Equilibrium of Force System

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Equilibrant :

A *force*, which is *equal, opposite* and *collinear to the resultant of a concurrent force system* is known as the *equilibrant of the concurrent force system.*

Equilibrant is the force which, when applied to a body acted by the concurrent force system, keeps the body in equilibrium.

Condition of Equilibrium for Various Force Systems:

A body is said to be in equilibrium when it is at rest or continues to be in uniform motion. According to Newton's first law of motion, *the body remains at rest or moves with uniform velocity if the resultant force acting on body is zero.*

When all these sums are zero for any force system its resultant is zero and the body on which the system acts is in equilibrium.

In simple words, when *the resultant of force system acting on a body is zero, the body is in equilibrium.*

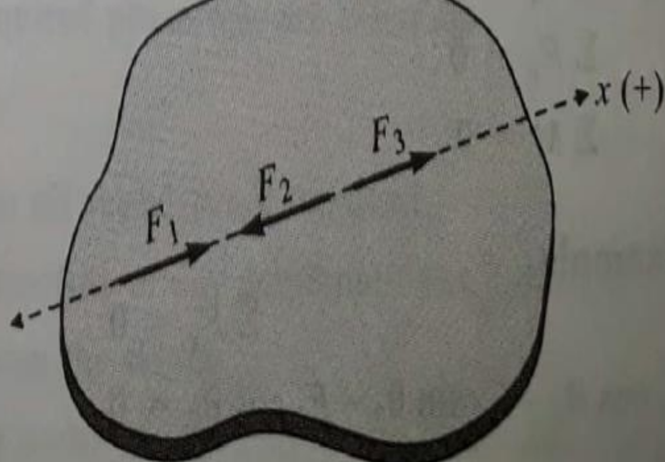
Thus, the resultant force R and resultant moment (couple) M_R both are zero, and we have the equilibrium equations

$$R = \Sigma F = 0 \quad \text{and} \quad M_R = \Sigma M = 0$$

1. **Equilibrium of Collinear Force System :** If forces are collinear then only one axis contains all the forces. Therefore, only one force equation in the direction of the force is required.

$$\Sigma F_x = 0$$

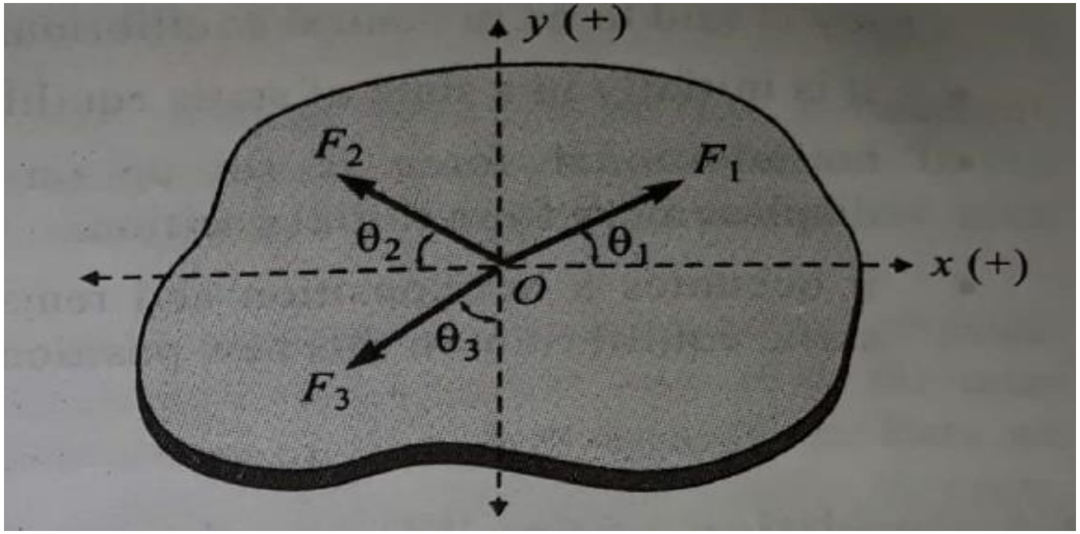
Example



$$\Sigma F_x = 0$$

$$F_1 - F_2 + F_3 = 0$$

2. Equilibrium of Concurrent Force System : If all the forces in coplanar force system are concurrent, then the following sets of equation can be used :

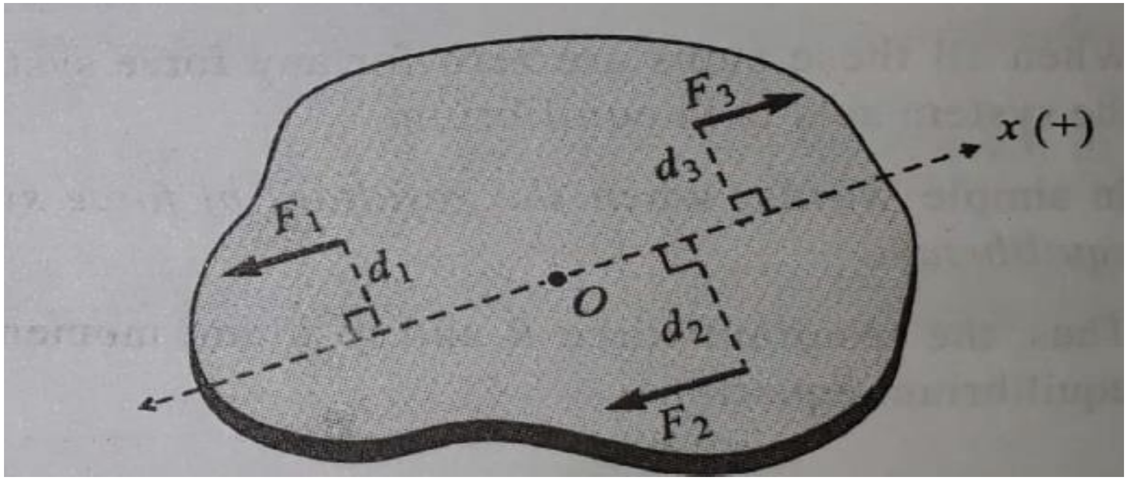


$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$

Example

$$\Sigma F_x = 0$$
$$F_1 \cos \theta_1 - F_2 \cos \theta_2 - F_3 \sin \theta_3 = 0$$
$$\Sigma F_y = 0$$
$$F_1 \sin \theta_1 + F_2 \sin \theta_2 - F_3 \cos \theta_3 = 0$$

3. Equilibrium of Parallel Force System : If all the forces in coplanar force system are parallel, then the following sets of equation can be used :

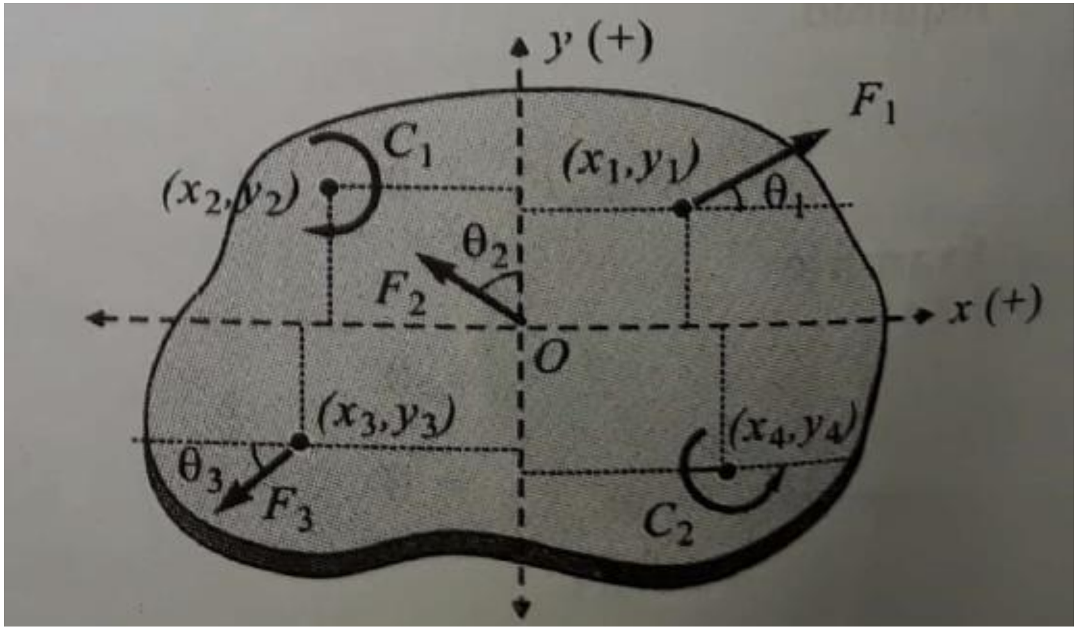


$$\Sigma F = 0$$
$$\Sigma M = 0$$

Example

$$\Sigma F_x = 0$$
$$-F_1 - F_2 + F_3 = 0$$
$$\Sigma M_O = 0$$
$$F_1 \times d_1 - F_2 \times d_2 - F_3 \times d_3 = 0$$

4. Equilibrium of General Force System : If all the forces and couples acting in a plane form general force system, then the following sets of equation can be used :



$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M = 0$$

Example

$$\Sigma F_x = 0$$

$$F_1 \cos \theta_1 - F_2 \sin \theta_2 - F_3 \cos \theta_3 = 0$$

$$\Sigma F_y = 0$$

$$F_1 \sin \theta_1 + F_2 \cos \theta_2 - F_3 \sin \theta_3 = 0$$

$$\Sigma M_O = 0$$

$$-F_1 \cos \theta_1 \times y_1 + F_1 \sin \theta_1 \times x_1 - F_3 \cos \theta_3 \times y_3 + F_3 \sin \theta_3 \times x_3 - C_1 + C_2 = 0$$

FREE BODY DIAGRAM (F.B.D.)

- *The **Free Body Diagram (F.B.D.)** is a sketch of the body showing all active and reactive forces that acts on it after removing all supports with consideration of geometrical angles and distance given.*

- **Importance of F.B.D.**

1. The sketch of F.B.D. is the key step that translates a physical problem into a form that can be analysed mathematically.

2. The F.B.D. is the sketch of a body, a portion of a body or two or more connected bodies completely isolated or free from all other bodies, showing the force exerted by all other bodies on the one being considered.

3. F.B.D. represents all active (applied) forces and reactive (reactions) forces. Forces acting on the body that are not provided by the supports are called *active force* (weight of the body and applied forces). *Reactive forces* are those that are exerted on a body by the supports to which it is attached.

4. F.B.D. helps in identifying known and unknown forces acting on a body.

5. F.B.D. helps in identifying which type of force system is acting on the body so by applying appropriate condition of equilibrium, the required unknowns are calculated.

Procedure for Drawing a F.B.D.

1. Draw a neat sketch of the body assuming that all supports are removed.

2. F.B.D. may consist of an entire assembled structure or any combination or part of it.

3. Show all the relevant dimensions and angles on the sketch.

4. Show all the active forces on corresponding point of application and insert their magnitude and direction, if known.

5. Show all the reactive forces due to each support.

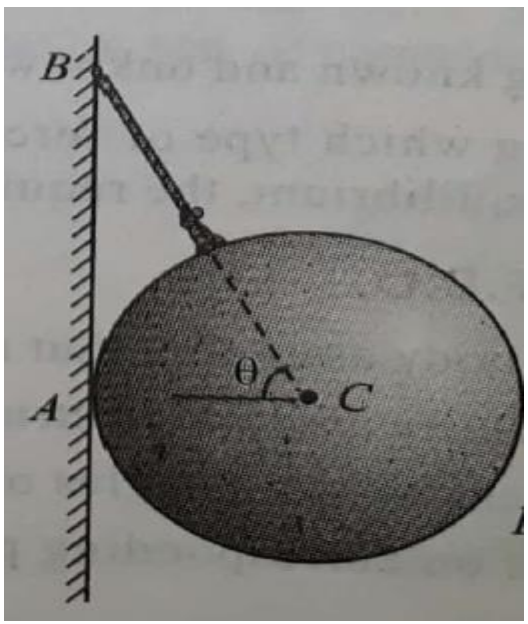
6. The F.B.D. should be legible and neatly drawn and of sufficient size to show dimensions, since this may be needed in computation of moments of forces.

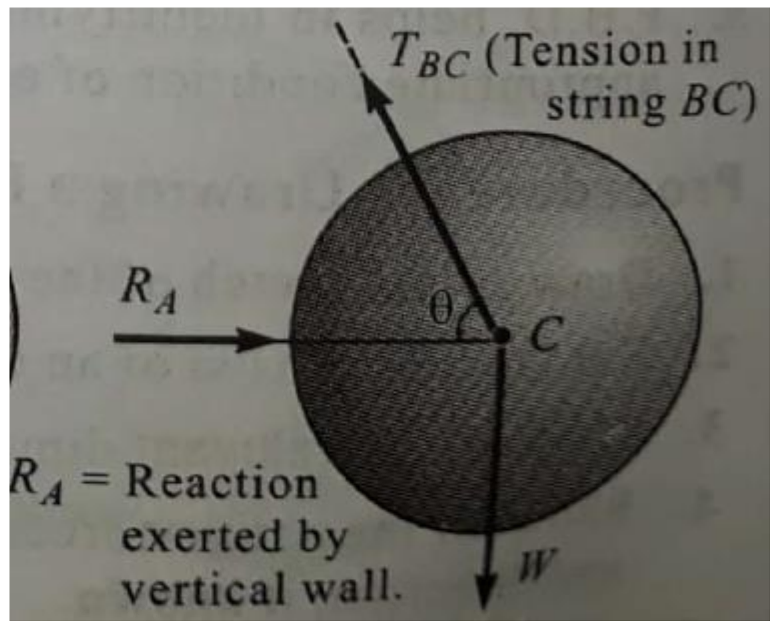
7. If the sense of reaction is unknown, it should be assumed. The solution will determine the correct sense. A positive result indicates that the assumed sense is correct, whereas a negative result means the assumed sense was incorrect, so the correct sense is opposite to the assumed sense.

8. Use principle of transmissibility wherever convenient.

Example 1:

A sphere having weight W is suspended by a string but rest against the vertical wall. Here the sphere is constrained to move downward by string and towards left due to vertical wall. The sphere not only pulls down on string BC but also pushes to the left against the vertical wall at A. In F.B.D. we remove string and vertical wall and replace by tension T and reaction R_A. Weight W is the active force.

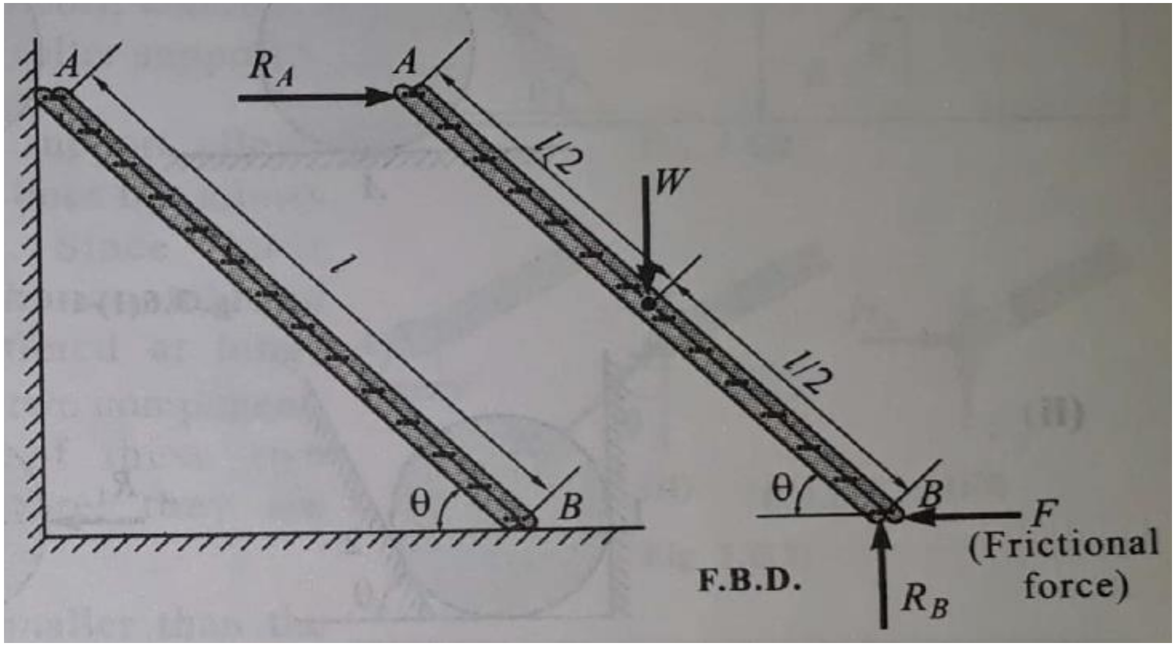




FBD

Example2:

Ladder having weight W resting against the rough horizontal floor and smooth vertical wall. R_A is reactive force exerted by vertical wall and R_B is the reactive force exerted by horizontal floor on ladder. F is the reactive frictional force between horizontal floor and ladder, weight W is the active force of ladder.

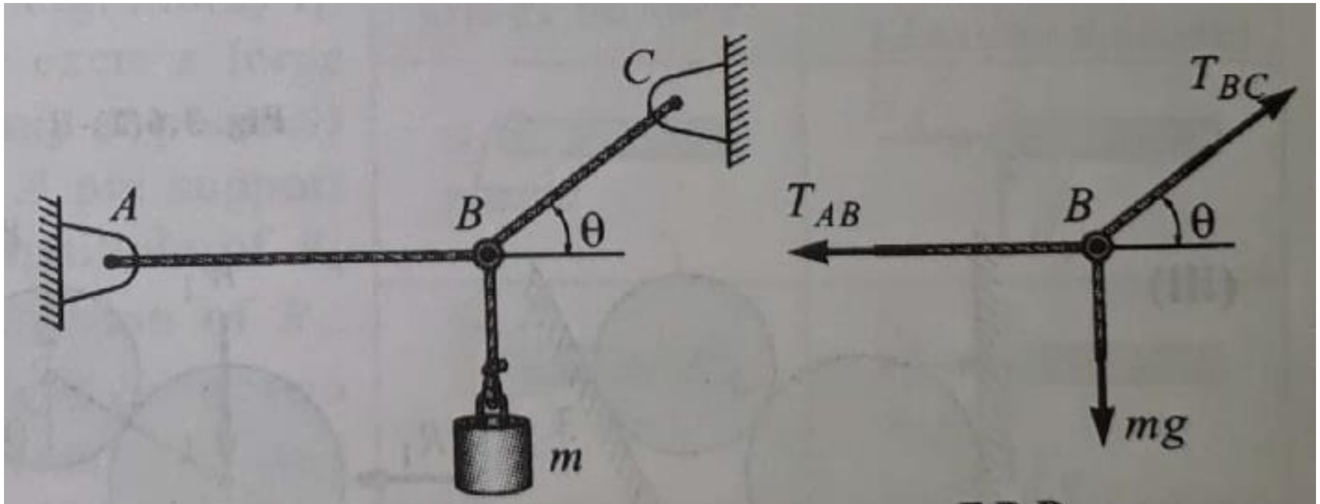


Example 3:

A block of mass m kg is suspended by ropes as shown in the figure.

$T_{AB} \Rightarrow$ Tension in rope AB

$T_{BC} \Rightarrow$ Tension in rope BC

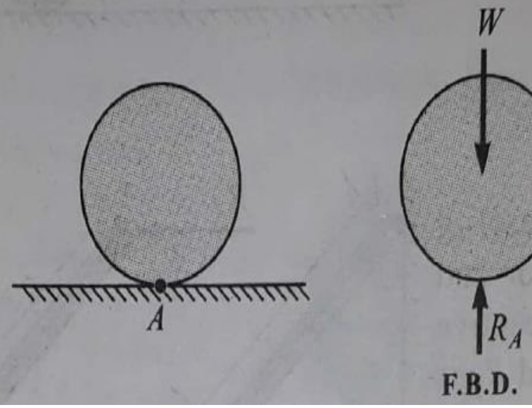


F.B.D.

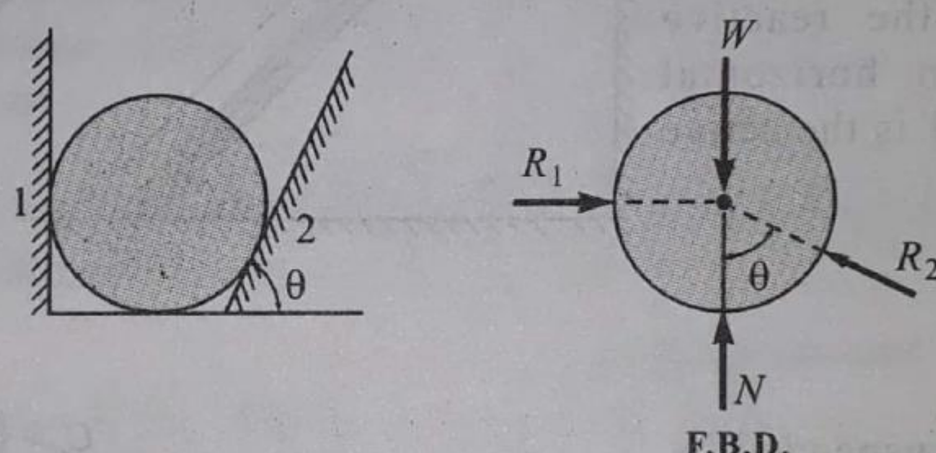
Types of Supports

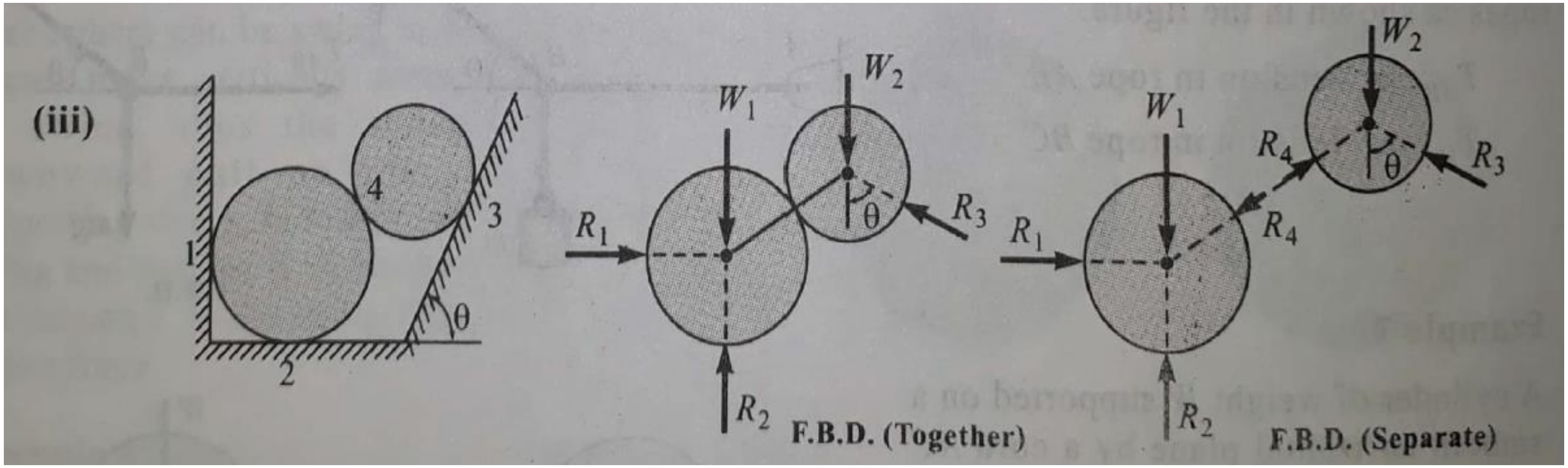
1. **Smooth Surface Contact :** When a body is in contact with a smooth (frictionless) surface at only one point, the reaction is a force normal to the surface, acting at the point of contact.

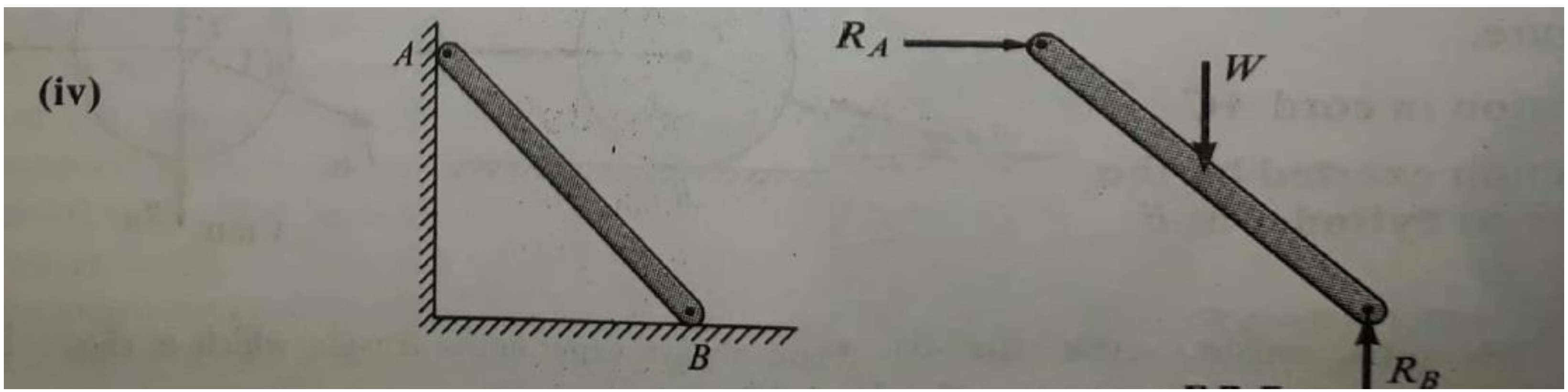
(i)



(ii)

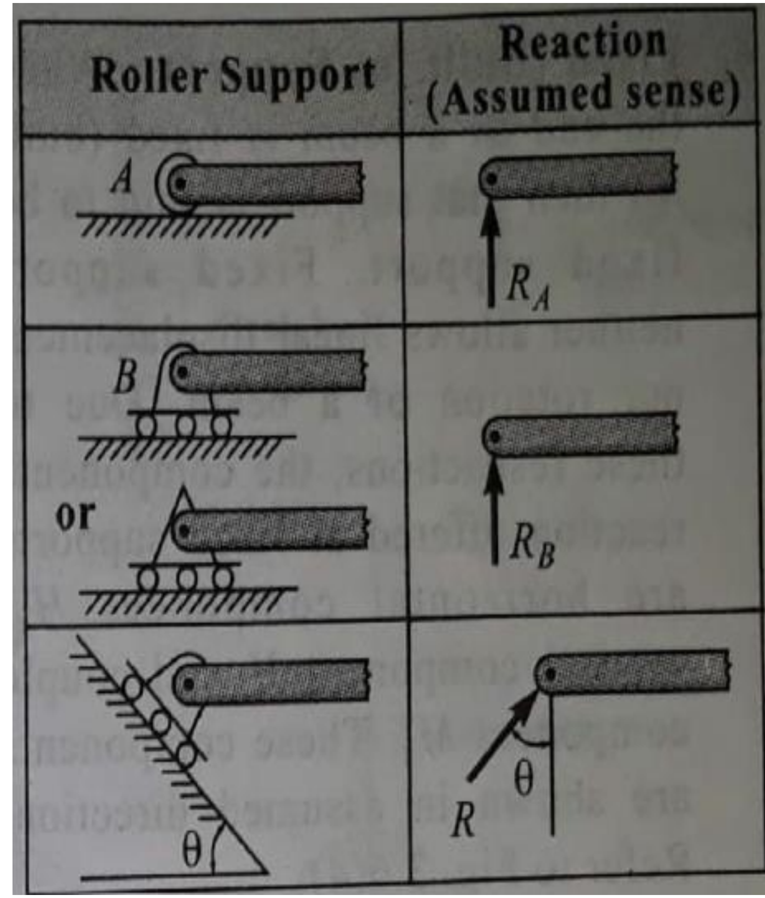



(iii)
1
2
3
4
θ
W_1
W_2
R_1
R_2
R_3
θ
F.B.D. (Together)
W_1
W_2
R_1
R_2
R_3
R_4
R_4
θ
F.B.D. (Separate)

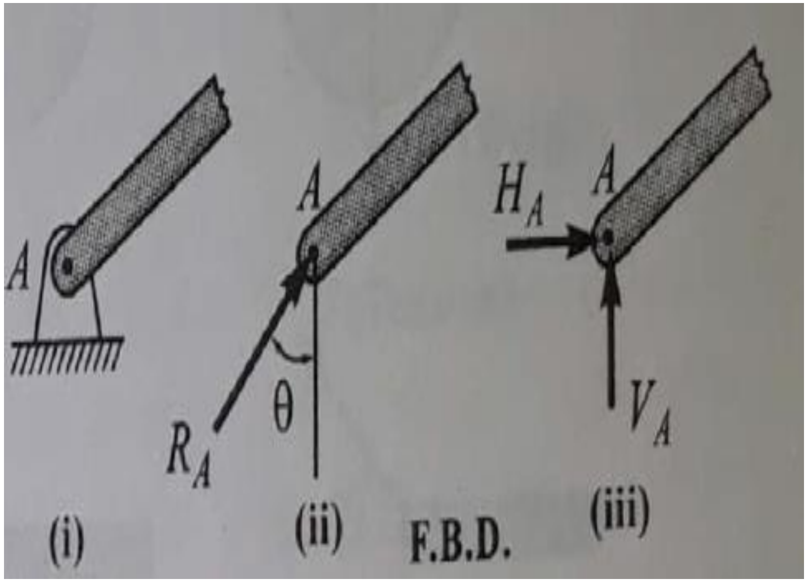

(iv)
A
B
R_A
W
R_B

FBD

2. **Roller Support :** A roller support is equivalent to a frictionless surface. It can only exert a force that is perpendicular to the supporting surface. The magnitude of the force is then the only unknown force introduced in a F.B.D. when the support is removed. The roller support is free to roll along the surface on which it rests. Since the linear displacement in normal direction to surface of roller is restricted, it offers a reaction in normal direction to surface of roller. For example, a sliding door slides smoothly with the help of a roller support, whereas a conveyer belt can move smoothly on roller support.

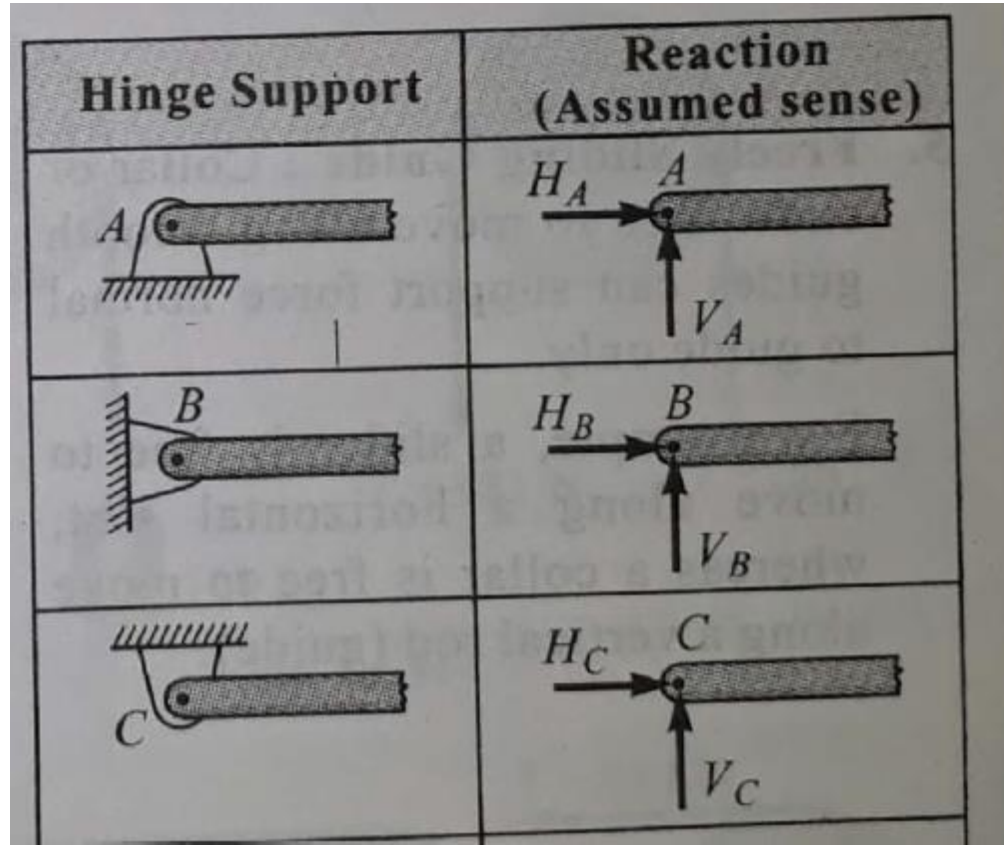


Hinge (Pin) Support : The hinge support allows free rotation about the pin end but it does not allows linear displacement of that end. Since linear displacements are restricted in horizontal and vertical directions, the reaction offered at hinge support (say R_A at θ) is resolved into two component, i.e., H_A and V_A. The direction of these two components are uncertain. Therefore, they are initially assumed in F.B.D.



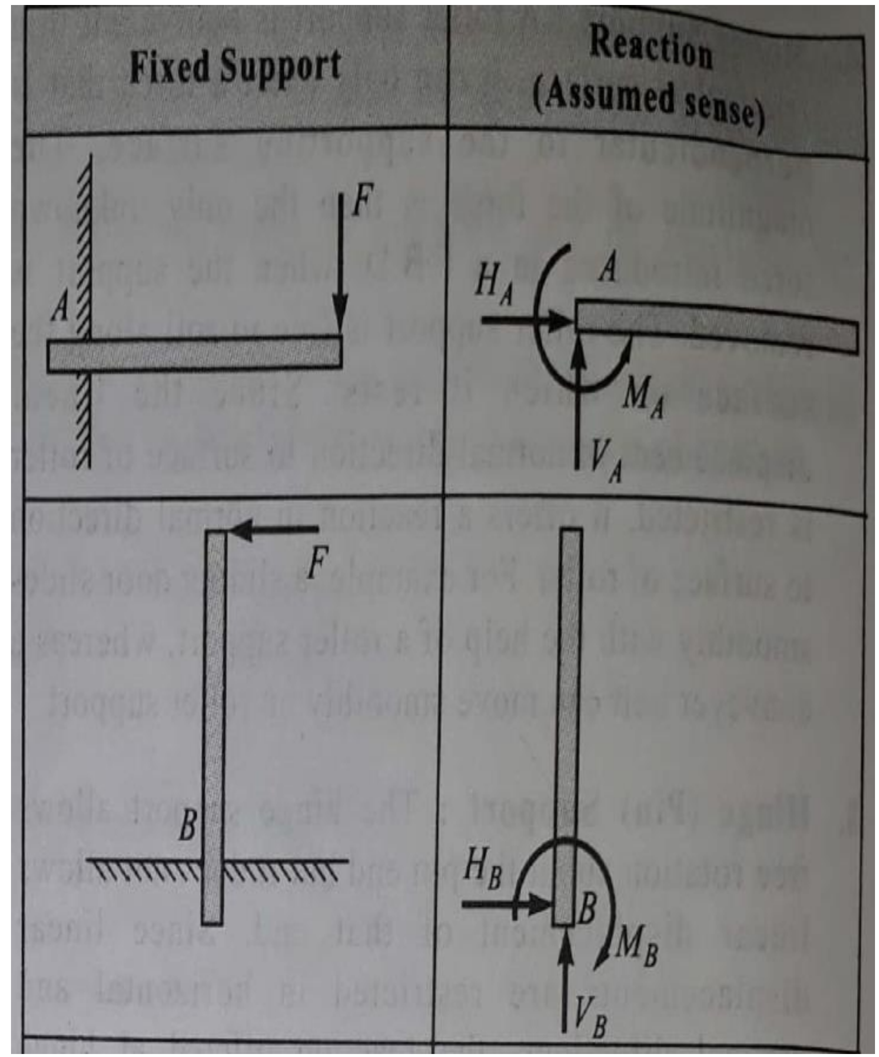
Introduces two unknowns, the magnitude of R_A and the angle Θ that specifies the direction of R_A

Reaction R_A at Θ can be resolved into two components i.e, Horizontal Component (H_A) and Vertical Component (V_A)



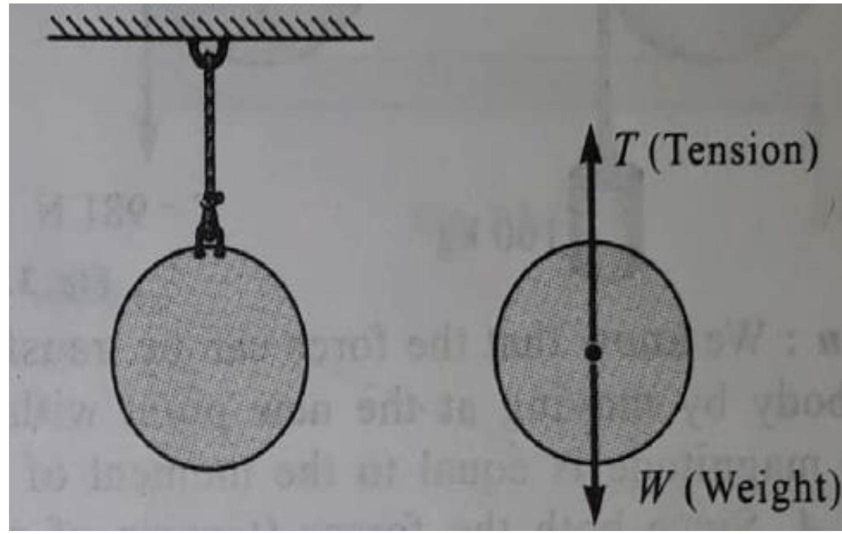
4. Fixed Support :

When the end of a beam is fixed then that support is said to be fixed support. Fixed support neither allows linear displacement nor rotation of a beam. Due to these restrictions, the components reaction offered at fixed supports are horizontal component H_A, vertical component V_A and couple component M_A. These components are shown in assumed direction.



5. String/ Cable/ Rope/Cord/Chain/Wire Support :

The force developed in rope is always a tension away from the body in the direction of rope. When one end of a rope is connected to a body then the rope is not to be considered as a part of the system and it is replaced by tension in F.B.D. as shown



EQUILIBRIUM OF TWO-FORCE SYSTEM

Two-Force Principle

If a body is in equilibrium and is acted upon by only two forces then these two forces must be equal in magnitude, opposite in direction and collinear.

Special case of two-force member

If a member (body) is a straight rod and subjected to two forces (pulling or pushing) having equal magnitude, opposite direction and collinear action, then the member is either in tension (pulling forces) or in compression (pushing forces).

Tension is represented by an arrow drawn away from joint or body whereas compression is represented by an arrow drawn towards the joint or body.

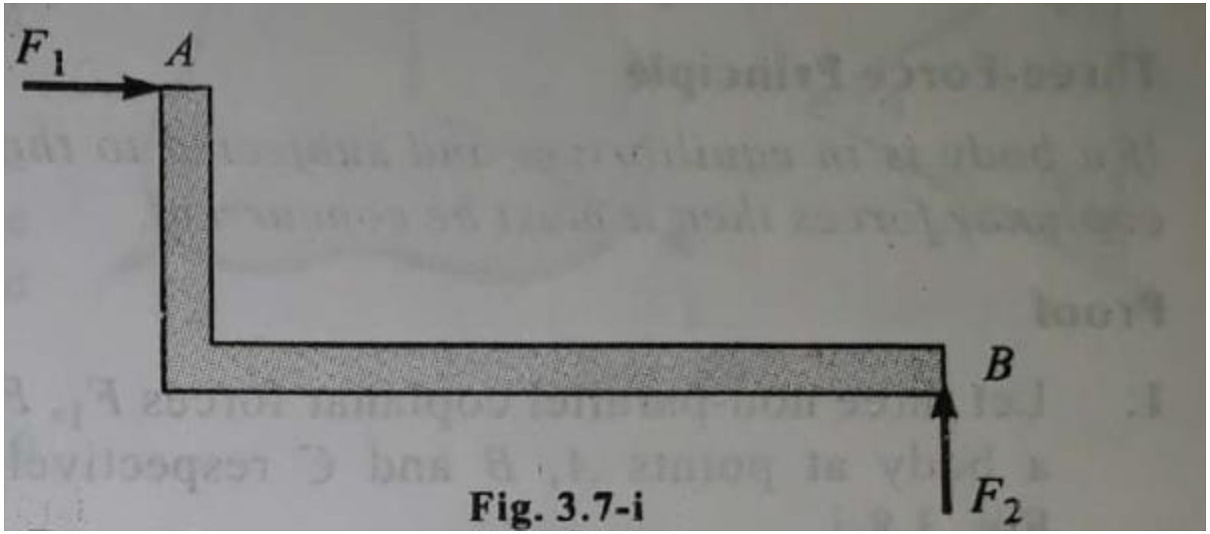


Fig. 3.7-i

Fig. 3.7- i and Fig. 3.7-iii are equivalent figures

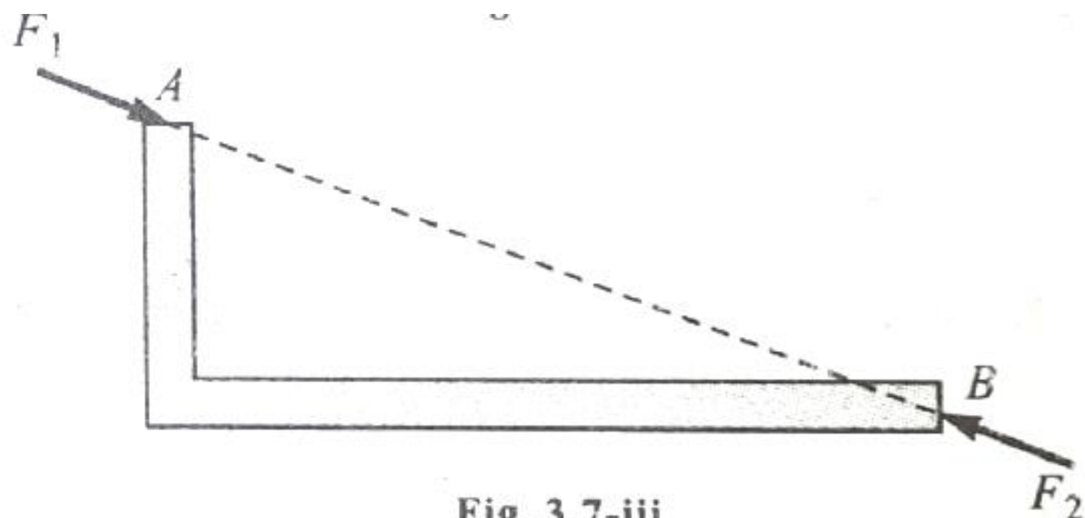


Fig. 3.7-iii

EQUILIBRIUM OF THREE-FORCE SYSTEM

If a body is in equilibrium and subjected to three coplanar forces then the force system acting on the body should be either concurrent or parallel force system.

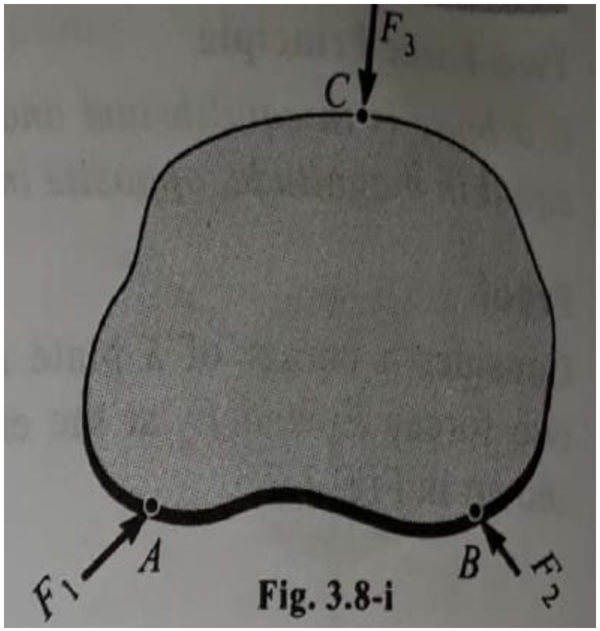


Fig. 3.8-i

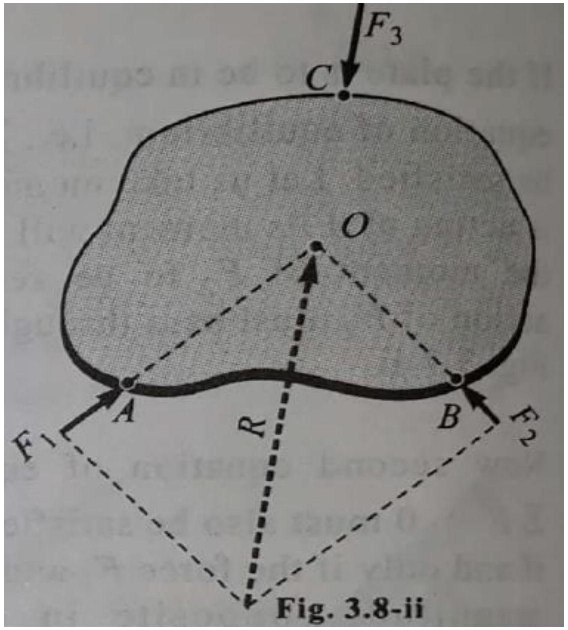


Fig. 3.8-ii

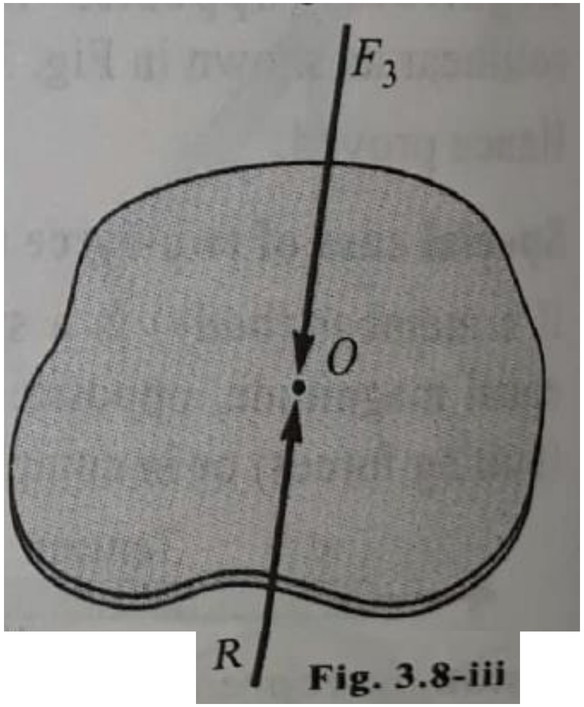


Fig. 3.8-iii

Note: In a three-force system, the resultant of any two forces becomes equal, opposite and collinear with the third force to attain the equilibrium.

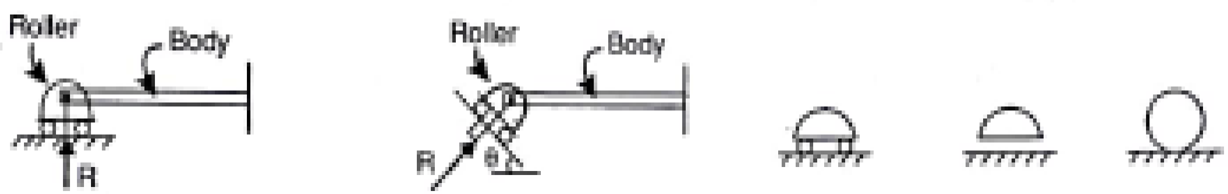
We have already studied different type of supports including roller support earlier.

Roller support can also be shown in this way :

Roller Support

A roller support is free to roll on a surface on which it rests. It offers a force reaction in a direction normal to the surface on which the roller is supported.

A roller support may be shown in any of the three symbols as shown in figure 3.4 (c).



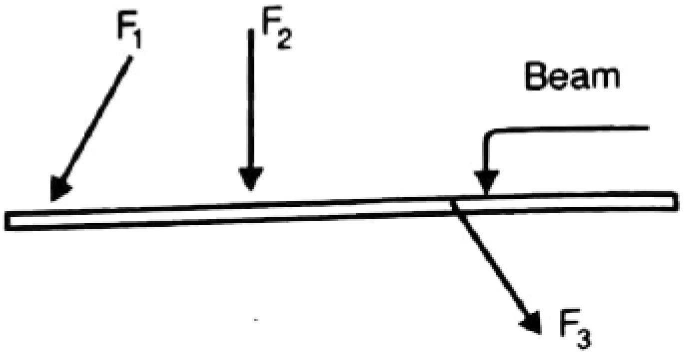
(a) Roller supported on horizontal surface

(b) Roller supported on inclined surface

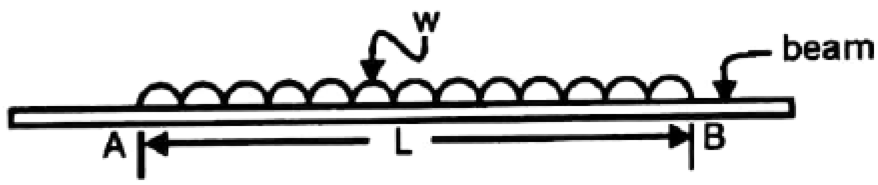
(c) Representation of roller

Types of Loads:

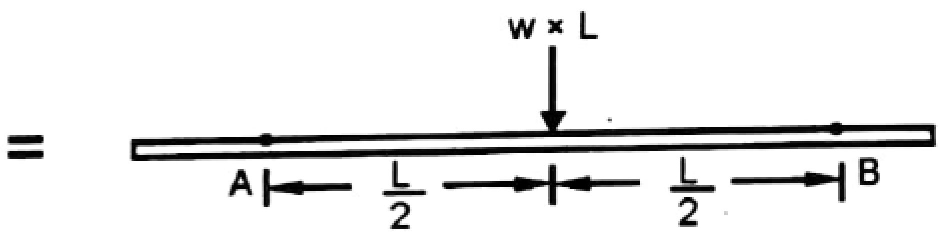
1. Point Load



2. Uniformly Distributed Load (U.D.L.)

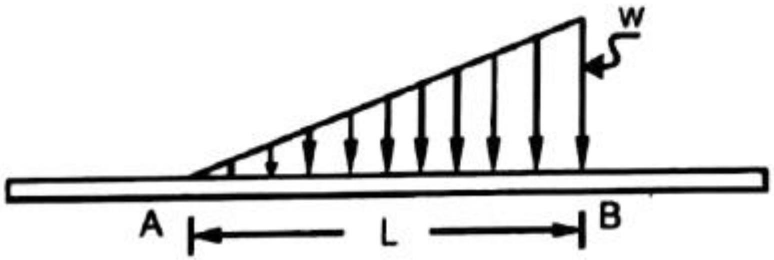


u.d.l spread over potion AB

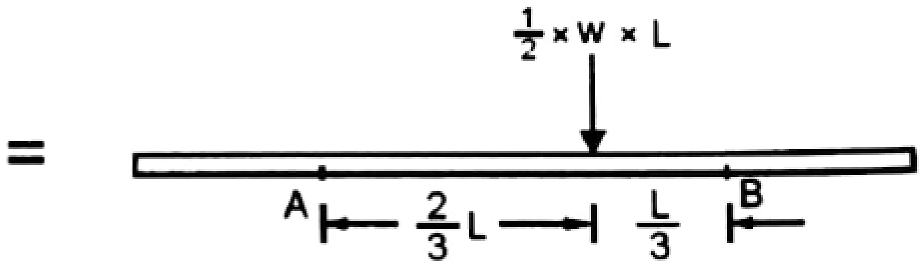


equivalent point load

3. Uniformly Varying Load (U.V.L.)

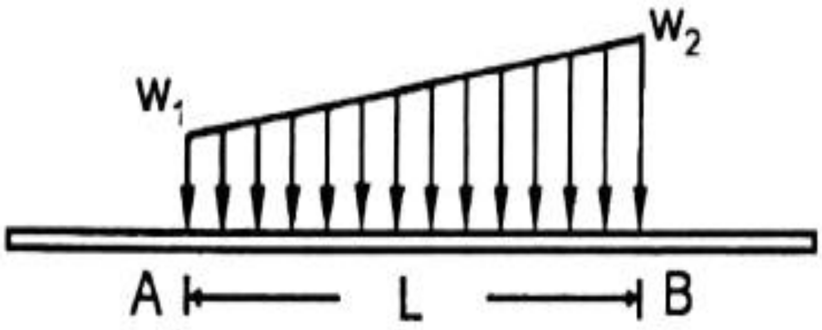


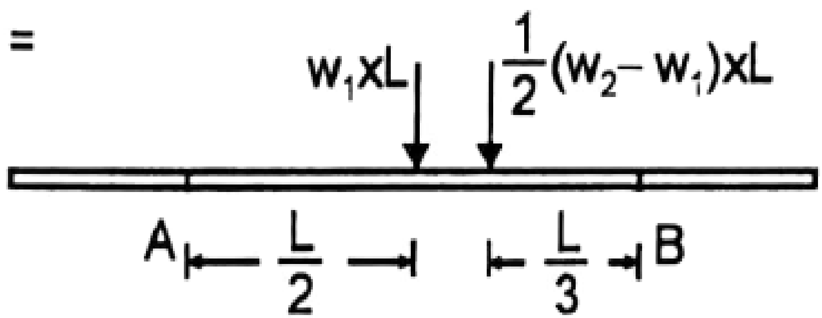
u.v.l. spread over potion AB



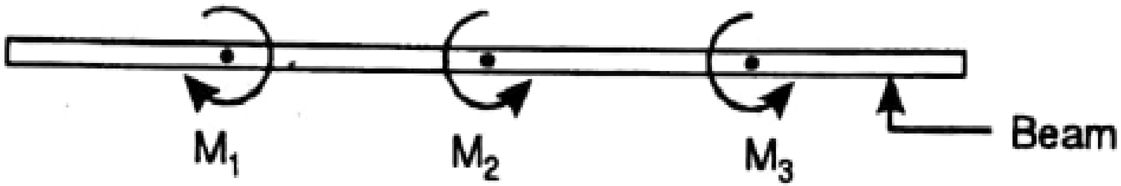
equivalent point load

4. Trapezoidal Load

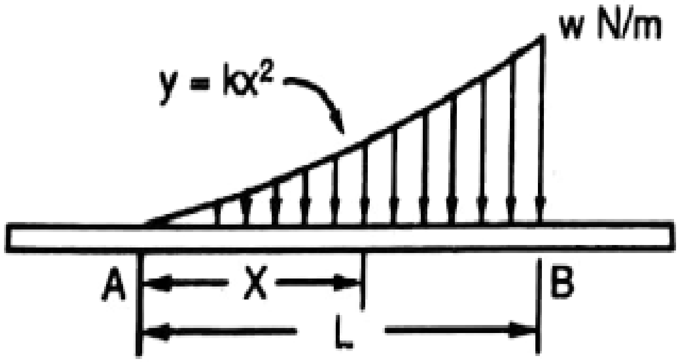




5. Couple Load

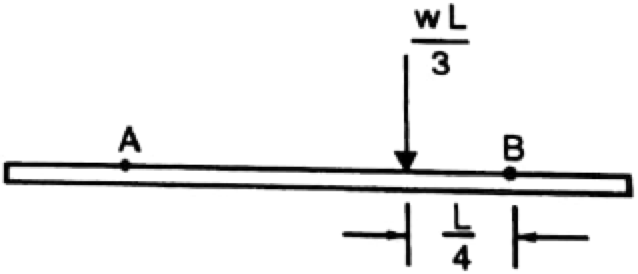


6. Varying Load



Varying load spread over portion AB

=



Equivalent point load