K J Somaiya College of Engineering, Mumbai-77

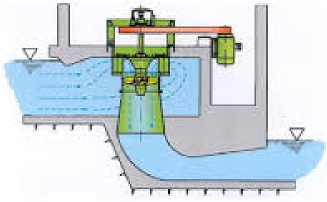
(Constituent college of Somaiya Vidyavihar University)

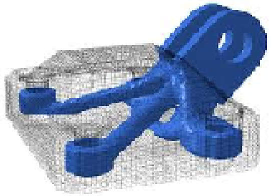
Course Code: 111U06C104

Course Title: Engineering Mechanics

Presented by: Chithra Biju Menon

- Engineering Mechanics
  - Fluid Mechanics
    - Statics
    - Dynamics
      - Kinematics
        - Ideal Fluids
        - Viscous Fluids
        - Compressible Fluids
      - Kinetics
  - Solid Mechanics
    - Mechanics of Rigid Bodies
      - Statics
      - Dynamics
        - Kinematics
        - Kinetics
    - Mechanics of Deformable Bodies
      - Strength of Materials
      - Theory of Elasticity
      - Theory of Plasticity





Classification of Engineering Mechanics

Kinematics of Particles and Rigid Bodies

2	**Kinematics of Particles and Rigid Bodies**		**11**	**CO 2**
	2.1	Variable motion, motion curves (a-t, v-t, s-t) (acceleration curves restricted to linear acceleration only), motion along plane curved path, velocity & acceleration in terms of rectangular components, tangential & normal component of acceleration, relative velocities.		
	2.2	Introduction to general plane motion, problems based on ICR method for general plane motion of bodies (up to 2 linkage mechanism and no relative velocity method)		

Difference between particle and rigid body??

Difference between kinetics and kinematics

Description of motion without regard to causes of motion

Kinetics or kinematics???

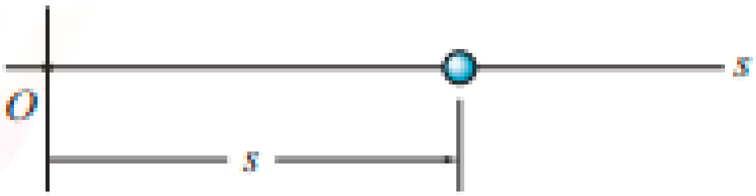
Kinematics of particles:

It is the study of geometry of translation motion (rectilinear and curvilinear) without reference to the cause of motion. Force and mass are not considered.

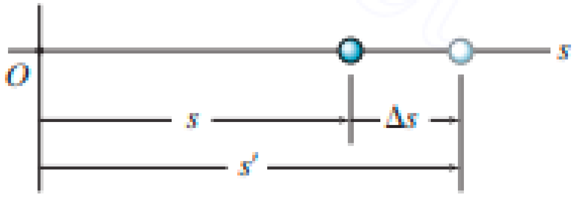
Rectilinear kinematics: Continuous motion

The kinematics of a particle is characterized by specifying, at any given instant, the particle’s

position, displacement, velocity, and acceleration.

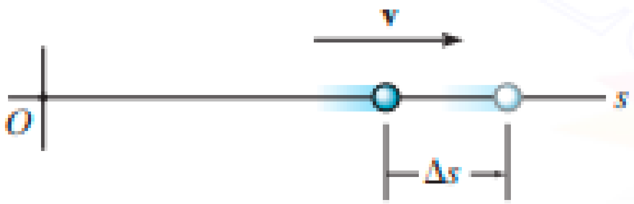


Position

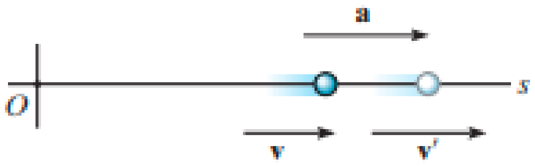


Displacement

$$dt = \frac{ds}{v} = \frac{dv}{a}$$



Velocity



Acceleration

$$a\,ds = v\,dv$$

$$v_{avg} = \frac{\Delta s}{\Delta t}$$

$$v = \frac{ds}{dt}$$

$$a_{avg} = \frac{\Delta v}{\Delta t}$$

$$a = \frac{dv}{dt}$$

When the acceleration is constant i.e. a = a_c

$$a = \frac{dv}{dt}$$

$$v = \frac{ds}{dt}$$

$$a\,ds = v\,dv$$

$$\int_{v_0}^{v} dv = \int_0^t a_c\,dt$$

$$v = v_0 + a_c t$$

Constant Acceleration

$$\int_{s_0}^{s} ds = \int_0^t (v_0 + a_c t)\,dt$$

$$s = s_0 + v_0 t + \tfrac{1}{2} a_c t^2$$

Constant Acceleration

$$\int_{v_0}^{v} v\,dv = \int_{s_0}^{s} a_c\,ds$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

Constant Acceleration

Problem 1

The car moves in a straight line such that for a short time its velocity is defined by $v = (3t^2 + 2t)$ m/s, where t is in seconds. Determine its position and acceleration when t = 3 s. When t = 0, s = 0.

Hint

$v = ds/dt$

$a = dv/dt$

Problem 1

The car moves in a straight line such that for a short time its velocity is defined by v = ($3t^2$ + 2t) ft/s, where t is in seconds. Determine its position and acceleration when t = 3 s. When t = 0, s = 0.

$$v = \frac{ds}{dt} = (3t^2 + 2t)$$

$$\int_0^s ds = \int_0^t (3t^2 + 2t)dt$$

$$s \Big|_0^s = t^3 + t^2 \Big|_0^t$$

$$s = t^3 + t^2$$

Problem 2

Acceleration of a ship, moving in a straight course varies directly as the square of the speed. If the speed drops from 3 m/s to 1.5 m/s in 1 minute, find the distance moved in this period.

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Problem 3

A particle starting from rest, moves in a straight line and its acceleration is given by $a = 50 - 36t^2$ m/s^2. Determine the velocity of the particle when it has travelled 52 m.

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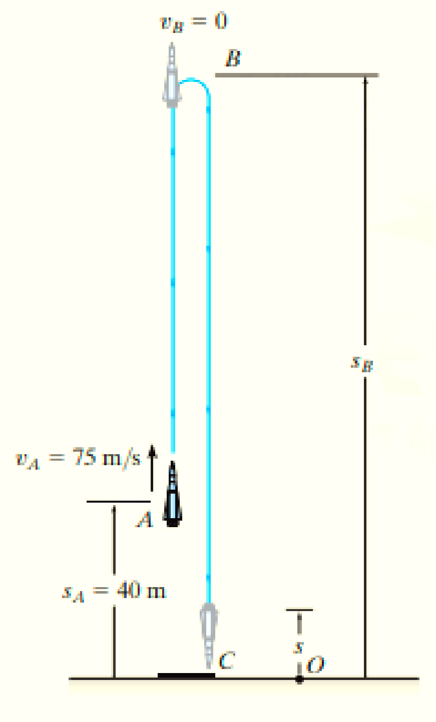
Problem 4



During a test a rocket travels upward at 75 m/s, and when it is 40 m from the ground its engine fails. Determine the maximum height s_B reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of 9.81 m/s^2 due to gravity. Neglect the effect of air resistance.

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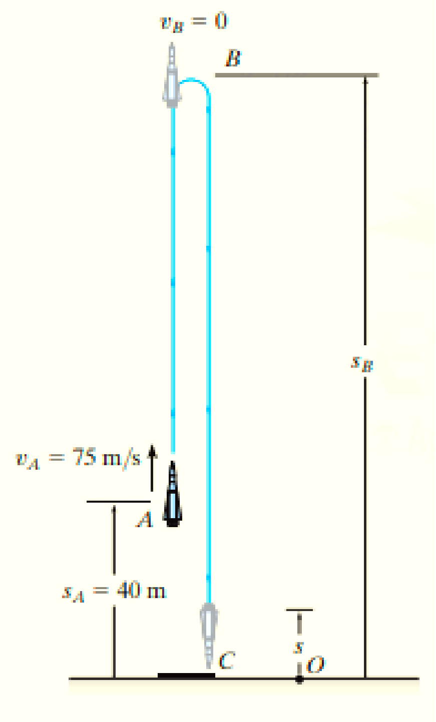

Problem 5

In Asian games, for a 100 m event an athlete accelerates uniformly from start to his maximum velocity in a distance of 4 m and runs the remaining distance with that velocity. If the athlete finishes the race in 10.4 sec, determine (i) his initial acceleration and (ii) his maximum velocity

Problem 5

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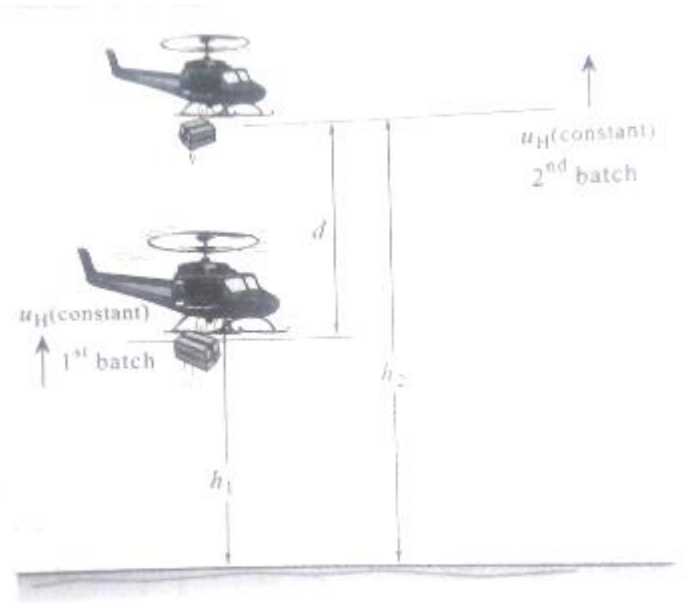
Problem 6

In a food relief area a helicopter going vertically up with a constant velocity drops first batch of food packets which takes 4 seconds to reach the ground. No sooner this batch reaches the ground, second batch of food products are released and this batch takes 5 seconds to reach the ground. From what height was the first batch released? Also determine the velocity with which the helicopter is moving up.



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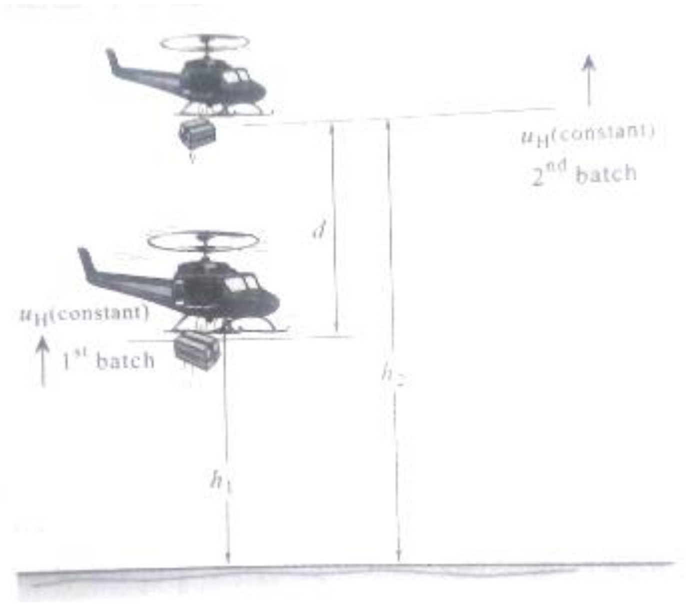

Problem 7

Tests reveal that a normal driver takes about 0.75 s before he or she can react to a situation to avoid a collision. It takes about 3 s for a driver having 0.1% alcohol in his system to do the same. If such drivers are traveling on a straight road at 44 ft/s and their cars can decelerate at 2 ft/s 2 , determine the shortest stopping distance d for each from the moment they see the pedestrians.

Moral: If you must drink, please don’t drive!

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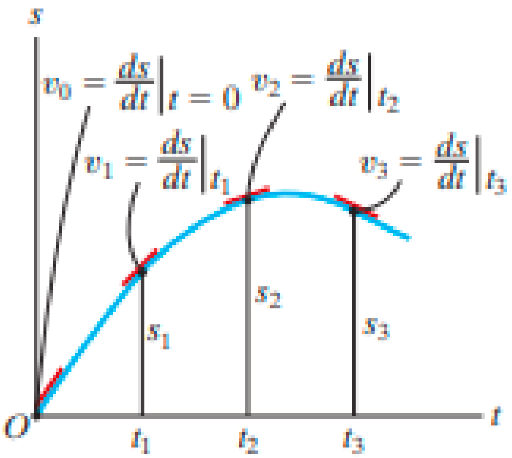
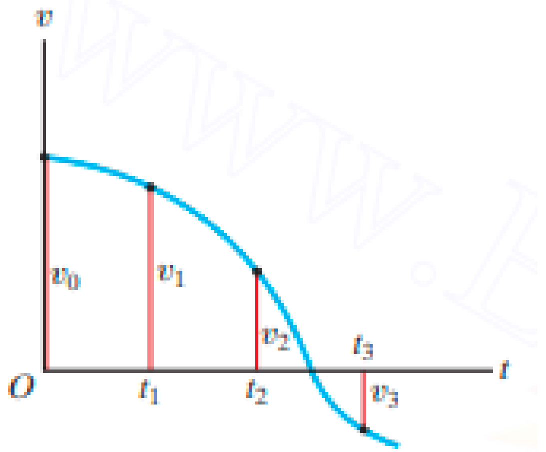
Rectilinear Kinematics: Erratic motion

- When a particle has erratic or changing motion then its position, velocity, and acceleration cannot be described by a single continuous mathematical function along the entire path.
- A series of functions will be required to specify the motion at different intervals.
- Hence, it is convenient to represent the motion as a graph.
- If a graph of the motion that relates any two of the variables s,v, a, t can be drawn, then this graph can be used to construct subsequent graphs relating two other variables since the variables are related by the differential relationships $v = ds/dt$, $a = dv/dt$, or $a\,ds = v\,dv$

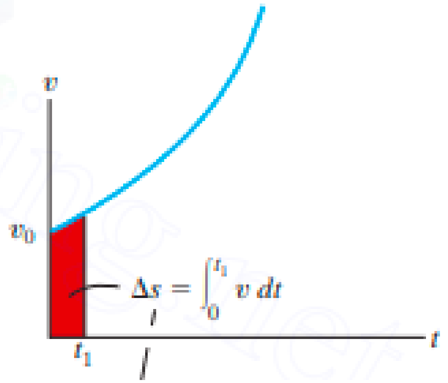
S-t, v-t graphs





$$\frac{ds}{dt} = v$$

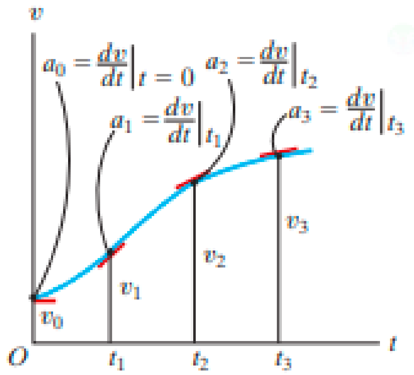
slope of s–t graph = velocity

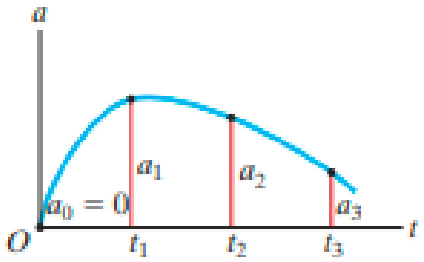


$$\Delta s = \int v\,dt$$

displacement = area under v–t graph

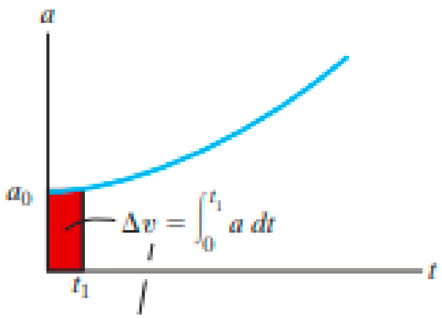
V-t, a-t graphs





$$\frac{dv}{dt} = a$$

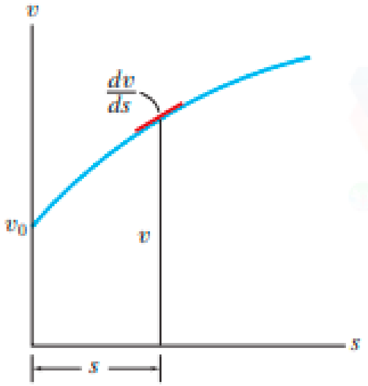
slope of v–t graph = acceleration



$$\Delta v = \int a\,dt$$

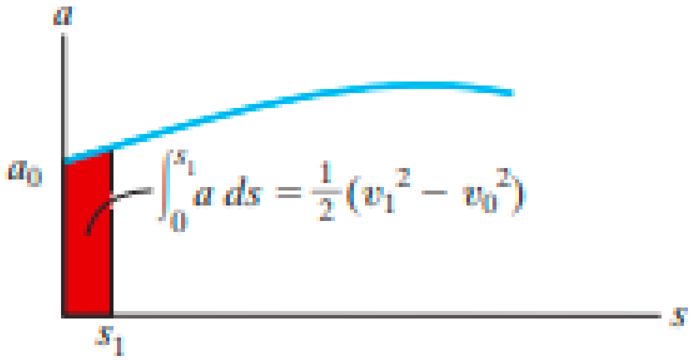
change in velocity = area under a–t graph

V-s, a-s graphs



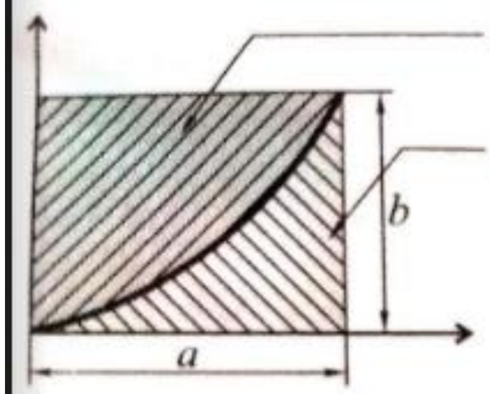
$$a = v\left(\frac{dv}{ds}\right)$$

acceleration = velocity times slope of v–s graph



$$\tfrac{1}{2}(v_1^2 - v_0^2) = \int_{s_0}^{s_1} a\,ds$$

area under a–s graph



$$A_1 = \frac{nab}{n+1}$$

$$A_2 = \frac{ab}{n+1}$$

$$A = A_1 + A_2 = ab$$

n = Curve of degree of polynomial

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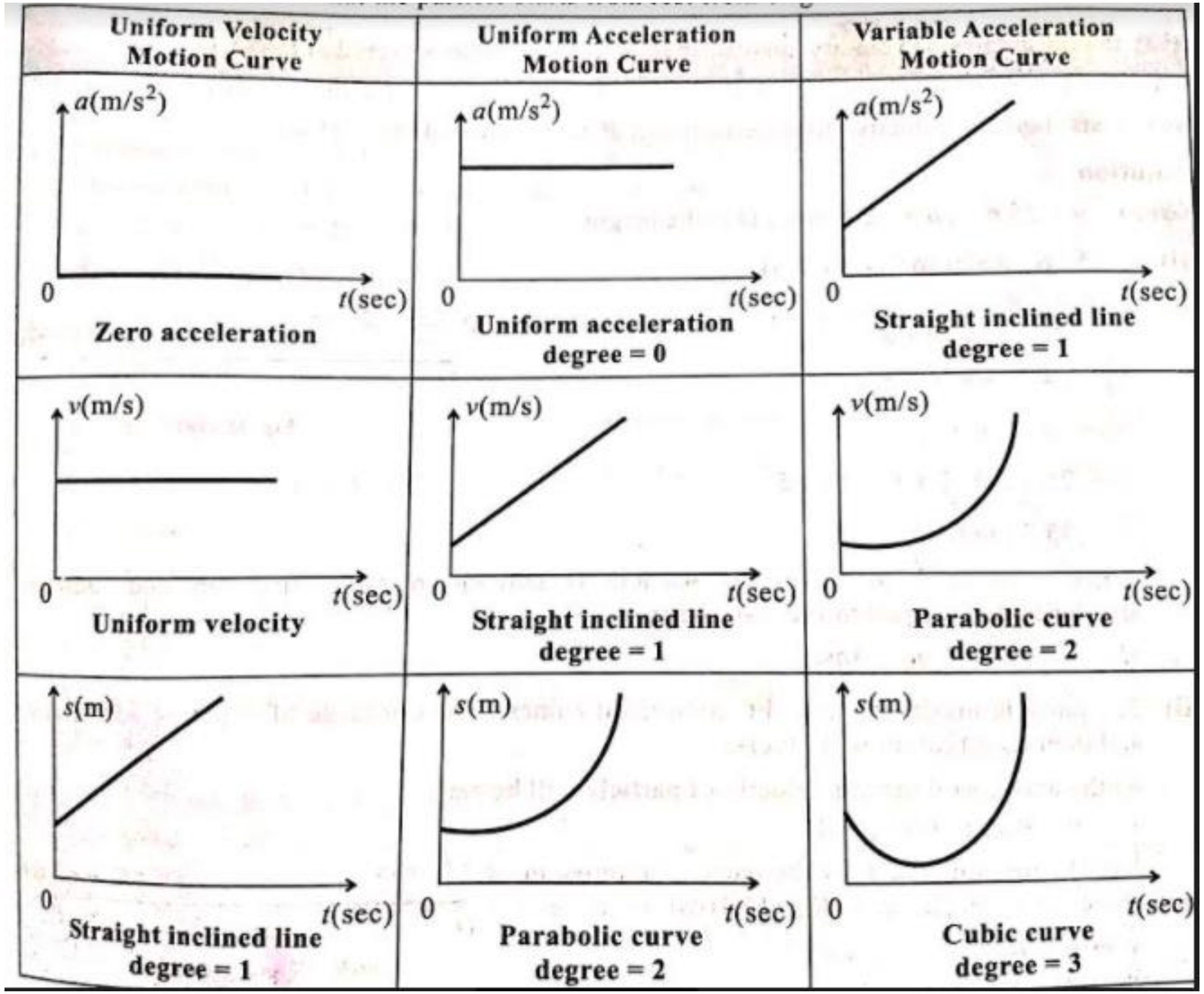
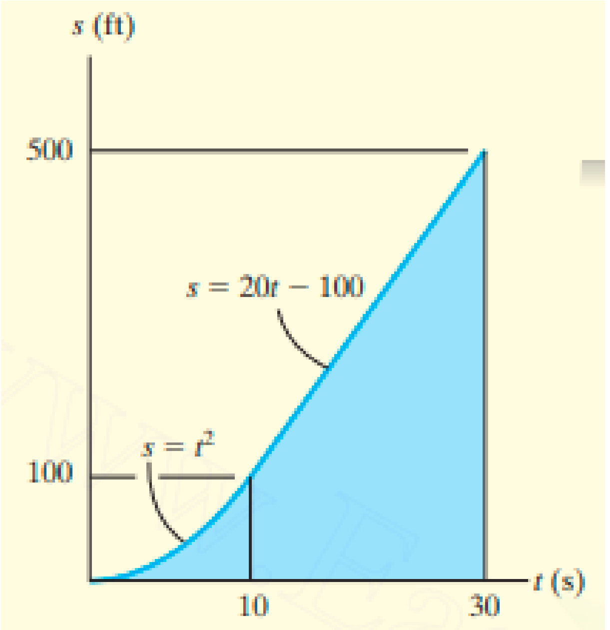
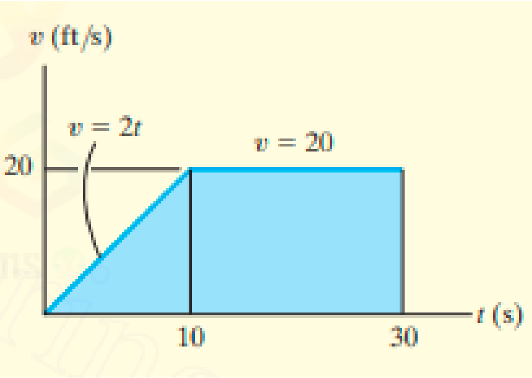
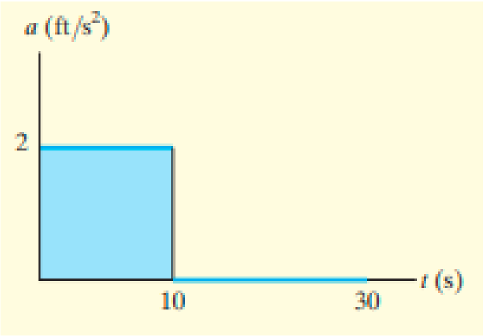
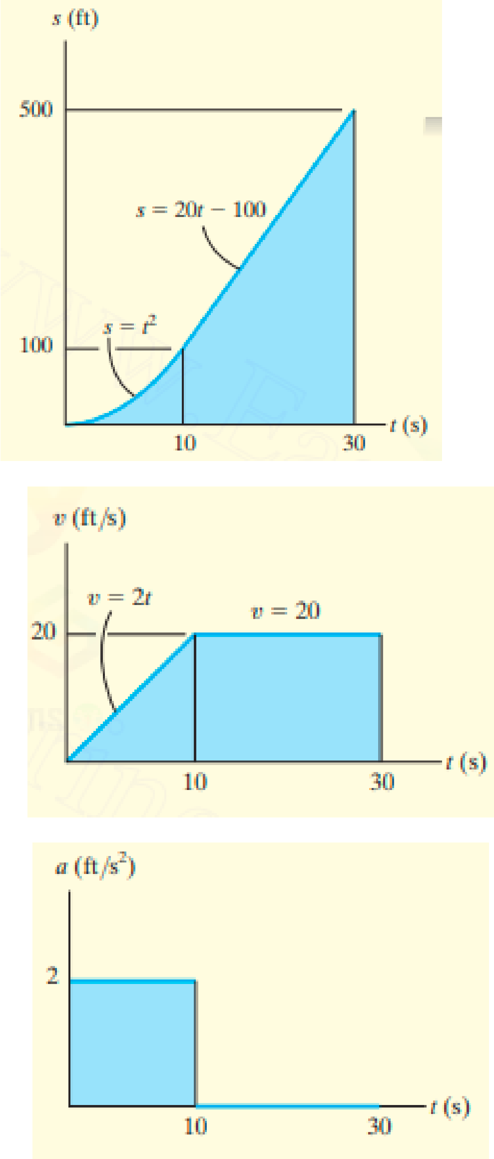

Uniform Velocity Motion Curve
Uniform Acceleration Motion Curve
Variable Acceleration Motion Curve
a(m/s²)
0
t(sec)
Zero acceleration
a(m/s²)
0
t(sec)
Uniform acceleration degree = 0
a(m/s²)
0
t(sec)
Straight inclined line degree = 1
v(m/s)
0
t(sec)
Uniform velocity
v(m/s)
0
t(sec)
Straight inclined line degree = 1
v(m/s)
0
t(sec)
Parabolic curve degree = 2
s(m)
0
t(sec)
Straight inclined line degree = 1
s(m)
0
t(sec)
Parabolic curve degree = 2
s(m)
0
t(sec)
Cubic curve degree = 3

A bicycle moves along a straight road such that its position is described by the graph shown in Fig. Construct the v-t and a–t graphs for 0 <=t <= 30 s.

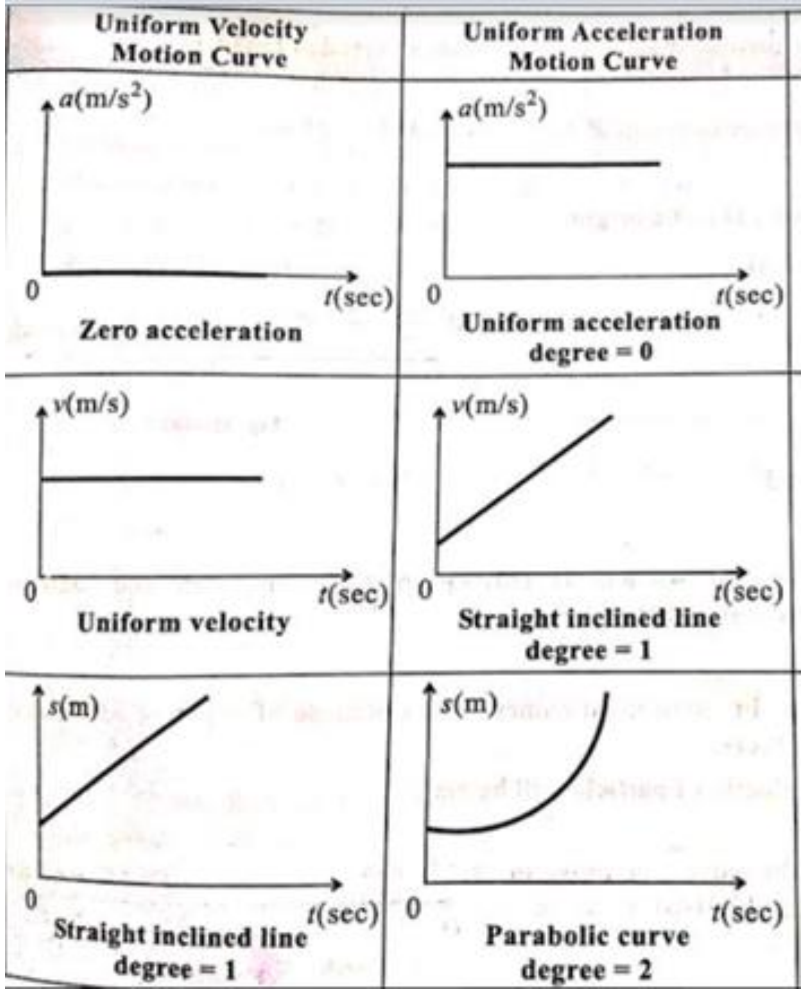






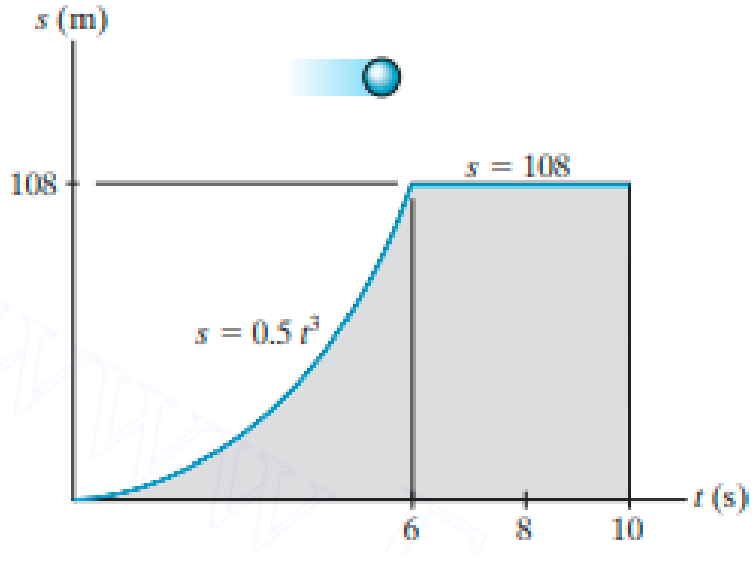


s (ft)
500
s = 20t − 100
s = t²
100
10
30
t (s)
v (ft/s)
v = 2t
v = 20
20
10
30
t (s)
a (ft/s²)
2
10
30
t (s)

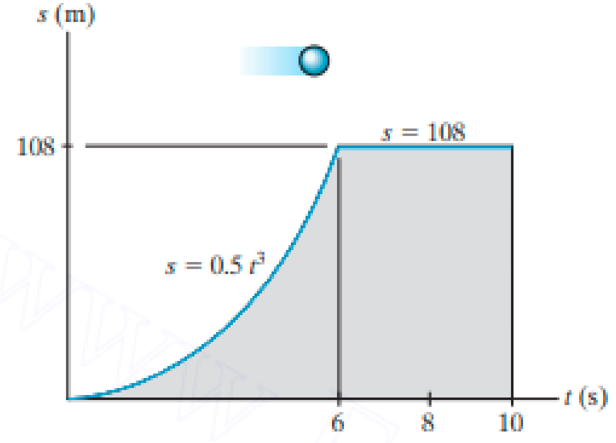


Uniform Velocity Motion Curve
Uniform Acceleration Motion Curve
a(m/s²)
0
t(sec)
Zero acceleration
a(m/s²)
0
t(sec)
Uniform acceleration degree = 0
v(m/s)
0
t(sec)
Uniform velocity
v(m/s)
0
t(sec)
Straight inclined line degree = 1
s(m)
0
t(sec)
Straight inclined line degree = 1
s(m)
0
t(sec)
Parabolic curve degree = 2

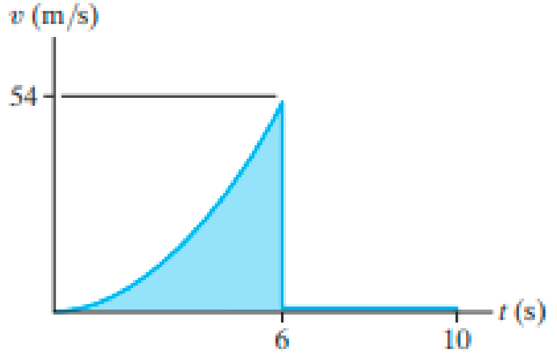
The particle travels along a straight track such that its position is described by the s-t graph. Construct the v-t graph for the same time interval.



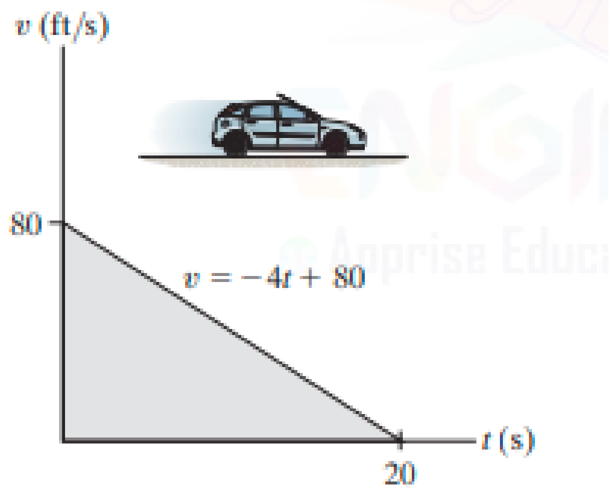
The particle travels along a straight track such that its position is described bv the s-t graph. Construct the v-t graph for the same time inter



$$v = \frac{ds}{dt} = \frac{d}{dt}\left(0.5t^3\right) = 1.5t^2$$
$$v = \frac{ds}{dt} = \frac{d}{dt}(108) = 0$$



F12–10. A van travels along a straight road with a velocity described by the graph. Construct the s-t and a-t graphs during the same period. Take s = 0 when t = 0



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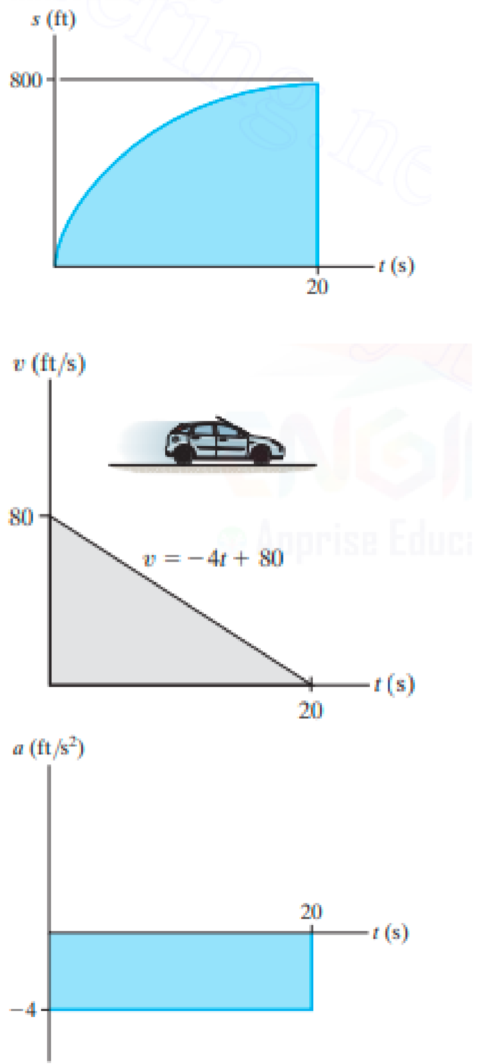


$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t (-4t + 80)\,dt$$

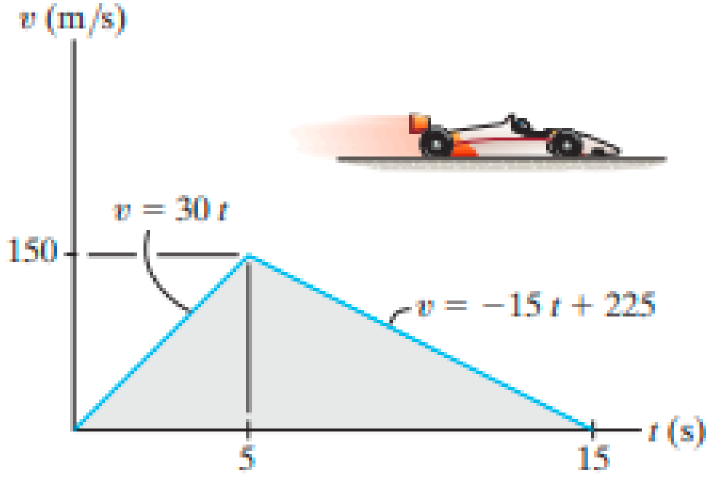
$$s = -2t^2 + 80t$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(-4t + 80) = -4 \text{ ft/s}^2 = 4 \text{ ft/s}^2 \leftarrow$$

Also,

$$a = \frac{\Delta v}{\Delta t} = \frac{0 - 80 \text{ ft/s}}{20 \text{ s} - 0} = -4 \text{ ft/s}^2$$

The dragster starts from rest and has a velocity described by the graph. Construct the s-t graph during the time interval 0<= t <=15 s. Also, determine the total distance traveled during this time interval.



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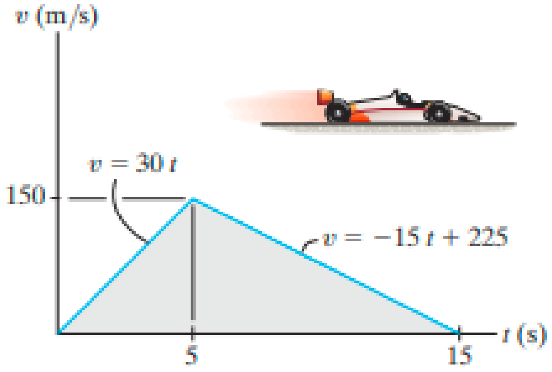
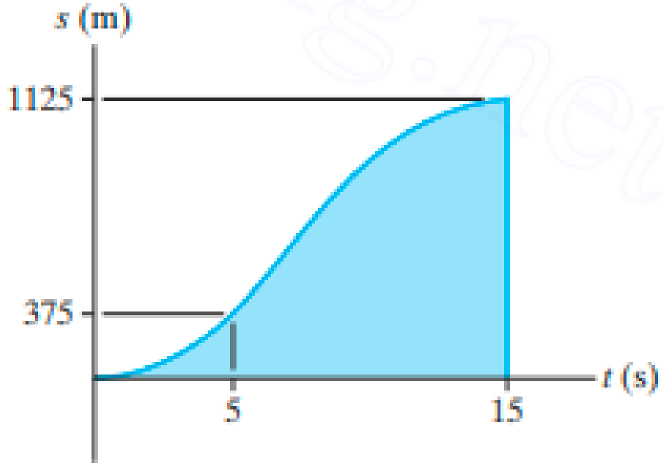


$0 \le t \le 5$ s,

$ds = v\,dt \qquad \int_0^s ds = \int_0^t 30t\,dt$

$s\big|_0^s = 15t^2\big|_0^t$

$s = (15t^2)$ m



$5\text{ s} < t \le 15$ s,

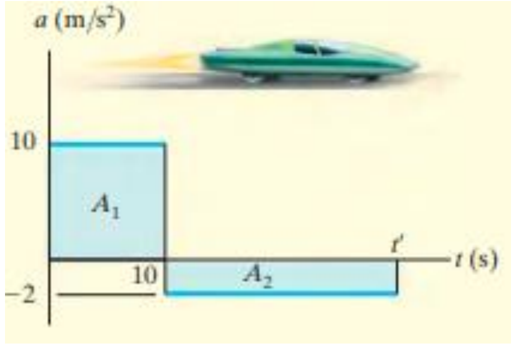
$(\overset{+}{\rightarrow})\; ds = v\,dt; \qquad \int_{375\text{ m}}^{s} ds = \int_{5\text{ s}}^{t} (-15t + 22$

$s = (-7.5t^2 + 225t - 562.5)$ m

$s = (-7.5)(15)^2 + 225(15) - 562.5$ m

$= 1125$ m

The car starts from rest and travels along a straight track such that it accelerates at 10 m/s^2 for 10 s, and then decelerates at 2 m/s^2. Draw the v-t and s–t graphs and determine the time t' needed to stop the car. How far has the car traveled?



$0 \le t < 10$ s; $a = (10)$ m/s^2; $\int_0^v dv = \int_0^t 10\,dt$, $v = 10t$

When $t = 10$ s, $v = 10(10) = 100$ m/s. Using this as the *initial condition* for the next time period, we have

$10 \text{ s} < t \le t'$; $a = (-2)$ m/s^2; $\int_{100 \text{ m/s}}^{v} dv = \int_{10 \text{ s}}^{t} -2\,dt$, $v = (-2t + 120)$ m/s

V= 0 → t'=60 s

$0 \le t \le 10$ s; $v = (10t)$ m/s; $\int_0^s ds = \int_0^t 10t\,dt$, $s = (5t^2)$ m

When $t = 10$ s, $s = 5(10)^2 = 500$ m. Using this *initial condition*,

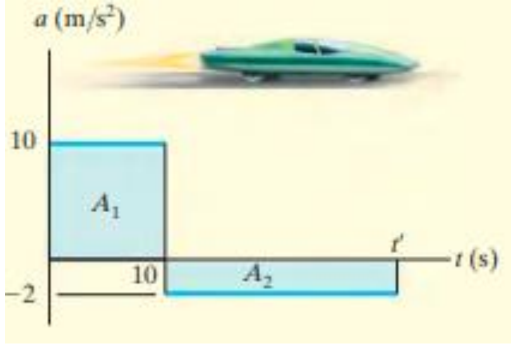
$10 \text{ s} \le t \le 60$ s; $v = (-2t + 120)$ m/s; $\int_{500 \text{ m}}^{s} ds = \int_{10 \text{ s}}^{t} (-2t + 120)\,dt$

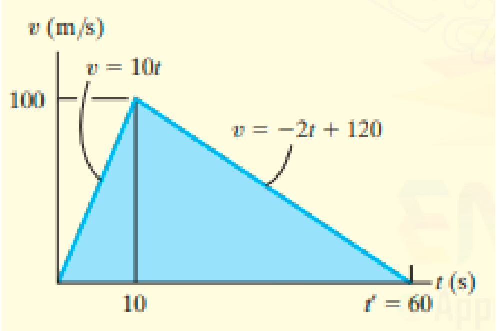
$$s - 500 = -t^2 + 120t - [-(10)^2 + 120(10)]$$

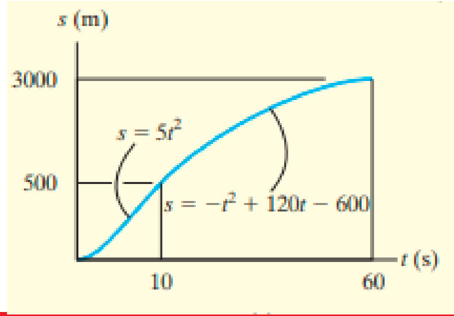
$$s = (-t^2 + 120t - 600) \text{ m}$$

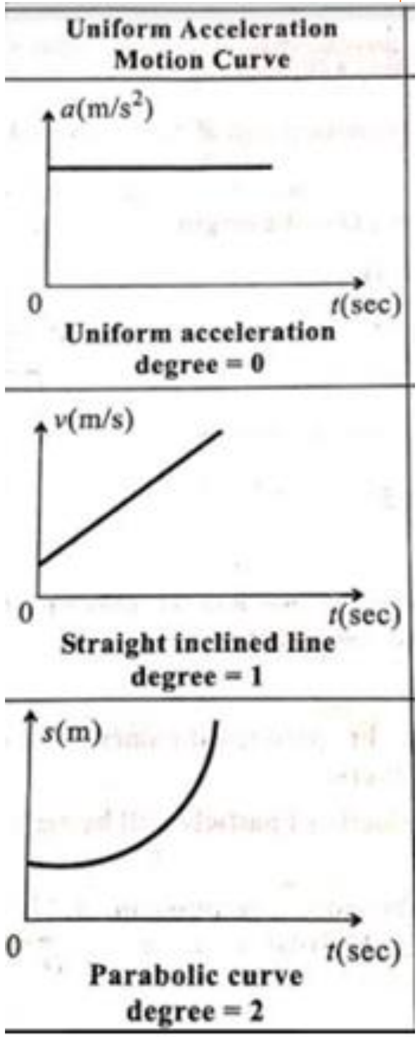
When $t' = 60$ s, the position is

$$s = -(60)^2 + 120(60) - 600 = 3000 \text{ m} \qquad \textit{Ans.}$$





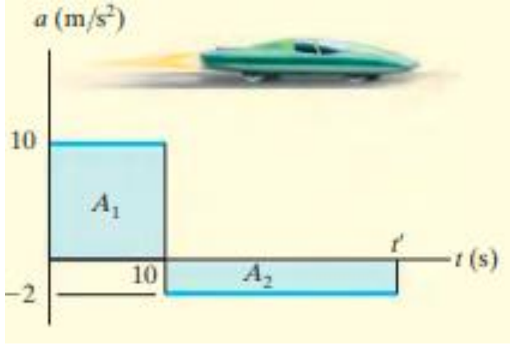


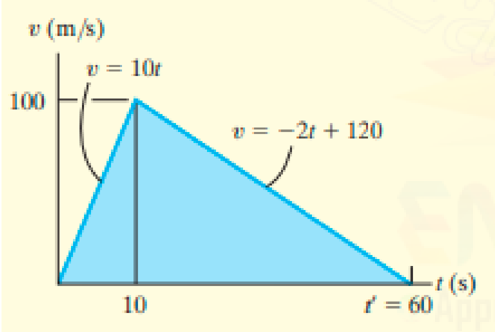


A more direct solution for t' is possible by realizing that the area under the a–t graph is equal to the change in the car's velocity. We require $\Delta v = 0 = A_1 + A_2$, Fig. 12–14*a*. Thus

$$0 = 10\ \text{m/s}^2(10\ \text{s}) + (-2\ \text{m/s}^2)(t' - 10\ \text{s})$$

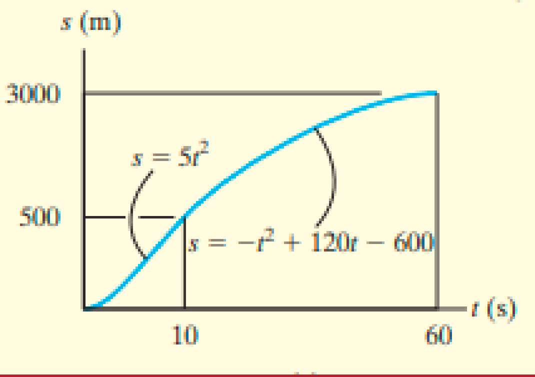
$$t' = 60\ \text{s} \qquad \textit{Ans.}$$



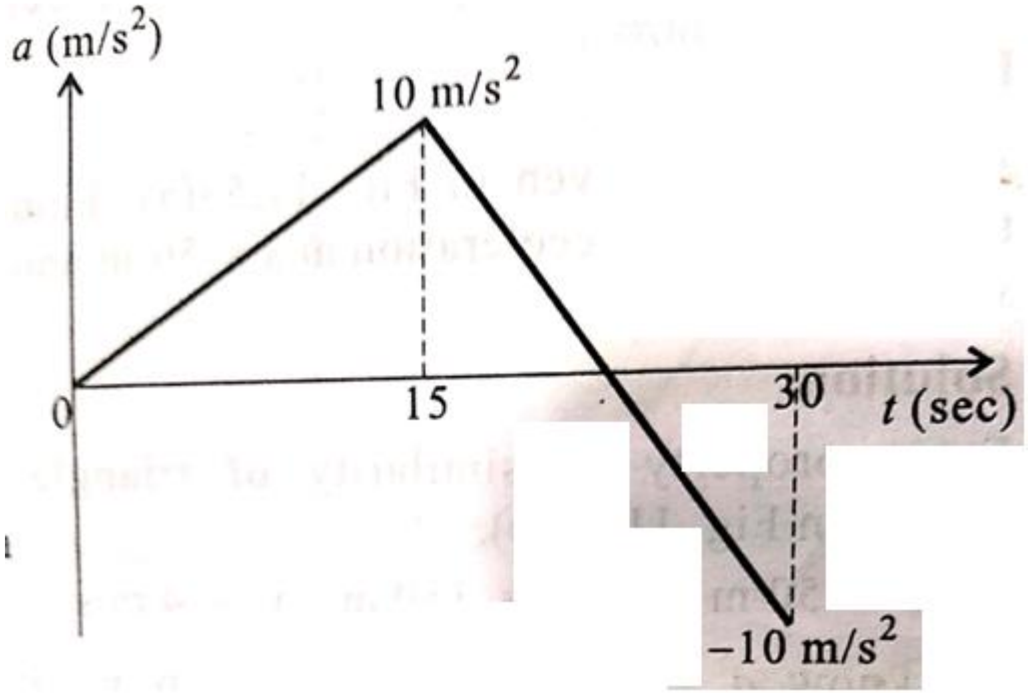


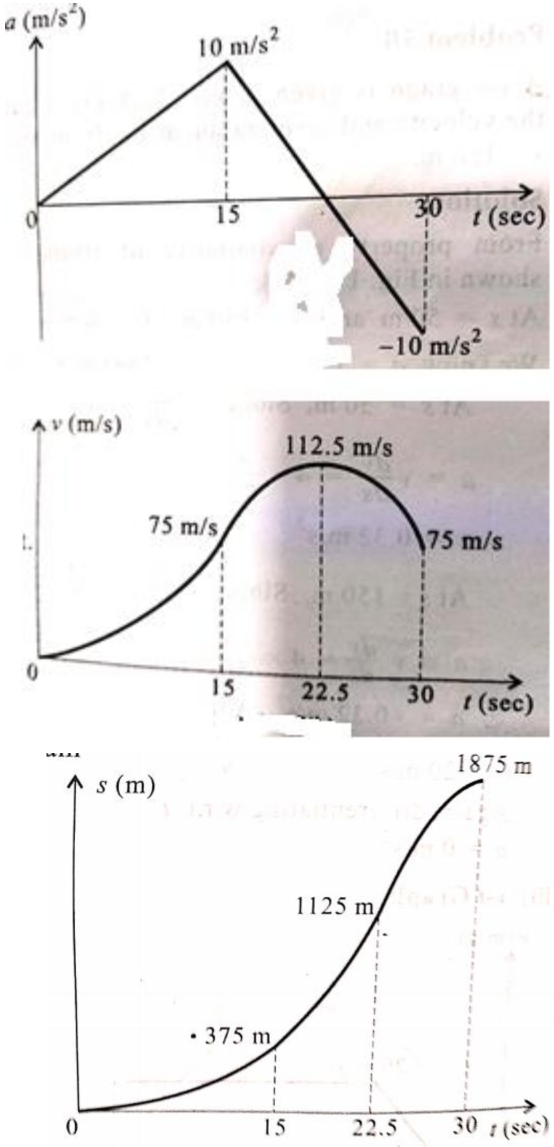
NOTE: A direct solution for s is possible when $t' = 60$ s, since the *triangular area* under the v–t graph would yield the displacement $\Delta s = s - 0$ from $t = 0$ to $t' = 60$ s. Hence,

$$\Delta s = \tfrac{1}{2}(60\ \text{s})(100\ \text{m/s}) = 3000\ \text{m} \qquad \textit{Ans.}$$

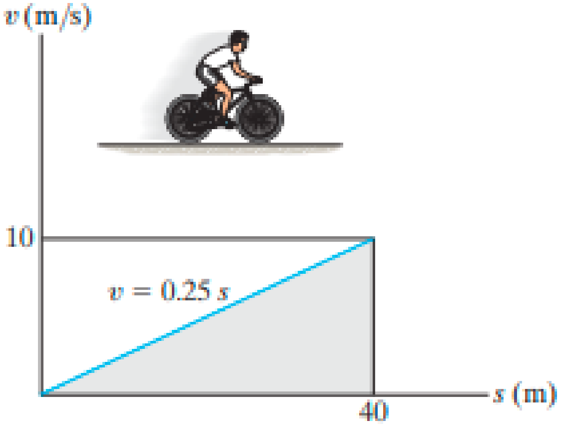


The a-t plot for motion of particle is given. Draw the v-t and s-t plots. Find the maximum velocity in the time interval and the distance travelled by the particle in the time interval




a (m/s^2)
10 m/s^2
0
15
30
t (sec)
−10 m/s^2
v (m/s)
112.5 m/s
75 m/s
75 m/s
0
15
22.5
30
t (sec)
s (m)
1875 m
1125 m
375 m
0
15
22.5
30
t (sec)

A bicycle travels along a straight road where its velocity is described by the v-s graph. Construct the a-s graph for the same interval



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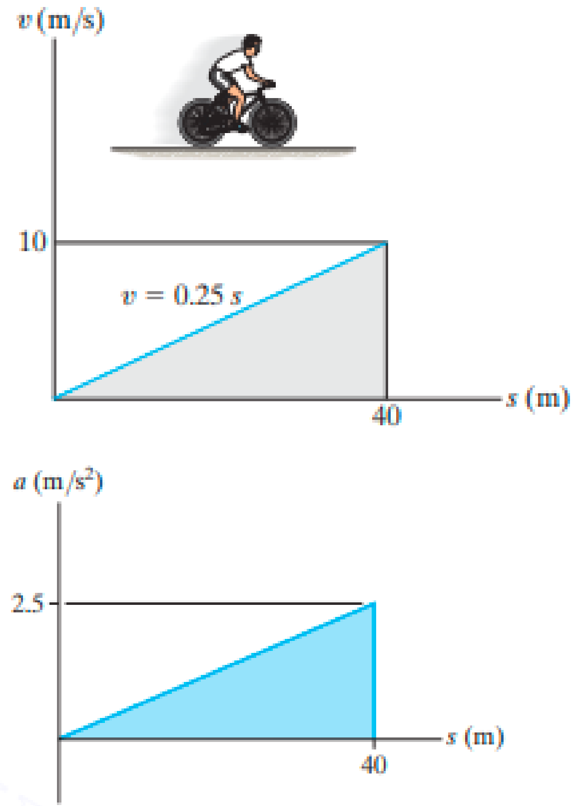
A bicycle travels along a straight road where its velocity is described by the v-s graph. Construct the a-s graph for the same interval

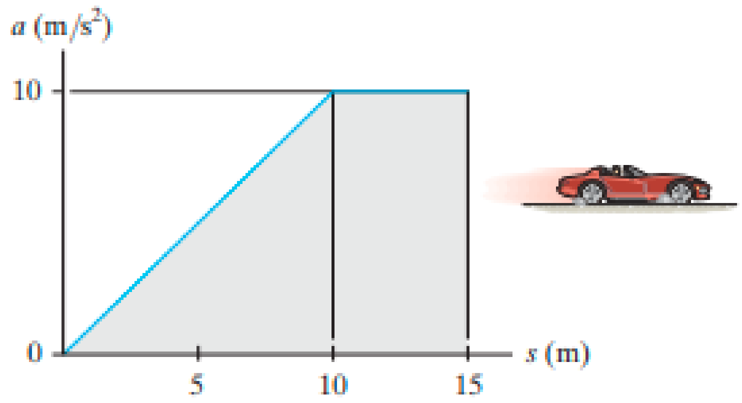
$$a\,ds = v\,dv$$

$$a = v\frac{dv}{ds} = 0.25s\frac{d}{ds}(0.25s) = 0.0625s$$

$$a\big|_{s=40\text{ m}} = 0.0625(40\text{ m}) = 2.5\text{ m/s}^2 \rightarrow$$



The sports car travels along a straight road such that its acceleration is described by the graph. Construct the v-s graph for the same interval and specify the velocity of the car when s = 10 m and s = 15 m.



The sports car travels along a straight road such that its acceleration is described by the graph. Construct the v-s graph for the same interval and specify the velocity of the car when s = 10 m and s = 15 m.

For $0 \leq s \leq 10$ m

$$a = s$$

$$\int_0^v v\,dv = \int_0^s s\,ds$$

$$v = s$$

at $s = 10$ m, $v = 10$ m

For $10 \text{ m} \leq s \leq 15$

$$a = 10$$

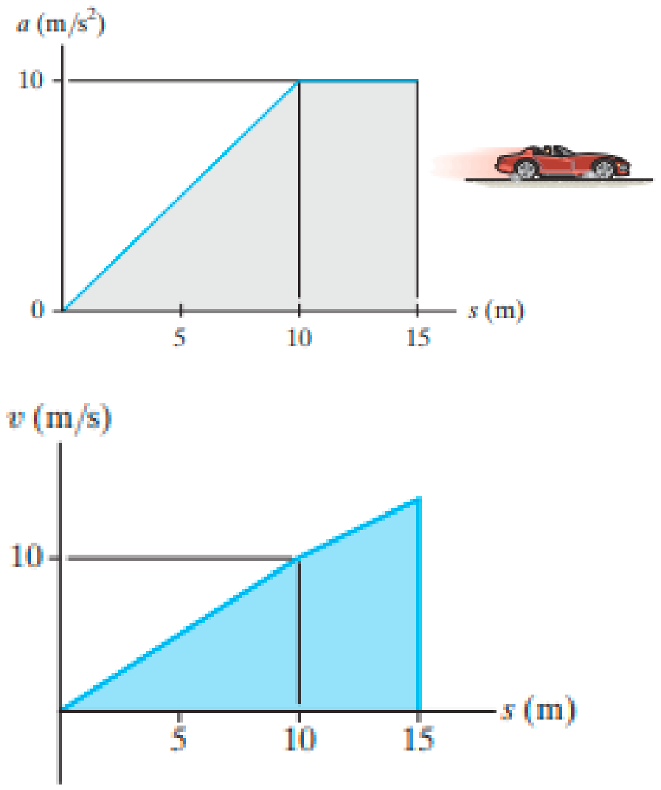
$$\int_{10}^v v\,dv = \int_{10}^s 10\,ds$$

$$\frac{1}{2}v^2 - 50 = 10s - 100$$

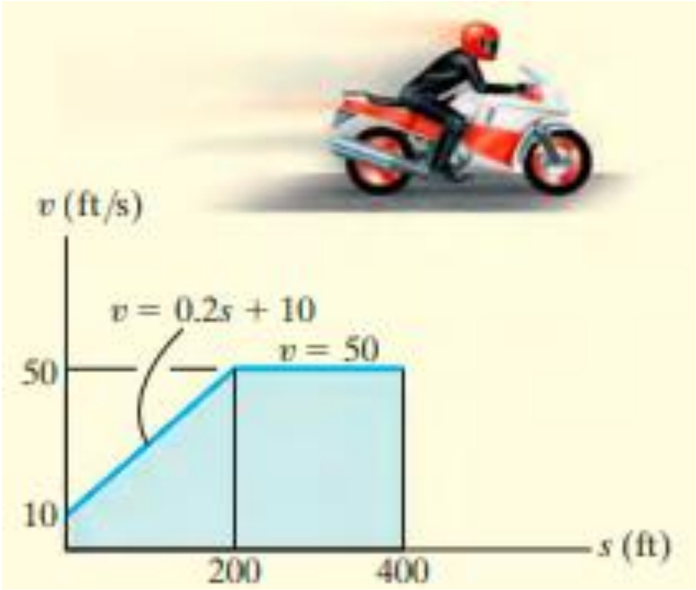
$$v = \sqrt{20s - 100}$$

at $s = 15$ m

$$v = 14.1 \text{ m/s}$$



The v–s graph describing the motion of a motorcycle is shown in Fig. Construct the a–s graph of the motion and determine the time needed for the motorcycle to reach the position s = 400 ft.



The v–s graph describing the motion of a motorcycle is shown in Fig. Construct the a–s graph of the motion and determine the time needed for the motorcycle to reach the position s = 400 ft.

$0 \le s < 200$ ft; $\qquad v = (0.2s + 10)$ ft/s

$$a = v\frac{dv}{ds} = (0.2s + 10)\frac{d}{ds}(0.2s + 10) = 0.04s + 2$$

200 ft $< s \le 400$ ft; $\qquad v = 50$ ft/s

$$a = v\frac{dv}{ds} = (50)\frac{d}{ds}(50) = 0$$

$0 \le s < 200$ ft; $\qquad v = (0.2s + 10)$ ft/s; $\qquad dt = \frac{ds}{v} = \frac{ds}{0.2s + 10}$

$$\int_0^t dt = \int_0^s \frac{ds}{0.2s + 10}$$

∫ 1/(ax+b) dx = 1/a ln|ax+b|

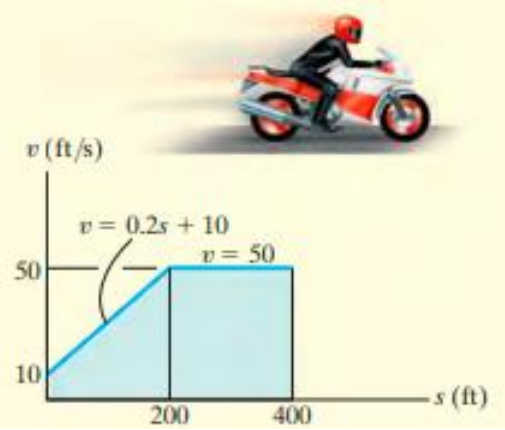
$$t = (5\ln(0.2s + 10) - 5\ln 10)\ \text{s}$$

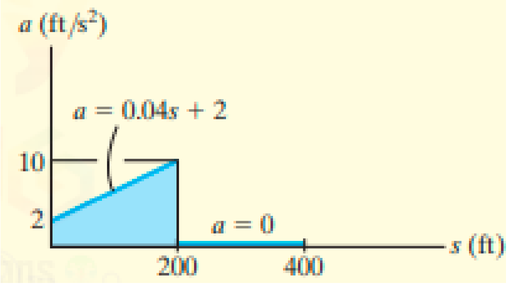
At $s = 200$ ft, $t = 5\ln[0.2(200) + 10] - 5\ln 10 = 8.05$ s. Therefore, using these initial conditions for the second segment of motion,

200 ft $< s \le 400$ ft; $\qquad v = 50$ ft/s; $\qquad dt = \frac{ds}{v} = \frac{ds}{50}$

$$\int_{8.05\ \text{s}}^{t} dt = \int_{200\ \text{m}}^{s} \frac{ds}{50};$$

$$t - 8.05 = \frac{s}{50} - 4; \quad t = \left(\frac{s}{50} + 4.05\right)\ \text{s}$$

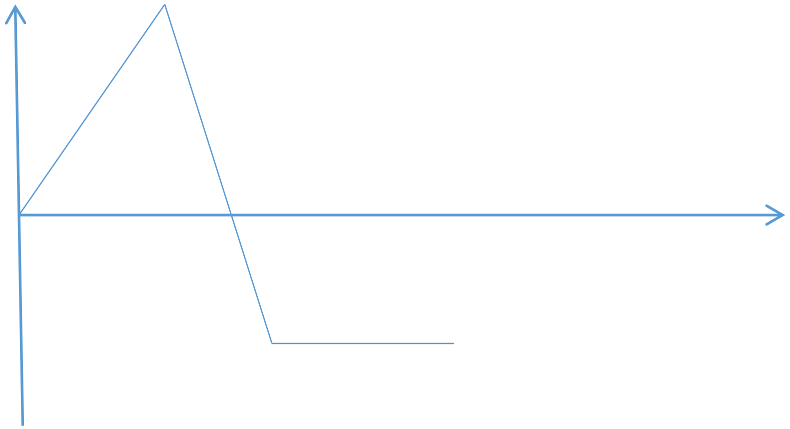




Therefore, at $s = 400$ ft,

$$t = \frac{400}{50} + 4.05 = 12.0\ \text{s}$$

V-t diagram of a particle moving in a straight line is shown. S= -25 m at t = 0, draw s-t and a-t diagram for 0<=t<=24



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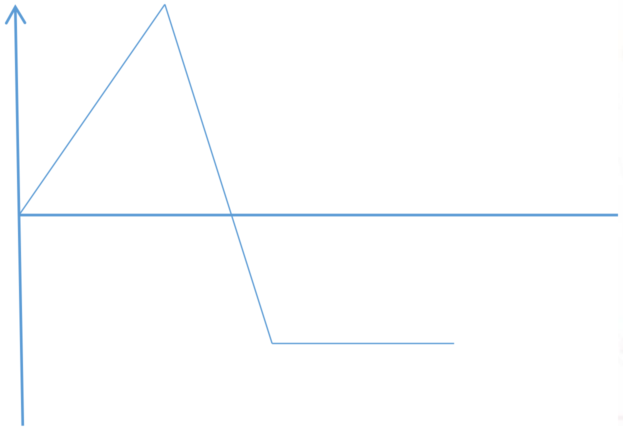
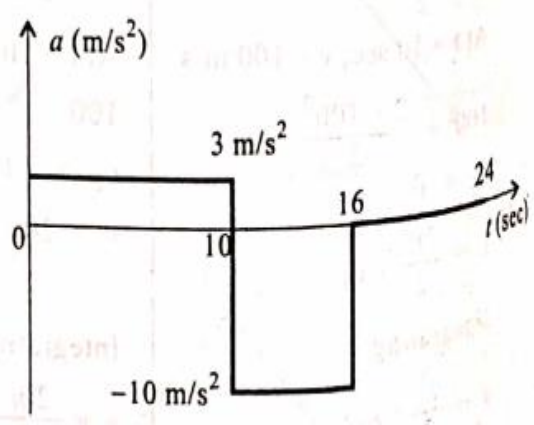
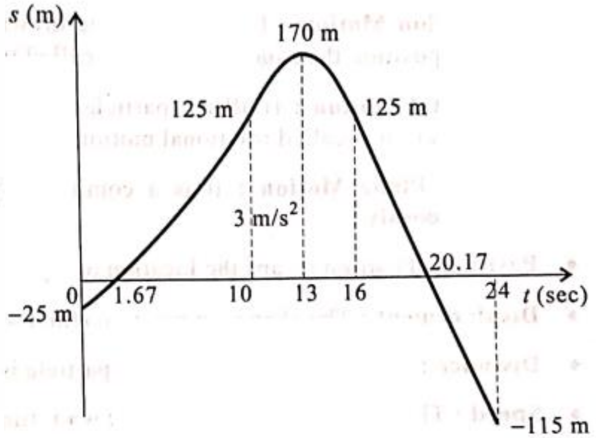
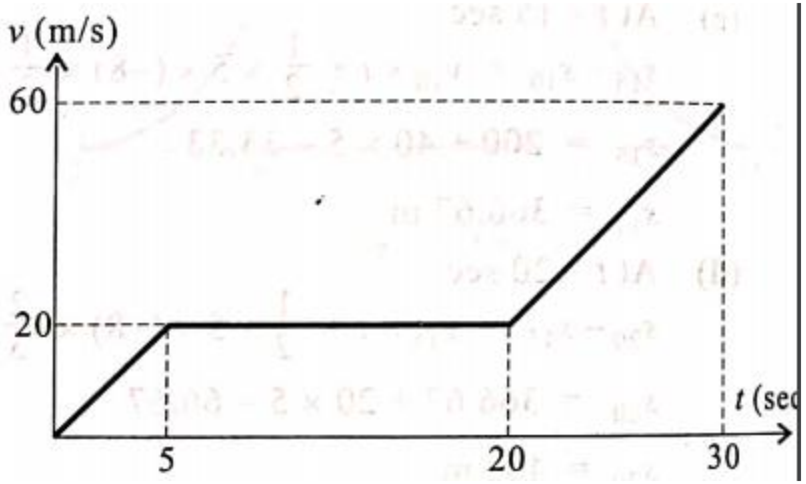






The motion of jet plane traveling along runway is defined by the v-t graph shown in figure. Construct the s-t and a-t graph for the motion. The plane starts from rest.



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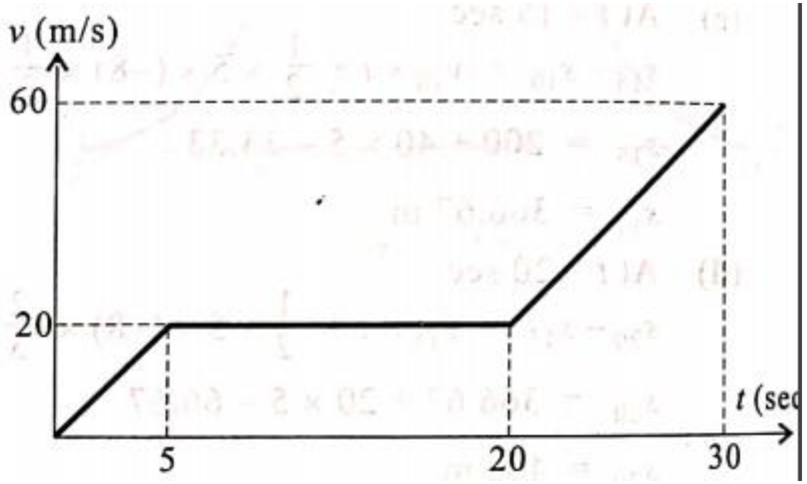
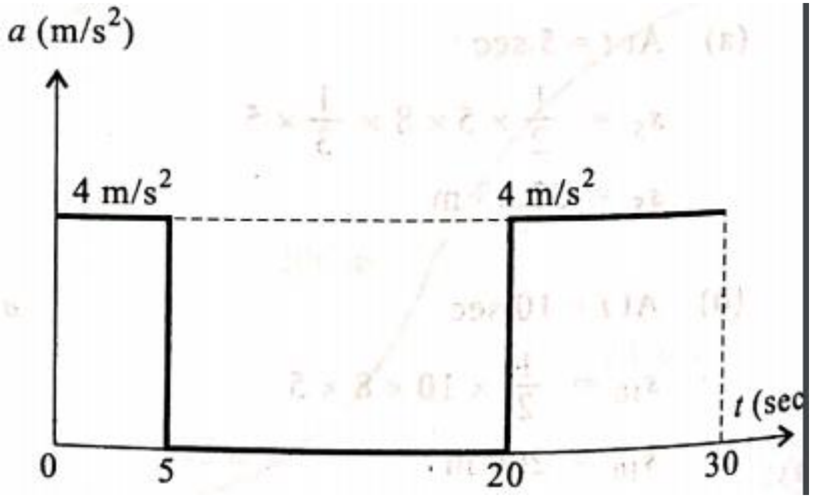
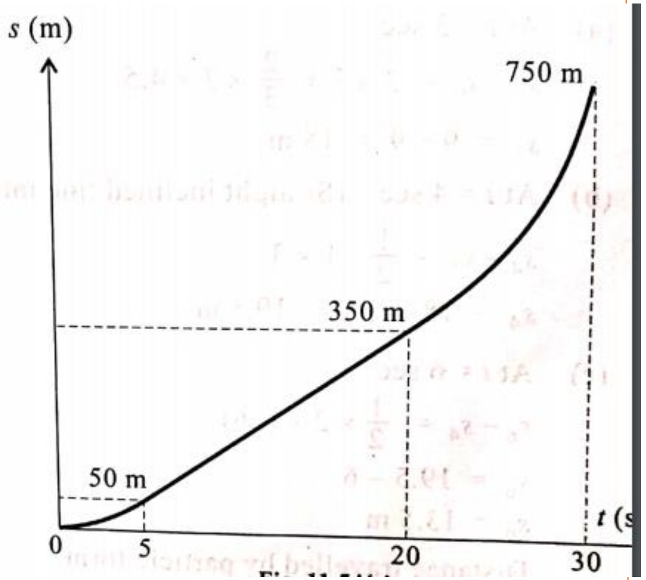




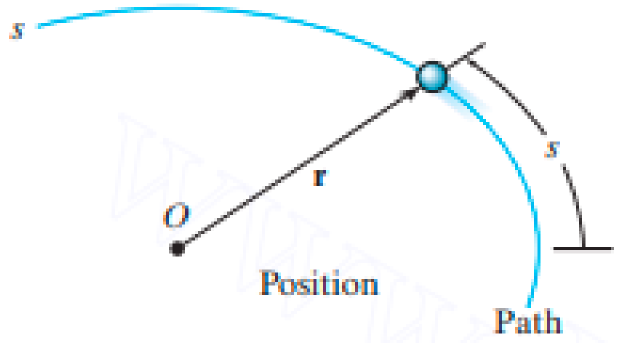


2	**Kinematics of Particles and Rigid Bodies**		**11**	**CO 2**
	2.1	Variable motion, motion curves (a-t, v-t, s-t) (acceleration curves restricted to linear acceleration only), motion along plane curved path, velocity & acceleration in terms of rectangular components, tangential & normal component of acceleration, relative velocities.		
	2.2	Introduction to general plane motion, problems based on ICR method for general plane motion of bodies (up to 2 linkage mechanism and no relative velocity method)		

General Curvilinear motion

- Curvilinear motion occurs when a particle moves along a curved path.
- Since this path is often described in three dimensions, vector analysis will be used to formulate the particle's position, velocity, and acceleration.

Position

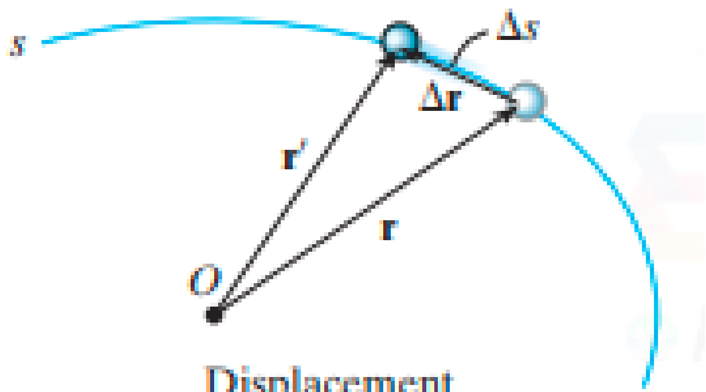


Consider a particle located at a point on a space curve defined by the path function s(t).

The position of the particle, measured from a fixed point O, will be designated by the position vector r = r(t).

Both the magnitude and direction of this vector will change as the particle moves along the curve.

Displacement



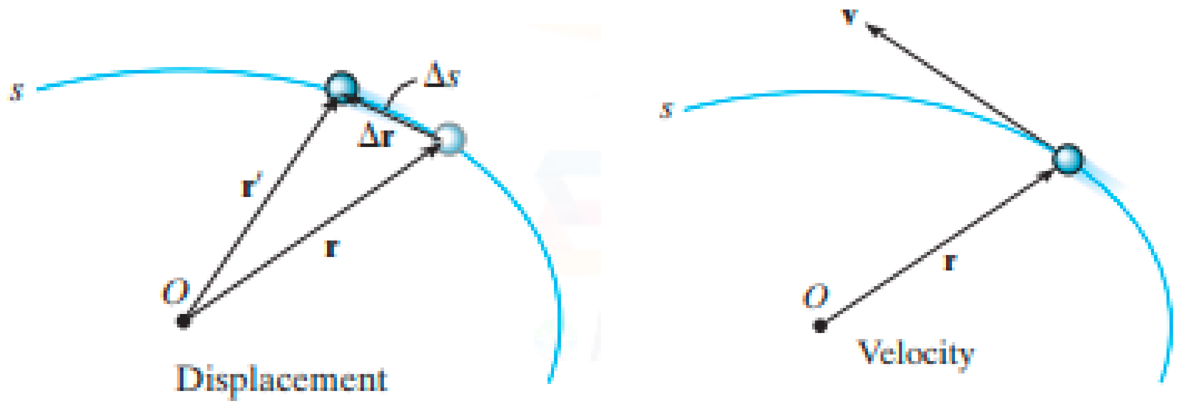
Displacement

Suppose that during a small time interval Δt the particle moves a distance Δs along the curve to a new position, defined by $r' = r + \Delta r$, The displacement Δr represents the change in the particle's position and is determined by vector subtraction; i.e. $\Delta r = r' - r$.

Velocity

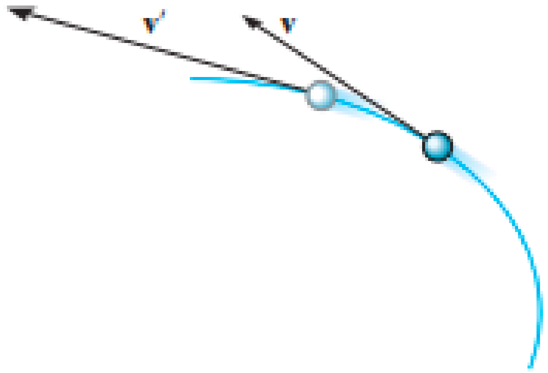
$$\mathbf{v}_{avg} = \frac{\Delta \mathbf{r}}{\Delta t}$$

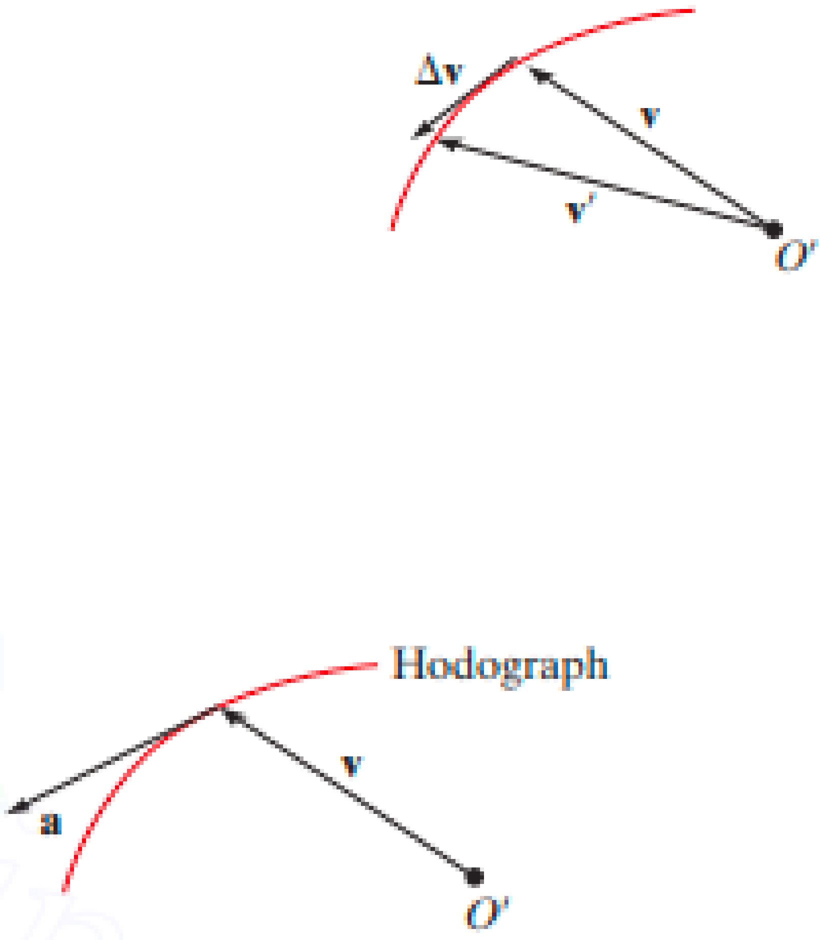
$$\mathbf{v} = \frac{d\mathbf{r}}{dt}$$



Direction is tangent to path curve. Magnitude is speed

Acceleration





$$a_{avg} = \frac{\Delta \mathbf{v}}{\Delta t}$$

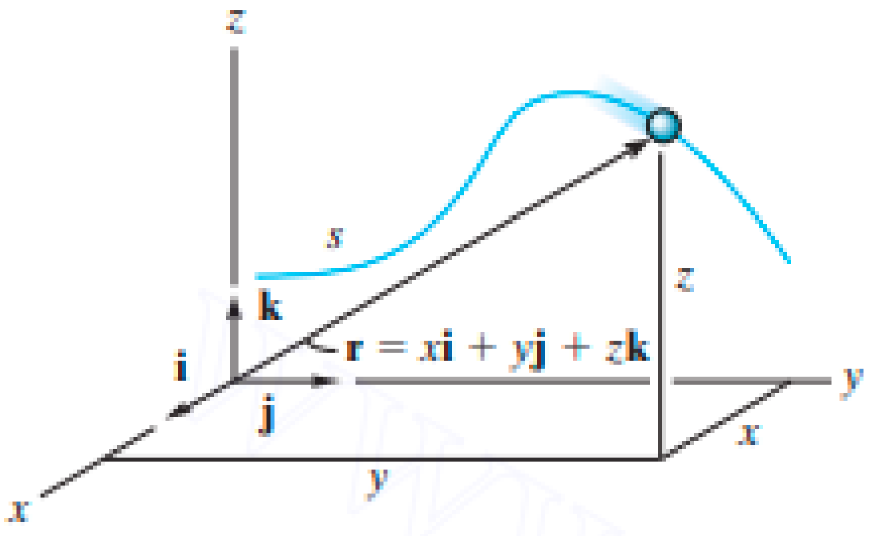
$$\mathbf{a} = \frac{d\mathbf{v}}{dt}$$

Analysis methods of curvilinear motion

- In rectilinear motion the displacement velocity and acceleration are always directed along the path of motion. In curvilinear motion it changes its direction instant to instant. Hence analysis of curvilinear motion is done using different component systems.

1. Rectangular component system
2. Normal and tangential component system

Rectangular component system



Position

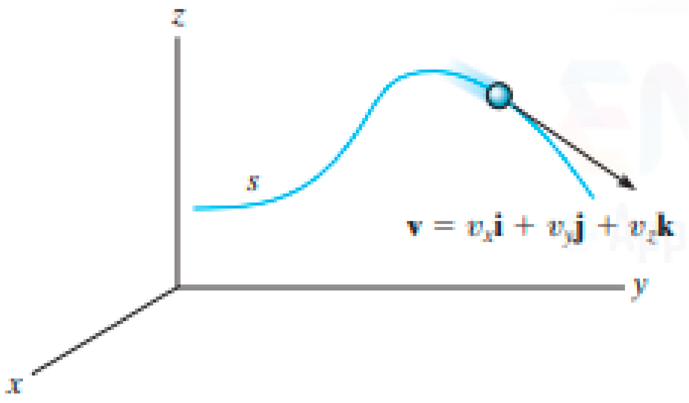
$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

And the *direction* of **r** is specified by the unit vector $\mathbf{u}_r = \mathbf{r}/r$.

Rectangular component system

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}(x\mathbf{i}) + \frac{d}{dt}(y\mathbf{j}) + \frac{d}{dt}(z\mathbf{k})$$



Velocity

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$$

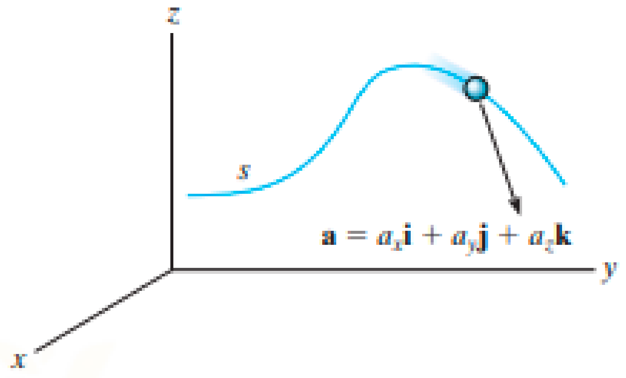
where

$$v_x = \dot{x} \quad v_y = \dot{y} \quad v_z = \dot{z}$$

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

and a *direction* that is specified by the unit vector $\mathbf{u}_v = \mathbf{v}/v$..

Rectangular component system



Acceleration

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$

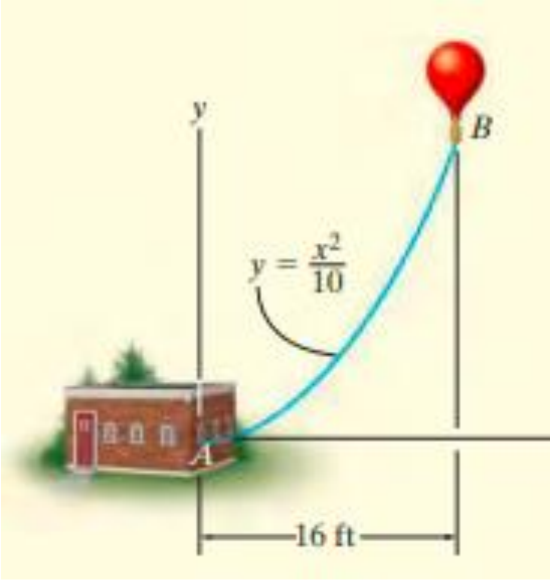
where

$$a_x = \dot{v}_x = \ddot{x}$$
$$a_y = \dot{v}_y = \ddot{y}$$
$$a_z = \dot{v}_z = \ddot{z}$$

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

and a *direction* specified by the unit vector $\mathbf{u}_a = \mathbf{a}/a$.

At any instant the horizontal position of the weather balloon in is defined by $x = (8t)$ ft, where t is in seconds. If the equation of the path is $y = x^2/10$, determine the magnitude and direction of the velocity and the acceleration when $t = 2$ s.



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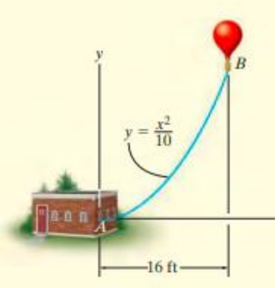


$$v_x = \dot{x} = \frac{d}{dt}(8t) = 8 \text{ ft/s} \rightarrow$$

$$v_y = \dot{y} = \frac{d}{dt}(x^2/10) = 2x\dot{x}/10 = 2(16)(8)/10 = 25.6 \text{ ft/s} \uparrow$$

When $t = 2$ s, the magnitude of velocity is therefore

$$v = \sqrt{(8 \text{ ft/s})^2 + (25.6 \text{ ft/s})^2} = 26.8 \text{ ft/s}$$

$$\theta_v = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{25.6}{8} = 72.6°$$

$$a_x = \dot{v}_x = \frac{d}{dt}(8) = 0$$

$$a_y = \dot{v}_y = \frac{d}{dt}(2x\dot{x}/10) = 2(\dot{x})\dot{x}/10 + 2x(\ddot{x})/10$$

$$= 2(8)^2/10 + 2(16)(0)/10 = 12.8 \text{ ft/s}^2 \uparrow$$

$$a = \sqrt{(0)^2 + (12.8)^2} = 12.8 \text{ ft/s}^2$$

$$\theta_a = \tan^{-1}\frac{12.8}{0} = 90°$$

For a short time, the path of the plane in is described by y = ($0.001x^2$) m. If the plane is rising with a constant upward velocity of 10 m/s, determine the magnitudes of the velocity and acceleration of the plane when it reaches an altitude of y = 100 m.

For a short time, the path of the plane in is described by $y = (0.001x^2)$ m. If the plane is rising with a constant upward velocity of 10 m/s, determine the magnitudes of the velocity and acceleration of the plane when it reaches an altitude of y = 100 m.

When $y = 100$ m, then $100 = 0.001x^2$ or $x = 316.2$ m. Also, due to constant velocity $v_y = 10$ m/s, so

$$y = v_y t; \qquad 100\text{ m} = (10\text{ m/s})\,t \qquad t = 10\text{ s}$$

$$y = 0.001x^2$$

$$v_y = \dot{y} = \frac{d}{dt}(0.001x^2) = (0.002x)\dot{x} = 0.002xv_x$$

$$10\text{ m/s} = 0.002(316.2\text{ m})(v_x)$$
$$v_x = 15.81\text{ m/s}$$

$$a_y = \dot{v}_y = (0.002\dot{x})\dot{x} + 0.002x(\ddot{x}) = 0.002(v_x^2 + xa_x)$$

When $x = 316.2$ m, $v_x = 15.81$ m/s, $\dot{v}_y = a_y = 0$,

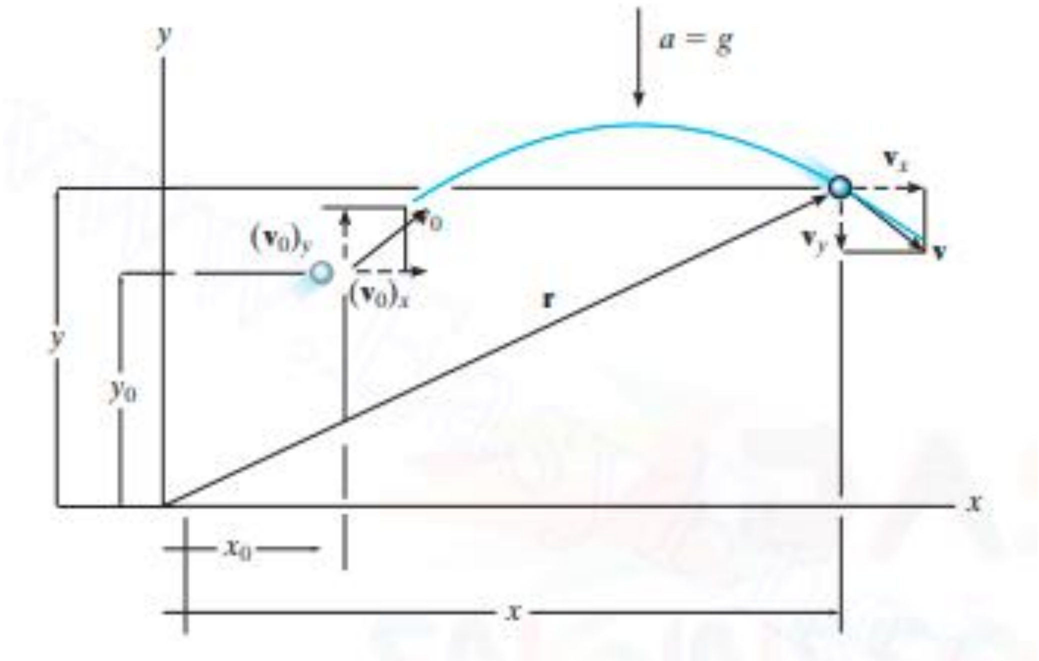
$$0 = 0.002\left[(15.81\text{ m/s})^2 + 316.2\text{ m}(a_x)\right]$$
$$a_x = -0.791\text{ m/s}^2$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-0.791\text{ m/s}^2)^2 + (0\text{ m/s}^2)^2}$$
$$= 0.791\text{ m/s}^2$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.81\text{ m/s})^2 + (10\text{ m/s})^2} = 18.7\text{ m/s}$$

Projectile motion

Kinematic analysis of projectile motion is often done in terms of its rectangular components



<u>Horizontal component</u>: (ax = 0m/s^2) a constant

$$v = v_0 + a_c t \qquad v_x = (v_0)_x$$
$$x = x_0 + v_0 t + \tfrac{1}{2} a_c t^2; \qquad x = x_0 + (v_0)_x t$$
$$v^2 = v_0^2 + 2a_c(x - x_0); \qquad v_x = (v_0)_x$$

Horizontal component of velocity remains constant.

<u>Vertical component</u> : (ay = 9.8 m/s2) a constant

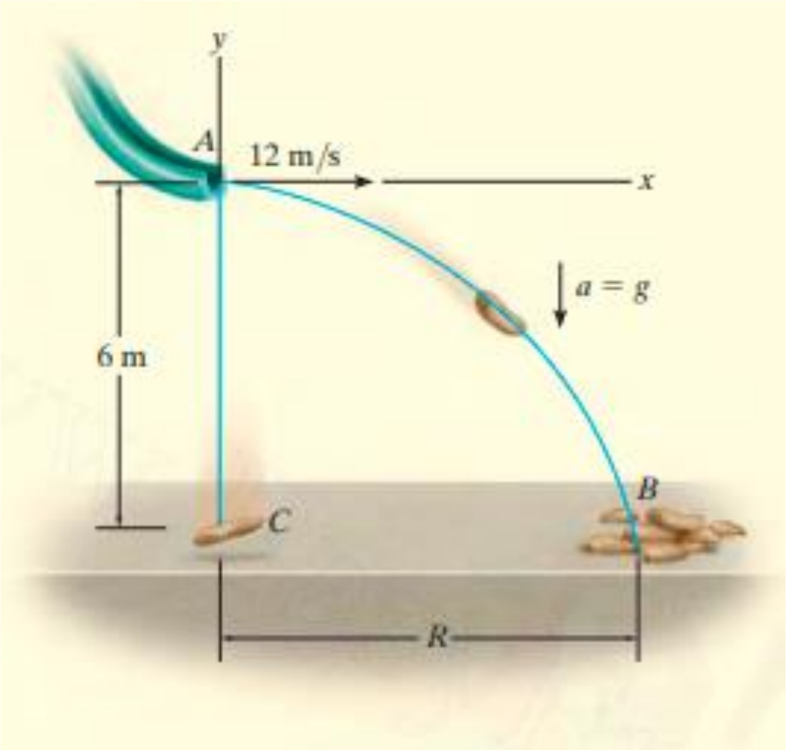
$$v = v_0 + a_c t; \qquad v_y = (v_0)_y - gt$$
$$y = y_0 + v_0 t + \tfrac{1}{2} a_c t^2; \qquad y = y_0 + (v_0)_y t - \tfrac{1}{2} g t^2$$
$$v^2 = v_0^2 + 2a_c(y - y_0); \qquad v_y^2 = (v_0)_y^2 - 2g(y - y_0)$$

Range: Distance from the point of projection to the point of target

Time of flight: Time taken by projectile to move from the point of projection to the point of target.

Maximum height : Height at which vertical component of velocity becomes zero

A sack slides off the ramp, shown in Fig. with a horizontal velocity of 12 m/s. If the height of the ramp is 6 m from the floor, determine the time needed for the sack to strike the floor and the range R where sacks begin to pile up



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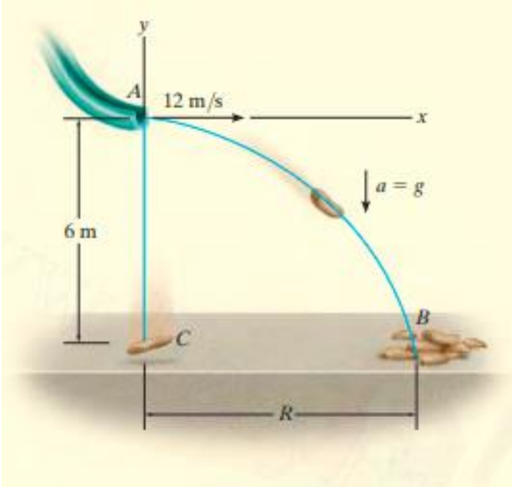


Choose the origin at A

Vertical motion:

$$y_B = y_A + (v_A)_y t_{AB} + \tfrac{1}{2} a_c t_{AB}^2$$

$$-6\text{ m} = 0 + 0 + \tfrac{1}{2}(-9.81\text{ m/s}^2)t_{AB}^2$$

$$t_{AB} = 1.11\text{ s}$$

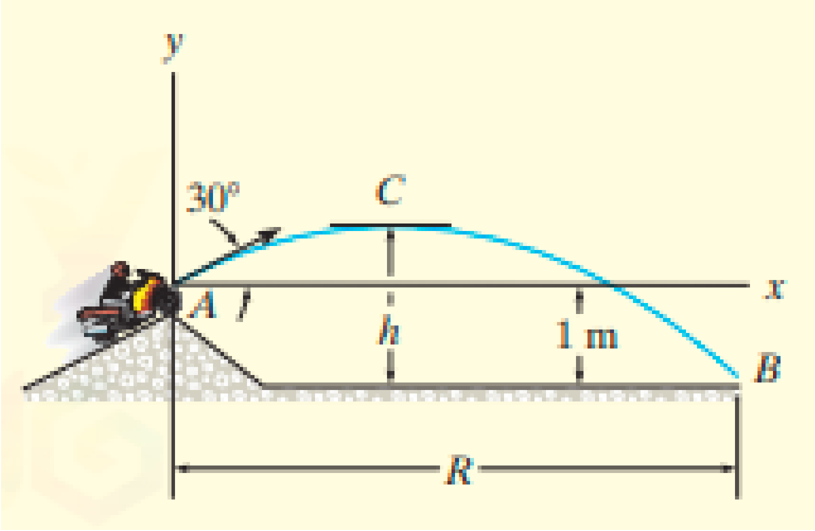
Horizontal motion:

$$x_B = x_A + (v_A)_x t_{AB}$$

$$R = 0 + 12\text{ m/s}\,(1.11\text{ s})$$

$$R = 13.3\text{ m}$$

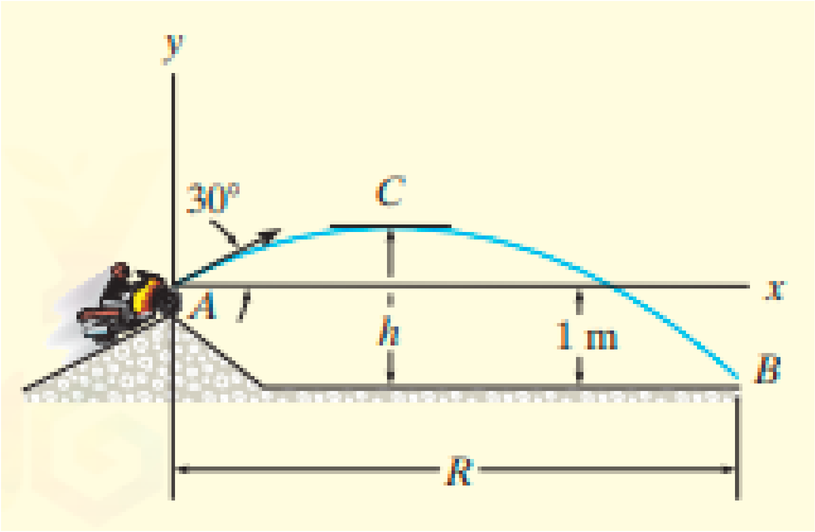
The track for a racing event was designed so that riders jump off the slope at 30°, from a height of 1 m. During a race it was observed that the rider shown in Fig. remained in mid air for 1.5 s. Determine the speed at which he was traveling off the ramp, the horizontal distance he travels before striking the ground, and the maximum height he attains. Neglect the size of the bike and rider.



The track for a racing event was designed so that riders jump off the slope at 30°, from a height of 1 m. During a race it was observed that the rider shown in Fig. remained in mid air for 1.5 s. Determine the speed at which he was traveling off the ramp, the horizontal distance he travels before striking the ground, and the maximum height he attains. Neglect the size of the bike and rider.

$$y_B = y_A + (v_A)_y t_{AB} + \tfrac{1}{2} a_c t_{AB}^2$$
$$-1\text{ m} = 0 + v_A \sin 30°(1.5\text{ s}) + \tfrac{1}{2}(-9.81\text{ m/s}^2)(1.5\text{ s})^2$$
$$v_A = 13.38\text{ m/s} = 13.4\text{ m/s}$$

$$x_B = x_A + (v_A)_x t_{AB}$$
$$R = 0 + 13.38 \cos 30°\text{ m/s}(1.5\text{ s})$$
$$= 17.4\text{ m}$$



A ball is thrown by a boy in the street is caught by another boy on a balcony 4 m above the ground and 18 m away after 2 sec as shown in fig. Calculate the initial velocity and the angle of projection.

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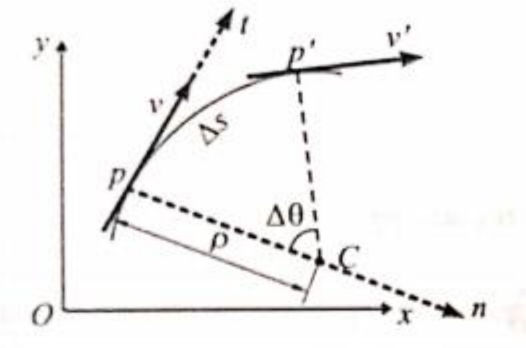
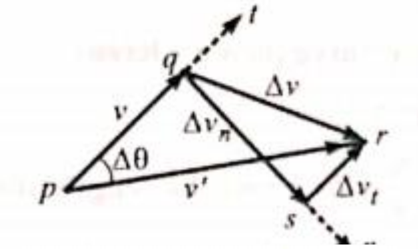
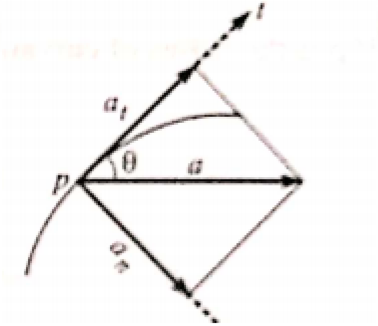
Tangential and normal component system







$$\Delta v = \overline{qr} = \overline{qs} + \overline{sr}$$

$$\Delta v = \Delta v_n + \Delta v_t$$

For acceleration, we have

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta v_t}{\Delta t} + \lim_{\Delta t \to 0} \frac{\Delta v_n}{\Delta t}$$

$$a = \frac{dv_t}{dt} + \frac{dv_n}{dt}$$

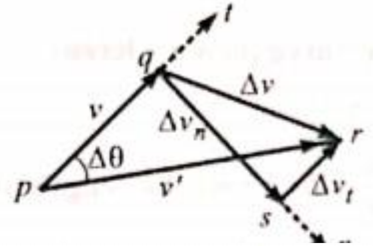
$$a = a_t + a_n$$

Magnitude of acceleration $a = \sqrt{a_t^2 + a_n^2}$

Direction $\tan\theta = \frac{a_n}{a_t}$

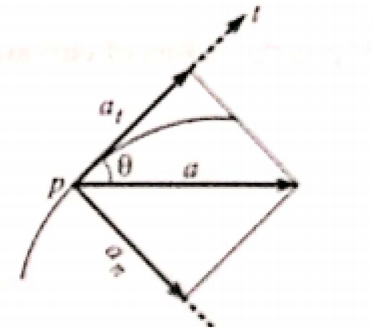
Component of Tangential Acceleration (a_t)

It can be noted that $\overline{sr}$ represents the change in the magnitude of the velocity v.

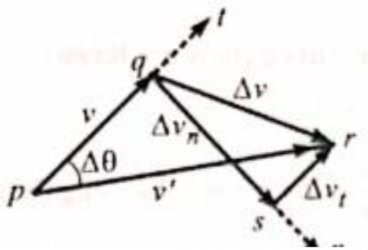


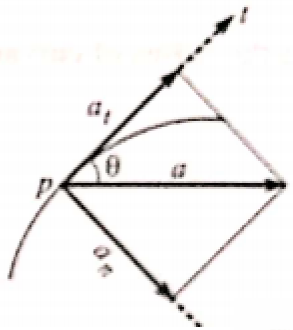
$$\therefore\ a_t = \lim_{\Delta t \to 0} \left(\frac{v' - v}{\Delta t} \right) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

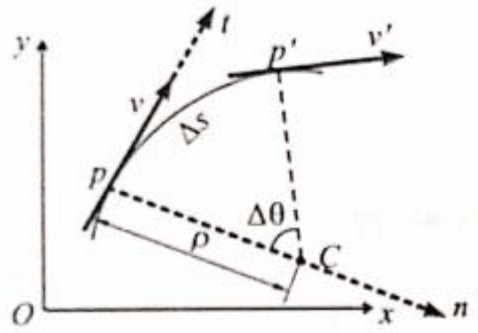
$$a_t = \frac{dv}{dt}$$



(Rate of change of speed of the particle)







Component of Normal Acceleration (a_n)

It can be noted that $\overline{qs}$ represents rate of change of direction of the velocity

$$qs \approx v\,\Delta\theta \text{ for a small change in the angle } \theta$$

$$\therefore\ \Delta v_n \approx v \cdot \Delta\theta$$

$$\therefore\ a_n = \lim_{\Delta t \to 0}\left(\frac{v \cdot \Delta\theta}{\Delta t}\right)$$

If ρ is the radius of curvature of the curve then we have

$$\Delta s = \rho\,\Delta d\theta$$

$$\Delta\theta = \frac{\Delta s}{\rho}, \ \Delta s \text{ being the length of the arc } pp'$$

$$\therefore\ a_n = \lim_{\Delta t \to 0}\left(\frac{v \cdot \Delta\theta}{\Delta t}\right) = \lim_{\Delta t \to 0}\frac{\Delta s}{\Delta t}\cdot\frac{v}{\rho}$$

$$\therefore\ a_n = \frac{v}{\rho}\cdot\frac{ds}{dt} \qquad \text{But } \frac{ds}{dt} = v$$

$$\therefore\ a_n = \frac{v^2}{\rho}$$

Component of normal acceleration is always directed towards the centre of curvature of the path. It is also called the *centripetal acceleration* (a_n).

The net acceleration of particle in vector form can be expressed as

$$a = a_t + a_n$$

where $a_t = \frac{dv}{dt}$ is responsible for changing the magnitude of speed and $a_n = \frac{v^2}{\rho}$ is responsible for changing the direction.

Magnitude of net acceleration $a = \sqrt{a_t^2 + a_n^2}$

and the direction $\tan\theta = \frac{a_n}{a_t}$

If particle is moving along curved path with uniform speed then component of tangential acceleration

$$a_t = \frac{dv}{dt} = 0$$

$\therefore$ Net acceleration $a = a_n = \frac{v^2}{\rho}$

If a_t is constant,

$$v = u + a_t t$$

$$s = ut + \frac{1}{2} a_t t^2$$

$$v^2 = u^2 + 2a_t s$$

where s is the distance covered along curved path.
u is initial speed and v is final speed and
a_t is the component of acceleration along tangential direction.

If equation of curve is given by $y = f(x)$ then at point $p(x, y)$ radius of curvature is calculated b the following relation:

$$\rho = \left| \frac{\left[1 + \left(\frac{dy}{dt}\right)^2\right]^{3/2}}{\frac{d^2y}{dt^2}} \right|$$

If data is given in rectangular components form then radius of curvature is calculated b following relation:

$$\rho = \left| \frac{(v_x^2 + v_y^2)^{3/2}}{v_x a_y - v_y a_x} \right|$$

moving in space curve then radius of curvature is calculated by following relatio

$$|\bar{v} \times \bar{a}| = \frac{v^3}{\rho}$$

Components of velocity along x and y axis is given by

$$v_x = v\cos\theta\ ,\quad v_y = v\sin\theta$$

$$\tan\theta = \frac{dy}{dx}$$

and component of acceleration along x and y axis are given by

$$a_x = a\cos(\theta + \alpha)$$

$$a_y = a\sin(\theta + \alpha)$$

$$\tan\alpha = \frac{a_n}{a_t}$$

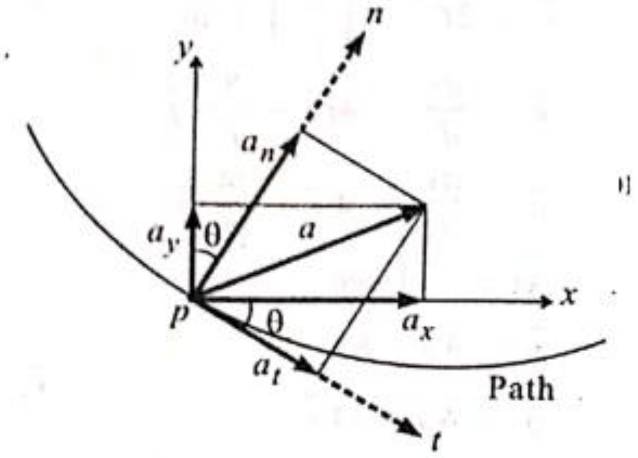
Relationship between rectangular components and tangential and normal components of acceleration.

$$a_x = a_n\sin\theta + a_t\cos\theta$$

$$a_y = a_n\cos\theta + a_t\sin\theta$$

$$a_n = a_x\sin\theta + a_y\cos\theta$$

$$a_t = a_x\cos\theta - a_y\sin\theta$$



Reference used for ppt:

Engineering Mechanics by R.C. Hibbeler

Engineering Mechanics by NH Dubey