B. It is acted upon by loads as shown. Find the support reactions.



Solution:



Applying Conditions of Equilibrium (COE) to the beam AB

$\Sigma M_A = 0$ ↺ + ve

$-(4 \times 5) - (12 \sin 30 \times 10) - (10 \times 12.5) - (10 \times 15) + (R_B \times 20) = 0$

$R_B = 17.75$ kN

$R_B = 17.75$ kN ↑ .... **Ans.**

$\sum F_X = 0 \quad \rightarrow + ve$

$H_A + 12 \cos 30 = 0$

$H_A = -10.39$ kN

$H_A = 10.39$ kN $\leftarrow$

$\sum F_Y = 0 \quad \uparrow + ve$

$V_A - 4 - 12 \sin 30 - 10 - 10 + 17.75 = 0$

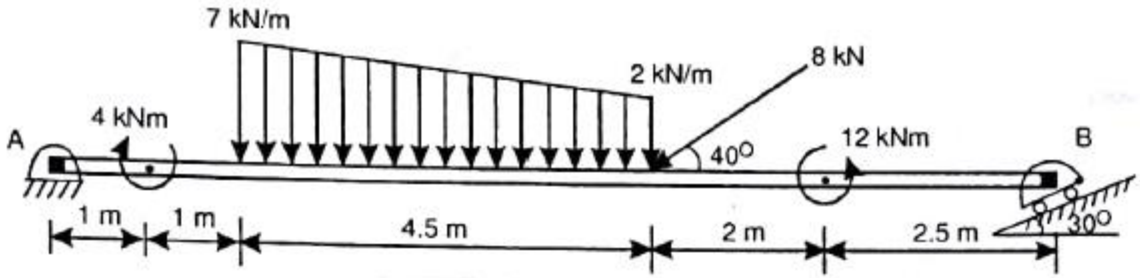
$V_A = 12.25$ kN $\uparrow$

Adding vectorially the components H_A and V_A, the reaction

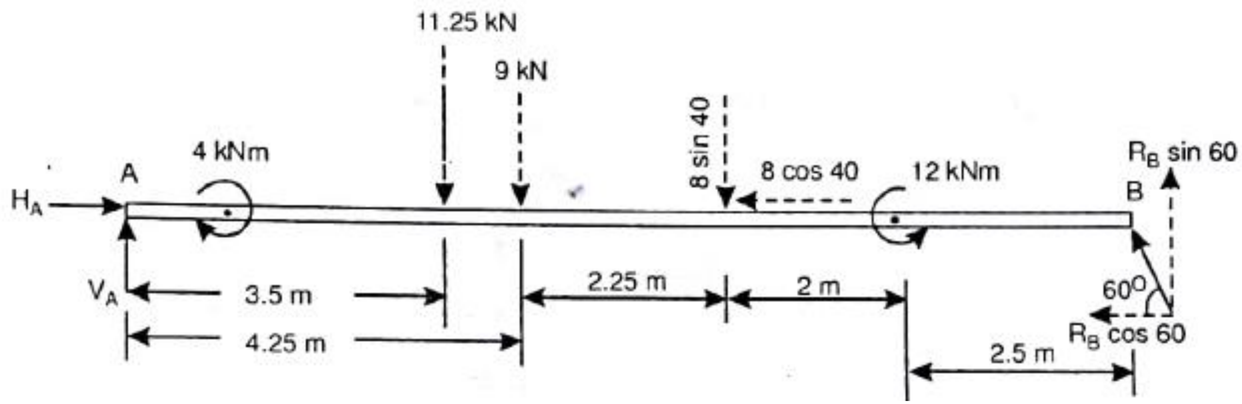
$R_A = 16.06$ kN $\theta = 49.69°$ .... **Ans.**

Note : Hinge reaction answers may also be written as $H_A = 10.39$ kN $\leftarrow$, $V_A = 12.25$ kN $\uparrow$

Ex. 3.2 The beam AB is loaded by forces and couples as shown. Find the reaction force offered by the supports to keep the system in equilibrium.



Solution:



Applying COE to the beam AB.

$\sum M_A = 0$ ↺ + ve

$-4 - (11.25 \times 3.5) - (9 \times 4.25) - (8 \sin 40 \times 6.5) + 12 + (R_B \sin 60 \times 11) = 0$

$R_B = 10.82$ kN

$\therefore \quad R_B = 10.82$ kN $\quad \theta = 60^\circ$ .... **Ans.**

$\sum F_X = 0 \quad \rightarrow$ + ve

$H_A - 8 \cos 40 - 10.82 \cos 60 = 0$

$H_A = 11.54$ kN

$H_A = 11.54$ kN $\rightarrow$

$\sum F_Y = 0 \quad \uparrow$ + ve

$V_A - 11.25 - 9 - 8 \sin 40 + 10.82 \sin 60 = 0$

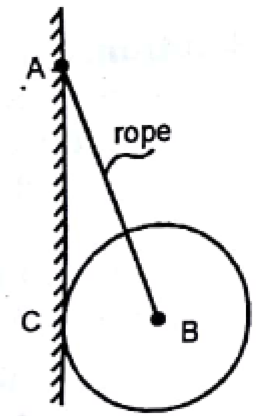
$V_A = 16.02$ kN

$V_A = 16.02$ kN $\uparrow$

Adding vectorially the components H_A and V_A the reaction

$R_A = 19.74$ kN, $\theta = 54.2^\circ$ .... **Ans.**

Ex. 3.7 A circular roller of weight 1000 N and radius 20 cm hangs by a rope AB of length 40 cm and rests against a smooth vertical wall at C as shown. Determine the tension in the rope and reaction at C.



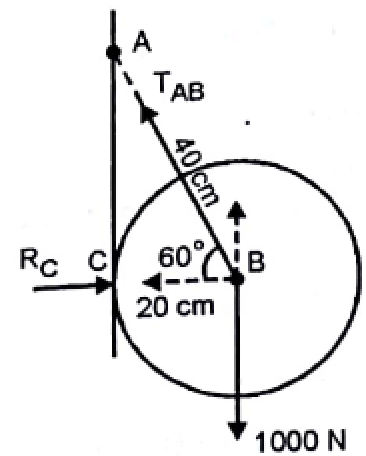
Solution

The roller is supported by a smooth surface at C and a rope AB.

Let R_C be the reaction at C and T_{AB} be the tension in the rope.

$\cos\theta = \frac{20}{40} \quad \therefore \quad \theta = 60°$

Applying COE to roller



$\Sigma F_y = 0 \quad \uparrow + ve$

$T_{AB} \sin 60 - 1000 = 0$

$\therefore \quad T_{AB} = 1154.7 \text{ N}$ .... **Ans.**

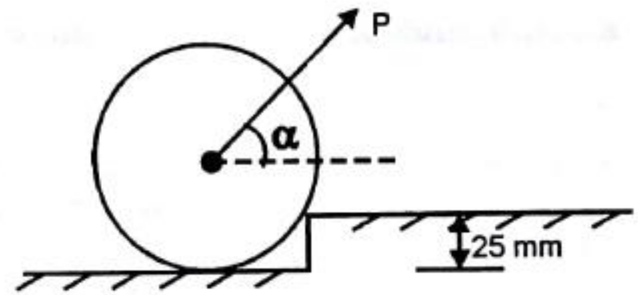
$\Sigma F_x = 0 \quad \rightarrow + ve$

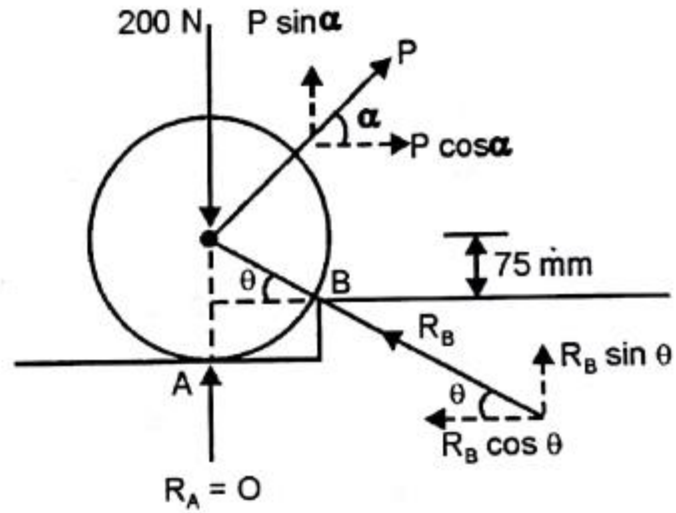
$R_C - T_{AB} \cos 60 = 0$

$\therefore \quad R_C - 1154.7 \cos 60 = 0$

or $\quad R_C = 577.35 \text{ N}$ .... **Ans.**

Ex. 3.8 A wheel of radius 100 mm and weight 200 N needs to be pulled over a 25 mm high kerb by applying a force P on a rope attached at the centre of the wheel.
Find the minimum force required to do so and the corresponding angle α.





Solution: The wheel gets a reaction R_A from smooth surface at A and reaction R_B from edge at B. For the condition that the wheel needs to be pulled over the obstruction, it would loose contact at A and hence $R_A = 0$

Applying COE'

$\sum F_x = 0$

$P \cos \alpha - R_B \cos 48.59 = 0$ .... (1)

$\sum F_y = 0$

$P \sin \alpha + R_B \sin 48.59 - 200 = 0$ .... (2)

From geometry

$$\sin\theta = \frac{75}{100}$$

$\therefore\ \theta = 48.59°$

Eliminating R_B from equation (1) and (2) we get

$$P \sin \alpha + 1.134 P \cos \alpha - 200 = 0$$

or $$P = \frac{200}{\sin\alpha + 1.134\cos\alpha} \quad \text{.........} (3)$$

For minimum value of P, $\frac{dP}{d\alpha} = 0$ $\therefore$ $\frac{dp}{d\alpha} = \frac{-200}{(\sin\alpha + 1.134\cos\alpha)^2} \times (\cos\alpha - 1.134\sin\alpha) = 0$

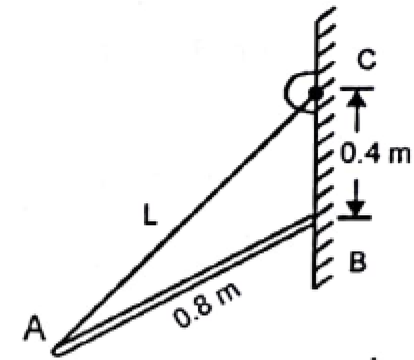
$\therefore$ $\cos\alpha - 1.134\sin\alpha = 0$ $\therefore$ $\tan\alpha = 0.8819$

or $\alpha = 41.41°$ .... **Ans.**

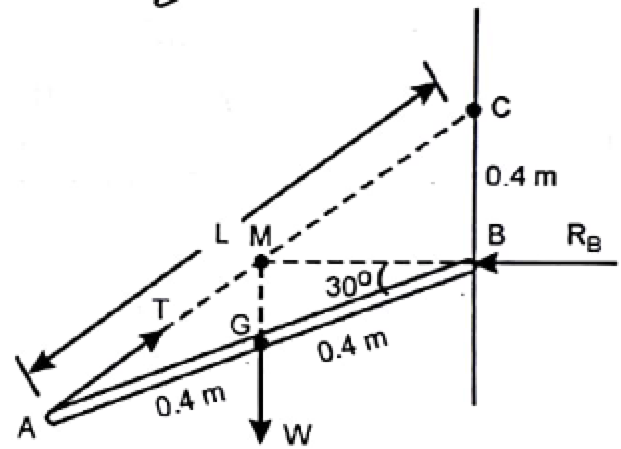
Substituting value of α in equation (3)

$$P = \frac{200}{\sin 41.41 + 1.134\cos 41.41}$$ or $P = 132.28$ N .... **Ans.**

Ex. 3.9 A uniform rod of weight W and length 0.8 m is held in equilibrium with one end resting against a smooth vertical wall, while the other end is supported by a rope. Determine the length L of the rope to be used.



Solution: Figure shows the FBD of the rod AB.



The external supports for the rod are

1) a smooth surface at B, giving reaction R_B ⊥ to the smooth surface.
2) a rope support at A, giving tension reaction force T.

The weight W acts through the rod's C.G. Since the system has three forces in equilibrium, the forces should be concurrent. Let M be the point of concurrence of R_B, T and W.

Δ CAB being similar to Δ MAG ∴ MG = 0.2 m

∴ In Δ BMG ∠ B = 30°

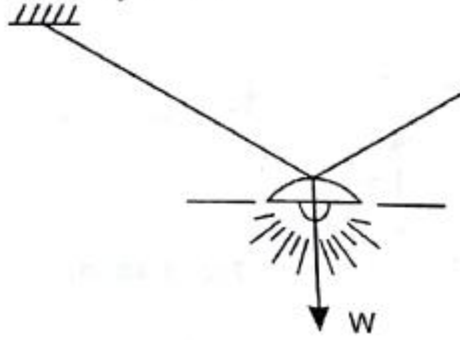
In Δ ABC ∠ B = 120°. Now using cosine rule

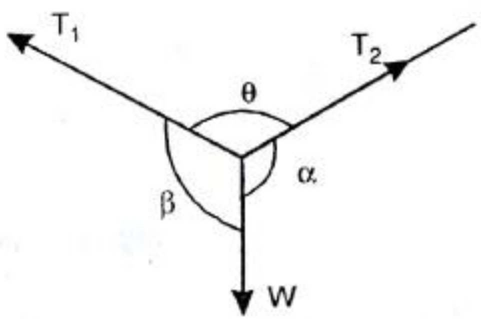
$L^2 = (0.8)^2 + (0.4)^2 - 2 \times 0.8 \times 0.4 \cos 120$

∴ L = 1.058 m .... **Ans.**

Lami's Theorem

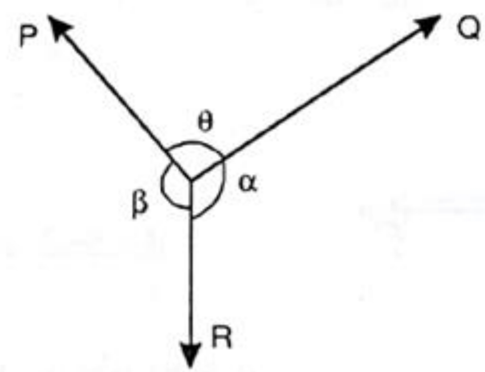
Lami's theorem deals with a particular case of equilibrium involving three forces only. It states "*If three concurrent forces act on a body keeping it in equilibrium, then each force is proportional to the sine of the angle between the other two forces*".

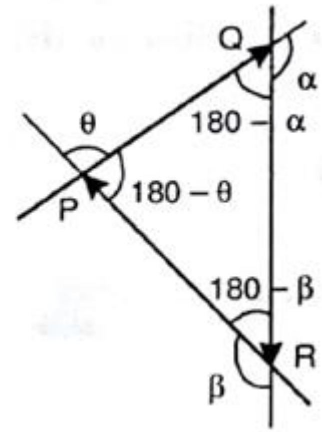




$$\frac{T_1}{\sin\alpha} = \frac{T_2}{\sin\beta} = \frac{W}{\sin\theta}$$

Proof: Let P, Q and R be the three concurrent forces in equilibrium as shown in Fig. 3.19 (a).





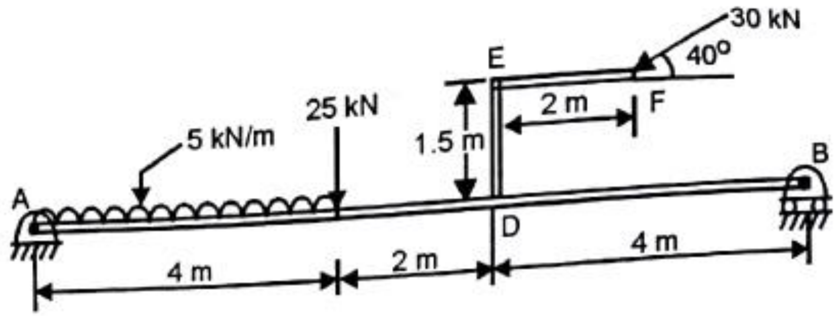
Applying sine rule we get

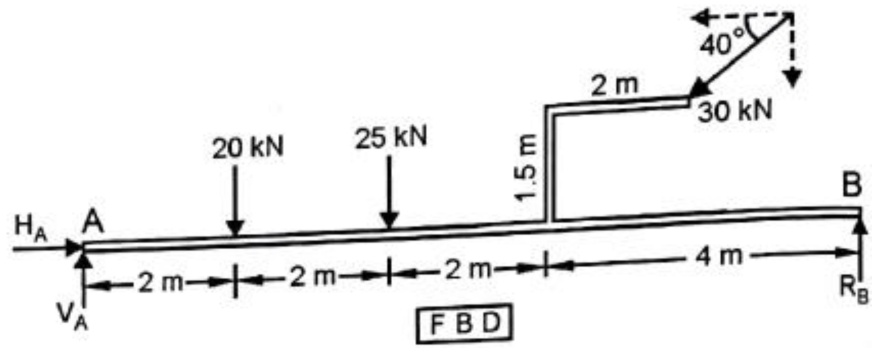
$$\frac{P}{\sin(180-\alpha)} = \frac{Q}{\sin(180-\beta)} = \frac{R}{\sin(180-\theta)}$$

$$\therefore \quad \frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\theta} \quad \text{.............. proved}$$

Problem:

Figure shows beam AB hinged at A and roller supported at B. The L shaped portion DEF is an extended part of beam AB. For the loading shown, find support reactions.





Solution: The beam AB is in equilibrium. It is supported by a hinge at A and roller at B.

The FBD is shown.

COE – Beam AB

$\Sigma M_A = 0$ ↺ + ve

$$-(20\times2)-(25\times4)-(30\sin40\times8)+(30\cos40\times1.5)+(R_B\times10)=0$$

∴ $R_B = 25.98\,\text{kN}$

Or $R_B = 25.98\,\text{kN}\uparrow$ .... **Ans.**

$\sum F_x = 0 \;\rightarrow\; +\text{ve}$

$$H_A - 30\cos 40 = 0$$

$\therefore \quad H_A = 22.98\,\text{kN}$

Or $\quad H_A = 22.98\,\text{kN} \rightarrow$ .... **Ans.**

$\sum F_y = 0 \;\uparrow\; +\text{ve}$

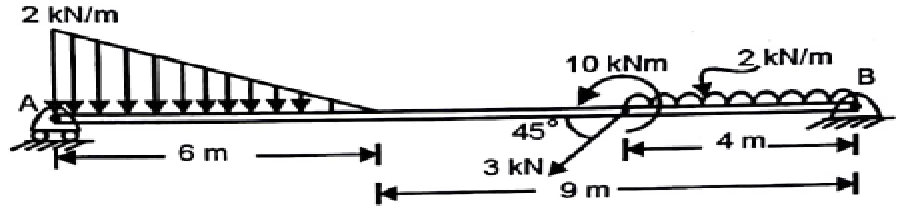
$$V_A - 20 - 25 - 30\sin 40 + 25.98 = 0$$

$\therefore \quad V_A = 38.3\,\text{kN}$

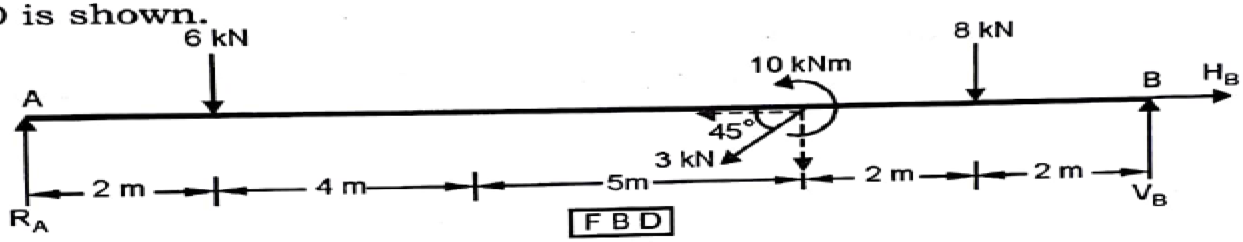
Or $\quad V_A = 38.3\,\text{kN} \uparrow$ .... **Ans.**

Problem:

Find the reactions at the supports of the beam AB loaded as shown.



Solution: The beam AB is in equilibrium. It is supported by a roller at A and a hinge at B. The FBD is shown.



COE – Beam AB

$\Sigma M_A = 0$ ↺ + ve

$-(6\times2)-(3\sin45\times11)-(8\times13)+10+(V_B\times15)=0$

$\therefore \quad V_B = 8.622\,\text{kN}$

Or $\quad V_B = 8.622\,\text{kN} \uparrow$ .... **Ans.**

$\Sigma F_x = 0 \rightarrow +\text{ ve}$

$-(3\cos45)+H_B = 0$

$\therefore \quad H_B = 2.12\,\text{kN}$

Or $\quad H_B = 2.12\,\text{kN} \rightarrow$ .... **Ans.**

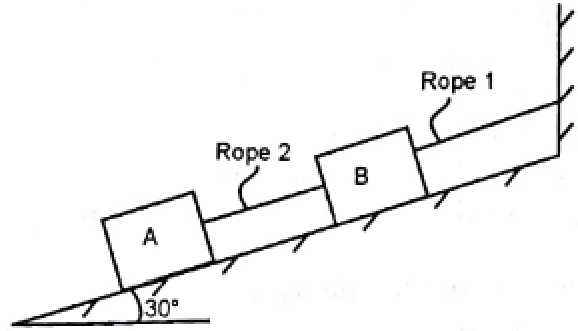
$\Sigma F_y = 0 \uparrow +\text{ ve}$

$R_A - 6 - 3\sin45 - 8 + 8.622 = 0$

$\therefore \quad R_A = 7.5\,\text{kN}$

Or $\quad R_A = 7.5\,\text{kN} \uparrow$ .... **Ans.**

Problem:

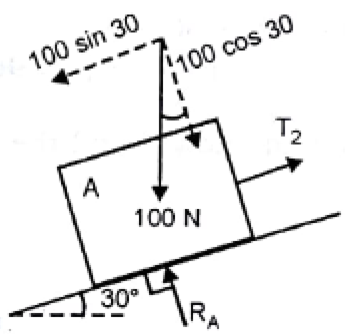


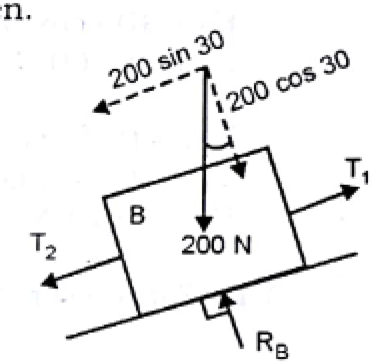
Block A of weight 100 N and B of weight 200 N are connected to each other as shown. They are held in equilibrium on a smooth slope and a rope tied parallel to the slope. Find external support reactions and tension in the connecting rope (2).

Solution: The system has two bodies viz blocks A and B which are externally supported by smooth surfaces offering reactions R_A and R_B and a rope (1) giving reaction T_1.

The bodies are internally connected by rope (2).

On isolating the blocks, the internal reaction tension T_2 can be seen.





Applying COE to block A

$\Sigma F_x = 0$

$T_2 - 100 \sin 30 = 0 \quad \therefore \; T_2 = 50$ N ... **Ans.**

$\Sigma F_y = 0$

$R_A - 100 \cos 30 = 0$

$\therefore \; R_A = 86.6$ N ... **Ans.**

Applying COE to block B

$\Sigma F_x = 0$

$T_1 - T_2 - 200 \sin 30 = 0$

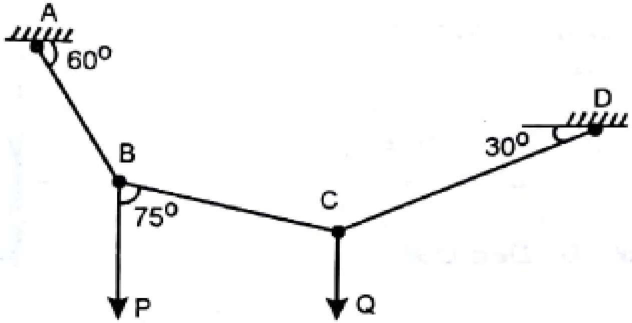
$\therefore \; T_1 - 50 - 100 = 0 \quad \therefore \; T_1 = 150$ N ... **Ans.**

$\Sigma F_y = 0$

$R_B - 200 \cos 30 = 0 \quad \therefore \; R_B = 173.2$ N ... **Ans.**

Problem:

A string ABCD carries two loads P and Q. If P = 50 kN, find force Q and tensions in different portions of the string.

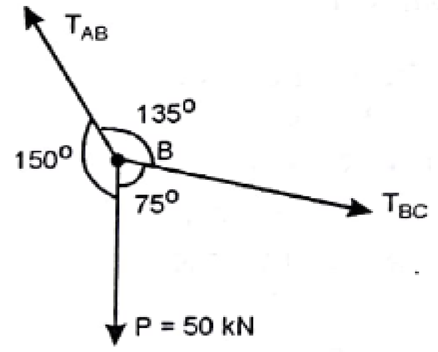


Solution: Isolating joint B of the string. Let T_{AB} and T_{BC} be the tensions in the string portions AB and BC respectively.
Using Lami's equation

$$\frac{T_{AB}}{\sin 75} = \frac{T_{BC}}{\sin 150} = \frac{50}{\sin 135}$$

$\therefore$ T_{AB} = 68.3 kN .... **Ans.**

T_{BC} = 35.35 kN ....**Ans.**



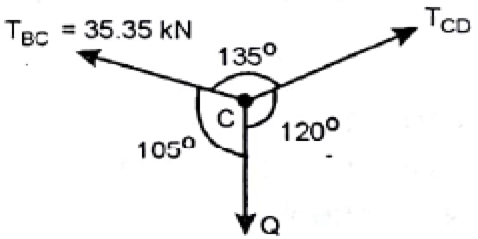
Now isolating joint C.
Let T_{CD} be the tension in portion CD.

Using Lami's equation

$$\frac{35.35}{\sin 120} = \frac{T_{CD}}{\sin 105} = \frac{Q}{\sin 135}$$

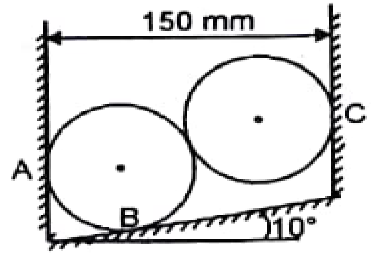
$\therefore$ T_{CD} = 39.43 kN .... **Ans.**

Q = 28.86 kN .... **Ans.**

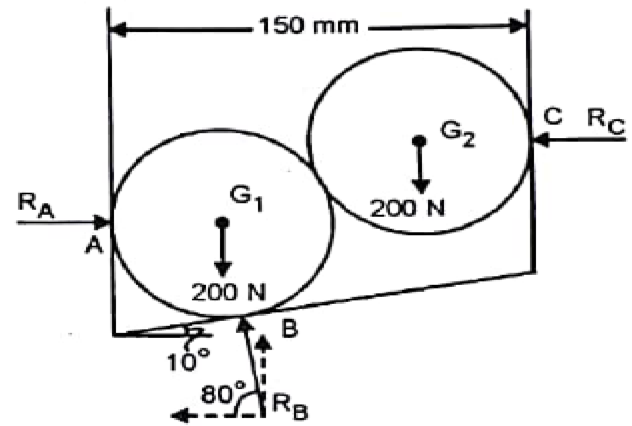


Problem:

Two cylinders each of diameter 100 mm and each weighing 200 N are placed as shown in figure. Assuming that all the contact surfaces are smooth find the reactions at A, B and C.



Solution: The system consists of two cylinders supported against three smooth surfaces at A, B and C. Let R_A, R_B and R_C be the reactions at three supports. The FBD of the system is shown.



Applying COE to the system

$\Sigma M_{G1} = 0$ ↺ +ve

$-(200 \times 50) + (R_C \times 86.6) = 0$

$\therefore \quad R_C = 115.47\,\text{N}$

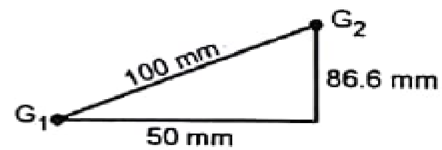
or $\quad R_C = 115.47\,\text{N} \leftarrow$ ……… **Ans.**

$\Sigma F_y = 0 \quad \uparrow +\text{ve}$

$R_B \sin 80 - 200 - 200 = 0$

$\therefore \quad R_B = 406.17\,\text{N}$

or $\quad R_B = 406.17\,\text{N}$, $\theta = 80°$ ……… **Ans.**



$\Sigma F_x = 0$

$R_A - R_B \cos 80 - R_C = 0$

$R_A - 406.17 \cos 80 - 115.47 = 0$

$\therefore \quad R_A = 186\,\text{N}$

or $\quad R_A = 186\,\text{N} \rightarrow$ ……… **Ans.**