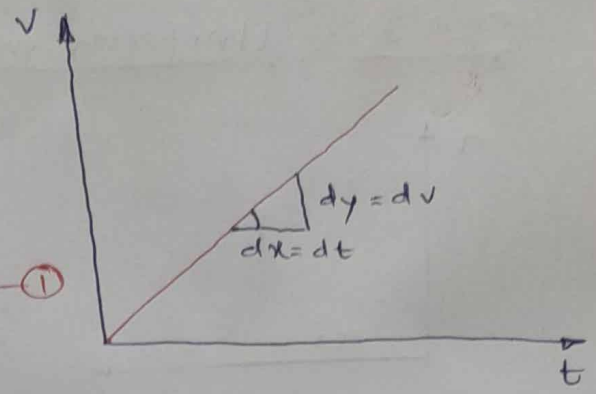
Motion Diagrams :

$a = \frac{dv}{dt}$

a = slope of velocity time dia.

a = slope of v-t diagram — ①

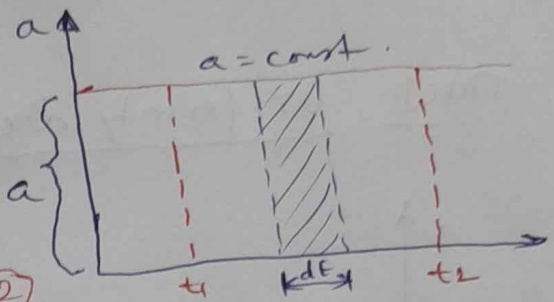


$dv = a\,dt$

$\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\,dt$

$(v_2 - v_1) = a(t_2 - t_1)$

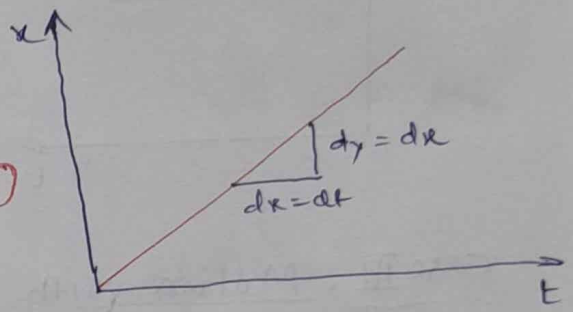
∴ **Δv = Area under a-t diagram** — ②



$v = \frac{dx}{dt}$

v = slope of position time diagram

v = slope of x-t diagram — ③

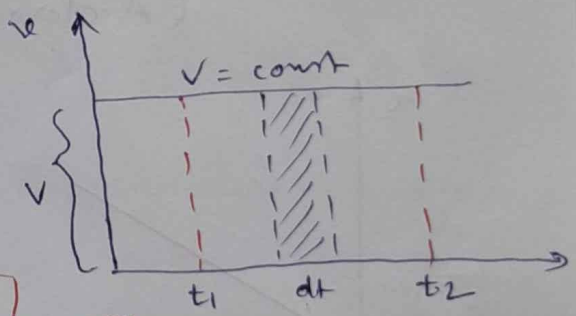


$dx = v \cdot dt$

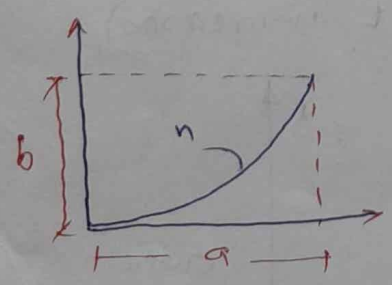
$\int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v\,dt$

$x_2 - x_1 = v(t_2 - t_1)$

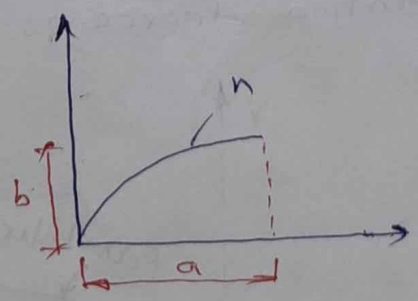
Δx = Area under v-t diagram — ④



⇒ n – degree of polynomial

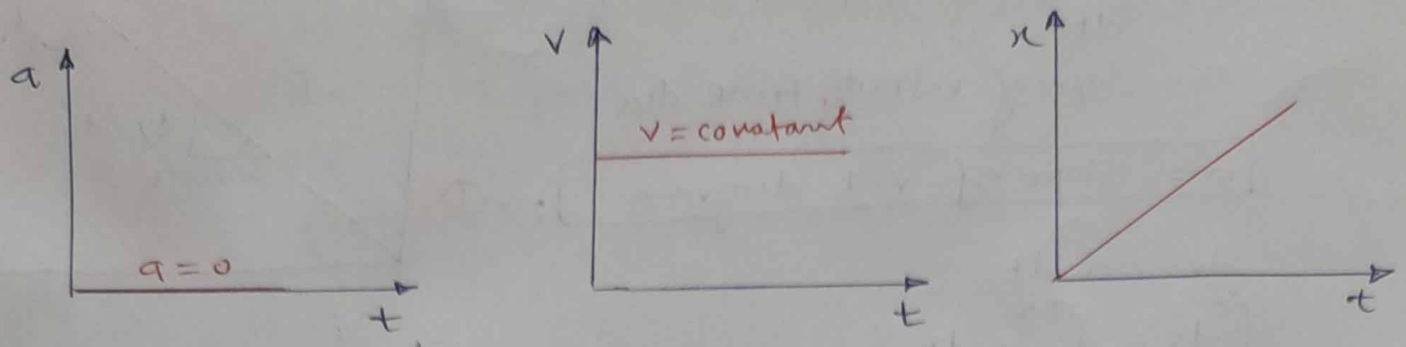


$\text{Area} = \frac{a \times b}{(n+1)}$

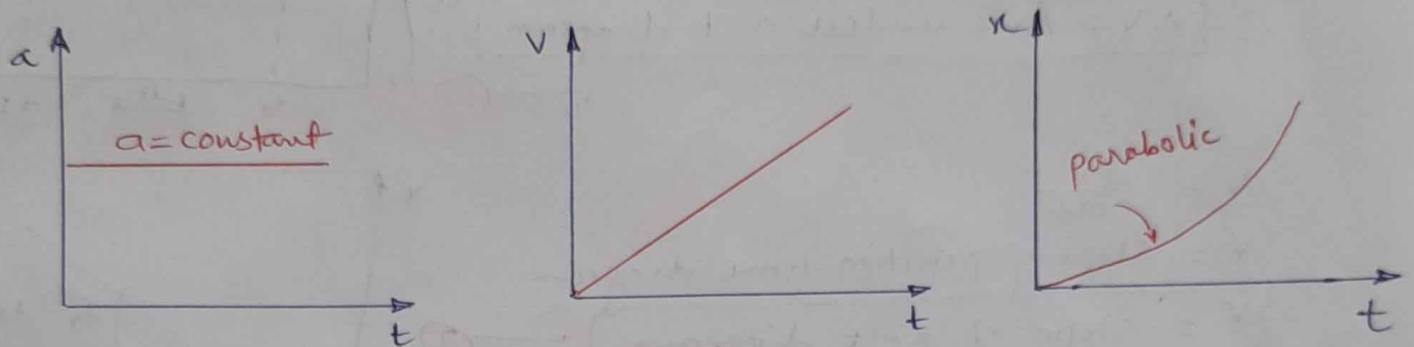


$\text{Area} = \frac{n(a \times b)}{(n+1)}$

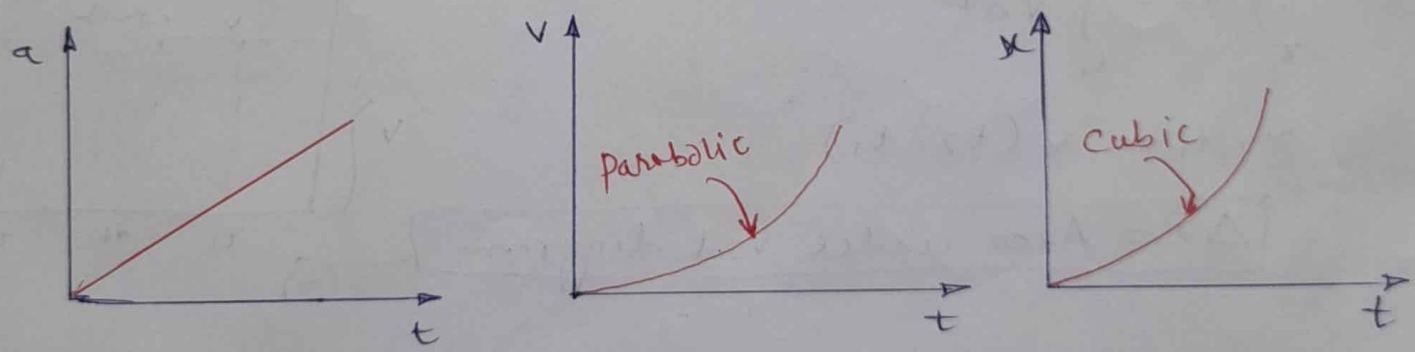
Case I :. **Uniform motion** (v = constant, $a = 0$)



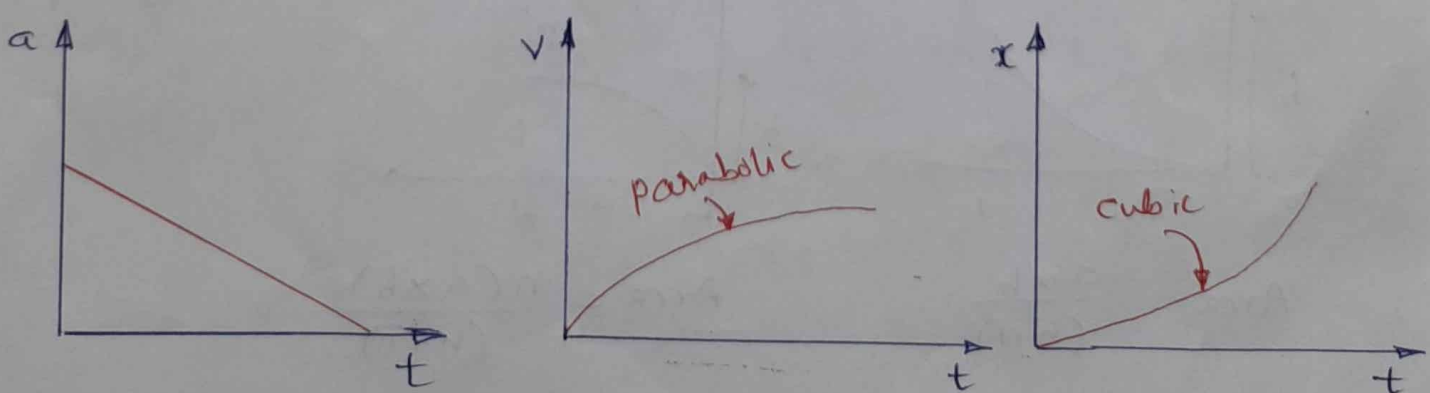
Case II : **Uniformly accelerated motion**



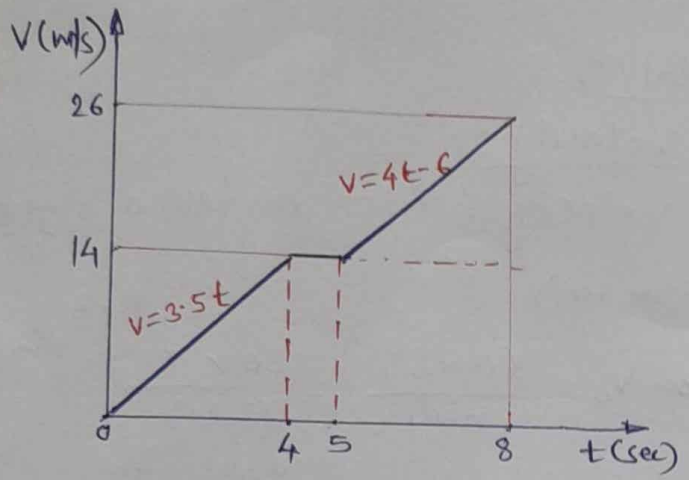
Case III : **Motion with variable acceleration**
(Acceleration increases as t increases)



Case IV : **Motion with variable acceleration**
(Acceleration decreases as t increases)



The race car starts from rest and travels along a straight road until the speed of 26 m/s and 8 sec as shown on v-t graph. The flat part of the graph is caused by shifting gears. Draw the a-t graph and determine the maxm acceleration of the car.



Soln :-

Acceleration :

a = slope of v-t diagram.

(0-4 sec)

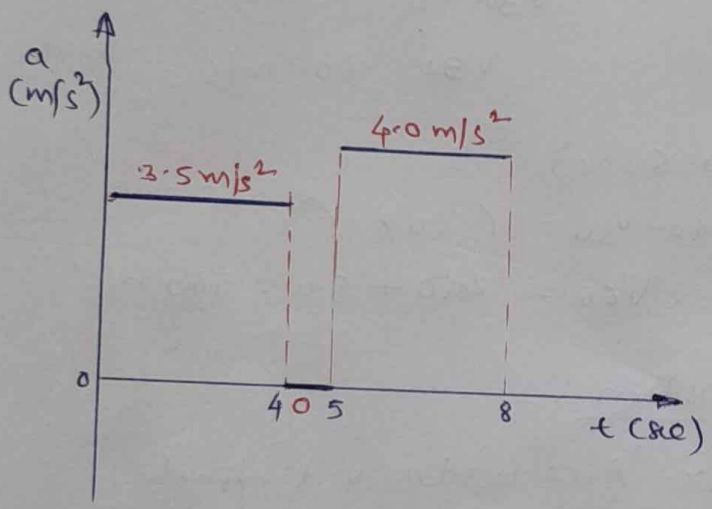
$$a = \frac{dv}{dt} = \frac{14-0}{4-0} = 3.5 \text{ m/s}$$

(4-5 sec)

$$a = \frac{dv}{dt} = \frac{14-14}{5-0} = 0 \text{ m/s}$$

(5-8 sec)

$$a = \frac{dv}{dt} = \frac{26-14}{8-5} = 4 \text{ m/s}$$



position :-

Δx = Area under v-t diagram

(0-4 sec)

$x_4 - x_0 = \frac{1}{2}(4 \times 14)$ $\quad x_0 = 0$

$\therefore x_4 = 28$ m.

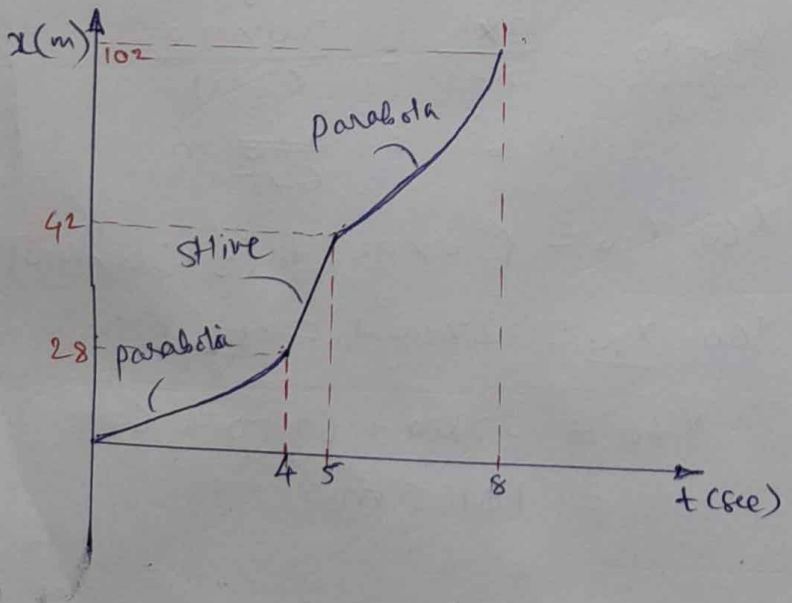
$x_5 - x_4 = (1 \times 14)$

$\therefore x_5 = 14 + 28 = 42$ m.

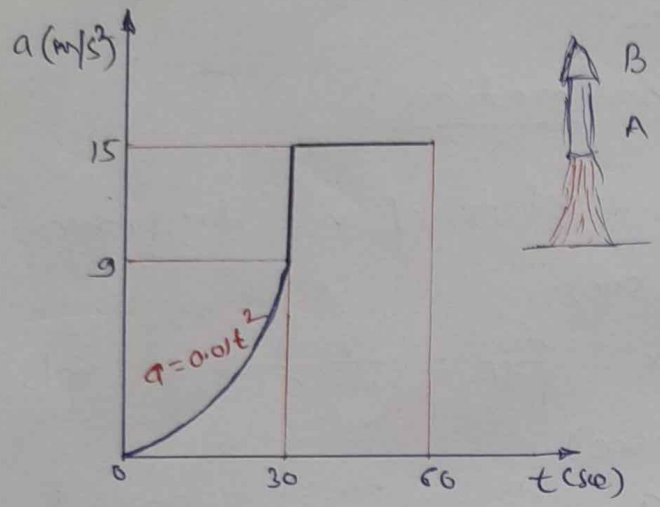
$x_8 - x_5 = \frac{1}{2}(3 \times 12) + (3 \times 14)$

$x_8 = 18 + 42 + 42$

$= 102$ m.



A two stage rocket is fired vertically from rest at $s=0$, with acceleration as shown. After 30 sec, the first stage A burns out and second stage B ignites. Plot v-t and s-t graphs which describes the motion for $0 \le t \le 60$ sec.



Sol?:

Velocity:

Velocity = Area under a-t graph

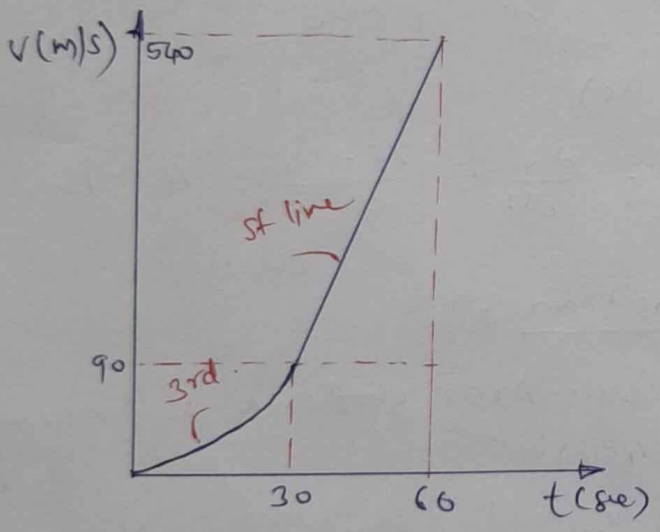
(0-30 sec) $n=2$

$$v_{30} - v_0 = \frac{(a \times b)}{n+1} = \frac{30 \times 9}{(2+1)}$$

$$v_{30} - v_0 = 90$$

$v_0 = 0$

$$v_{30} = 90 \text{ m/s}.$$



(30-60 sec)

$$v_{60} - v_{30} = (30 \times 15)$$

$$v_{60} = 450 + 90 = 540 \text{ m/s}.$$

position:-

Δx = Area under v-t graph.

$$x_{30} - x_0 = \frac{a * b}{n+1} = \frac{30 \times 90}{(3+1)}$$

$$x_{30} = \cancel{400}\ 675 \text{ m}.$$

$$x_{60} - x_{30} = (30 \times 90) + \left(\tfrac{1}{2} \times 30 \times 450\right)$$

$$x_{60} - x_{30} = 2700 + 6750$$

$$\therefore x_{60} = 2700 + 6750 + 675$$

$$= 10125 \text{ m}.$$