Engineering Mechanics

Module 3.1 – Centroid of Wires, Laminas and Solids

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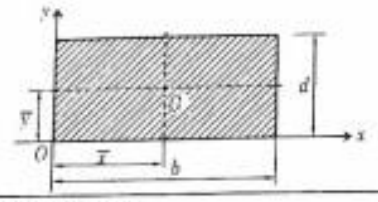
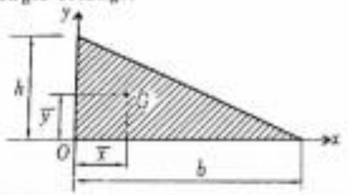
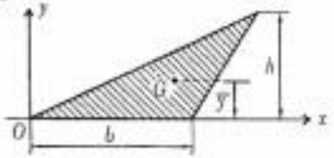
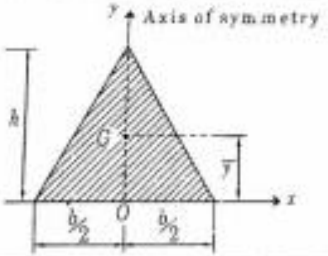
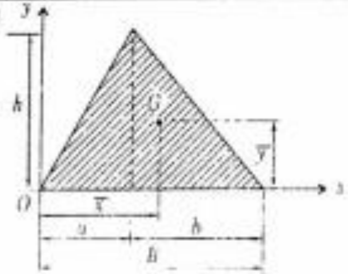
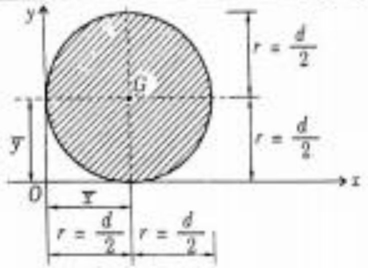
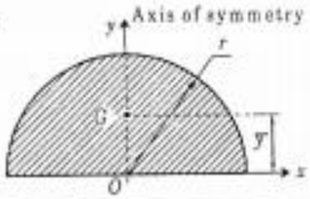
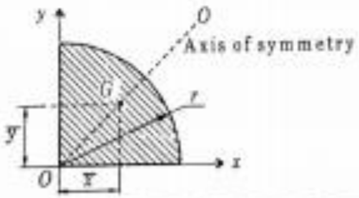
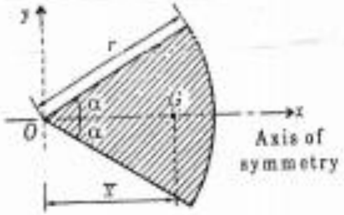
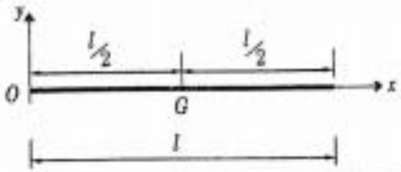
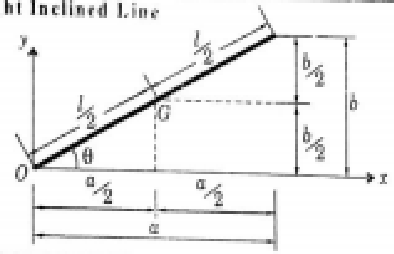
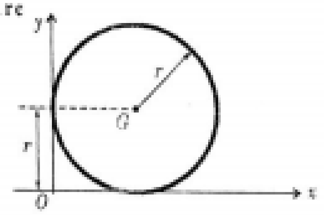
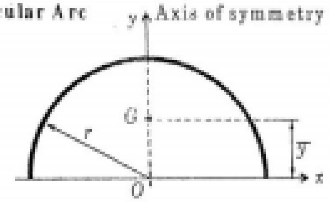
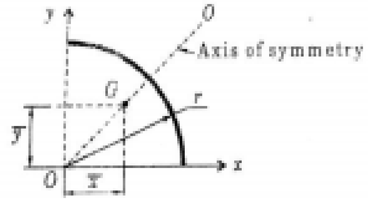
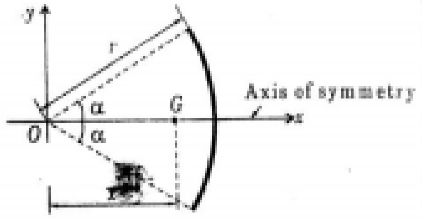
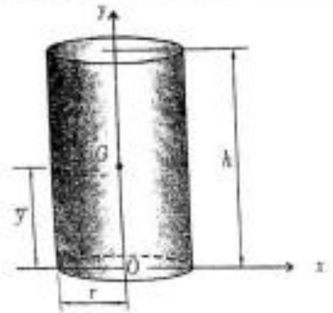
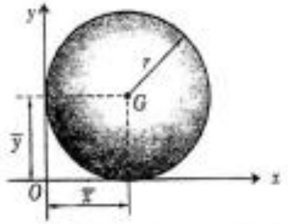
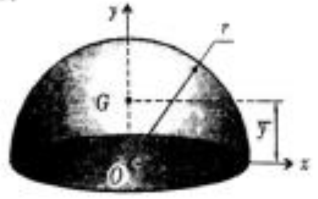
Table on Centroid of Laminas, Wires or Rods and Solid Bodies

Sr. No.	Plane Figure	Area	$\bar{x}$	$\bar{y}$
1.	Rectangle 	bd	$\frac{b}{2}$	$\frac{d}{2}$
2.	Right Angle Triangle 	$\frac{1}{2}bh$	$\frac{b}{3}$	$\frac{h}{3}$
	Any Triangle 	$\frac{1}{2}bh$	—	$\frac{h}{3}$
	Symmetrical Triangle 	$\frac{1}{2}bh$	0	$\frac{h}{3}$
	Unsymmetrical Triangle 	$\frac{1}{2}(a+b)h = \frac{1}{2}Bh$	$\frac{2a+b}{3} = \frac{a+B}{3}$	$\frac{h}{3}$

Plane Figure	Area	$\bar{x}$	$\bar{y}$
Circle 	πr^2 or $\frac{\pi d^2}{4}$	r or $\frac{d}{2}$	r or $\frac{d}{2}$
Semi-circle 	$\frac{\pi r^2}{2}$	0	$\frac{4r}{3\pi}$
Semi-circle 	$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
Sector 	$r^2\alpha$ α measured in radians	$\frac{2r \sin \alpha}{3\alpha}$	0
Straight Horizontal Line 	Length l	$\frac{l}{2}$	0

Lines/ Wires/ Rods/ Arcs	Length	$\bar{x}$	$\bar{y}$
1. Straight Inclined Line 	l	$\frac{a}{2} = \frac{l}{2}\cos\theta$	$\frac{b}{2} = \frac{l}{2}\sin\theta$
2. Circular Arc 	$2\pi r$	r	r
3. Semi-circular Arc 	πr	0	$\frac{2r}{\pi}$
4. Semi-circle 	$\frac{\pi r}{2}$	$\frac{2r}{\pi}$	$\frac{2r}{\pi}$
5. Sector of an Arc 	$2r\alpha'$ α' measured in radians	$\frac{r\sin\alpha}{\alpha'}$	0

Centroid of Common Solid Bodies

Solid Bodies	Volume	$\bar{x}$	$\bar{y}$
Cylinder 	$\pi r^2 h$	0	$\frac{h}{2}$
Right Circular Cone 	$\frac{1}{3}\pi r^2 h$	0	$\frac{h}{4}$
Sphere 	$\frac{4}{3}\pi r^3$	r	r
Hemi-sphere 	$\frac{2\pi r^3}{3}$	0	$\frac{3r}{8}$

Centroid of Composite Area / Rods (Lines or Wires)

An area/rod made up of number of regular plane areas/rods are known as composite area/rods. To locate the centroid of a composite area/rod, we adopt the following procedures :

(1) Study the given figure properly and select suitable coordinate axes if axes are not specified.

At the time of choosing the axis, check the symmetry of the figure.

(a) If composite figure is symmetrical about x-axis, we find without calculation.

(b) If composite figure is symmetrical about y-axis we find without calculation.

(c) If composite figure is symmetrical about x-axis and y-axis, then centroid lies on intersection of these two axis.

(2) Divide the composite figure into different parts having known areas in case of laminas and known lengths in case of rods.

(3) Mark the centroids G_1, G_2, G_3, ... on the individual areas/rods and find their coordinates from the reference axes.

(4) Prepare the table containing areas or lengths, distance of individual centroid from reference axis etc. ....

(5) To find out the coordinates of centroid, we use the following :

(a) **For Areas**

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3 + \ldots}{A_1 + A_2 + A_3 + \ldots} = \frac{\Sigma A_i x_i}{\Sigma A_i} \qquad \bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3 + \ldots}{A_1 + A_2 + A_3 + \ldots} = \frac{\Sigma A_i y_i}{\Sigma A_i}$$

Here $A_1, A_2, \ldots$ = Areas of individual components.

$x_1, x_2, \ldots$ = Distance of individual centroid from y-axis.

$y_1, y_2, \ldots$ = Distance of individual centriod from x-axis.

(b) **For Lines**

$$\bar{x} = \frac{l_1 x_1 + l_2 x_2 + l_3 x_3 + \ldots}{l_1 + l_2 + l_3 + \ldots} = \frac{\Sigma l_i x_i}{\Sigma l_i} \qquad \bar{y} = \frac{l_1 y_1 + l_2 y_2 + l_3 y_3 + \ldots}{l_1 + l_2 + l_3 + \ldots} = \frac{\Sigma l_i y_i}{\Sigma l_i}$$

$$\bar{z} = \frac{l_1 z_1 + l_2 z_2 + l_3 z_3 + \ldots}{l_1 + l_2 + l_3 + \ldots} = \frac{\Sigma l_i z_i}{\Sigma l_i}$$

Here $l_1, l_2, \ldots$ = Length of individual rods.

(c) **For Solid Bodies [with Constant Density]**

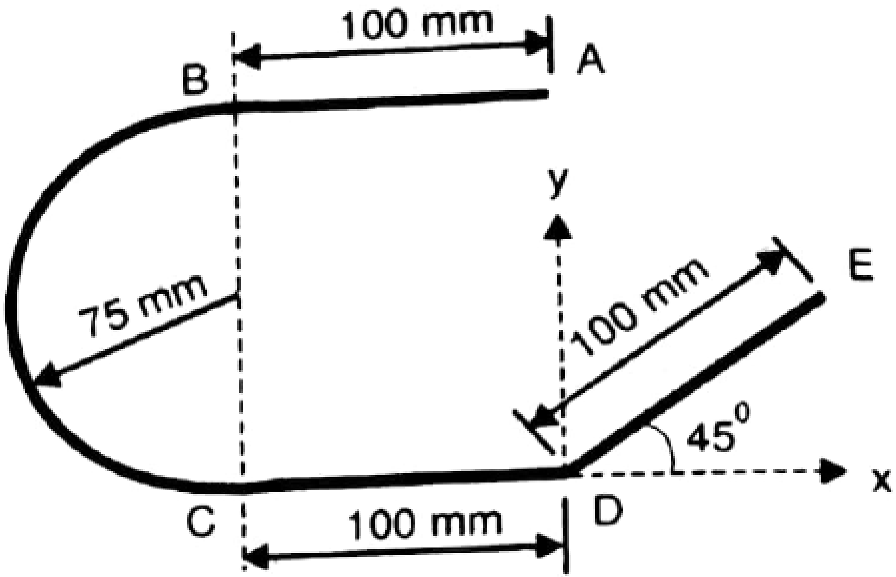
$$\bar{x} = \frac{V_1 x_1 + V_2 x_2 + V_3 x_3 + \ldots}{V_1 + V_2 + V_3 + \ldots} = \frac{\Sigma V_i x_i}{\Sigma V_i} \qquad \bar{y} = \frac{V_1 y_1 + V_2 y_2 + V_3 y_3 + \ldots}{V_1 + V_2 + V_3 + \ldots} = \frac{\Sigma V_i y_i}{\Sigma V_i}$$

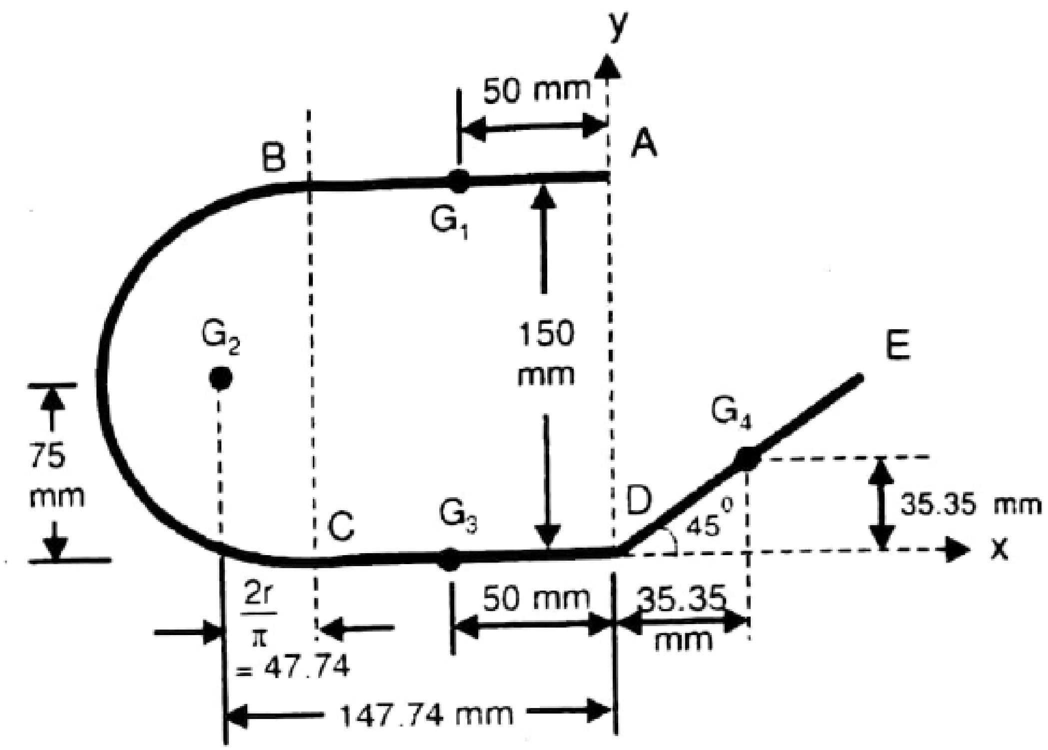
$$\bar{z} = \frac{V_1 z_1 + V_2 z_2 + V_3 z_3 + \ldots}{V_1 + V_2 + V_3 + \ldots} = \frac{\Sigma V_i z_i}{\Sigma V_i}$$

Here $V_1, V_2, \ldots$ = Volume of individual body.

Problem:

A uniform wire is bent into a shape shown. Calculate position of C.G. of the wire.





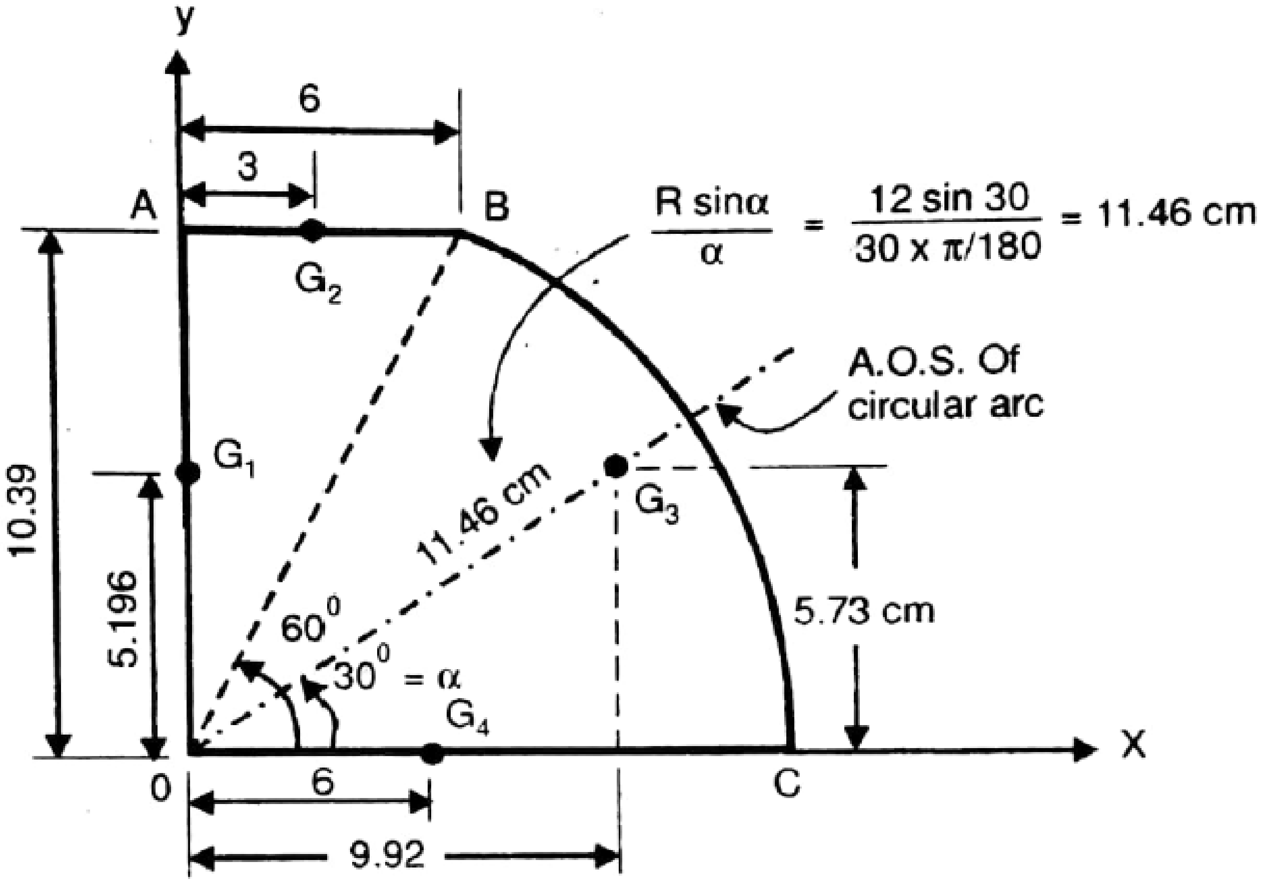
PART	LENGTH L_i, mm	Co-ordinates		$L_i.X_i$ mm^2	$L_i.Y_i$ mm^2
		X_i (mm)	Y_i (mm)		
AB St. horizontal	100	– 50	150	– 5000	15000
BC Semi circular arc	$\pi \times 75 = 235.62$	– 147.74	75	– 34812	17671
CD St. horizontal	100	– 50	0	– 5000	0
DE St. inclined	100	35.35	35.35	3535	3535
	$\Sigma L_i = 535.62$			$\Sigma L_i.X_i = -41277$	$\Sigma L_i.Y_i = 36206$

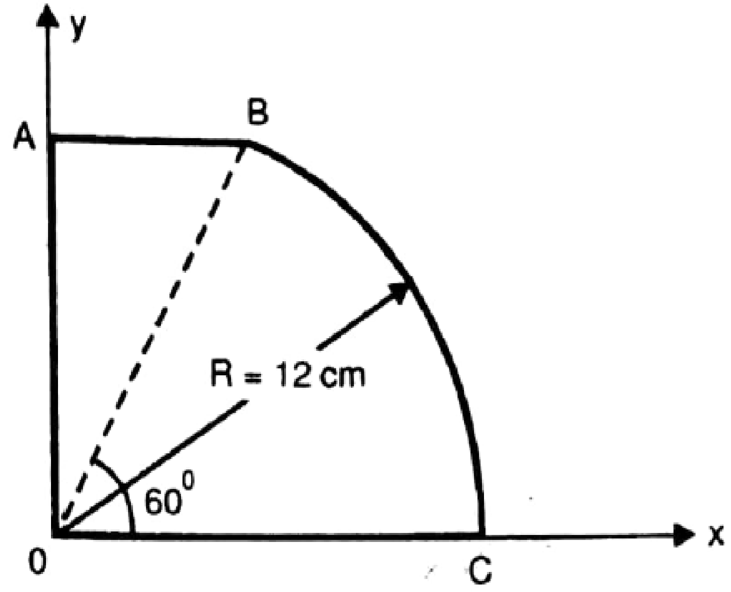
Using $\bar{X} = \frac{\Sigma L_i X_i}{\Sigma L_i} = \frac{-41277}{535.62} = -77.06$ mm and $\bar{Y} = \frac{\Sigma L_i Y_i}{\Sigma L_i} = \frac{36206}{535.62} = 67.59$ mm

$$(\bar{X}, \bar{Y}) = (-77.06, 67.59) \text{ mm}$$

Problem:

A thin homogeneous wire of uniform cross-section is bent into shape OABCO as shown. Find its centroid.





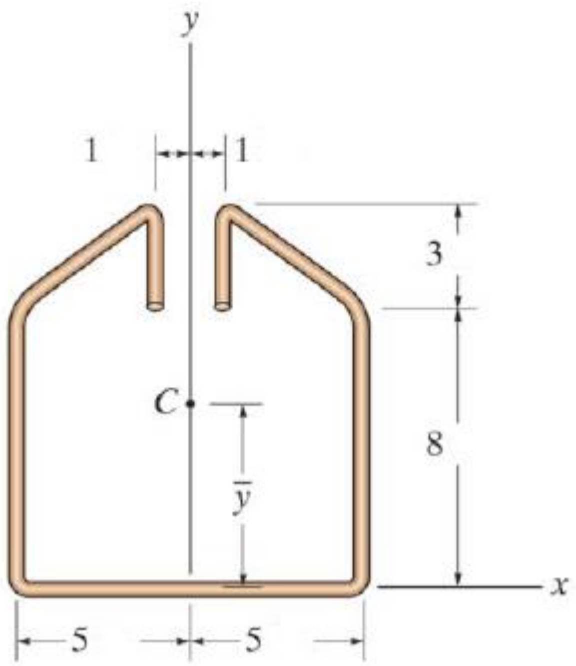
PART	LENGTH L_i, cm	Co-ordinates (cm)		$L_i.X_i$ cm^2	$L_i.Y_i$ cm^2
		X_i	Y_i		
OA St. vertical	10.39	0	5.196	0	53.98
AB St. horizontal	6	3	10.39	18	62.34
BC Circular arc	$2r\alpha = 2\times12\left[30\times\frac{\pi}{180}\right]$ $= 12.56$	9.92	5.73	124.6	71.96
CO St. horizontal	12	6	0	72	0
	$\Sigma L_i = 40.95$			$\Sigma L_i.X_i = 214.6$	$\Sigma L_i.Y_i = 188.28$

Using $\bar{X} = \frac{\Sigma L_i X_i}{\Sigma L_i} = \frac{214.6}{40.95} = 5.24$ cm and $\bar{Y} = \frac{\Sigma L_i Y_i}{\Sigma L_i} = \frac{188.28}{40.95} = 4.59$ cm

$\therefore$ the co-ordinates of centroid of the bent up wire are, $(\bar{X}, \bar{Y}) = (5.24, 4.59)$ cm ...**Ans.**

Problem:

Locate the centroid for the wire. Neglect the thickness of the material and slight bends at the corners.

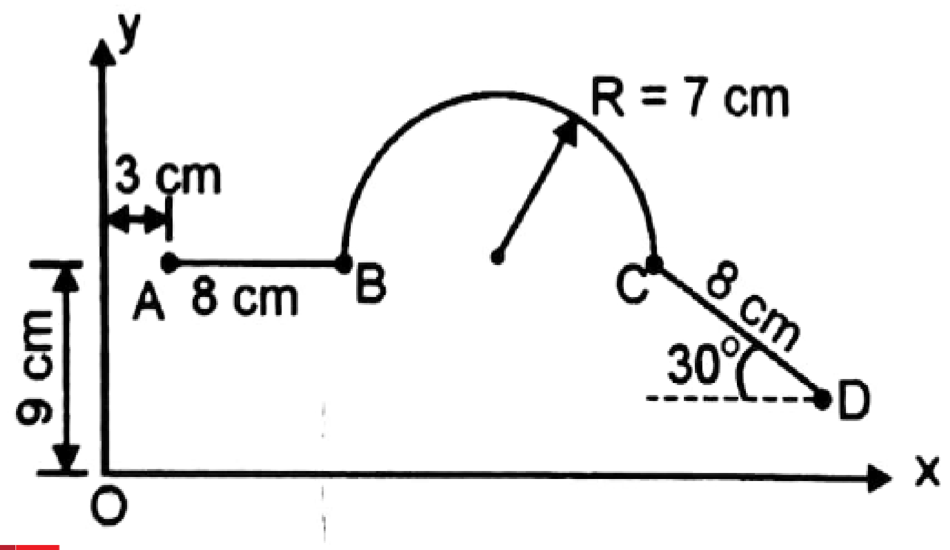


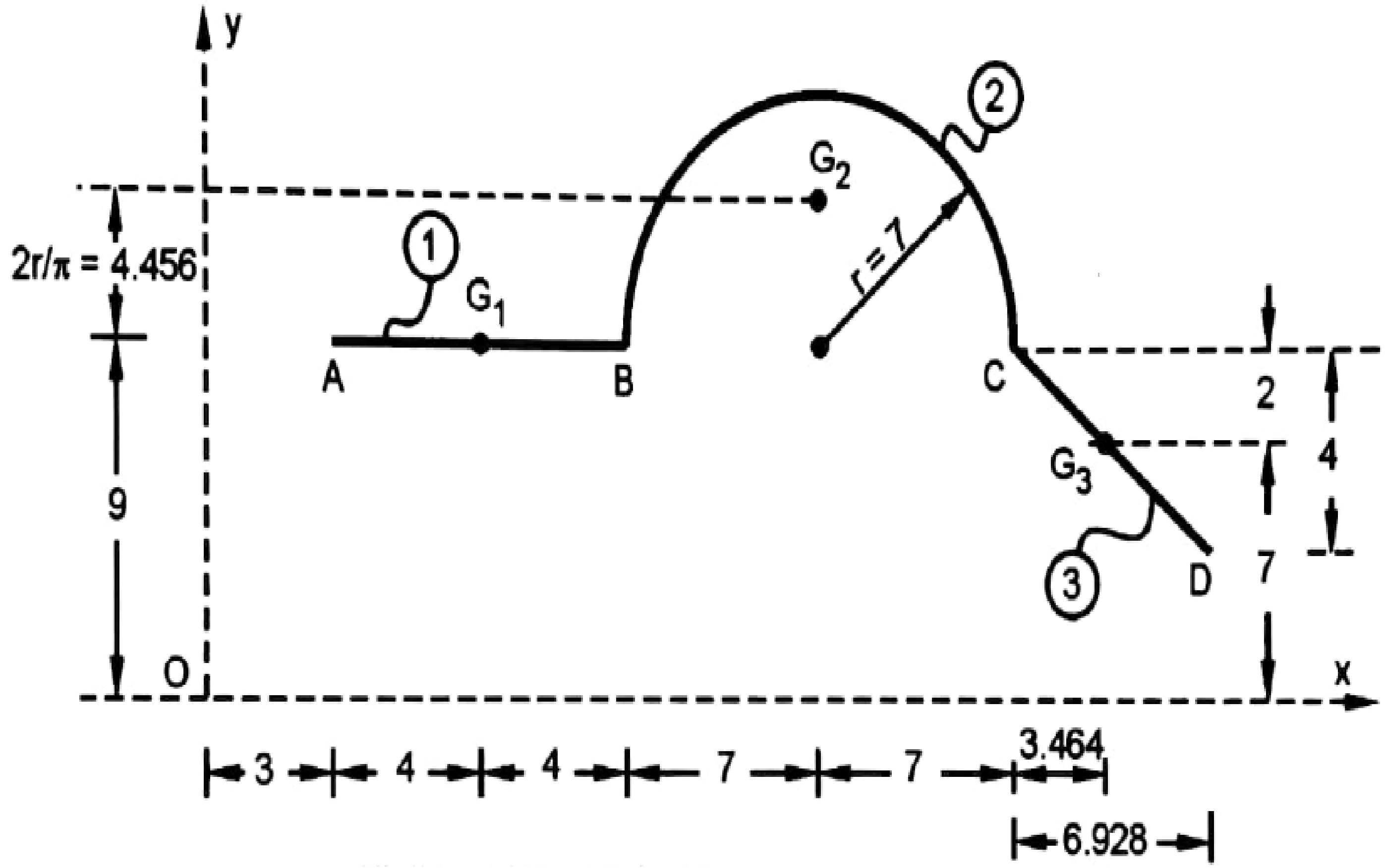
piece	L	$\tilde{y}$	$\tilde{y}$ L
①	5	0	0
②	8	4	32
③	5	9.5	47.5
④	3	9.5	28.5
	21		108

$$\bar{y} = \frac{108}{21} \approx 5.14$$

Due to symmetry, we only have to find $\bar{y}$ and so I am only going to consider the right ½ of the wire. I will break it up into the 4 pieces.

Problem: Find the centroid of the bent wire as shown in figure.



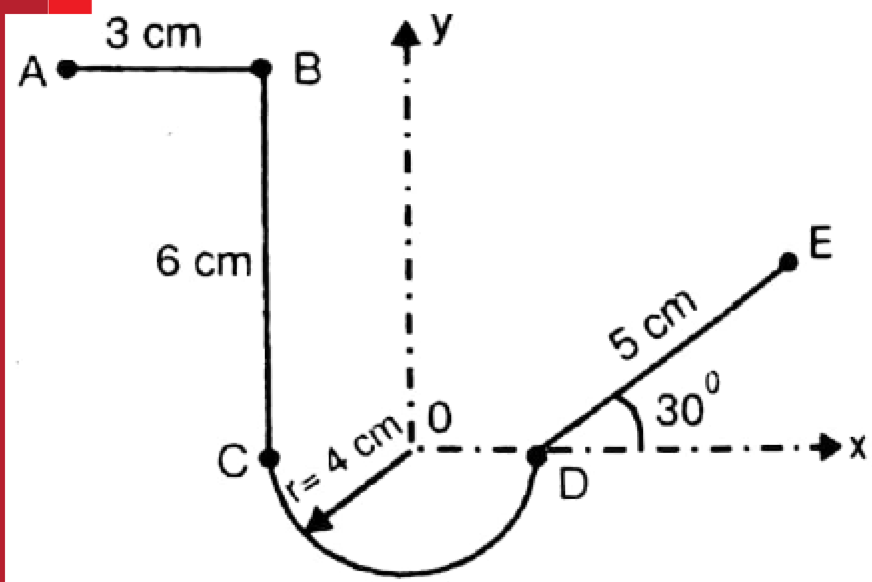


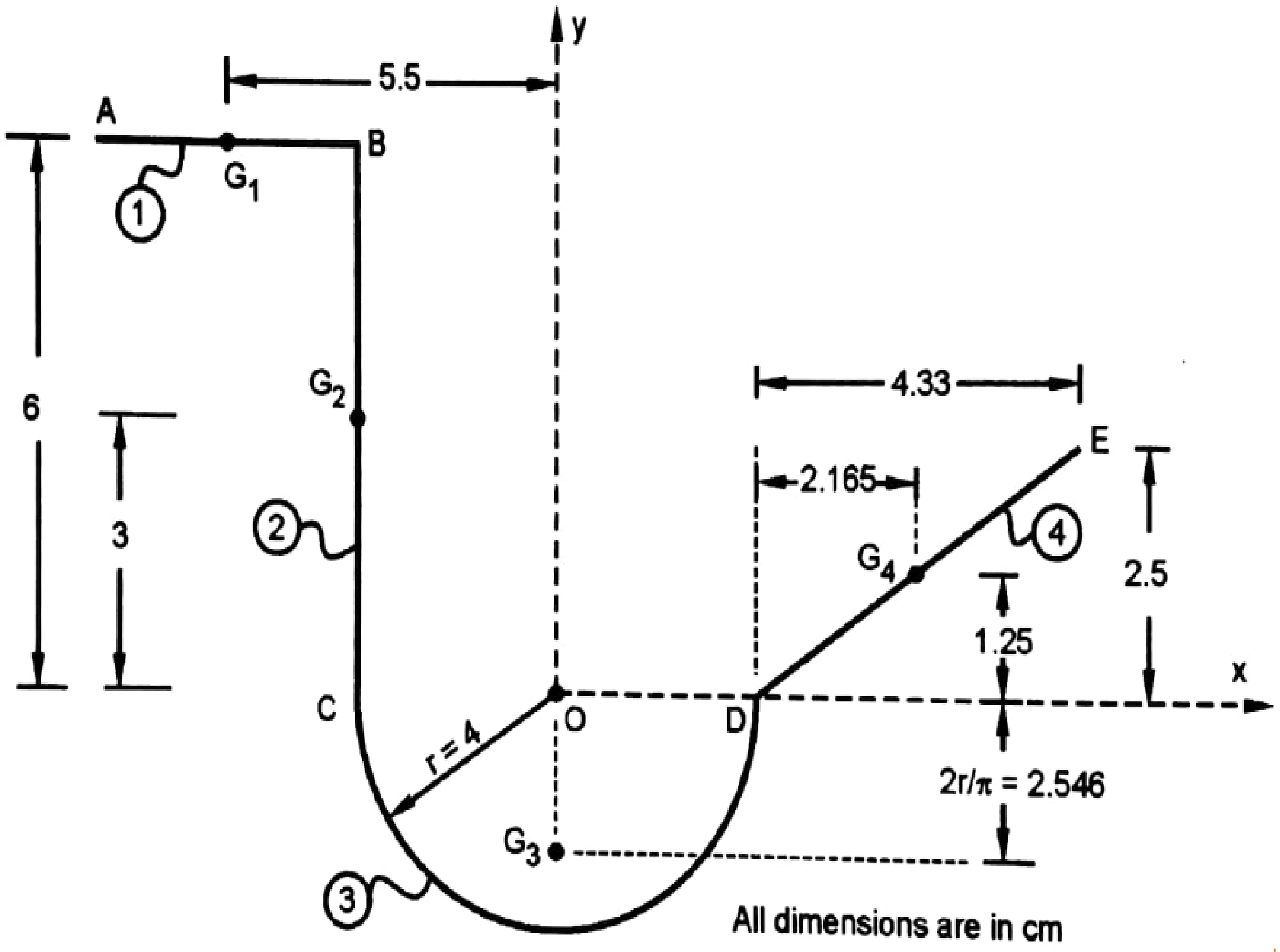
Part	Length L cm	Co–ordinates		L x cm²	L y cm²
		x cm	y cm		
1. AB	8	7	9	56	72
2. BC	$\pi \times 7 = 22$	18	13.456	396	296
3. CD	8	28.464	7	227.7	56
	$\Sigma L = 38$			$\Sigma Lx = 679.7$	$\Sigma Ly = 424$

Using $\overline{X} = \frac{\Sigma Lx}{\Sigma L} = \frac{679.7}{38} = 17.886$ cm and $\overline{Y} = \frac{\Sigma Ly}{\Sigma L} = \frac{424}{38} = 11.158$ cm

$\therefore \quad \overline{X}, \overline{Y} = (17.886\ ,\ 11.158)$ cm .... **Ans.**

Problem: A bent up wire ABCDE is as shown in figure. Locate its centre of gravity.



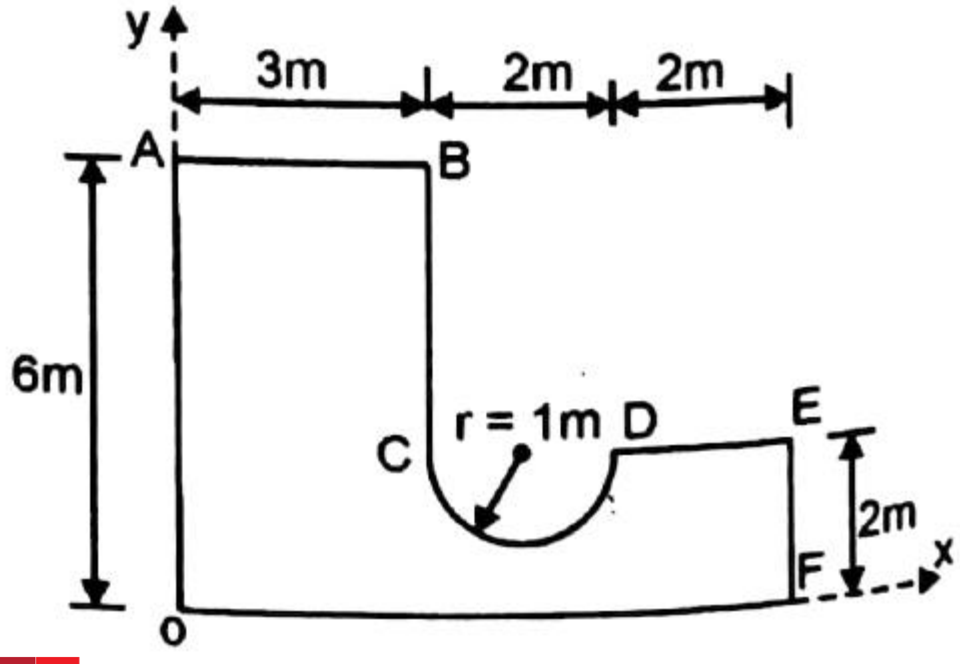


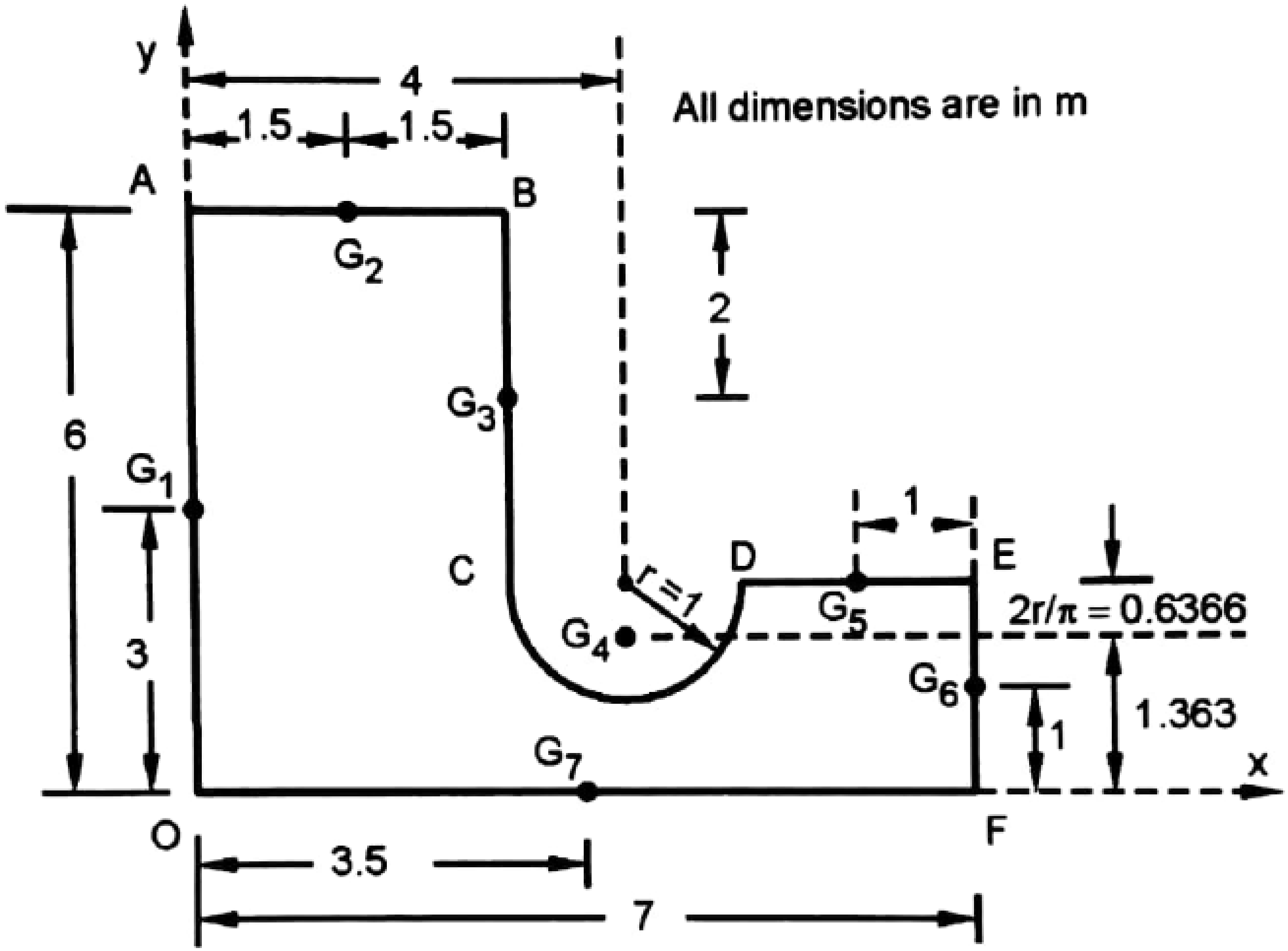
Part	Length L cm	Co–ordinates		L x cm²	L y cm²
		x cm	y cm		
1. AB	3	– 5.5	6	– 16.5	18
2. BC	6	– 4	3	– 24	18
3. CD	$\pi \times 4 =$ 12.566	0	– 2.546	0	– 32
4. DE	5	6.165	1.25	30.825	6.25
	$\Sigma L =$ 26.566			$\Sigma L x =$ – 9.675	$\Sigma L y =$ 10.25

Using $\bar{X} = \frac{\Sigma L x}{\Sigma L} = \frac{-9.675}{26.566} = -0.364$ cm and $\bar{Y} = \frac{\Sigma L y}{\Sigma L} = \frac{10.25}{26.566} = 0.3858$ cm

$\therefore \quad \bar{X}, \bar{Y} = (-0.364, \; 0.3858)$ cm .... **Ans.**

Problem: **Find the centroid of the bent wire ABCDEFOA as shown in figure.**



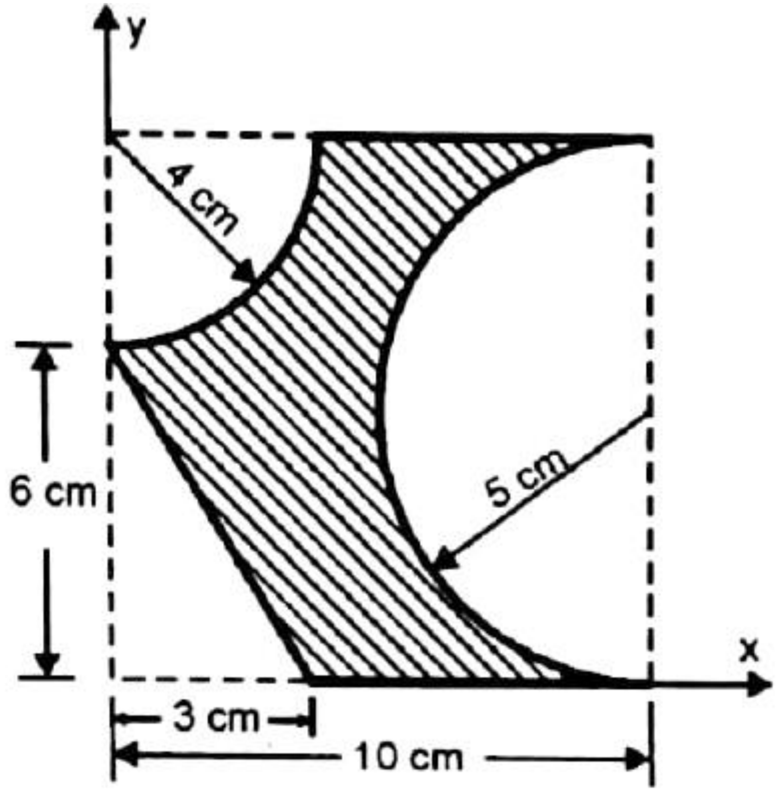


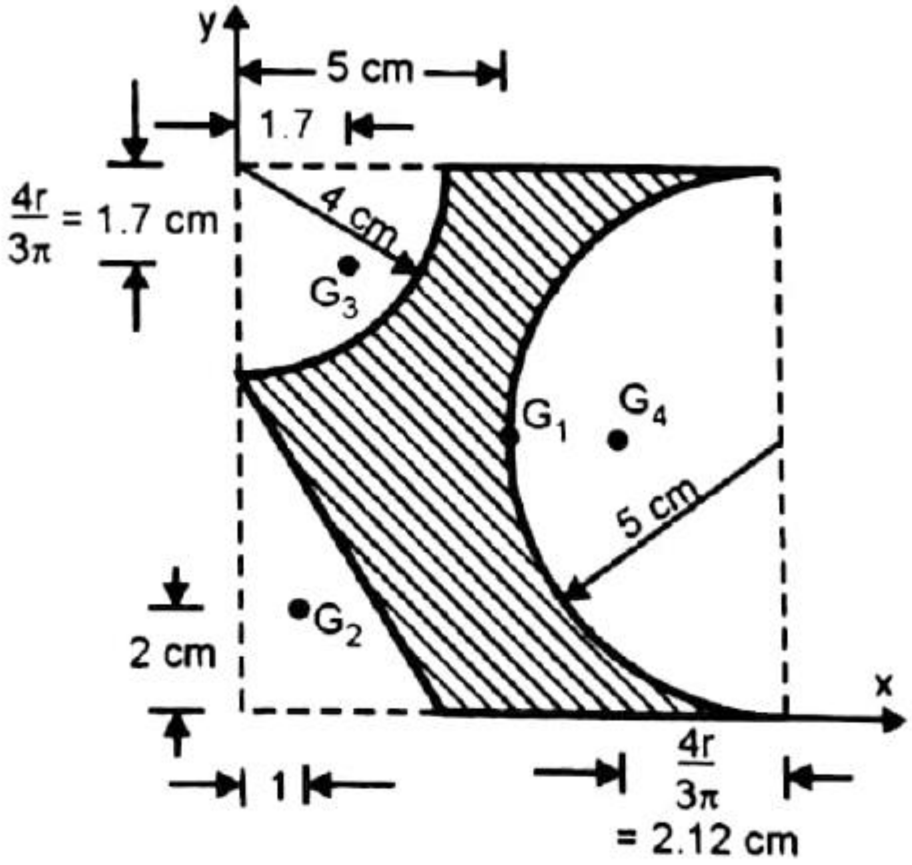
Part	Length L m	Co–ordinates x m	Co–ordinates y m	L x m^2	L y m^2
1. OA	6	0	3	0	18
2. AB	3	1.5	6	4.5	18
3. BC	4	3	4	12	16
4. CD	$\pi \times 1 = 3.14$	4	1.363	12.56	4.28
5. DE	2	6	2	12	4
6. EF	2	7	1	14	2
7. OF	7	3.,5	0	24.5	0
	$\Sigma L = 27.14$			$\Sigma L x = 79.56$	$\Sigma L y = 62.28$

Using $\bar{X} = \frac{\Sigma L x}{\Sigma L} = \frac{79.56}{27.14} = 2.931$ m and $\bar{Y} = \frac{\Sigma L y}{\Sigma L} = \frac{62.28}{27.14} = 2.295$ m

$\therefore \quad \bar{X}, \bar{Y} = (2.931,\ 2.295)$ m .... **Ans.**

Problem: Find the centroid of shaded area as shown.





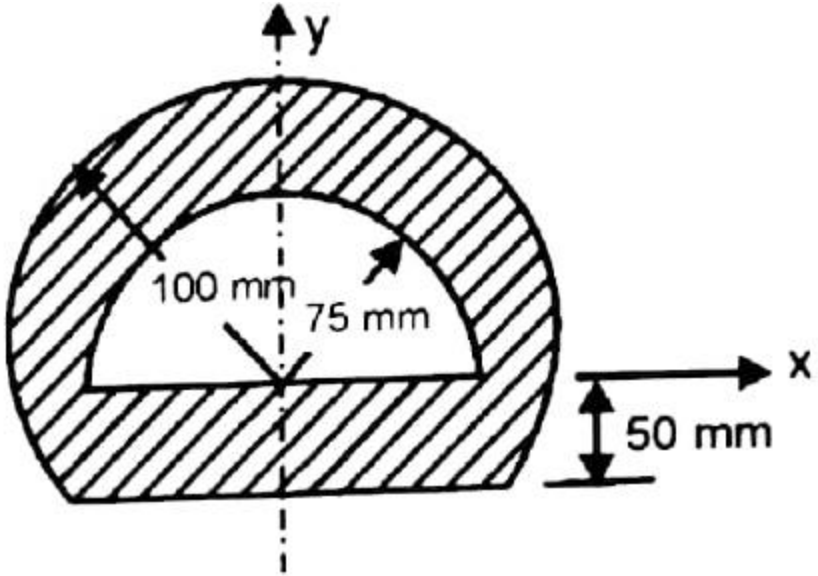
Part	Area (A_i) cm^2	x_i cm	y_i cm	A_ix_i cm^3	A_iy_i cm^3
1. Square	100	5	5	500	500
2. Rt. Triangle	- 9	1	2	- 9	- 18
3. Quarter-circle	- 12.57	1.697	8.302	- 21.32	- 104.33
4. Semi-circle	- 39.27	7.878	5	- 309.37	- 196.35
	ΣA_i = 39.16			ΣA_ix_i = 160.31	ΣA_iy_i = 181.32

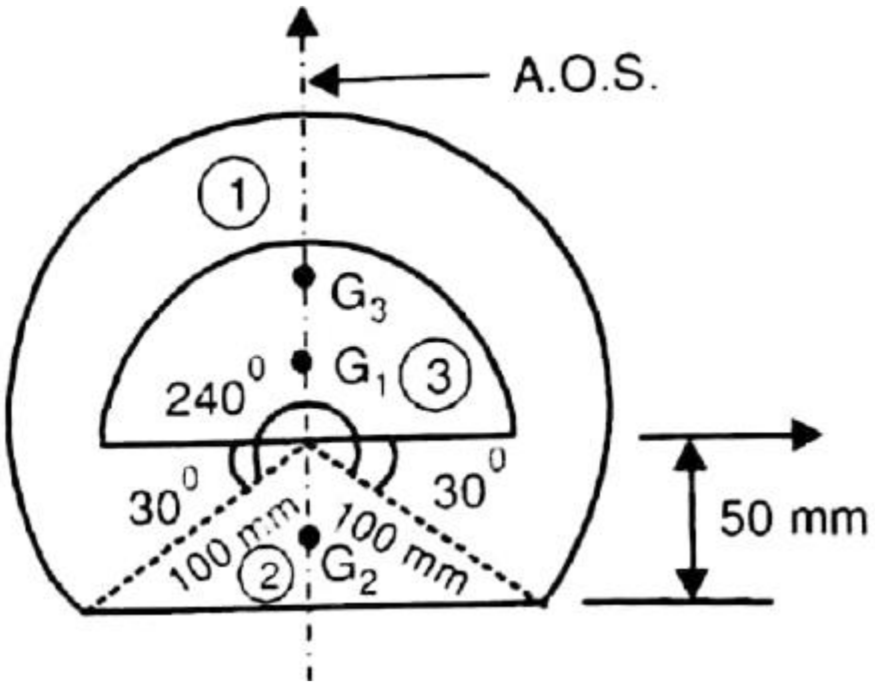
$$\bar{X} = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{160.31}{39.16} = 4.09 \text{ cm} \quad \text{and} \quad \bar{Y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{181.32}{39.16} = 4.63 \text{ cm}$$

$$\therefore \quad \bar{X}, \bar{Y} = (4.09, 4.63) \text{ cm}$$ .... **Ans.**

Problem:

A semi-circular section is removed from the plane area as shown. Find centroid of the remaining shaded area.





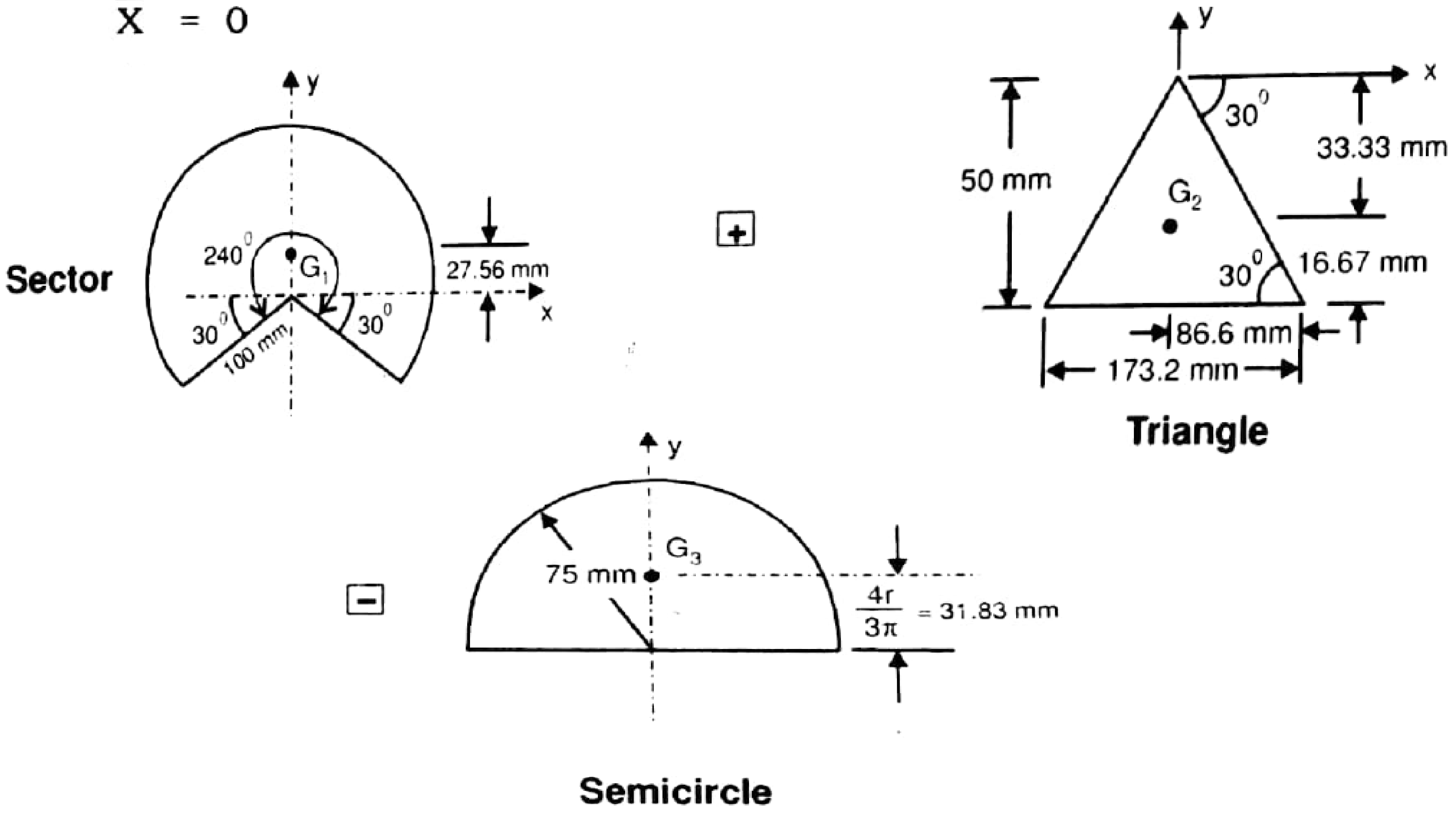
here $\alpha = \frac{240}{2} = 120° = 2.094$ radians

$\frac{2}{3} \times \frac{r \sin\alpha}{\alpha} \quad \therefore \quad \frac{2}{3} \times \frac{100 \sin 120}{2.094} = 27.56$ mm

$\text{Area} = A_1 = r^2\pi = (100)^2 \times 2.094 = 20944 \text{ mm}^2$

A.O.S., which is the y-axis, we have

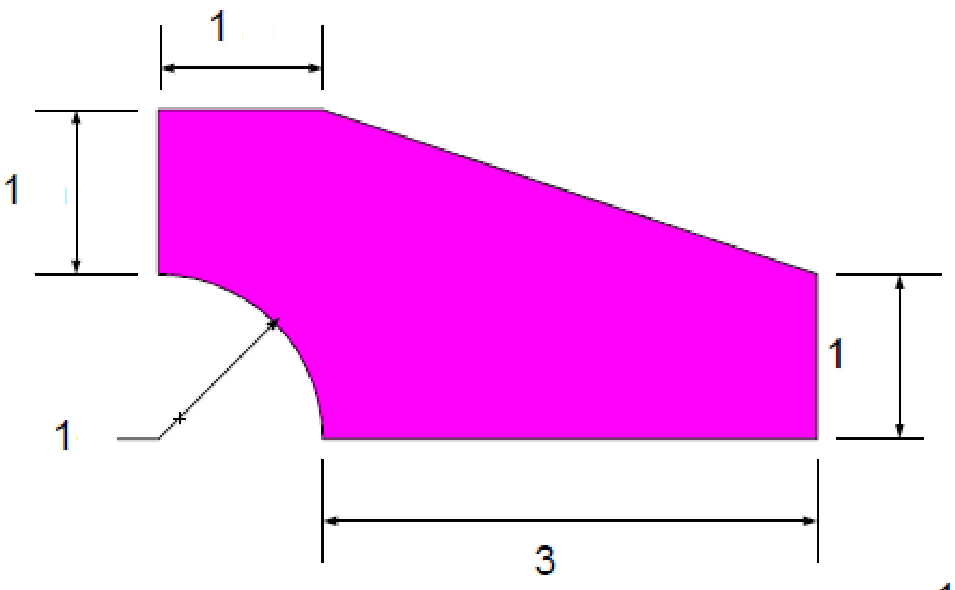
$\bar{X} = 0$



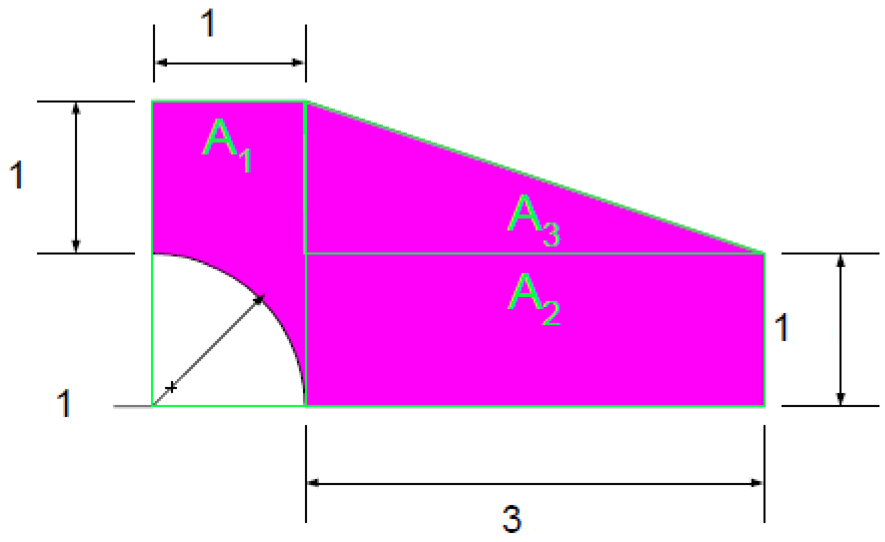
PART	AREA A_i, mm^2	Y_i mm	A_iY_i mm^3
1. SECTOR	20944	27.56	577217
2. TRIANGLE	$\frac{1}{2} \times 173.2 \times 50 = 4330$	– 33.33	– 144319
3. SEMI-CIRCLE	$-\frac{1}{2} \pi (75)^2 = -8835.7$	31.83	– 281241
	$\Sigma A_i = 16438.3$		$\Sigma A_i.Y_i = 151657$

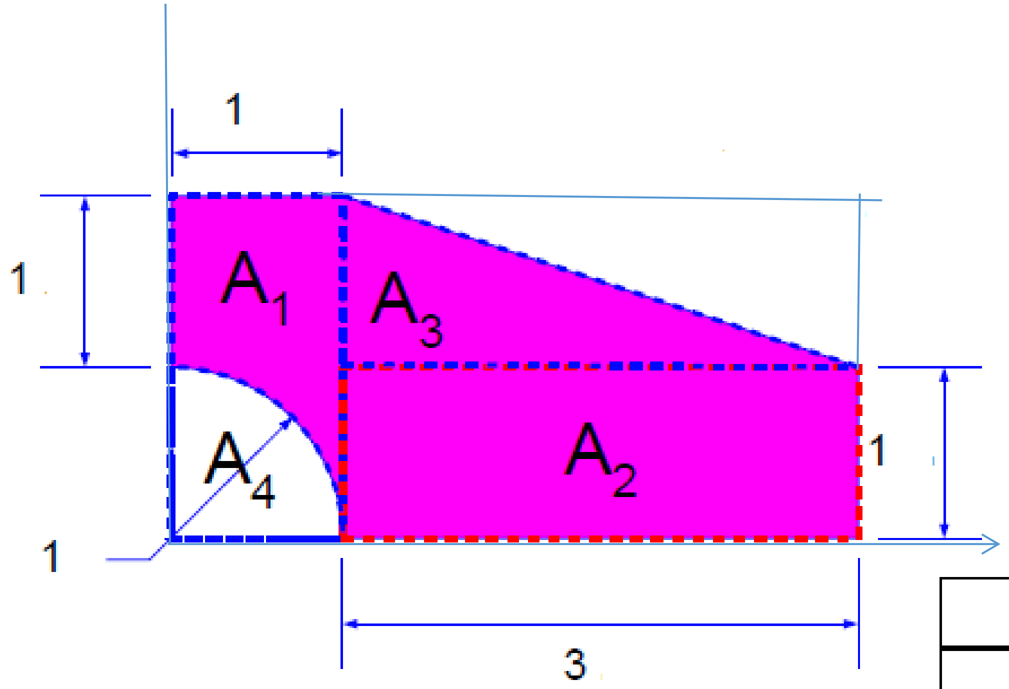
Using $\bar{Y} = \frac{\Sigma A_i Y_i}{\Sigma A_i} = \frac{151657}{16438.3} = 9.22$ mm $\quad \therefore (\bar{X}, \bar{Y}) = (0, 9.22)$ mm .... **Ans.**

Problem: For the plane area shown, determine the first moments with respect to the *x* and *y* axes and the location of the centroid. (All dimensions are in cm)



$$\bar{x} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i} \qquad \bar{y} = \frac{\sum_{i=1}^{n} y_i A_i}{\sum_{i=1}^{n} A_i}$$

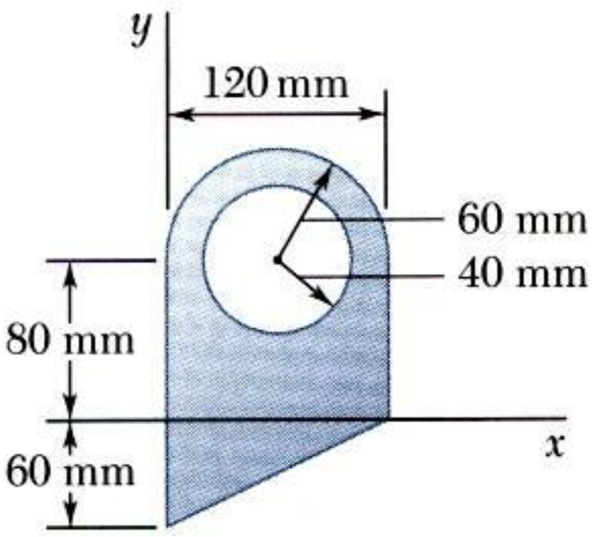




ID	Area	x_i
A_1	2	0.5
A_2	3	2.5
A_3	1.5	2
A_4	-0.7854	0.42441

ID	Area	x_i	x_i*Area	y_i	y_i*Area
A_1	2	0.5	1	1	2
A_2	3	2.5	7.5	0.5	1.5
A_3	1.5	2	3	1.333333	2
A_4	-0.7854	0.42441	-0.33333	0.42441	-0.33333
	5.714602		11.16667		5.166667
	x_{bar}	1.9541	y_{bar}	0.904117	

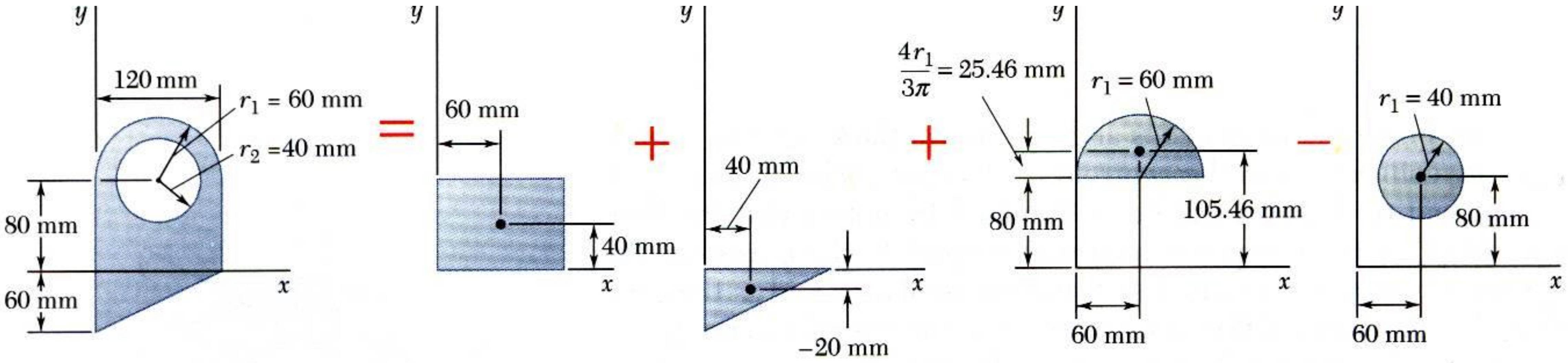
Problem: For the plane area shown, determine the first moments with respect to the *x* and *y* axes and the location of the centroid. (All dimensions are in cm)



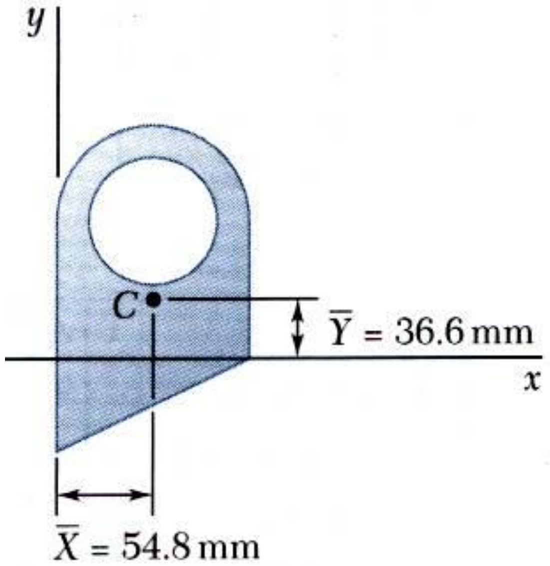
SOLUTION:

- Divide the area into a triangle, rectangle, and semicircle with a circular cutout.
- Calculate the first moments of each area with respect to the axes.
- Find the total area and first moments of the triangle, rectangle, and semicircle.
- Subtract the area and first moment of the circular cutout.
- Compute the coordinates of the area centroid by dividing the first moments by the total area.

Problem:



Component	A, mm²	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm³	$\bar{y}A$, mm³
Rectangle	$(120)(80) = 9.6 \times 10^3$	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$	40	−20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655 \times 10^3$	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle	$-\pi(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma\bar{x}A = +757.7 \times 10^3$	$\Sigma\bar{y}A = +506.2 \times 10^3$



$$\bar{X} = \frac{\sum \bar{x} A}{\sum A} = \frac{+757.7 \times 10^3 \, mm^3}{13.828 \times 10^3 \, mm^2}$$

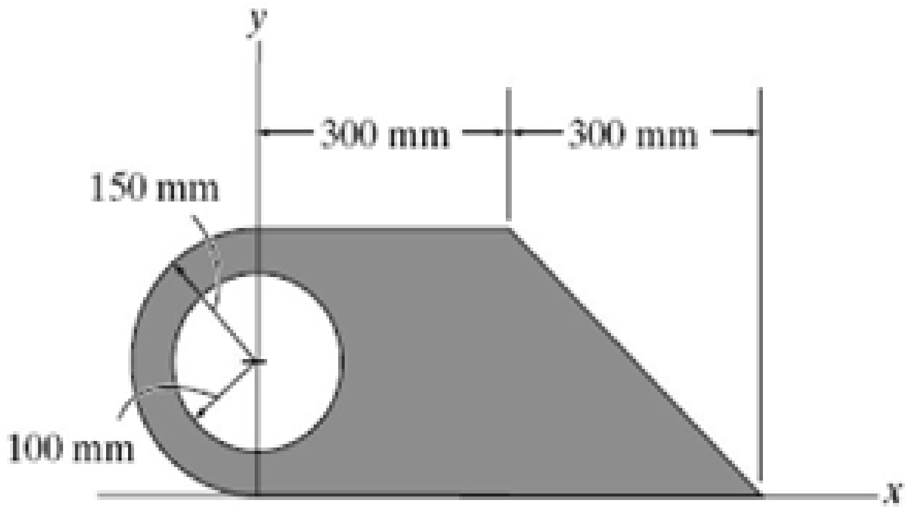
$$\boxed{\bar{X} = 54.8 \, mm}$$

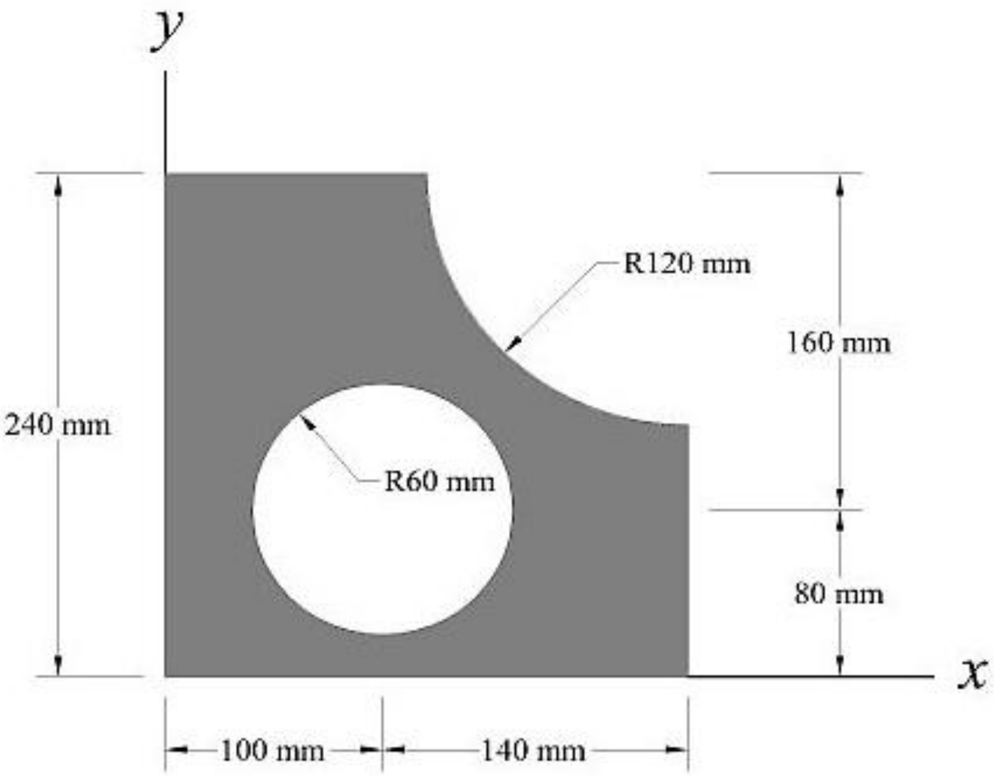
$$\bar{Y} = \frac{\sum \bar{y} A}{\sum A} = \frac{+506.2 \times 10^3 \, mm^3}{13.828 \times 10^3 \, mm^2}$$

$$\boxed{\bar{Y} = 36.6 \, mm}$$

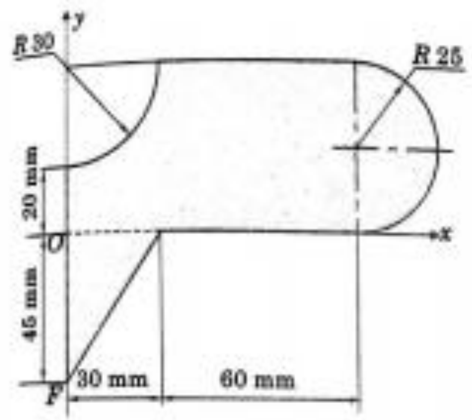
Problem for Practice:

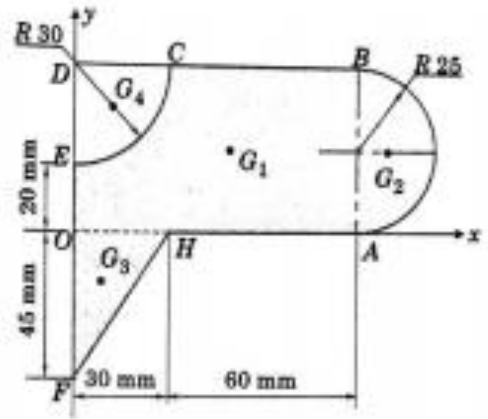
Determine the centroid of given plane laminas





Problem:





Solution : Divide the shaded areas into 4 parts and mark the centriod of the respective area. Prepare the table as follows.

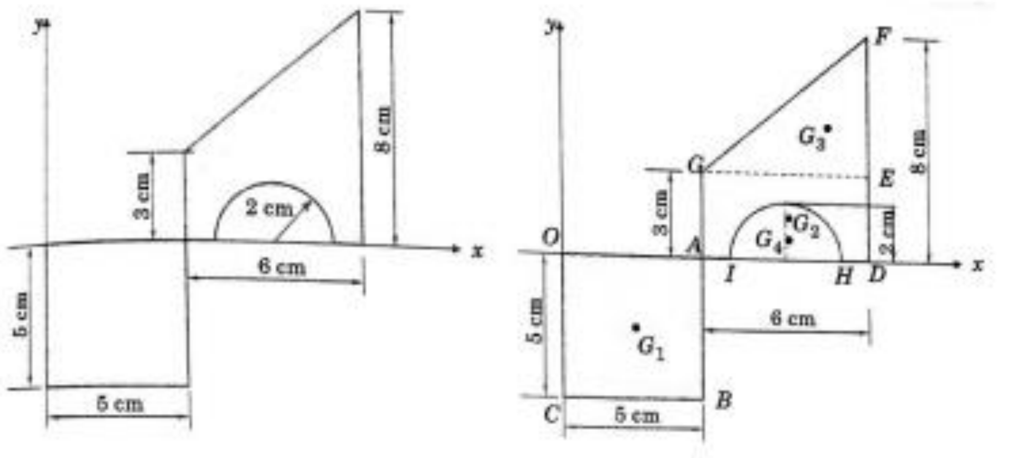
Component	Area A_i (mm²)	$\bar{x}_{Gi}$ (mm)	$\bar{y}_{Gi}$ (mm)	$A_i\bar{x}_{Gi}$ (mm³)	$A_i\bar{y}_{Gi}$ (mm³)
(1) Rectangle *OABD*	$90 \times 50 = 4500$	$\frac{90}{2} = 45$	$\frac{50}{2} = 25$	202500	112500
(2) Semicircle *ABA*	$\frac{\pi}{2} \times 25^2 = 981.747$	$90 + \frac{4 \times 25}{3\pi} = 100.61$	25	98773.566	24543.675
(3) Triangle *OHF*	$\frac{1}{2} \times 30 \times 45 = 675$	$\frac{1}{3} \times 30 = 10$	$-\frac{1}{3} \times 45 = -15$	6750	−10125
(4) Quarter Circle *DCE*	$-\frac{\pi \times 30^2}{4} = -706.858$	$\frac{4 \times 30}{3\pi} = 12.733$	$50 - \frac{4 \times 30}{3\pi} = 37.267$	−9000.423	−26342.477

$$\Sigma A_i = 5449.889 \text{ mm}^2, \quad \Sigma A_i\bar{x}_{Gi} = 299039 \text{ mm}^3, \quad \Sigma A_i\bar{y}_{Gi} = 100576.198 \text{ mm}^3$$

$$\bar{x} = \frac{\Sigma A_i\bar{x}_{Gi}}{\Sigma A_i} = \frac{299039}{5449.889} = 54.868 \text{ mm}, \quad \bar{y} = \frac{\Sigma A_i\bar{y}_{Gi}}{\Sigma A_i} = \frac{100576.198}{5449.889} = 18.455 \text{ mm}$$

Centroid $G[\bar{x}, \bar{y}] = [54.868, 18.455]$ mm ... *Ans.*

Problem:



Solution : Divide the shaded area into 4 parts and mark the centroid of the respective area. Prepare the table as follows.

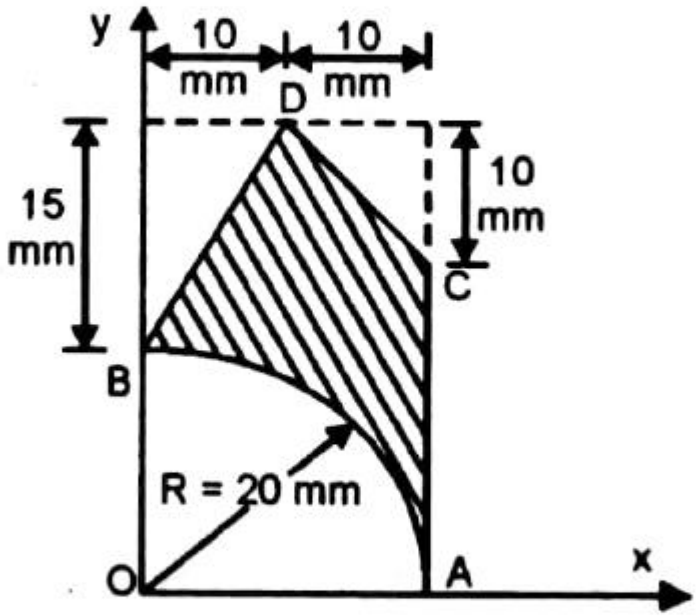
Component	Area A_i (cm²)	$\bar{x}_{Gi}$ (cm)	$\bar{y}_{Gi}$ (cm)	$A_i\bar{x}_{Gi}$ (cm³)	$A_i\bar{y}_{Gi}$ (cm³)
(1) Rectangle *OABC*	$5 \times 5 = 25$	$\frac{5}{2} = 2.5$	$-\frac{5}{2} = -2.5$	62.5	−62.5
(2) Rectangle *ADEG*	$6 \times 3 = 18$	$5 + \frac{5}{2} = 2.5$	$\frac{3}{2} = 1.5$	144	27
(3) Triangle *GEF*	$\frac{1}{2} \times 6 \times 5 = 15$	$5 + \frac{2}{3} \times 6 = 9$	$3 + \frac{1}{3} \times (8-3) = 4.667$	135	70.005
(4) Semicircle *IHI*	$\frac{-\pi \times 25^2}{2} = -6.283$	$5 + 1 + 2 = 8$	$\frac{4 \times 2}{3\pi} = 0.849$	−50.264	−5.334

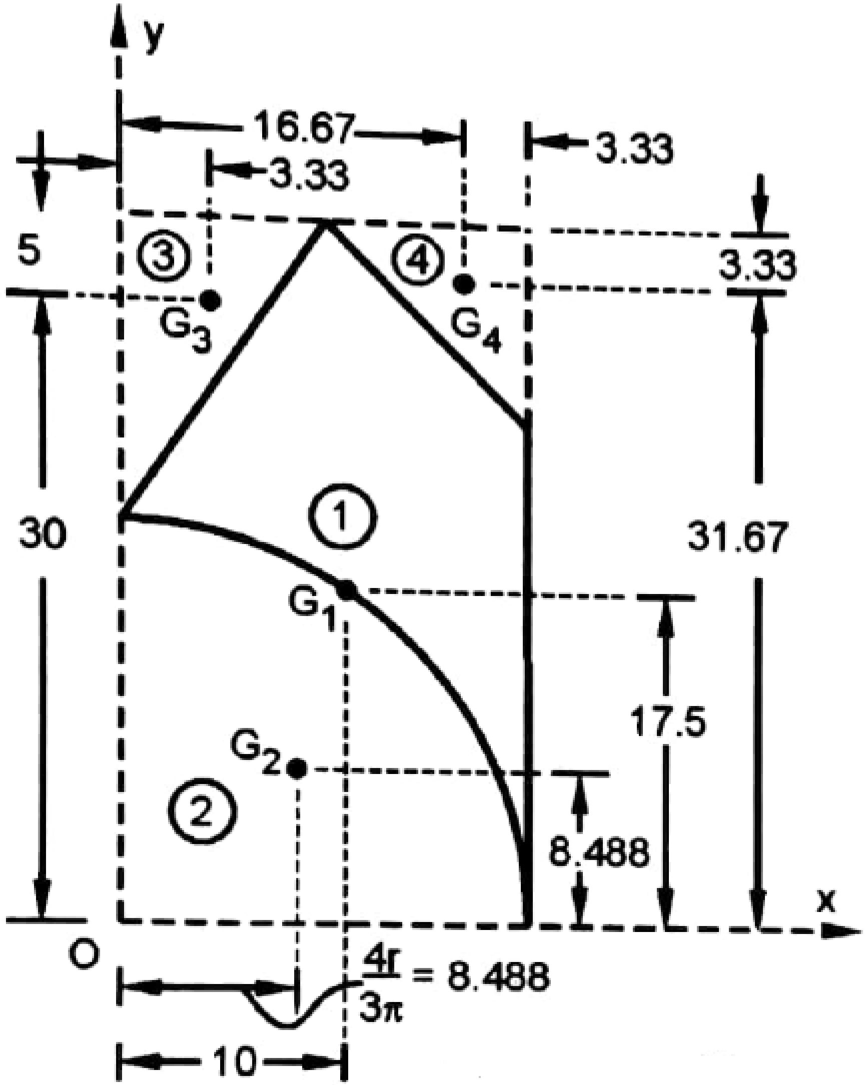
$$\Sigma A_i = 51.717 \text{ cm}^2, \quad \Sigma A_i\bar{x}_{Gi} = 291.236 \text{ cm}^3, \quad \Sigma A_i\bar{y}_{Gi} = 29.171 \text{ cm}^3$$

$$\bar{x} = \frac{\Sigma A_i\bar{x}_{Gi}}{\Sigma A_i} = \frac{291.236}{51.717} = 5.631 \text{ cm}, \qquad \bar{y} = \frac{\Sigma A_i\bar{y}_{Gi}}{\Sigma A_i} = \frac{29.171}{51.717} = 0.564 \text{ cm}$$

Centroid $G[\bar{x}, \bar{y}] = [5.631, 0.564]$ cm ... *Ans.*

Problem: Find centroid of shaded plane area.



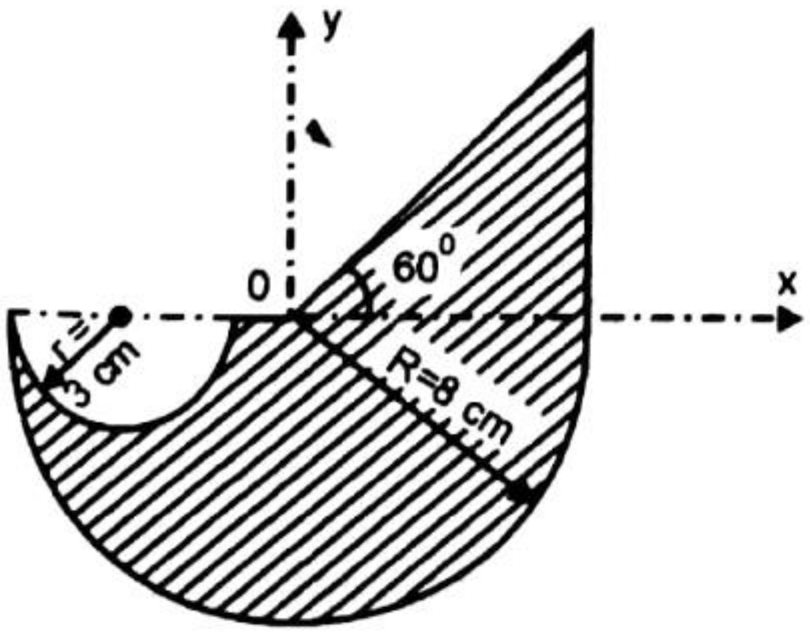


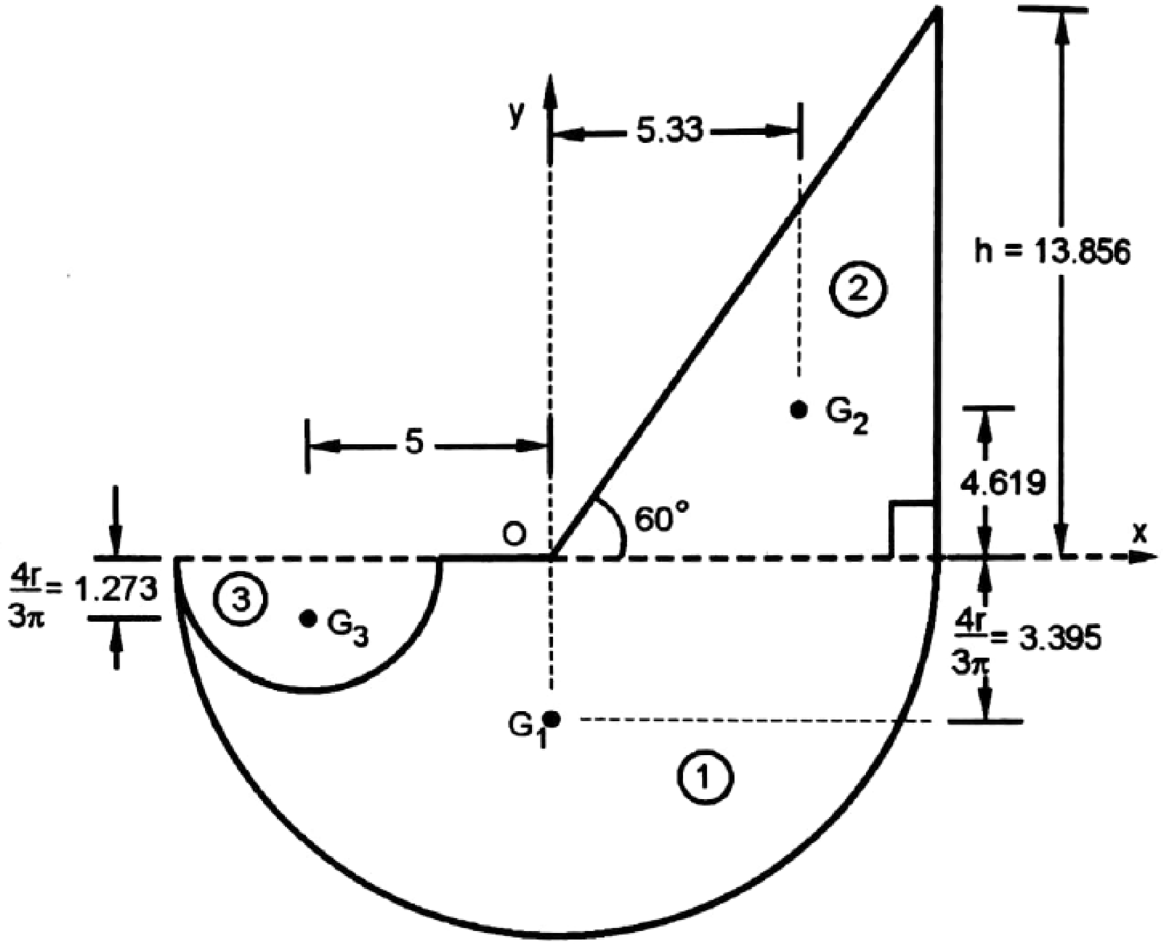
Part	Area A mm^2	Co-ordinates		A x mm^3	A y mm^3
		x mm	y mm		
1. Rectangle	$20 \times 35 = 700$	10	17.5	7000	12250
2. Qt. circle	$-(\pi \times 20^2)/4 = -314.16$	8.488	8.488	− 2666.6	− 2666.6
3. Rt. Triangle	$-(\frac{1}{2} \times 10 \times 15) = -75$	3.33	30	− 249.8	− 2250
4. Rt. Triangle	$-(\frac{1}{2} \times 10 \times 10) = -50$	16.67	31.67	− 833.5	− 1583.5
	$\Sigma A =$ 260.84			$\Sigma A x =$ 3250	$\Sigma A y =$ 5750

Using $\overline{X} = \frac{\Sigma A x}{\Sigma A} = \frac{3250}{260.84} = 12.46$ mm and $\overline{Y} = \frac{\Sigma A y}{\Sigma A} = \frac{5750}{260.84} = 22.04$ mm

$\therefore \quad \overline{X}, \overline{Y} = (12.46 \,,\ 22.04)$ mm .... **Ans.**

Problem: Find the centroid of shaded portion as shown.





Part	Area A cm^2	Co-ordinates		A x cm^3	A y cm^3
		x cm	y cm		
1. Semicircle	$(\pi \times 8^2)/2 =$ 100.53	0	– 3.395	0	– 341.3
2. Rt. Triangle	$(\frac{1}{2} \times 8 \times 13.856)$ = 55.42	5.33	4.619	295.4	256
3. Semicircle	$-(\pi \times 3^2)/2$ = – 14.137	– 5	– 1.273	70.68	18
	$\Sigma A =$ 141.81			$\Sigma A x =$ 366	$\Sigma A y =$ – 67.33

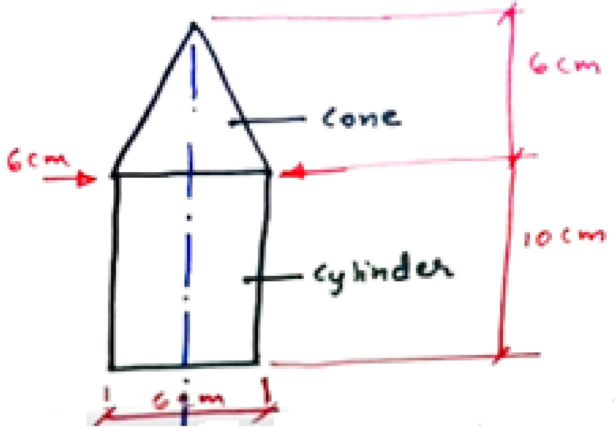
Using $\bar{X} = \frac{\Sigma A x}{\Sigma A} = \frac{366}{141.81} = 2.581$ cm and $\bar{Y} = \frac{\Sigma A y}{\Sigma A} = \frac{-67.33}{141.81} = -0.474$ cm

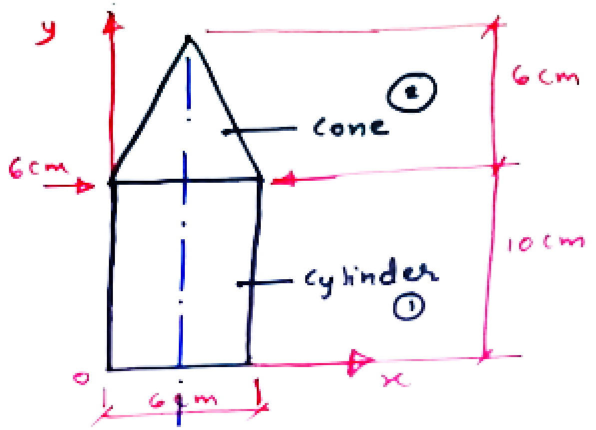
$\therefore \quad \bar{X}, \bar{Y} = (2.581, -0.474)$ cm .... **Ans.**

Problem:

Problem on Composite solid [Cylinder & cone].

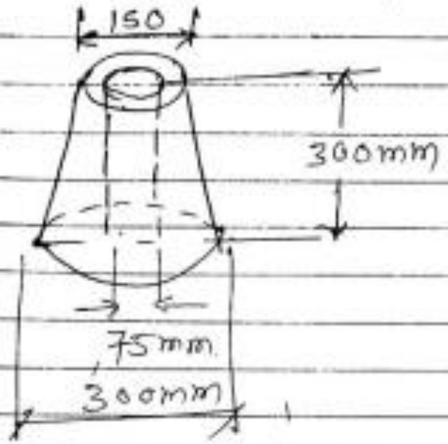
A solid cone having base diameter 6cm & height 6cm is kept co-axially on a solid cylinder having 6cm diameter and 10 cm high. Find C.G. of the combination.

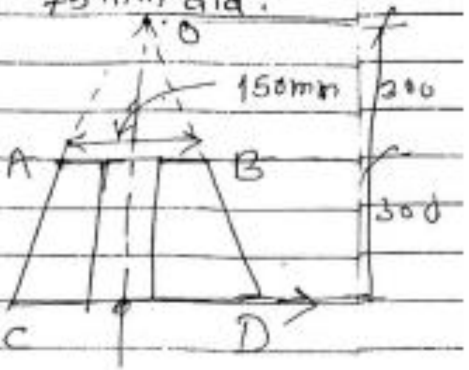




Problem:

1] The frustum of solid circular cone has an axial hole of 75 mm dia. Det. C.G. of body.





tem	Volume	y_i
ght circular one OCD	$\frac{1}{3}\pi R^2 H$	$\frac{H}{4} = \frac{600}{4}$
ght circular one OAB	$-\frac{1}{3}\pi R^2 h$	$300 + \frac{300}{4}$
rcular hole	$-\pi r^2 h$	$\frac{H}{2} = \frac{300}{2}$

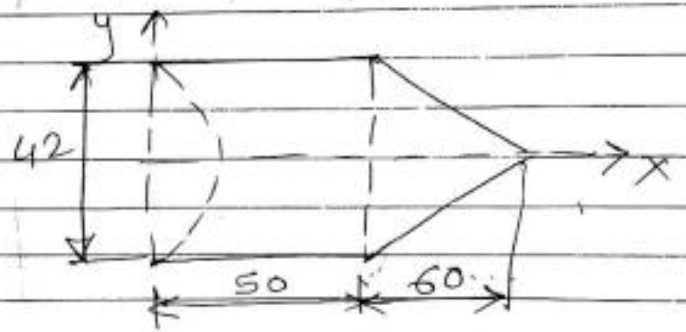
$$\bar{y} = \frac{\sum V_i y_i}{\sum V} = 119\ mm$$

Problem:

(2) A cylinder with hemispherical cavity & a conical cap is shown in fig.
(a) Find centroid of composite volume
(b) Locate the centre of mass of composite volume if the cylinder is made of steel & cap is made of Al

$\rho_{st} = 7870 \ kg/m^3$

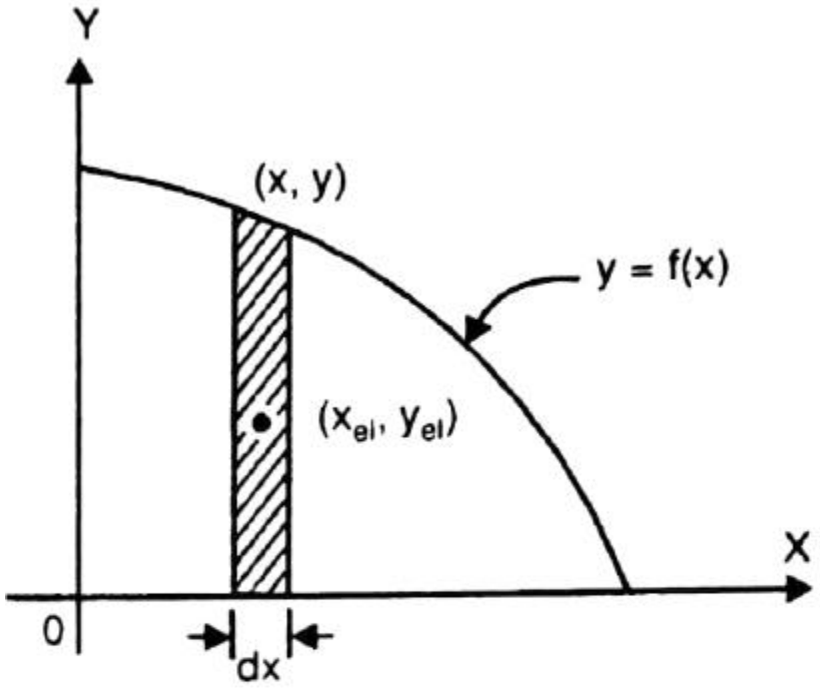
$\rho_{Al} = 2770 \ kg/m^3$



Sr No.	Elements	V_i	x_i	$V_i x_i$
1	cylinder	$\pi R^2 H$	$50/2$	
2	cone	$\frac{1}{3}\pi R^2 H$	$50 + 60/4$	
3	hemisphere	$\frac{2}{3}\pi R^3$	$\frac{3 \times 21}{8}$ $\left(\frac{3R}{8}\right)$	

$$\bar{x} = \frac{\sum V_i x_i}{\sum V} = 43.56 \ cm$$

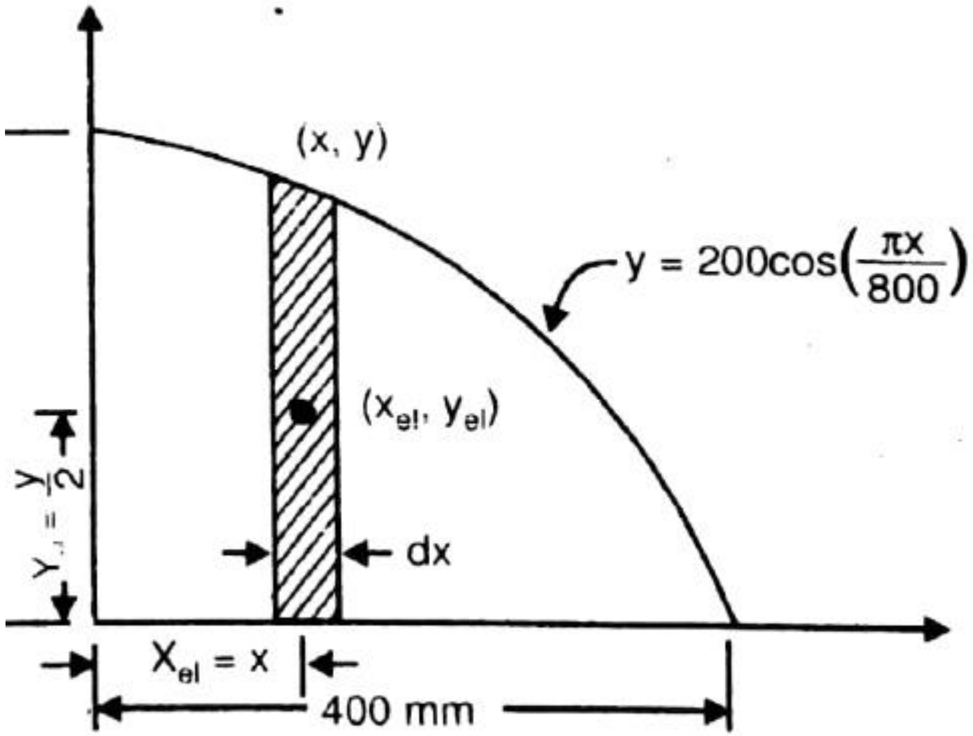
Centroids by Integration: to locate the centroid of figures bounded by curves.



$$\bar{X} = \frac{\int x_{el}.dA}{\int dA} \quad , \quad \bar{Y} = \frac{\int y_{el}.dA}{\int dA}$$

Problem:

Determine the centroid of the plane area as shown in figure.



Solution:

$$\text{Total area} = \int dA = \int y\,dx$$

$$= \int_0^{400} 200\cos\left(\frac{\pi x}{800}\right) dx$$

$$= 200\left(\frac{\sin\left(\frac{\pi x}{800}\right)}{\frac{\pi}{800}}\right)_0^{400} = \frac{160000}{\pi}\sin\left(\frac{\pi}{2} - \sin 0\right)$$

$$= \frac{160000}{\pi} = 50930 \text{ mm}^2.$$

If (x_{el}, y_{el}) are the co-ordinates of the centroid of the element, then

$$x_{el} = x \quad \text{and} \quad y_{el} = y/2$$

Now, $\int x_{el}.dA = \int_0^{400} x.\, ydx$ -------- since $x_{el} = x$ and $dA = ydx$

$$= \int_0^{400} x.\, 200\cos\left(\frac{\pi x}{800}\right)dx$$

$$= 200\left(x.\frac{\sin\left(\frac{\pi x}{800}\right)}{\frac{\pi}{800}} - \int 1.\sin\frac{\left(\frac{\pi x}{800}\right)dx}{\pi/800}\right)_0^{400}$$

$$= 200\left[\frac{800}{\pi}.x.\sin\left(\frac{\pi x}{800}\right) - \frac{800}{\pi}.\left(\frac{-\cos\left(\frac{\pi x}{800}\right)dx}{\pi/800}\right)\right]_0^{400}$$

$$= 200\left\{\left(\frac{800}{\pi}.400\sin\left(\frac{\pi}{2}\right) + \frac{800^2}{\pi^2}.\cos\left(\frac{\pi}{2}\right)\right) - \left(0 + \frac{800^2}{\pi^2}\cos 0\right)\right\}$$

$$= 200[101859 - 64845.5]$$

$$= 7402721 \text{ mm}^3$$

Also, $\int y_{el}.dA = \int_0^{400} \frac{y}{2} . ydx$

$$= \frac{1}{2}\int_0^{400}\left(200\cos\left(\frac{\pi x}{800}\right)\right)^2 dx = 20000\int_0^{400}\frac{1+\cos\left(\frac{\pi x}{400}\right)}{2}dx$$

$$= 10000\left(x + \frac{\sin\left(\frac{\pi x}{400}\right)}{\pi/400}\right)_0^{400}$$

$$= 10000\left\{\left(400 + \frac{400}{\pi}(\sin\pi)\right) - 0\right\} = 4\times10^6 \ \ mm^3$$

Using
$$\overline{X} = \frac{\int x_{el}.dA}{\int dA} = \frac{7402721}{50930} = 145.35 \ \ mm$$

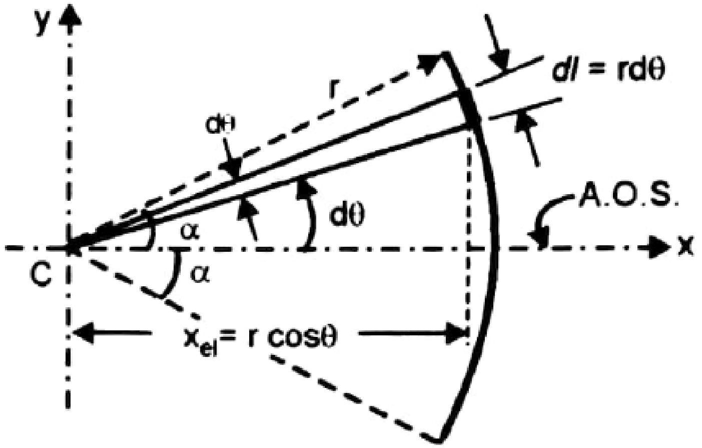
$$\overline{Y} = \frac{\int y_{el}.dA}{\int dA} = \frac{4\times10^6}{50930} = 78.54 \ \ mm$$

$\therefore$ Centroid of the plane area has co-ordinates ($\overline{X}$, $\overline{Y}$) = (145.35, 78.54) mm......**Ans.**

Show that the centroid of an arc of radius 'r' is located at $\frac{r.\sin\alpha}{\alpha}$ from the centre along axis of symmetry.

Solution: Consider a circular arc which subtends an angle 2α. Let the A.O.S. be x-axis. Let us take an element of length dL situated at an angle θ from the x-axis. Let the element subtend an angle $d\theta$ at the centre.

$\therefore \quad dL = r d\theta$



$$\text{Total length } L = \int dL = \int_{-\alpha}^{\alpha} r\, d\theta = r[\theta]_{-\alpha}^{\alpha}$$

$$\text{or} \quad \int dL = 2r\alpha \quad \text{------------ (1)}$$

The centroid of the element is on the element itself

$$\therefore \quad x_{el} = r\cos\theta$$

$$\therefore \quad \int x_{el}.dL = \int_{-\alpha}^{\alpha} r\cos\theta . r d\theta = r^2[\sin\theta]_{-\alpha}^{\alpha} = 2r^2 \sin\alpha \quad \text{------------ (2)}$$

$$\text{Using} \quad \bar{X} = \frac{\int x_{el}.dL}{\int dL} = \frac{2r^2\sin\alpha}{2r.\alpha} \quad \text{or} \quad \boxed{\bar{X} = \frac{r\sin\alpha}{\alpha}} \quad \text{----- Proved.}$$