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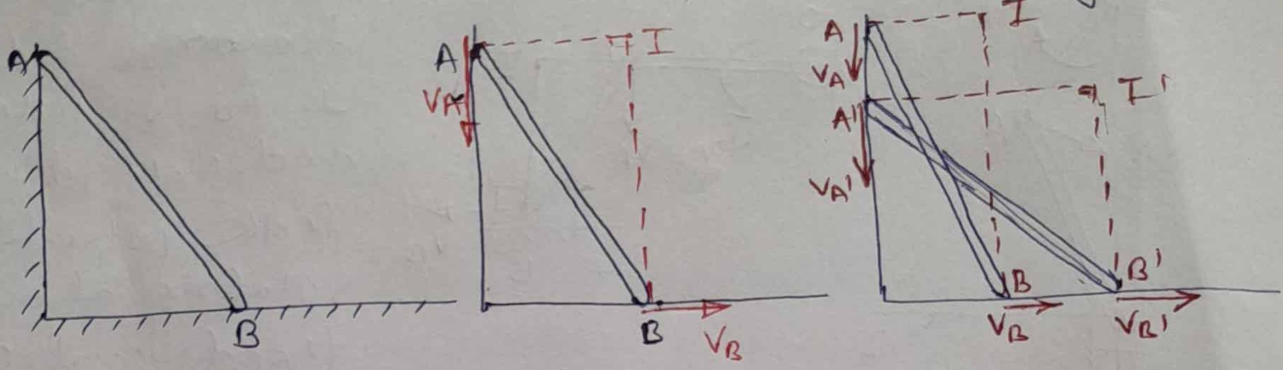
Instantaneous Centre of Rotation

General plane motion: It is the combination of translation motion and rotational motion together.

eg:

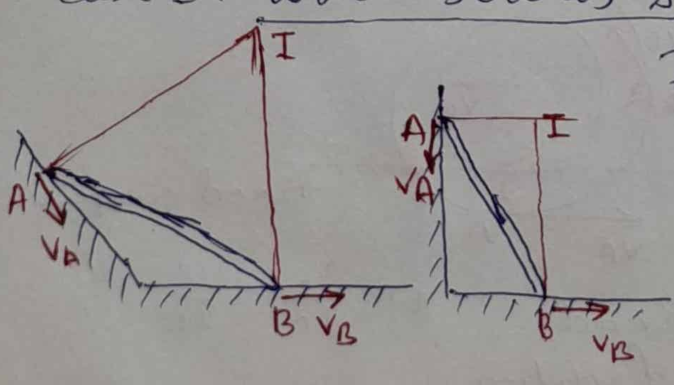


General plane motion can be converted to pure rotation about an arbitrary point called instantaneous centre of rotation

Note:
* ICR is a point of zero velocity
* ICR may lie within the body or outside the body
* ICR changes from instant to instant and not a fixed point.
* To locate ICR, directions of velocities of any two points in the rigid body are sufficient.
* ICR is a imaginary point.

Techniques used for solving ICR problems

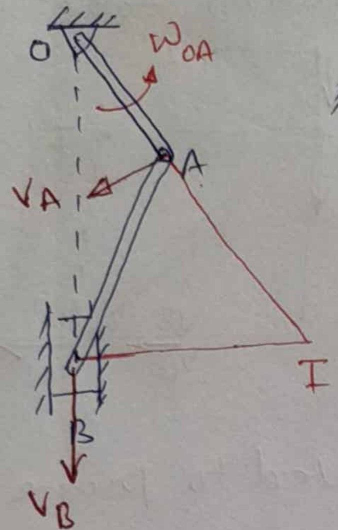
case I: When bodies slides on two surfaces

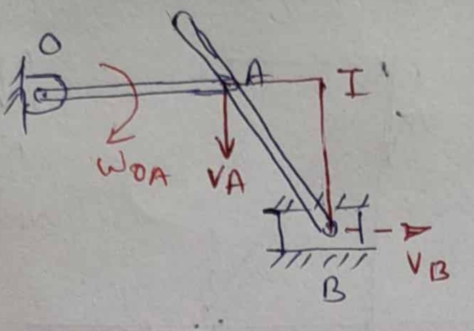


ICR is located by drawing lines ⊥ to plane on which two points slides.

$$\omega = \frac{V_A}{IA} = \frac{V_B}{IB}$$

Case II: When one part of body slides and other part

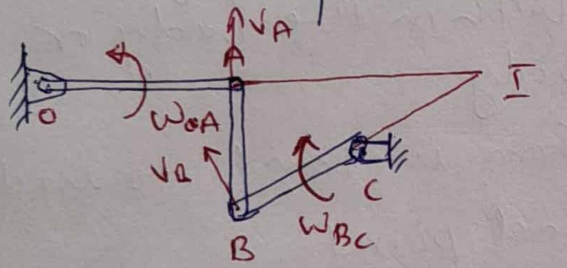


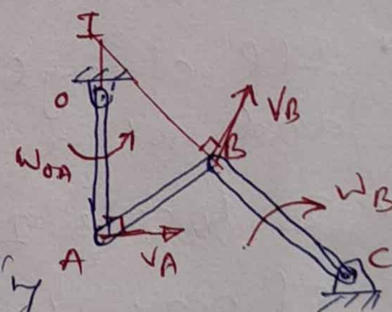


ICR is located by drawing a line ⊥ to sliding surface and extending the link which is rotating about a fixed centre 'O'

$$\omega = \frac{V_A}{I_A} = \frac{V_B}{I_B}$$

Case III: When two links of system rotate about two separate hinged points.

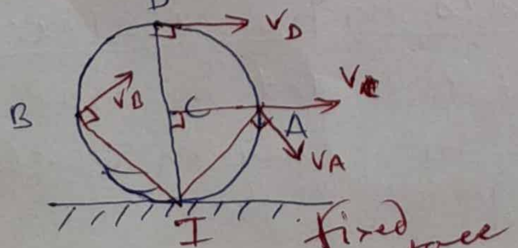


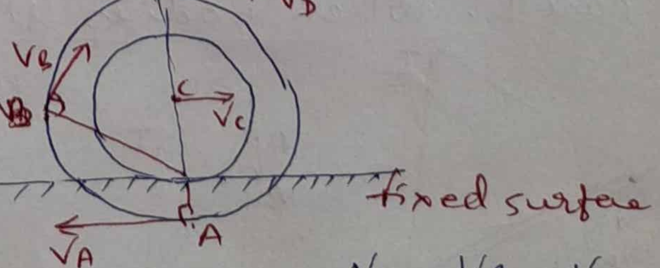


ICR is located by extending two links

$$\omega = \frac{V_A}{I_A} = \frac{V_B}{I_B}$$

Case IV: When body rolls on fixed surface

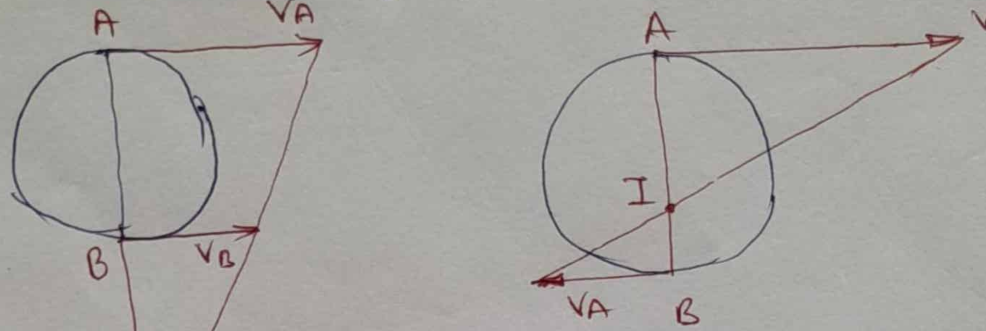




$$\omega = \frac{V_A}{I_A} = \frac{V_B}{I_B} = \frac{V_C}{I_C} = \frac{V_D}{I_D}$$

Point of contact with fixed surface becomes inst. centre of rotation.

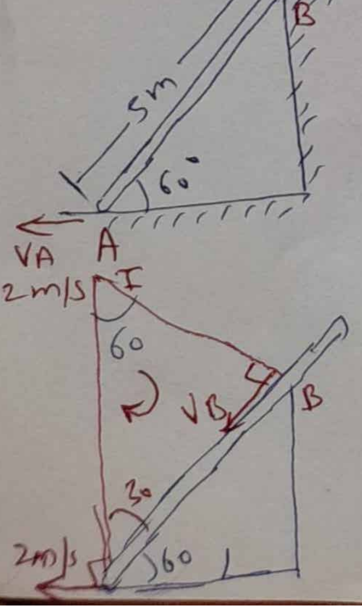
Case V: When body lies betn two moving surfaces



Here pt A & B are in contact with two moving surfaces which are moving with velocities V_A & V_B. In such cases ICR lies on the line AB or extension of line AB and on the line joining tips of velocity vector A & B

$$\omega = \frac{V_A}{IA} = \frac{V_B}{IB}$$

eg: Velocity of point on the rod is 2m/s at the instant shown in fig. Locate the instantaneous centre of rotation and determine the velocity of pt B on the rod.



Given: $V_A = 2m/s$

$$\omega = \frac{2}{IA} = \frac{V_B}{IB} \quad \text{--- (1)}$$

From ΔIAB, $\cos 30 = \frac{5}{AI}$ ∴ $AI = 5.7735\ m$

$\sin 30 = \frac{BI}{AI} \Rightarrow BI = 2.886\ m$

sub. the values in eqn (1)

$$\omega = \frac{2}{5.7735} = \frac{V_B}{2.886} \Rightarrow \omega = 0.346\ rad/s$$

$V_B = 1\ m/s$ ans

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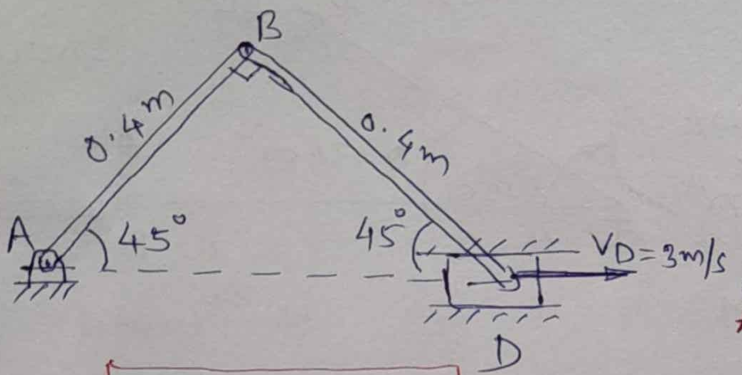
(2) Block D shown in fig moves with a speed of 3 m/s. Determine the angular velocity of link BD and AB and velocity of point B at given instant shown. Take length of AB = BD = 0.4 m.



Given figure

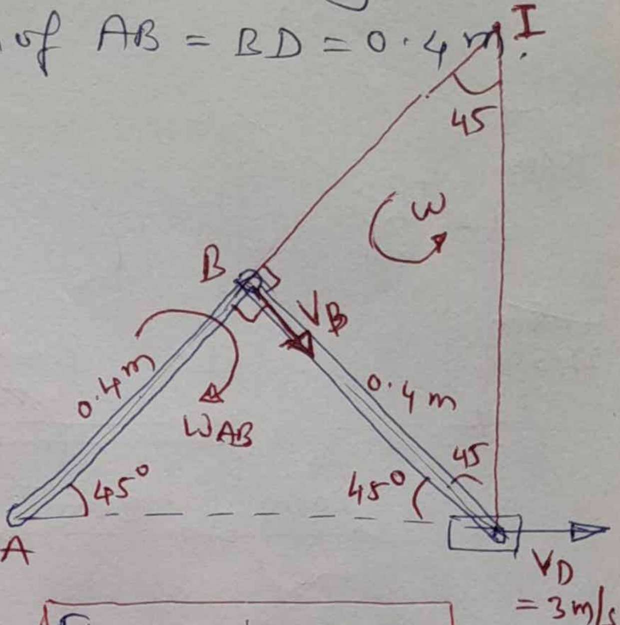


Fig showing ICR

Soln: ICR is located by drawing ⊥ slider (D) and extension of link AB

$$V_B = AB \times W_{AB} = 0.4\,\omega_{AB}$$

$$V_B = 0.4\,W_{AB} = IB \times \omega \quad \text{—— (I)}$$

$$V_D = ID \times \omega$$

$$3 = ID \times \omega \quad \text{—— (II)}$$

From fig; △ABD & △DBI (similar triangles)

with $AB = BI = 0.4$ $\quad AD = ID$

$$AD = \sqrt{0.4^2 + 0.4^2} = 0.5659\ m$$

$$\therefore ID = 0.5659\ m$$

sub ID in eqn (II) $3 = 0.5659 \times \omega$

$\omega = 5.30$ rad/s (↻)

from (I) $0.4\,W_{AB} = 0.4 \times 0.5659$

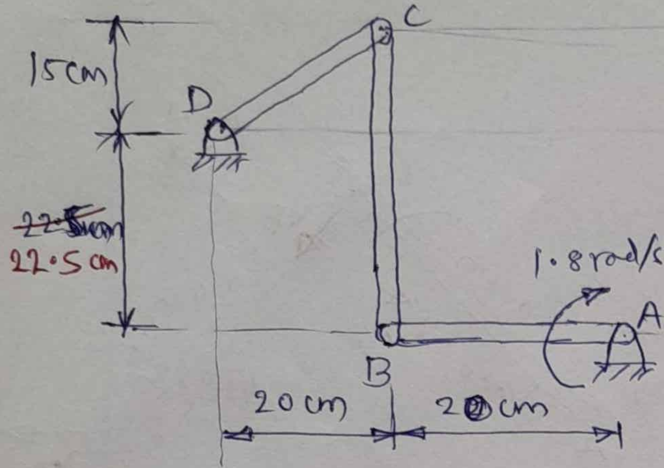
$\therefore W_{AB} = 5.3$ rad/sec (↺)

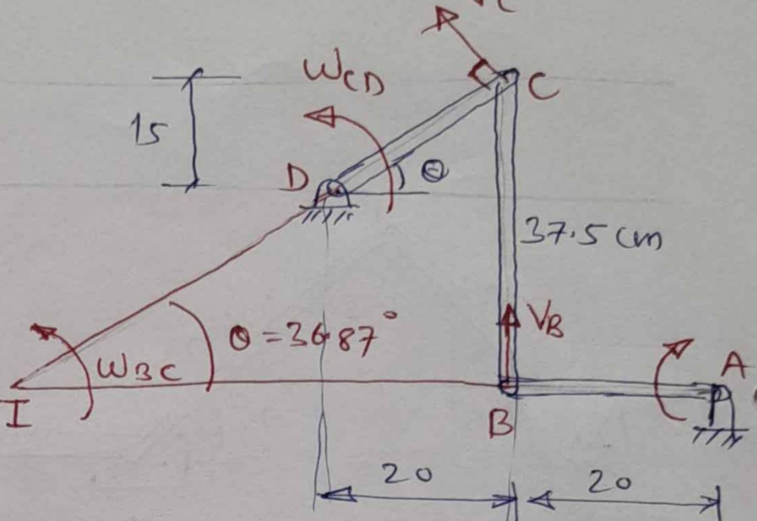
$\therefore V_B = 5.3 \times 0.4$

$= 2.12$ m/s

↘45°

③ In the position shown in fig, the rod AB is horizontal and has angular velocity 1.8 rad/sec in clockwise direction. Determine the angular velocities of BC and CD.





Soln: $\tan\theta = \frac{15}{20} \Rightarrow \theta = 36.87°$

$\tan 36.87 = \frac{BC}{IB} = \frac{37.5}{IB}$

$\therefore IB = 50$ cm

$IC = \sqrt{IB^2 + BC^2} = 62.5$ cm

Rod AB: (Performs rotational motion about A)

$V_B = (AB)(\omega_{AB}) = 20 \times 1.8 = 36$ cm/sec.

Rod BC: (Perform General plane motion)

At given instant point I is ICR

$V_B = (IB)(\omega_{BC})$

$36 = 50 \times \omega_{BC}$

$\therefore \omega_{BC} = 0.72$ rad/sec (↺)

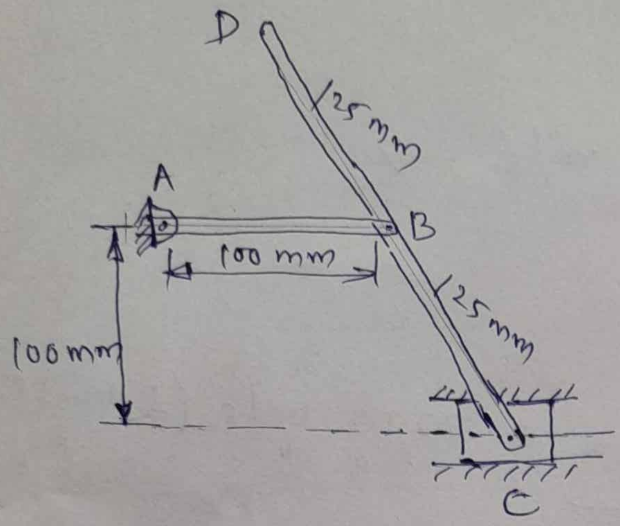
$V_C = IC \times \omega_{BC} = 62.5 \times 0.72$

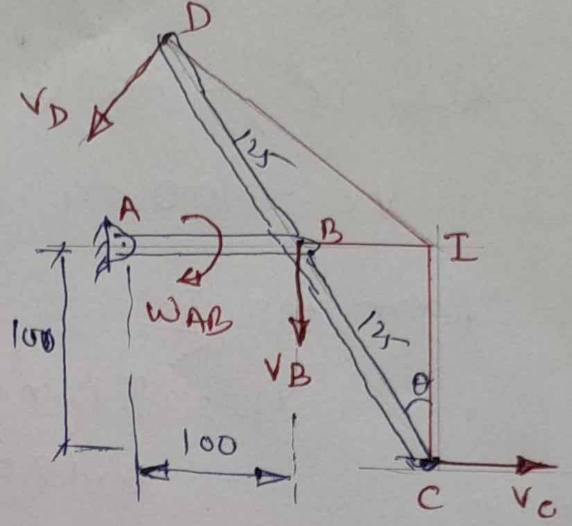
$V_C = 45$ cm/sec

RoD CD: $V_C = CD \times \omega_{CD}$

$45 = 25 \times \omega_{CD} \Rightarrow \omega_{CD} = 1.8$ rad/sec (↺)

(4) At position shown in fig, the crank AB has an angular velocity of 3 rad/sec clockwise. Find the velocity of slider C and point D at this moment.





Soln :-

(Crank AB) : Performing rotational motion about A

$V_B = AB \times W_{AB} = 0.1 \times 3 = 0.3\ m/s$ (↓)

Also $V_B = 0.3 = IB \times \omega$ —— (I)

$V_C = IC \times \omega = 0.1 \times \omega$ —— (II)

Join pt I and D

$\therefore V_D = ID \times \omega$ —— (III)

Find IB & ID

From Δ IBC, we have

$\cos\theta = IC/BC = 100/125$ $\theta = 36.87°$

$\sin\theta = IB/BC =$

$\sin 36.87° = \frac{IB}{125}$ $\therefore IB = 75\ mm = 0.075\ m$

Applying cosine Rule to Δ ICD

$ID^2 = IC^2 + CD^2 - 2(IC)(CD)\cos\theta$

$= 100^2 + 250^2 = 2(100)(250)\cos 36.870$

$= 180.278\ mm$

$= 0.18027\ mm$

Sub IB in eqn (I)

$0.3 = IB \times \omega = 0.075 \times \omega$

$\therefore \omega = 4$ rad/sec

Sub IC & ω in eqn (II)

$V_C = 0.100 \times 4 = 0.4$ m/s →

from eqn III

$V_D = ID \times \omega$

$= 0.1803 \times 4 = 0.7212$ m/s (⊥ to ID)

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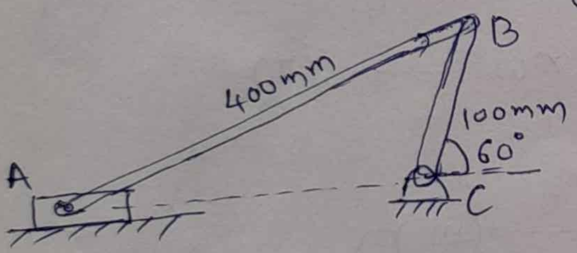
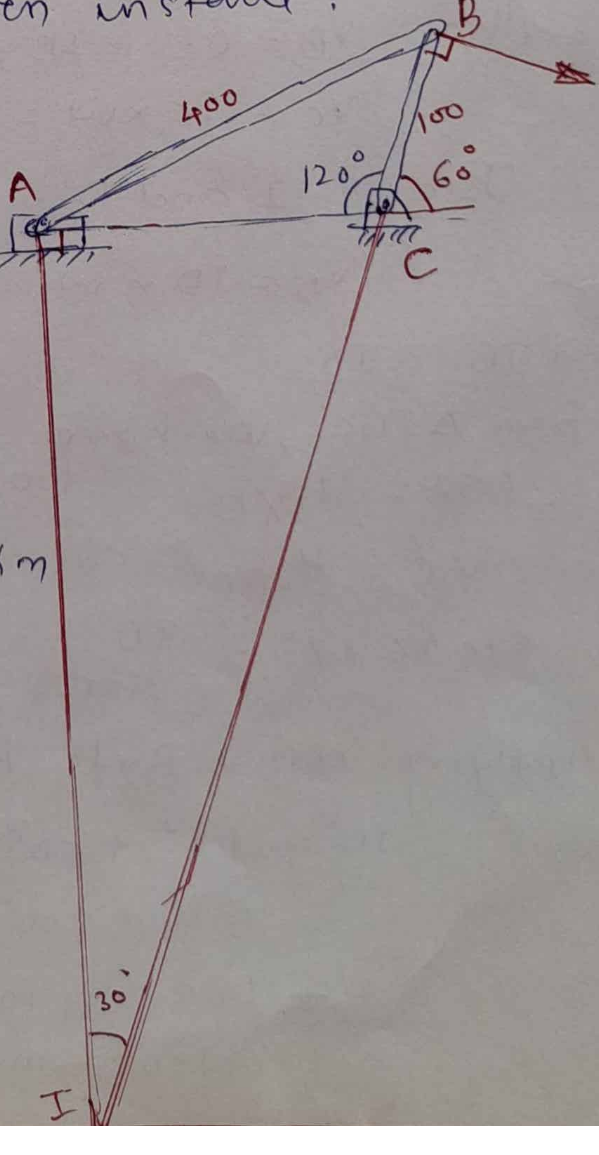
(5) The crank BC of a slider crank mechanism is rotating at constant speed of 30 rpm clockwise. Determine the velocity of the cross head A at the given instant.





Solution:

Crank BC:

performs rotational motion about pt C

$V_B = (BC) \times \omega_{BC}$

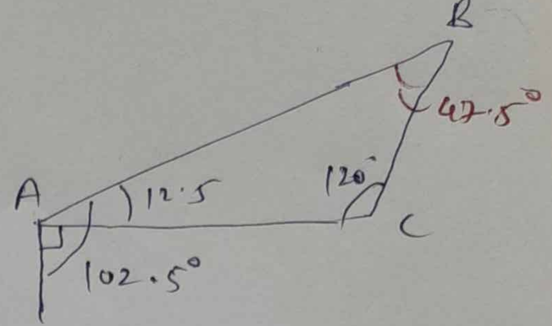
$= 100 \times \left[30 \times \frac{2\pi}{60}\right]$

$= 100\pi$ mm/sec

$= 314.15$ mm/s.

$= 0.314$ m/s.

In Δ ABC, by sine rule



$$\frac{400}{\sin 120} = \frac{100}{\sin\theta}$$

$\therefore \theta = 12.5°$

In Δ ABI by sine rule

$$\frac{IA}{\sin 47.5} = \frac{400}{\sin 30} = \frac{IB}{\sin 102.5}$$

$\therefore IA = 589.82$ mm

$IB = 781.04$ mm

<u>Link AB</u> (Perform general plane motion)

At given instant pt I is the ICR

$$V_B = IB \times (\omega_{AB})$$

$$\omega_{AB} = \frac{V_B}{I_B}$$

$$= \frac{314.15}{781.04}$$

$\omega_{AB} = 0.402$ rad/sec (Q)

$V_A = IA\ (\omega_{AB})$

$= 589.82 \times 0.402$

$= 237.12$ mm/sec