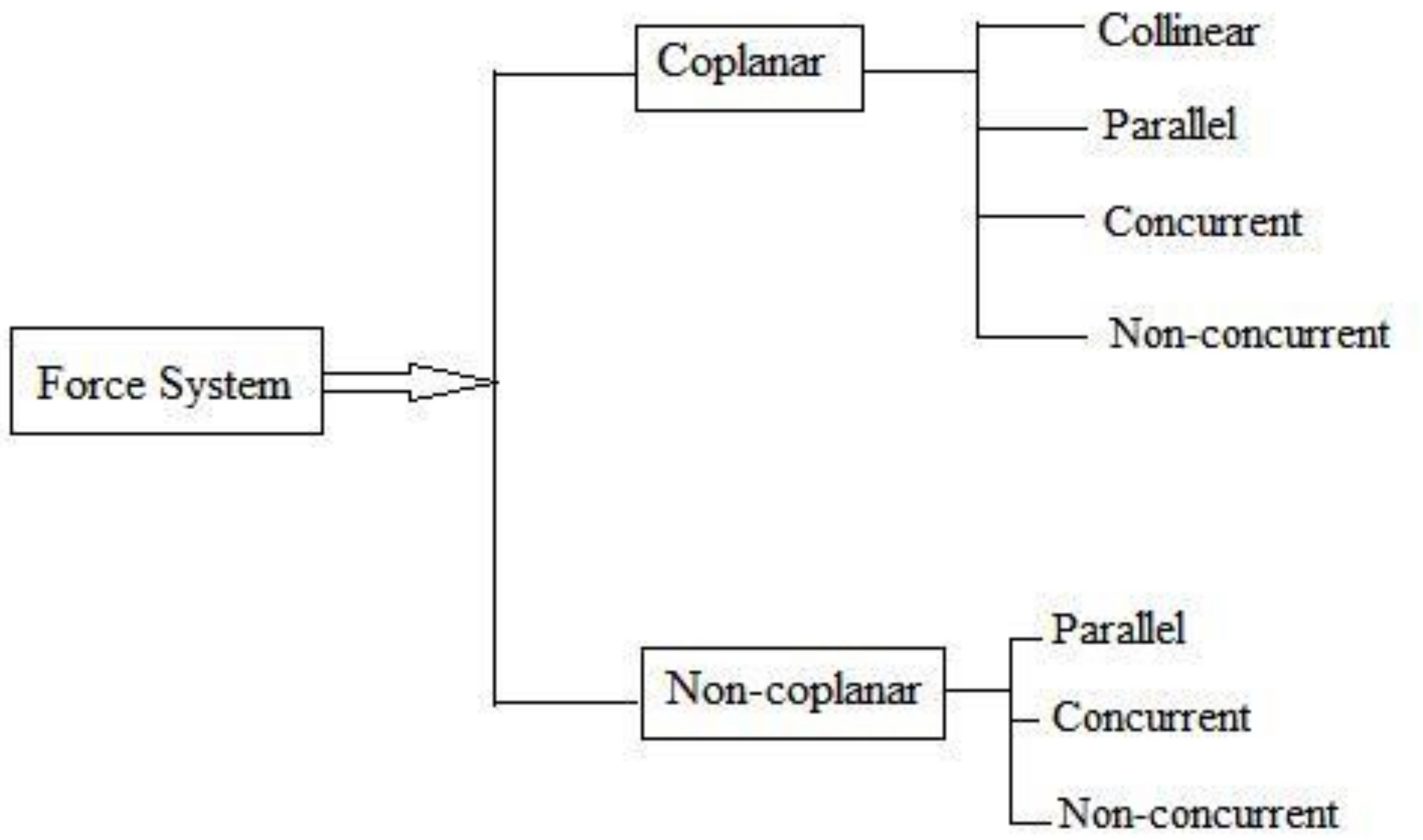
K J SOMAIYA COLLEGE OF ENGINEERING, MUMBAI-77
(CONSTITUENT COLLEGE OF SOMAIYA VIDYAVIHAR UNIVERSITY)

Module 1.2 – Forces in Space

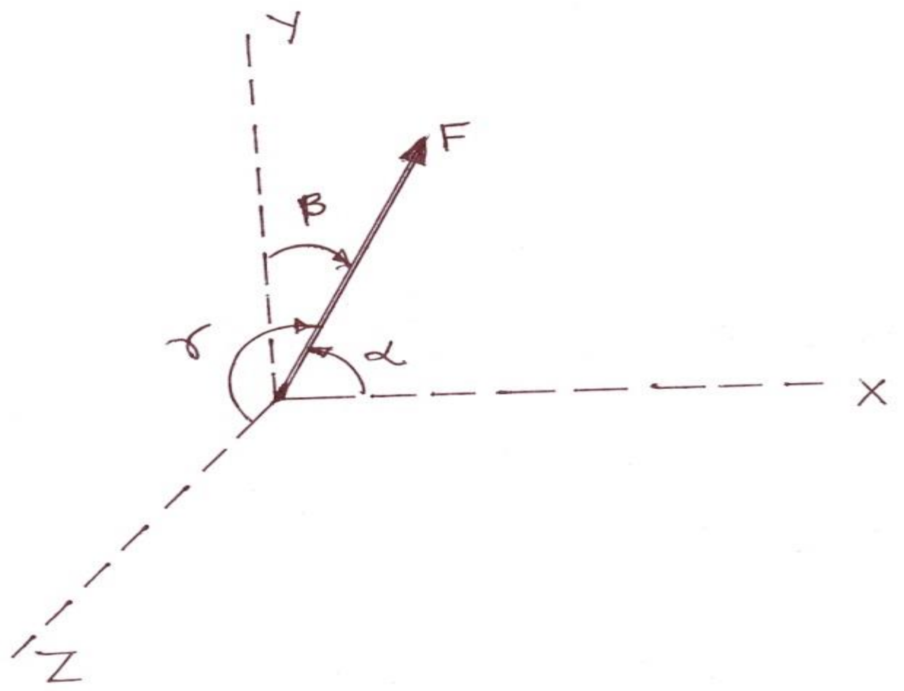
Presented by: Prof. Abhishek P. S. Bhadauria

Classification of system of forces



A Force in space: The force system which is acting in different planes is called as non-coplanar force system or space forces.

A Force is said to be in space if its line of action makes an angle α, β and γ with respect to rectangular co-ordinate axes X, Y and Z respectively as shown the Fig.

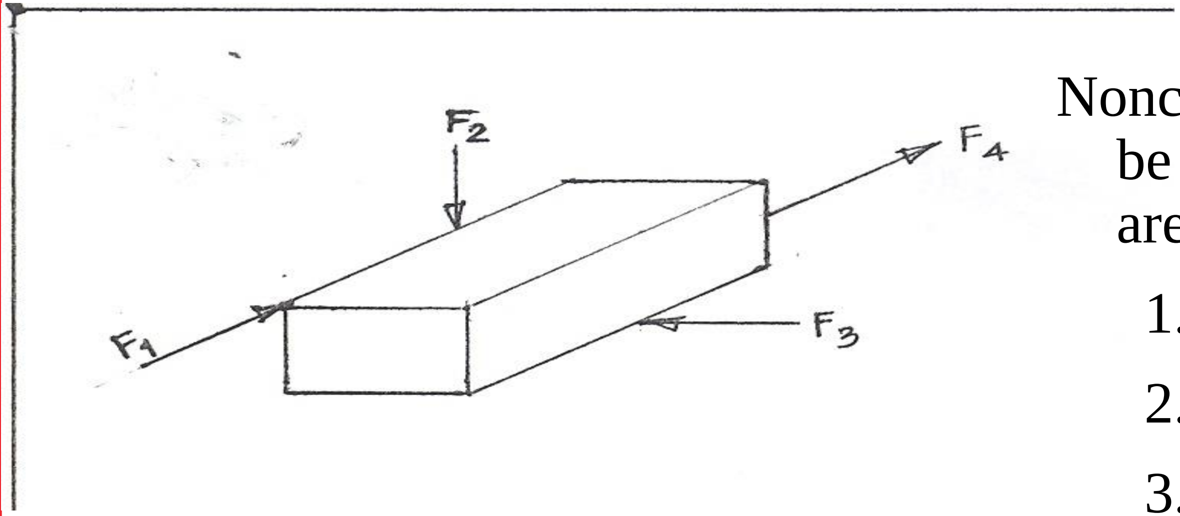


Forces in space

Noncoplanar system of forces (Forces in Space) and Their Classifications

System of forces which do not lie in a single plane is called non-coplanar system of forces(Forces in space).

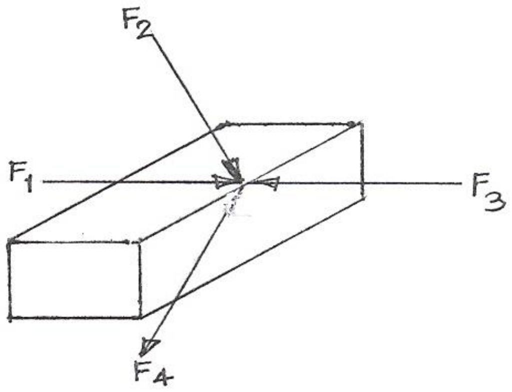
A typical noncoplanar system of forces (forces in space) is shown in the Fig. below

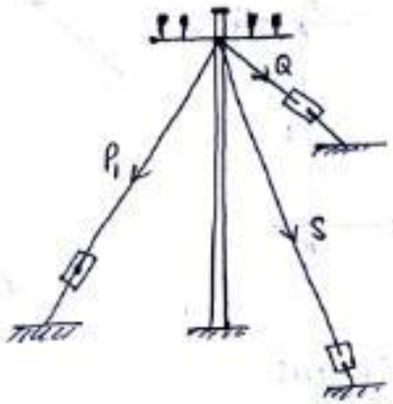


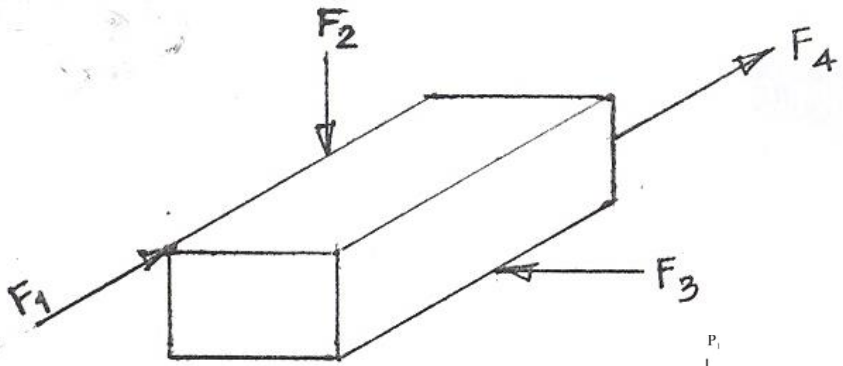
Noncoplanar system of forces(Forces in space) can be broadly classified into three categories. They are

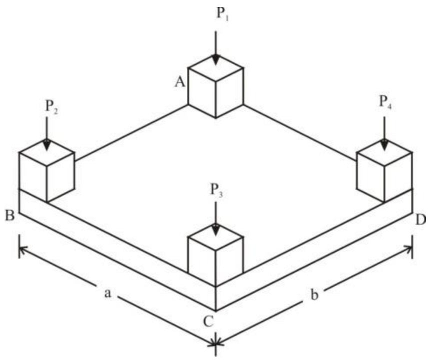
1. Concurrent noncoplanar system of forces
2. Non-concurrent noncoplanar system of forces
3. Noncoplanar parallel system of forces

1. Concurrent noncoplanar system of forces: Forces which meet at a point with their lines of action do not lie in a plane are called "Concurrent noncoplanar system of forces". A typical system of Concurrent noncoplanar system of forces is shown in the Fig.





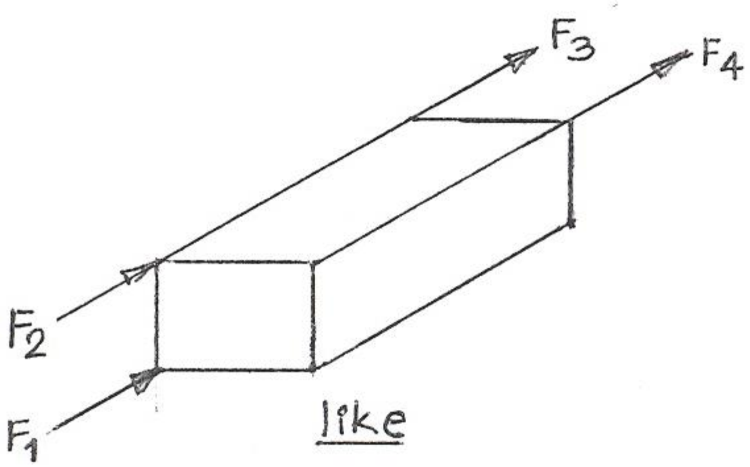


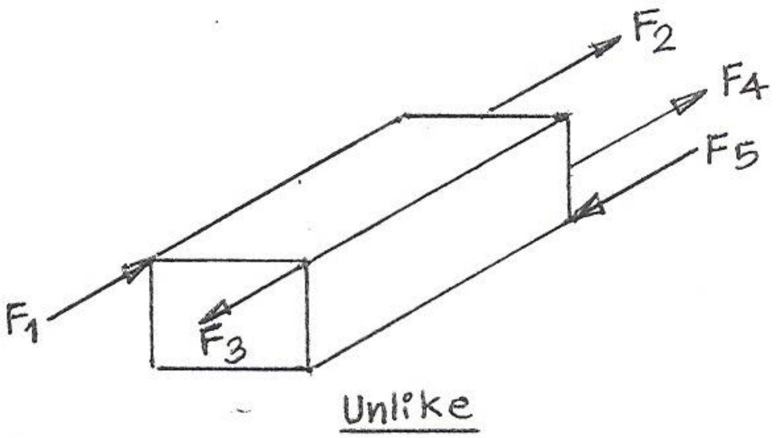


Non- Coplanar and non-Concurrent force system

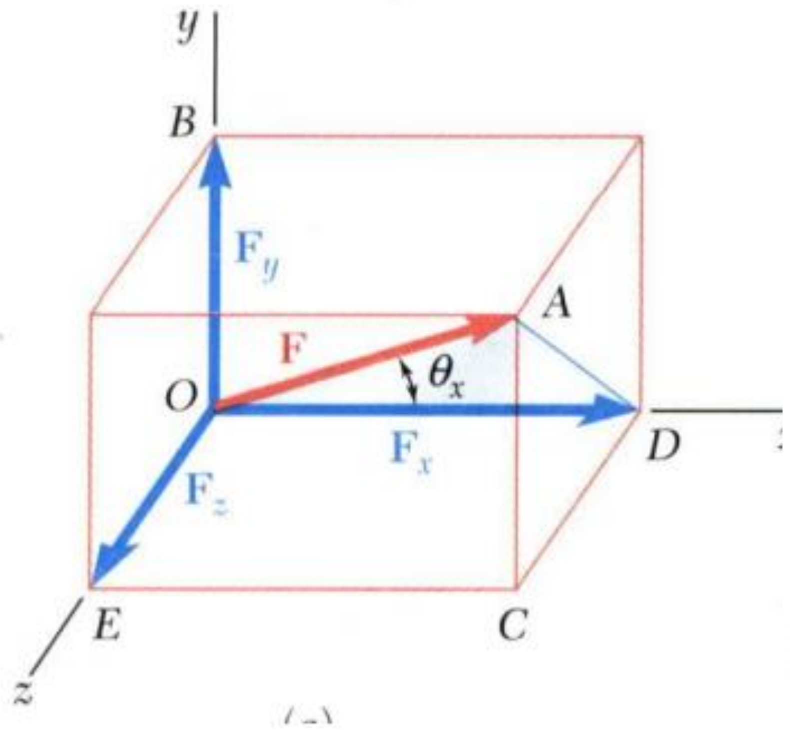
2. Non-concurrent noncoplanar system of forces: Forces which do not meet at a point and their lines of action do not lie in a plane, such forces are called "Non-concurrent noncoplanar system of forces". A typical system of non-concurrent noncoplanar system of forces is shown in the Fig.

3. **<u>Noncoplanar parallel system of forces</u>:** If lines of action of all the forces in a system are parallel and they do not lie in a plane such a system is called Non-coplanar parallel system of forces. If all the forces are pointing in one direction then they are called Like parallel forces otherwise they are called unlike parallel forces as shown in the Fig.





Rectangular components of a force in space



- With the angles between **F** and the axes,

$$F_x = F\cos\theta_x \quad F_y = F\cos\theta_y \quad F_z = F\cos\theta_z$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$= F\left(\cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}\right)$$

$$= F\vec{\lambda}$$

$$\vec{\lambda} = \cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}$$

- $\vec{\lambda}$ is a **unit vector** along the line of action of **F**; $\cos\theta_x$, $\cos\theta_y$, and $\cos\theta_z$ are the **direction cosines**

Now applying Pythagorean theorem to the triangles OAB and OCD

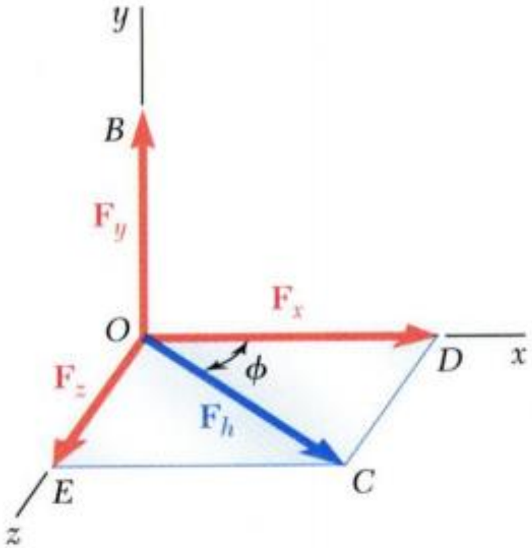
$F^2 = (OA)^2 = OB^2 + BA^2 = F_y^2 + F_h^2$ ----------------(1)

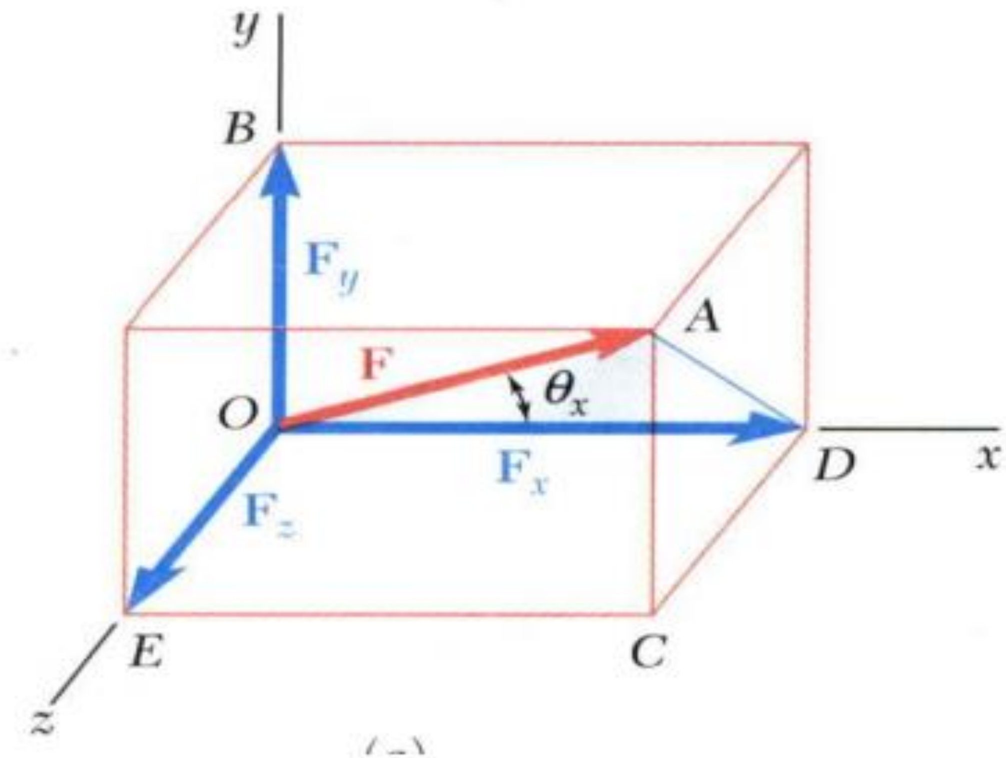
$F_h^2 = OC^2 = OD^2 + DC^2 = F_x^2 + F_z^2$ ----------------(2)

Substituting equation (2) into the equation (1), we get

$F^2 = F_x^2 + F_y^2 + F_z^2$

$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$ -----(3)





From the above Figure (Fig.)

$$F_x = F \cos\theta_x, \quad F_y = F \cos\theta_y, \quad F_z = F \cos\theta_z \quad \text{------------(4)}$$

Where $\theta_x, \theta_y, \theta_z$ are the angles formed by the force F with X, Y, Z axes respectively.
F_x, F_y, F_z are the rectangular components of the force F in the directions of X, Y, Z axes respectively.

$$\cos\theta_x = F_x/F; \quad \cos\theta_y = F_y/F; \quad \cos\theta_z = F_z/F$$

Substituting equation (4) into the equation (3), we get

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

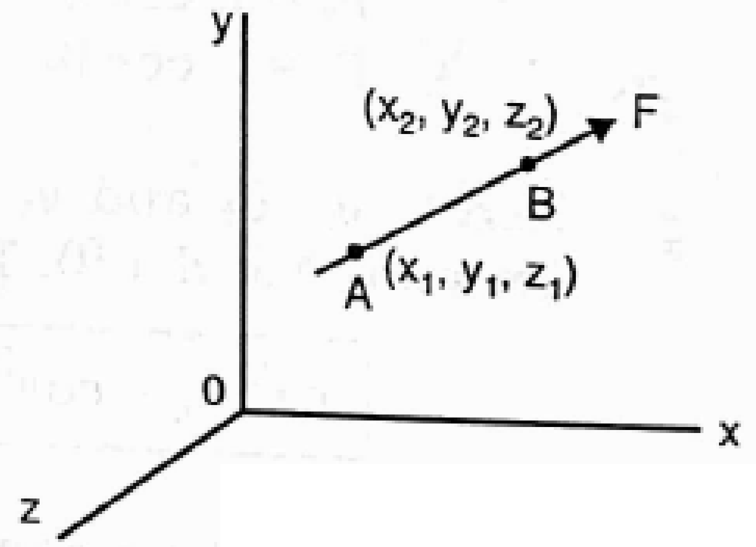
$$F = \sqrt{F^2\cos^2\theta_x + F^2\cos^2\theta_y + F^2\cos^2\theta_z}$$

$$F^2 = F^2 (\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z)$$

$$\mathbf{1 = \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z} \quad \text{-------------(5)}$$

Force in vector form

Fig. shows a force of magnitude F in space passing through A (x_1, y_1, z_1) and B (x_2, y_2, z_2). The force in vector form is



$$\bar{F} = F.\hat{e}_{AB}$$

$$\bar{F} = F\left(\frac{(x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}\right)$$

$$\bar{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j} + F_z\,\mathbf{k}$$

... Force in vector form

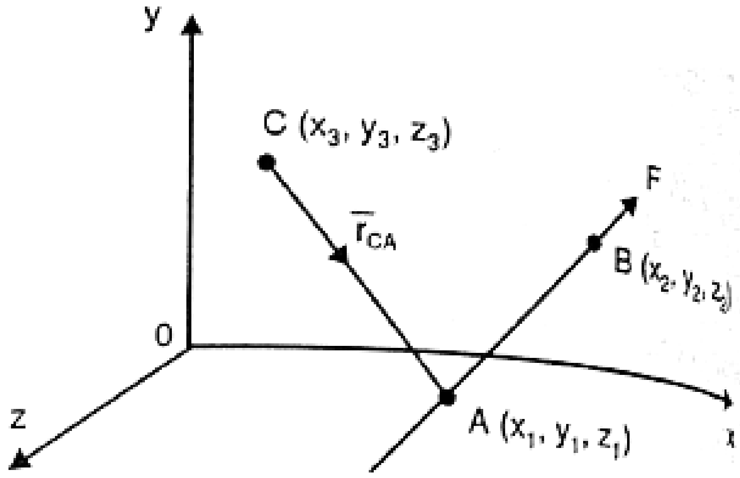
Note: **i**, **j** *and* **k** *printed in bold type denote unit vectors along the x, y and z axis respectively.*

Moment of a force

Step 1: Put the force in vector form i.e.

$$\overline{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j} + F_z\,\mathbf{k}$$

Step 2: Find the position vector extending from the moment centre to any point on the force i.e. $\bar{r} = r_x\,\mathbf{i} + r_y\,\mathbf{j} + r_z\,\mathbf{k}$



Step 3: Perform the cross product of the position vector and the force vector to get the moment vector i.e.

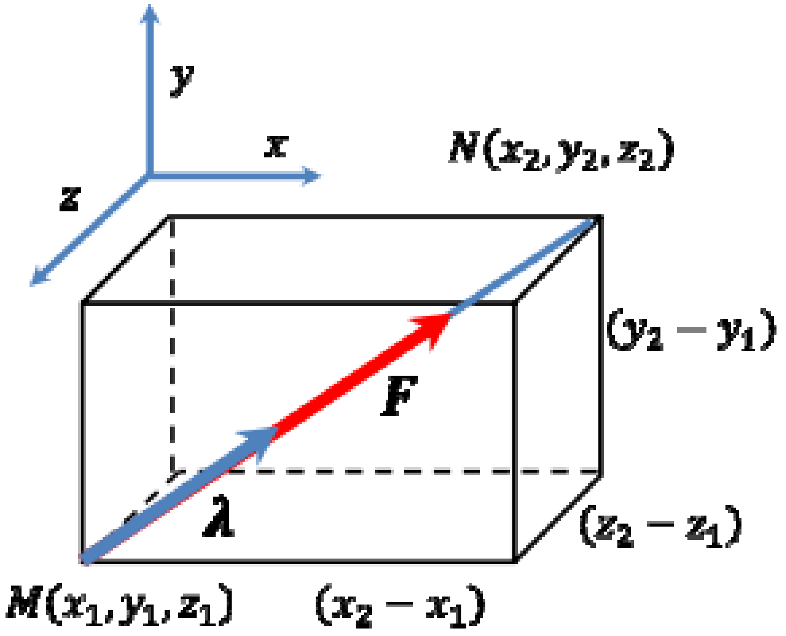
$$\overline{M}^{F}_{point} = \bar{r} \times \overline{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Rectangular Components in Space

Direction of the force is defined by the location of two points

$M(x_1, y_1, z_1)$ and $N(x_2, y_2, z_2)$



$\boldsymbol{d}$ is the vector joining M and N

$$\boldsymbol{d} = d_x\boldsymbol{i} + d_y\boldsymbol{j} + d_z\boldsymbol{k}$$

$$d_x = (x_2 - x_1) \qquad d_y = (y_2 - y_1)$$

$$d_z = (z_2 - z_1)$$

$$\boldsymbol{F} = F\boldsymbol{\lambda}$$

$$= F\left(\frac{d_x\boldsymbol{i} + d_y\boldsymbol{j} + d_z\boldsymbol{k}}{d}\right)$$

$$F_x = F\frac{d_x}{d} \qquad F_y = F\frac{d_y}{d} \qquad F_z = F\frac{d_z}{d}$$

Resultant of concurrent forces in Space:-

Resolve all the forces into their rectangular components in X, Y and Z axes directions. Adding algebraically all the horizontal components in the x direction gives

$$R_x = \sum F_x,$$

Similarly adding algebraically all the components in y and z directions yield the following relations

$$R_y = \sum F_y,$$

$$R_z = \sum F_z$$

Thus magnitude of resultant

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

Angles θ_x, θ_y, θ_z resultant forms with the axes of coordinates are obtained by

$$Cos\theta_x = \frac{R_x}{R}; Cos\theta_y = \frac{R_y}{R}; Cos\theta_z = \frac{R_z}{R}$$

Problem:

The direction of a force is given by $\theta_x = 66°$ and $\theta_y = 140°$. If $F_z = -4$ N determine i) θ_z ii) the magnitude of force iii) the other components.

Solution:

Using $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

$$\cos^2 66 + \cos^2 140 + \cos^2\theta_z = 1$$

$$\cos^2\theta_z = 0.2477$$

$$\therefore \quad \cos\theta_z = \pm 0.4977$$

$$\therefore \quad \theta_z = 60.14° \quad \text{or} \quad \theta_z = 119.85°$$

Since $F_z = -4$ N it implies that the force component is directed towards the negative direction of the z axis.

$\therefore \quad \theta_z = 119.85°$ .... **Ans.**

using $F_z = F \cos \theta_z$

$-4 = F \cos 119.85$

$\therefore \quad F = 8.036$ N .... **Ans.**

using $F_y = F \cos \theta_y$

$= 8.036 \cos 140$

$\therefore \quad F_y = 6.156$ N .... **Ans.**

using $F_x = F \cos \theta_x$

$= 8.036 \cos 66$

$\therefore \quad F_x = 3.269$ N .... **Ans.**

Problem:

- A force of magnitude 50 KN is acting at point A (2,3,4) m towards point B (6, -2, -3) m. Find the moment of the given force about a point D (-1, 1, 2) m

Solution: The force in vector form is

$$\bar{F} = F.\hat{e}_{AB}$$

$$= 50\left(\frac{4\,\mathbf{i} - 5\,\mathbf{j} - 7\,\mathbf{k}}{\sqrt{4^2 + 5^2 + 7^2}}\right)$$

$$= 21.08\,\mathbf{i} - 26.35\,\mathbf{j} - 36.89\,\mathbf{k} \quad \text{kN}$$

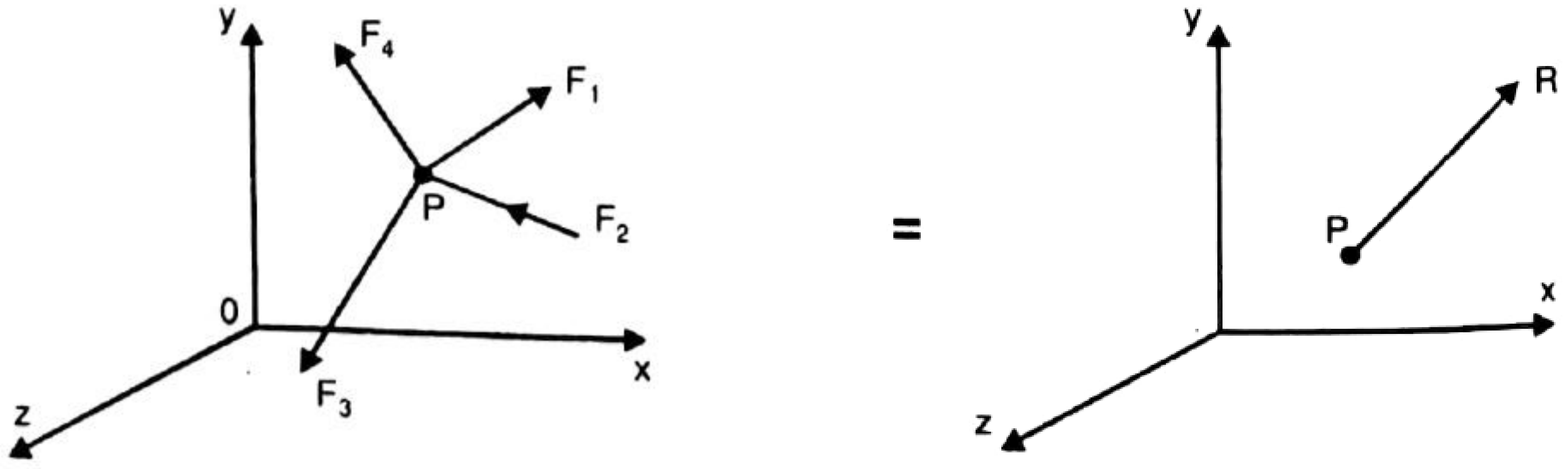
$$\overline{M}_D^F = \bar{r}_{DA} \times \bar{F} \qquad \text{Here } \bar{r}_{DA} = 3\,\mathbf{i} + 2\,\mathbf{j} + 2\,\mathbf{k} \text{ m}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & 2 \\ 21.08 & -26.35 & -36.89 \end{vmatrix}$$

$$\therefore \quad \overline{M}_D^F = -21.08\,\mathbf{i} + 152.8\,\mathbf{j} - 121.2\,\mathbf{k} \text{ kNm}$$

.... **Ans.**

Resultant of Concurrent Space Force System:



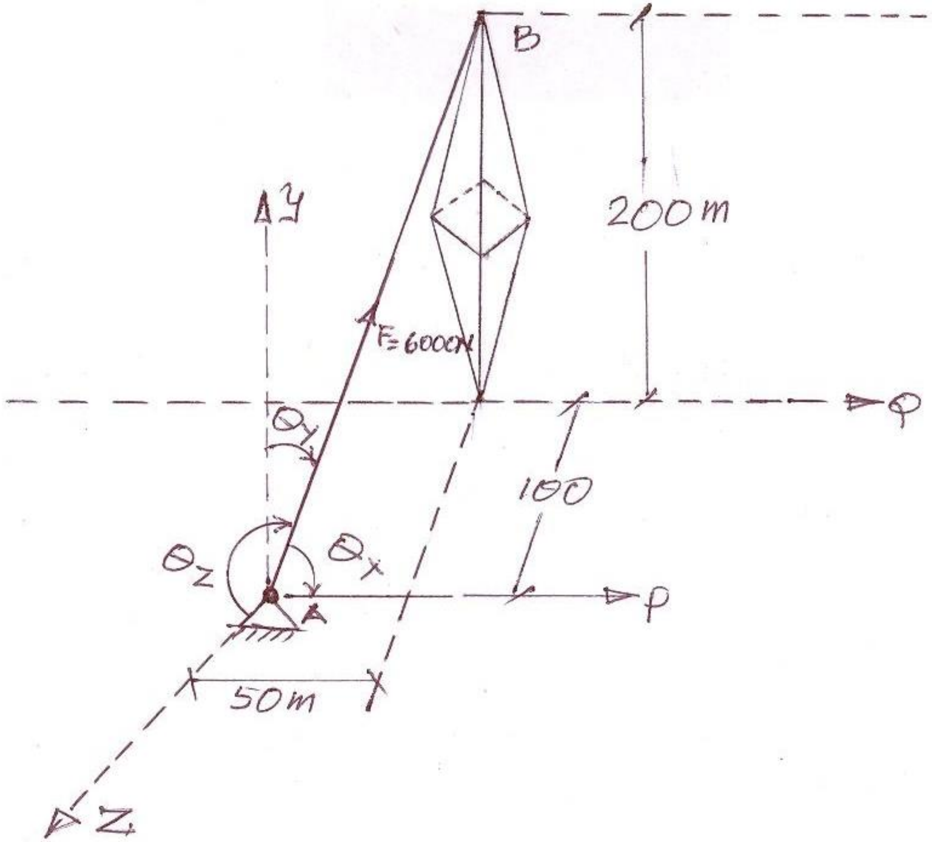
$$\overline{R} = \overline{F}_1 + \overline{F}_2 + \overline{F}_3 + \overline{F}_4$$

Problems:

(1) A tower guy wire is anchored by means of a bolt at A is shown in the following Figure. The tension in the wire is 6000N. Determine

(a) The components F_x, F_y, F_z of the forces acting on the bolt.

(b) The angles θ_x, θ_y, θ_z defining the direction of the force.



Solution: (a) Here $d_x = 50m$, $d_y = 200m$, $d_z = -100m$

Total distance A to B

$$d = \sqrt{d_x^2 + d_y^2 + d_z^2}$$

$$= \sqrt{(50)^2 + (200)^2 + (-100)^2}$$

$$= 229.13 \text{ m}$$

Using the equation, $F_x/d_x = F_y/d_y = F_z/d_z = F/d$

$\therefore F_x = d_x \cdot (F/d) = (50 \times 6000)/229.13 = 1309.3 \text{ N}$

$F_y = d_y \cdot (F/d) = (200 \times 6000)/229.13 = 5237.20 \text{ N}$

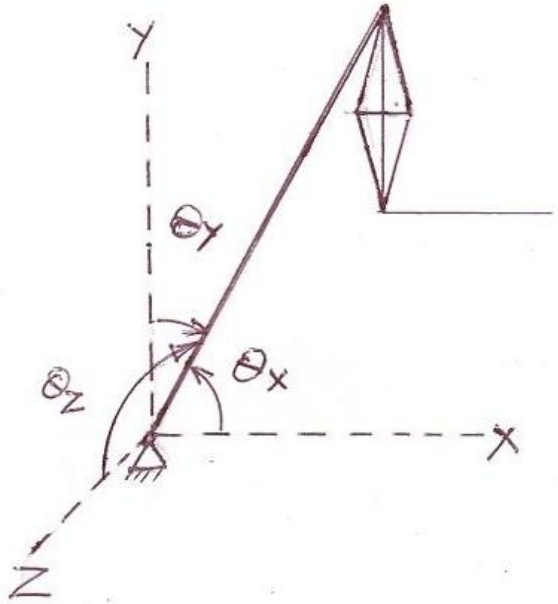
$F_z = d_z \cdot (F/d) = (-100 \times 6000)/229.13 = -2618.6 \text{ N}$

(b) Directions of the force:

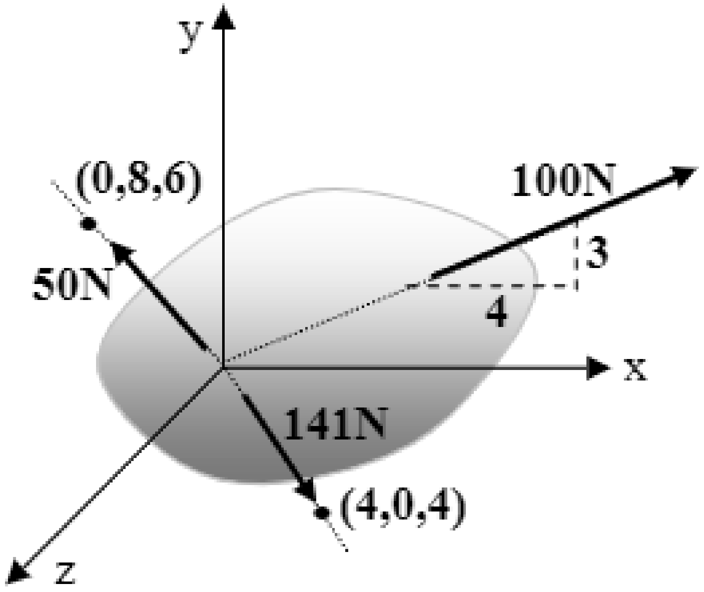
$$\cos\theta_x = d_x/d\ ,\quad \theta_x = \cos^{-1}(50/229.13) = 77.4°$$

$$\theta_y = \cos^{-1}(d_y/d) = \cos^{-1}(200/229.13) = 29.2°$$

$$\theta_z = \cos^{-1}(d_z/d) = \cos^{-1}(-100/229.13) = 115.88°$$



Example: Find the resultant of the three concurrent forces (passing through origin) shown in the figure. The 100N force lies in the X-Y plane.



Let us first represent these forces as vectors in rectangular Cartesian coordinate system as follows:

$$\bar{F}_1 = 100\left(\left(\frac{4}{\sqrt{4^2+3^2}}\right)\hat{i} + \left(\frac{3}{\sqrt{4^2+3^2}}\right)\hat{j} + 0\hat{k}\right) = \left(80\hat{i} + 60\hat{j} + 0\hat{k}\right)\text{N}$$

$$\bar{F}_2 = 50\left(\left(\frac{0}{\sqrt{0^2+8^2+6^2}}\right)\hat{i} + \left(\frac{8}{\sqrt{0^2+8^2+6^2}}\right)\hat{j} + \left(\frac{6}{\sqrt{0^2+8^2+6^2}}\right)\hat{k}\right) = \left(0\hat{i} + 40\hat{j} + 30\hat{k}\right)\text{N}$$

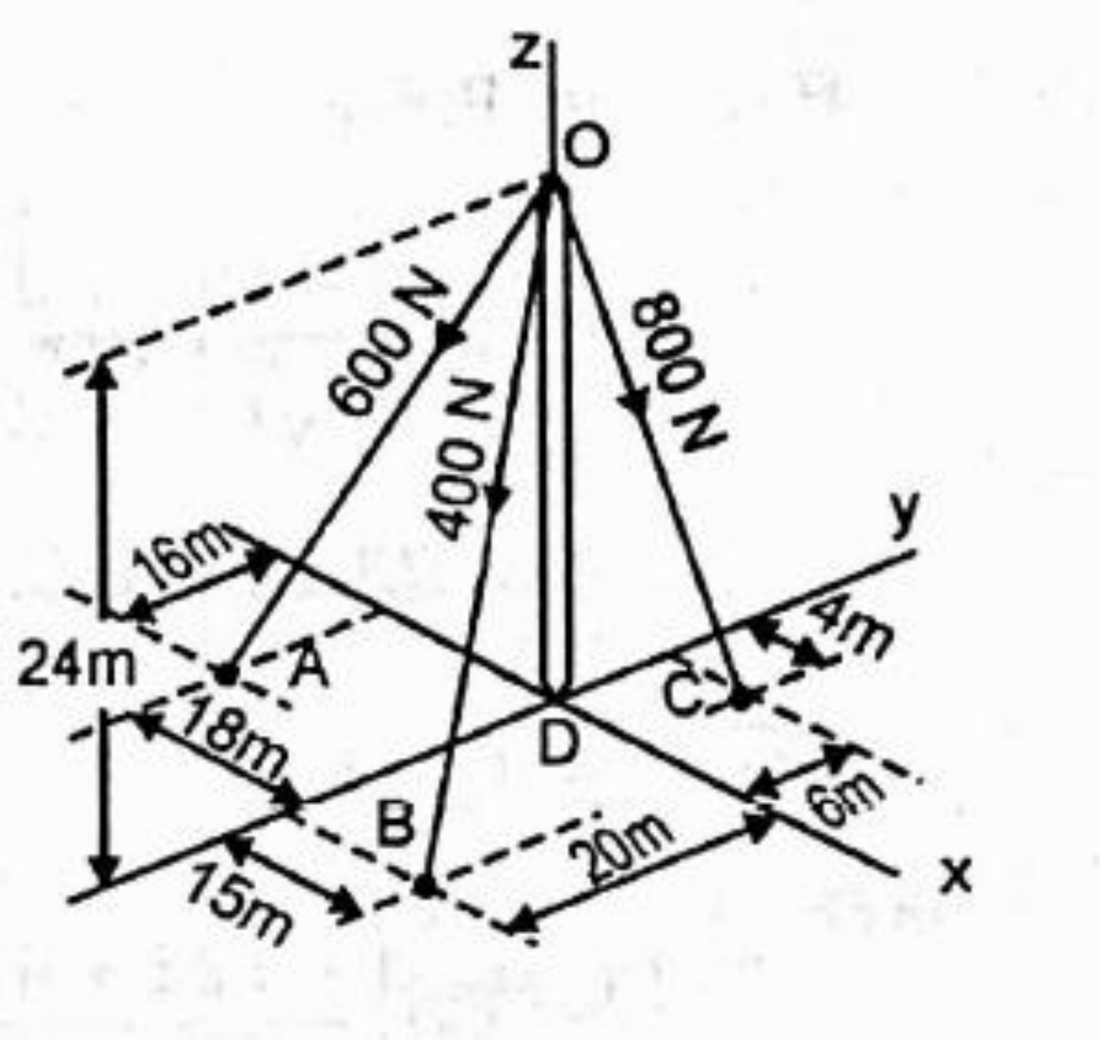
$$\bar{F}_3 = 141\left(\left(\frac{4}{\sqrt{4^2+0^2+4^2}}\right)\hat{i} + \left(\frac{0}{\sqrt{4^2+0^2+4^2}}\right)\hat{j} + \left(\frac{4}{\sqrt{4^2+0^2+4^2}}\right)\hat{k}\right) = \left(100\hat{i} + 0\hat{j} + 100\hat{k}\right)\text{N}$$

$$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 = 180\hat{i} + 100\hat{j} + 130\hat{k}\ \text{N}$$

$$= \sqrt{180^2+100^2+130^2}\left(\left(\frac{180}{\sqrt{180^2+100^2+130^2}}\right)\hat{i} + \left(\frac{100}{\sqrt{180^2+100^2+130^2}}\right)\hat{j} + \left(\frac{130}{\sqrt{180^2+100^2+130^2}}\right)\hat{k}\right)\text{N}$$

$$= 244\left(0.74\hat{i} + 0.41\hat{j} + 0.53\hat{k}\right)\text{N}$$

The tower is held in place by three cables. If the force of each cable acting on the tower is as shown in figure, determine the resultant.



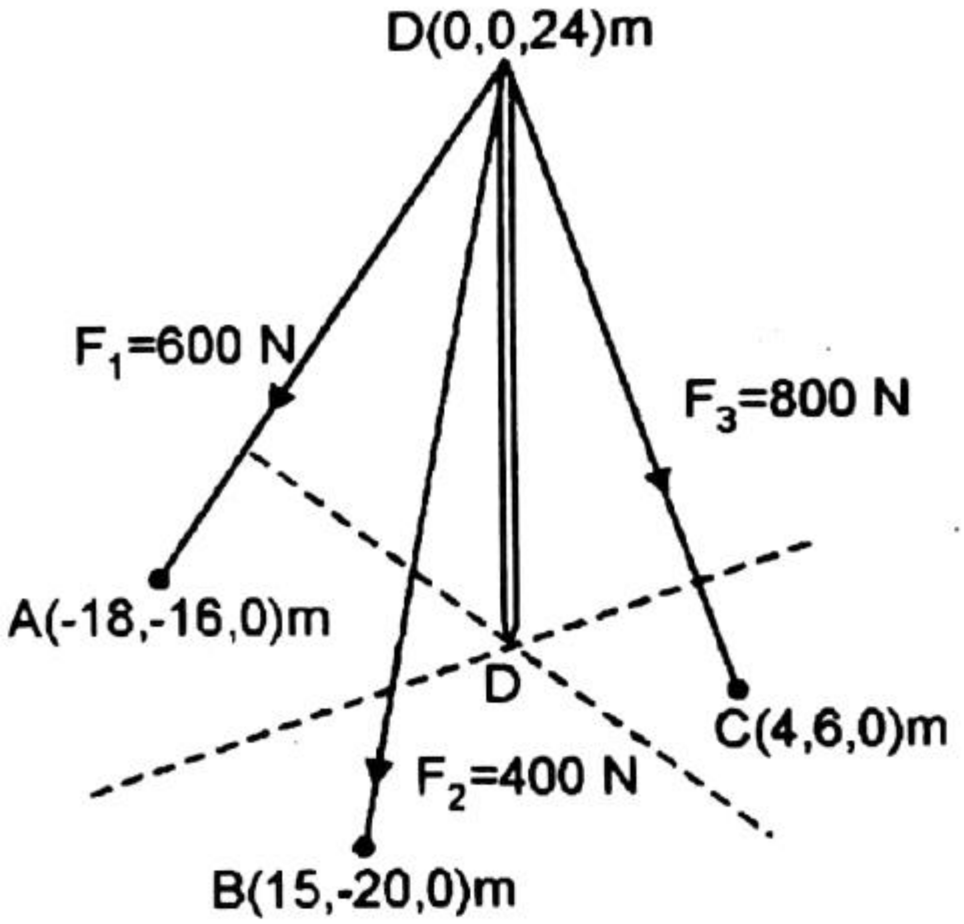
Solution: This is a concurrent space force system of 3 forces at O

Let $\bar{F}_1$ be the force in cable OA

$$\therefore \quad \bar{F}_1 = F_1 \cdot \hat{e}_{OA}$$

$$= 600\left[\frac{-18\mathbf{i} - 16\mathbf{j} - 24\mathbf{k}}{\sqrt{18^2 + 16^2 + 24^2}}\right]$$

$$= -317.6\mathbf{i} - 282.4\mathbf{j} - 423.5\mathbf{k}\ \text{N}$$



Let $\overline{F}_2$ be the force in cable OB

$$\therefore \quad \overline{F}_2 = F_2 . \hat{e}_{OB}$$

$$= 400\left[\frac{15\mathbf{i} - 20\mathbf{j} - 24\mathbf{k}}{\sqrt{15^2 + 20^2 + 24^2}}\right]$$

$$= -173.1\mathbf{i} - 230.8\mathbf{j} - 277\mathbf{k} \text{ N}$$

Let $\overline{F}_3$ be the force in cable OC

$$\therefore \quad \overline{F}_3 = F_3 . \hat{e}_{OC}$$

$$= 800\left[\frac{4\mathbf{i} + 6\mathbf{j} - 24\mathbf{k}}{\sqrt{4^2 + 6^2 + 24^2}}\right]$$

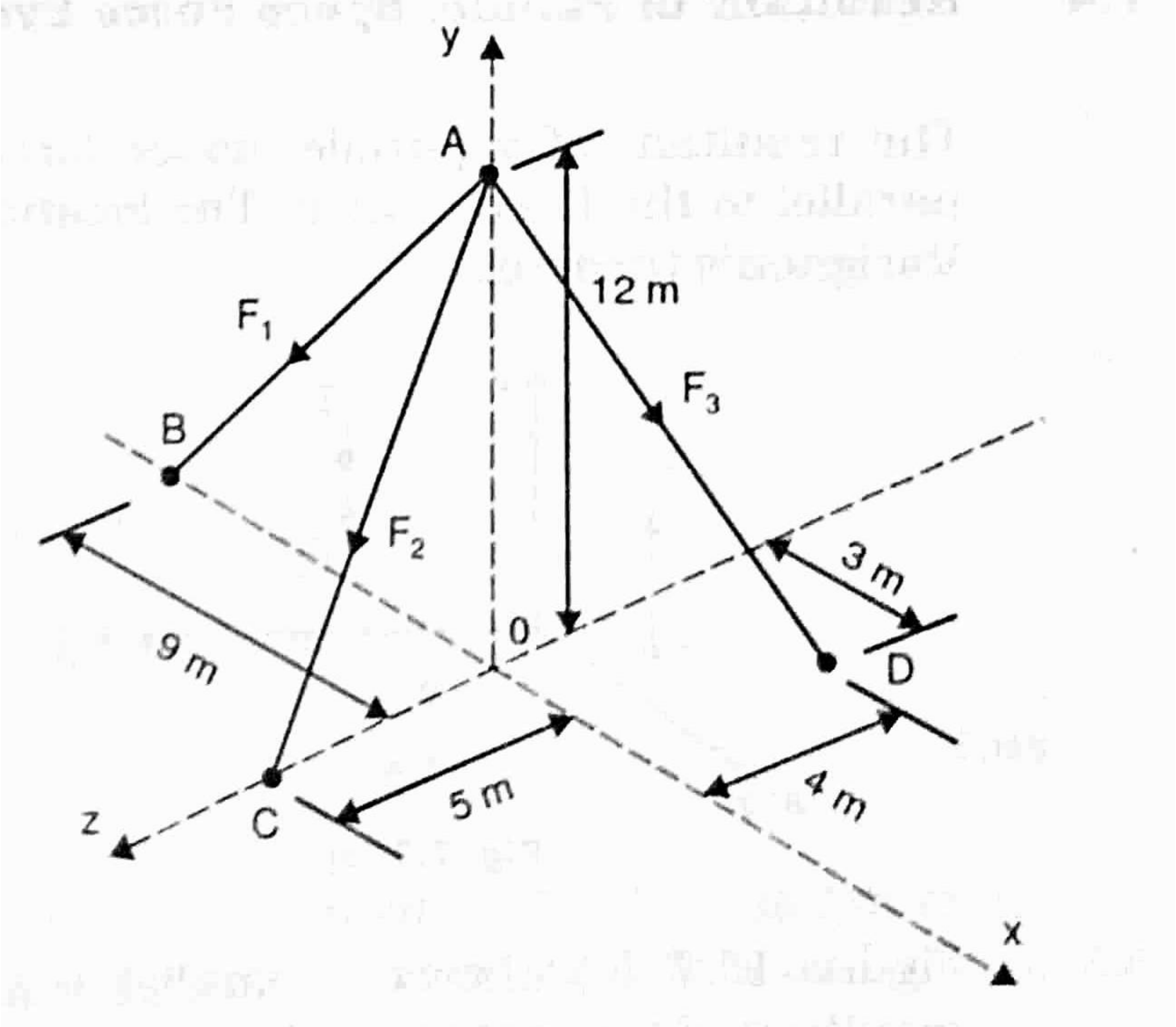
$$= 127.7\mathbf{i} + 191.5\mathbf{j} - 766.2\mathbf{k} \text{ N}$$

Resultant force $\overline{R} = \overline{F}_1 + \overline{F}_2 + \overline{F}_3$

$$\therefore \quad \overline{R} = -16.8\mathbf{i} - 321.7\mathbf{j} - 1466.7\mathbf{k} \text{ N}$$

.... **Ans.**

The resultant of the three concurrent space forces at A is $\bar{R}$ = -788j N. Find the magnitude of F1, F2 and F3 force.



Solution:

From the figure the coordinates are, A (0, 12, 0) m, B (– 9, 0, 0) m, C (0, 0, 5) m and D (3, 0, – 4) m.

Putting the forces in vector form.

$$\bar{F}_1 = F_1.\hat{e}_{AB}$$

$$= F_1\left(\frac{-9\,\mathbf{i} - 12\,\mathbf{j}}{\sqrt{9^2 + 12^2}}\right)$$

$$= F_1(-0.6\,\mathbf{i} - 0.8\,\mathbf{j}) \text{ N}$$

$$\bar{F}_2 = F_2.\hat{e}_{AC}$$

$$= F_2\left(\frac{-12\,\mathbf{j} + 5\,\mathbf{k}}{\sqrt{12^2 + 5^2}}\right)$$

$$= F_2(-0.923\,\mathbf{j} + 0.385\,\mathbf{k}) \text{ N}$$

$$\bar{F}_3 = F_3.\hat{e}_{AD}$$

$$= F_3\left(\frac{3\,\mathbf{i} - 12\,\mathbf{j} - 4\,\mathbf{k}}{\sqrt{3^2 + 12^2 + 4^2}}\right)$$

$$= F_3\,(0.231\,\mathbf{i} - 0.923\,\mathbf{j} - 0.308\,\mathbf{k})\ \text{N}$$

The resultant of the forces at A is $\bar{R} = -788\,\mathbf{j}$ N

$$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3$$

$$0\,\mathbf{i} - 788\,\mathbf{j} + 0\,\mathbf{j} = F_1(-0.6\,\mathbf{i} - 0.8\,\mathbf{j}) + F_2(-0.923\,\mathbf{j} + 0.385\,\mathbf{k}) + F_3(0.231\,\mathbf{i} - 0.923\,\mathbf{j} - 0.308\,\mathbf{k})$$

$$0\,\mathbf{i} - 788\,\mathbf{j} + 0\,\mathbf{j} = (-0.6\,F_1 + 0.231\,F_3)\,\mathbf{i} + (-0.8\,F_1 - 0.923\,F_2 - 0.923\,F_3)\,\mathbf{j} + (0.385\,F_2 - 0.308\,F_3)\,\mathbf{k}$$

Equating the coefficients

$$-0.6\ F_1 - 0.231\ F_3 = 0 \qquad \text{..............} \quad (1)$$

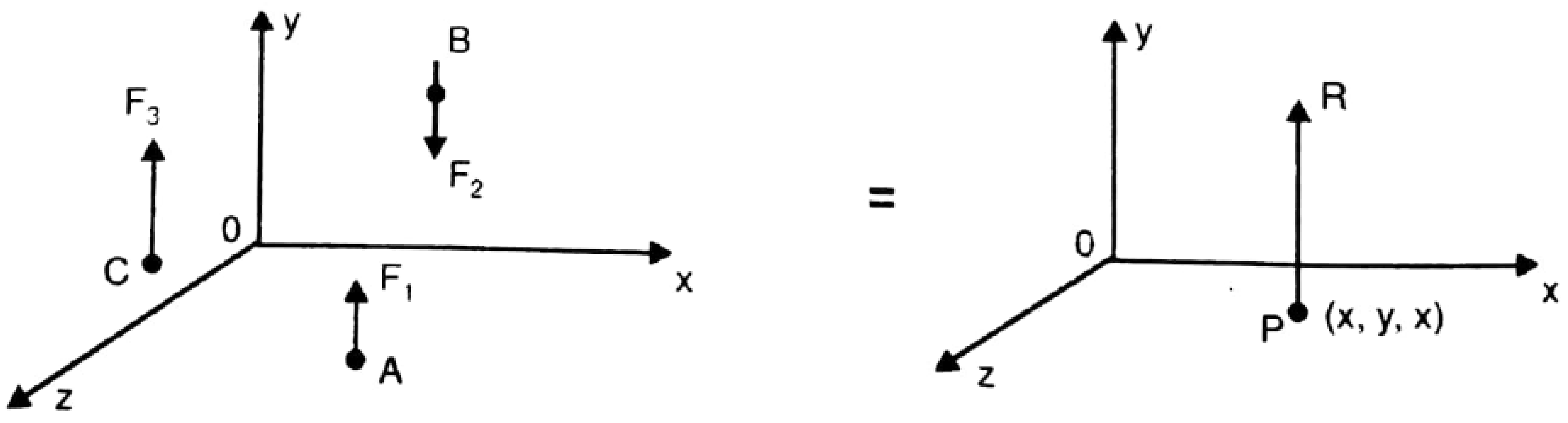
$$-0.8\ F_1 - 0.923\ F_2 - 0.923\ F_3 = -788 \qquad \text{..............} \quad (2)$$

$$0.385\ F_2 - 0.308\ F_3 = 0 \qquad \text{..............} \quad (3)$$

Solving equations (1), (2) and (3) we get,

$F_1 = 154$ N, $\quad F_2 = 320$ N, $\quad F_3 = 400$ N .... **Ans.**

Resultant of Parallel Space Force System:



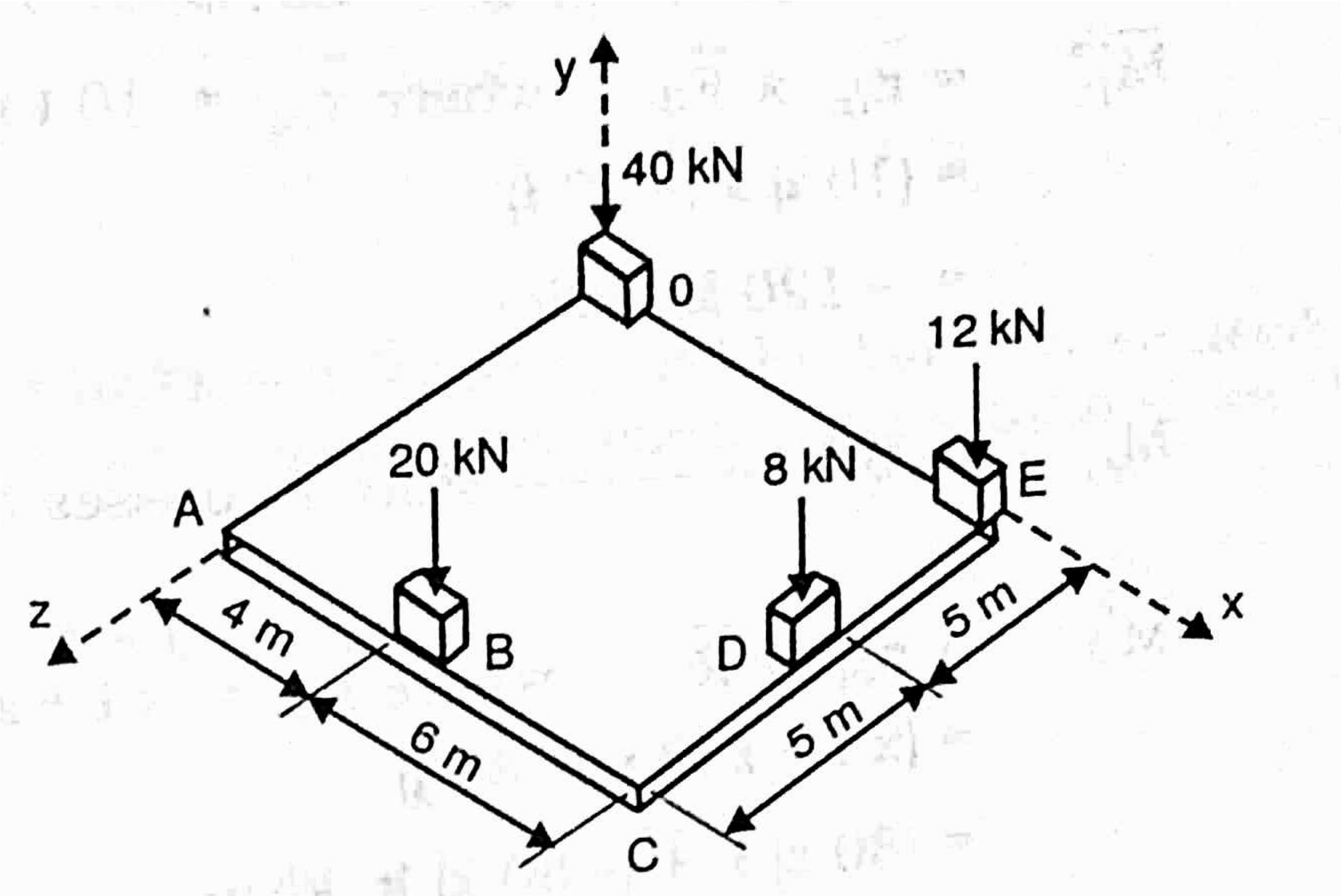
$$\overline{R} = \overline{F}_1 + \overline{F}_2 + \overline{F}_3$$

Location of Resultant → Found out by using Varignon's Theorum

$$\Sigma \overline{M}_O^F = \overline{M}_O^R$$

Problem:

A square foundation mat supports the four columns as shown in figure. Determine the magnitude and point of application of the resultant of the four loads.



Solution:

O (0, 0, 0), B (4; 0, 10), D (10, 0, 5), E (10, 0, 0)

Putting the forces in vector form

Let $F_1 = 20$ kN

$\therefore$ $\bar{F}_1 = -20\,\mathbf{j}$ kN since it is parallel to y axis and directed downwards.

Similarly

Let $F_2 = 8$ kN

$\therefore$ $\bar{F}_2 = -8\,\mathbf{j}$ kN

Let $F_3 = 12$ kN

$\therefore$ $\bar{F}_3 = -12\,\mathbf{j}$ kN

Let $F_4 = 40$ kN

$\therefore$ $\bar{F}_4 = -40\,\mathbf{j}$ kN

The resultant $\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F}_4$

$\bar{R} = (-20\,\mathbf{j}) + (-8\,\mathbf{j}) + (-12\,\mathbf{j}) + (-40\,\mathbf{j})$

$\therefore \quad \bar{R} = -80\,\mathbf{j}$ kN

Point of application of resultant:

Let the resultant act at a point P (x, 0, z) in the plane of the foundation mat.

$\bar{M}_O^{F_1} = \bar{r}_{OB} \times \bar{F}_1$ where $\bar{r}_{OB} = 4\,\mathbf{i} + 10\,\mathbf{k}$ m

$= (4\,\mathbf{i} + 10\,\mathbf{k}) \times (-20\,\mathbf{j})$

$= 200\,\mathbf{i} - 80\,\mathbf{k}$ kNm

$$\overline{M}_O^{F_2} = \bar{r}_{OD} \times \overline{F}_2 \quad \text{where } \bar{r}_{OD} = 10\,\mathbf{i} + 5\,\mathbf{k}$$

$$= (10\,\mathbf{i} + 5\,\mathbf{k}) \times (-8\,\mathbf{j})$$

$$= 40\,\mathbf{i} - 80\,\mathbf{k} \quad \text{kNm}$$

$$\overline{M}_O^{F_3} = \bar{r}_{OE} \times \overline{F}_3 \quad \text{where } \bar{r}_{OE} = 10\,\mathbf{i}\ \text{m}$$

$$= (10\,\mathbf{i}) \times (-12\,\mathbf{j})$$

$$= -120\,\mathbf{k} \quad \text{kNm}$$

$\overline{M}_O^{F_4} = 0$ ----------- since F_4 passes through O

$$\overline{M}_O^{R} = \bar{r}_{OP} \times \overline{R} \quad \text{where } \bar{r}_{OP} = x\,\mathbf{i} + z\,\mathbf{k}$$

$$= (x\,\mathbf{i} + z\,\mathbf{k}) \times (-80\,\mathbf{j})$$

$$= (80\,z)\,\mathbf{i} + (-80\,x)\,\mathbf{k} \quad \text{kNm}$$

Using Varignon's theorem

$$\Sigma \overline{M}_O^F = \Sigma \overline{M}_O^R$$

$$\overline{M}_O^{F_1} + \overline{M}_O^{F_2} + \overline{M}_O^{F_3} + \overline{M}_O^{F_4} = \overline{M}_O^R$$

$$(200\ \mathbf{i} - 80\ \mathbf{k}) + (40\ \mathbf{i} - 80\ \mathbf{k}) + (-120\ \mathbf{k}) = (80\ z)\ \mathbf{i} + (-80x)\ \mathbf{k}$$

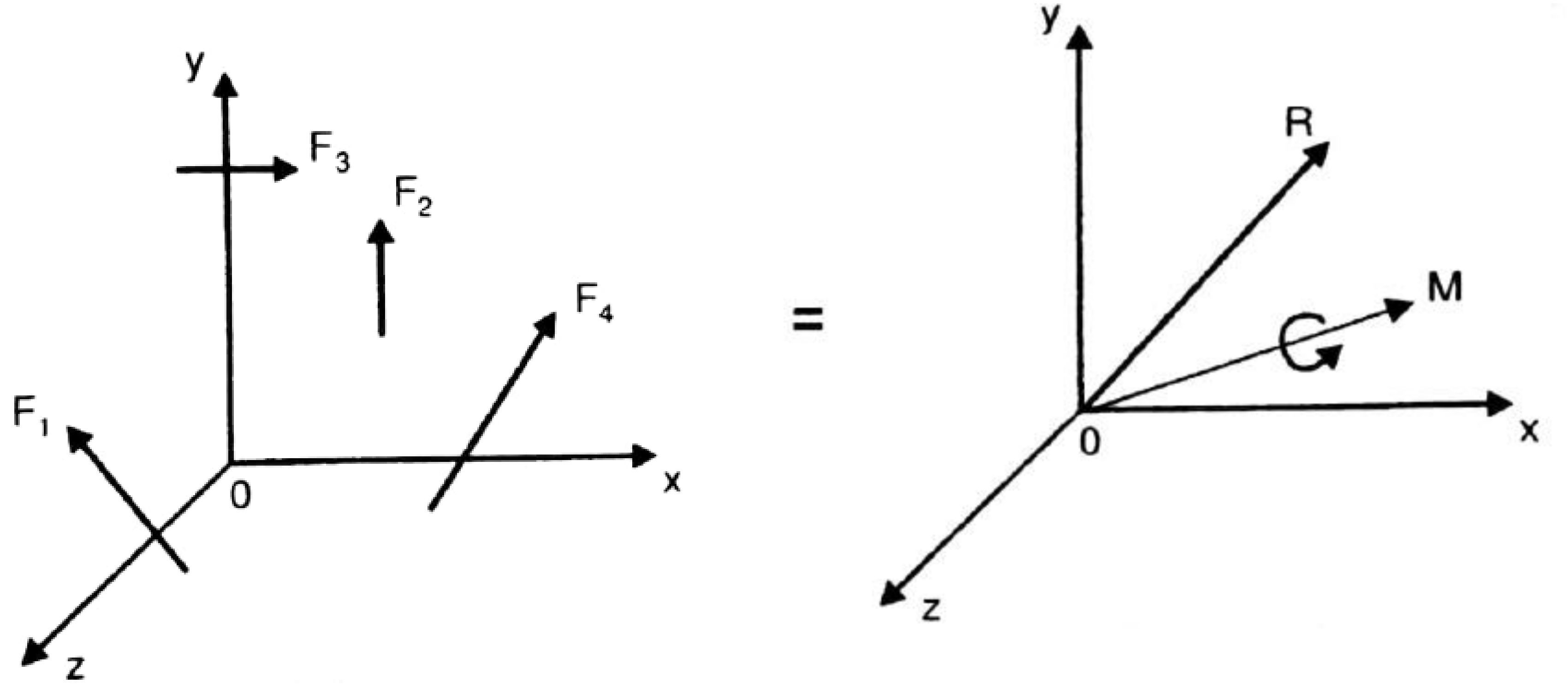
$$\therefore \quad 240\ \mathbf{i} - 280\ \mathbf{k} = (80\ z)\ \mathbf{i} + (-80\ x)\ \mathbf{k}$$

equating the coefficients

$$240 = 80\ z$$
$$z = 3\ \text{m}$$
$$-280 = -80\ x$$
$$x = 3.5\ \text{m}$$

$\therefore$ The resultant $\overline{R} = -80\ \mathbf{j}$ kN passes through point P (3.5, 0, 3) m .... **Ans.**

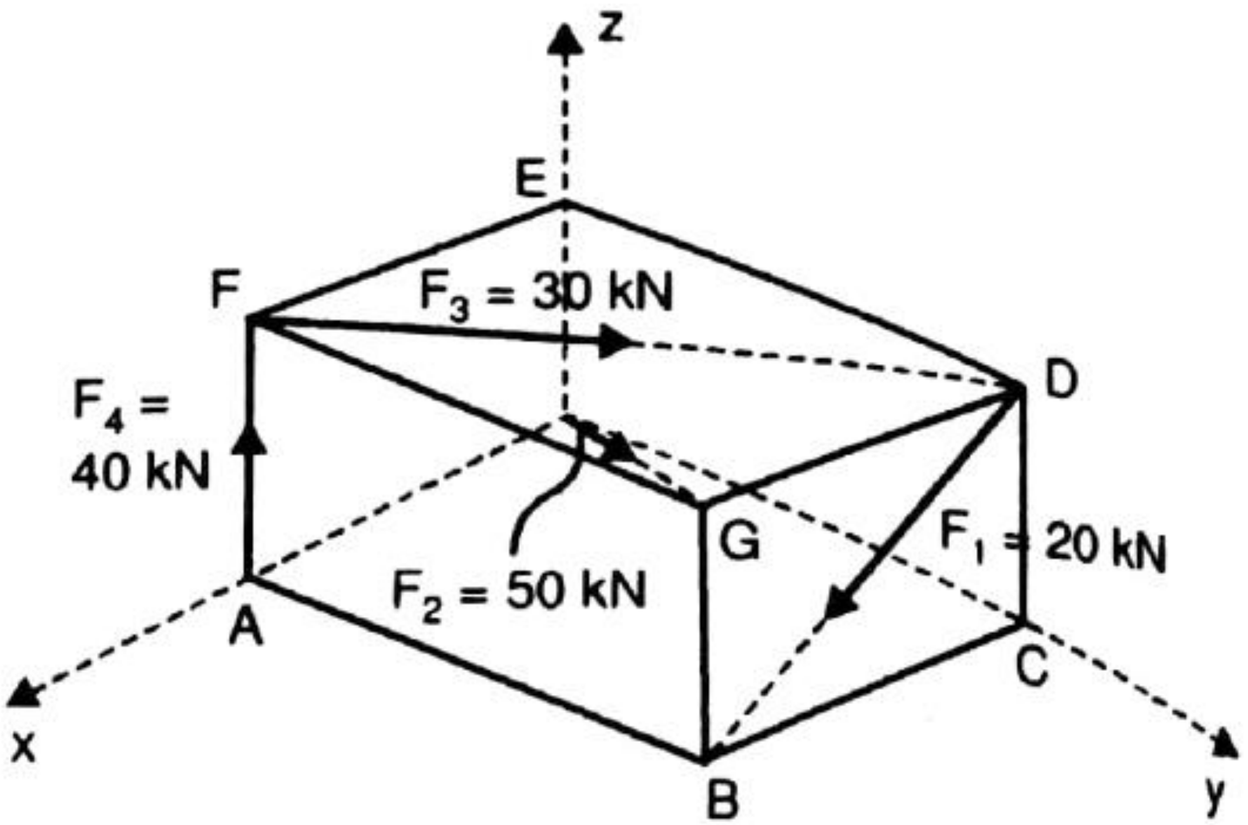
Resultant of General Space Force System:



$$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F}_4$$

$$\bar{M} = \bar{M}_O^{F_1} + \bar{M}_O^{F_2} + \bar{M}_O^{F_3} + \bar{M}_O^{F_4}$$

Determine the resultant force and couple moment about the origin of the force system shown in figure. L(OA) = 4 m, L(OC) = 5 m, L(OE) = 3 m



Solution:

A (4, 0, 0), B (4, 5, 0),

C (0, 5, 0), D (0, 5, 3), F (4, 0, 3), G (4, 5, 3) and O (0, 0, 0).

$$\bar{F}_1 = F_1.\hat{e}_{DB}$$

$$= 20\left(\frac{4\,\mathbf{i} - 3\,\mathbf{k}}{\sqrt{4^2 + 3^2}}\right)$$

$$= 16\,\mathbf{i} - 12\,\mathbf{k} \ \text{kN}$$

$$\bar{F}_2 = F_2.\hat{e}_{OG}$$

$$= 50\left(\frac{4\,\mathbf{i} + 5\,\mathbf{j} + 3\,\mathbf{k}}{\sqrt{4^2 + 5^2 + 3^2}}\right)$$

$$= 28.28\,\mathbf{i} + 35.35\,\mathbf{j} + 21.21\,\mathbf{k} \ \text{kN}$$

$$\overline{F}_3 = F_3.\hat{e}_{FD}$$

$$= 30\left(\frac{-4\mathbf{i} + 5\mathbf{j}}{\sqrt{4^2 + 5^2}}\right)$$

$$= -18.74\,\mathbf{i} + 23.42\,\mathbf{j} \quad kN$$

$\overline{F}_4 = 40\,\mathbf{k}$ kN since the force is parallel to z axis.

The resultant force $\overline{R} = \overline{F}_1 + \overline{F}_2 + \overline{F}_3 + \overline{F}_4$

$$\overline{R} = (16\,\mathbf{i} - 12\,\mathbf{k}) + (28.28\,\mathbf{i} + 35.35\,\mathbf{j} + 21.21\,\mathbf{k}) + (-18.74\,\mathbf{i} + 23.42\,\mathbf{j}) + (40\,\mathbf{k})$$

$$\therefore \overline{R} = 25.54\,\mathbf{i} + 58.77\,\mathbf{j} + 49.21\,\mathbf{k} \quad kN$$

.........**Ans.**

$$\overline{M}_O^{F_1} = \bar{r}_{OD} \times \overline{F}_1 \qquad \text{where } \bar{r}_{OD} = 5\,\mathbf{j} + 3\,\mathbf{k} \text{ m}$$

$$= (5\,\mathbf{j} + 3\,\mathbf{k}) \times (16\,\mathbf{i} - 12\,\mathbf{k})$$

$$= -60\,\mathbf{i} + 48\,\mathbf{j} - 80\,\mathbf{k} \text{ kNm}$$

$$\overline{M}_O^{F_2} = 0 \qquad \text{since } F_2 \text{ passes through O}$$

$$\overline{M}_O^{F_3} = \bar{r}_{OD} \times \overline{F}_3 \qquad \text{where } \bar{r}_{OD} = 5\,\mathbf{j} + 3\,\mathbf{k} \text{ m}$$

$$= (5\,\mathbf{i} + 3\,\mathbf{k}) \times (-18.74\,\mathbf{i} + 23.42\,\mathbf{j})$$

$$= -70.26\,\mathbf{i} - 56.22\,\mathbf{j} + 93.7\,\mathbf{k} \text{ kNm}$$

117.1 k

$$\overline{M}_O^{F_4} = \bar{r}_{OA} \times \overline{F}_4 \qquad \text{where } \bar{r}_{OA} = 4\,\mathbf{i} \text{ m}$$

$$= (4\,\mathbf{i}) \times (40\,\mathbf{k})$$

$$= -160\,\mathbf{j} \text{ kNm}$$

The resultant moment at the origin is

$$\overline{M}_O = \overline{M}_O^{F_1} + \overline{M}_O^{F_2} + \overline{M}_O^{F_3} + \overline{M}_O^{F_4}$$

117·1

$$= (-60\,\mathbf{i} + 48\,\mathbf{j} - 80\,\mathbf{k}) + 0 + (-70.26\,\mathbf{i} - 56.22\,\mathbf{j} + 93.7\,\mathbf{k}) + (-160\,\mathbf{j})$$

37·1

$$\therefore\ \overline{M}_O = -130.26\,\mathbf{i} - 168.2\,\mathbf{j} + 13.7\,\mathbf{k} \text{ kNm}$$ .... **Ans.**

The resultant force and couple moment at the origin is

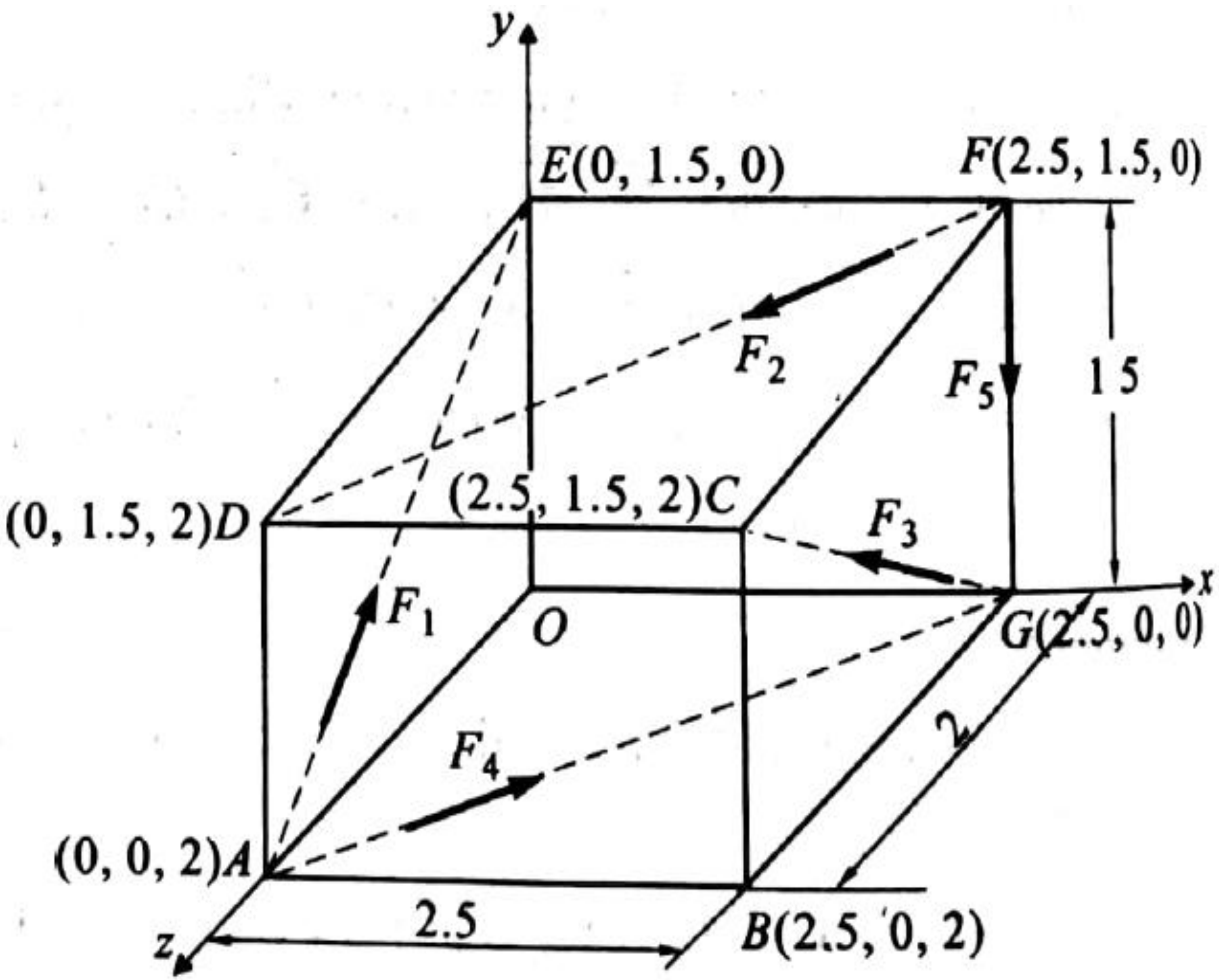
$$\overline{R} = 25.54\,\mathbf{i} + 58.77\,\mathbf{j} + 49.21\,\mathbf{k} \text{ kN}$$

$$\overline{M}_O = -130.26\,\mathbf{i} - 168.2\,\mathbf{j} + 13.7\,\mathbf{k} \text{ kNm}$$ .... **Ans.**

37·1

Problem

Force $F_1 = 1$ kN, $F_2 = 3$ kN, $F_3 = 2$ kN, $F_4 = 5$ kN and $F_5 = 2$ kN act along the line joining the corners of the parallel piped whose sides are 2.5 m, 2 m and 1.5 m as shown in Fig. 4.19. Find the resultant force and the moment of the resultant couple at the origin O.



Solution

Given : $F_1 = 1 \text{ kN} \quad (A \rightarrow E)$;

$F_2 = 3 \text{ kN} \quad (F \rightarrow D)$; $F_3 = 2 \text{ kN} \quad (G \rightarrow C)$;

$F_4 = 5 \text{ kN} \quad (A \rightarrow G)$; $F_5 = 2 \text{ kN} \quad (F \rightarrow G)$.

(i) Coordinates : $O(0, 0, 0)$; $A(0, 0, 2)$; $B(2.5, 0, 2)$; $C(2.5, 1.5, 2)$; $D(0, 1.5, 2)$; $E(0, 1.5, 0)$; $F(2.5, 1.5, 0)$; $G(2.5, 0, 0)$.

(ii) Force vector

$$\bar{F}_1 = (F_1)(\bar{e}_{AE}) = (1)\left[\frac{1.5\,\mathbf{j} - 2\,\mathbf{k}}{\sqrt{(1.5)^2 + 2^2}}\right] \qquad \therefore \quad \bar{F}_1 = 0.6\,\mathbf{j} - 0.8\,\mathbf{k}$$

$$\bar{F}_2 = (F_2)(\bar{e}_{FD}) = (3)\left[\frac{-2.5\,\mathbf{i} + 2\,\mathbf{k}}{\sqrt{(2.5)^2 + 2^2}}\right] \qquad \therefore \quad \bar{F}_2 = -2.34\,\mathbf{i} + 1.87\,\mathbf{k}$$

$$\bar{F}_3 = (F_3)(\bar{e}_{GC}) = (2)\left[\frac{1.5\,\mathbf{j} + 2\,\mathbf{k}}{\sqrt{(1.5)^2 + 2^2}}\right] \quad \therefore \quad \bar{F}_3 = 1.2\,\mathbf{j} + 1.6\,\mathbf{k}$$

$$\bar{F}_4 = (F_4)(\bar{e}_{AG}) = (5)\left[\frac{2.5\,\mathbf{i} - 2\,\mathbf{k}}{\sqrt{(2.5)^2 + 2^2}}\right] \quad \therefore \quad \bar{F}_4 = 3.90\,\mathbf{i} - 3.12\,\mathbf{k}$$

$$\bar{F}_5 = (F_5)(\bar{e}_{FG}) = (2)\left[\frac{-1.5\,\mathbf{j}}{\sqrt{(1.5)^2}}\right] \quad \therefore \quad \bar{F}_5 = -2\,\mathbf{j}$$

(iii) Position vectors

$\bar{r}_1 = 2\,\mathbf{k}$; $\bar{r}_2 = 1.5\,\mathbf{j} + 2\,\mathbf{k}$; $\bar{r}_3 = 2.5\,\mathbf{i}$; $\bar{r}_4 = 2\,\mathbf{k}$; $\bar{r}_5 = 2.5\,\mathbf{i}$

(iv) Moment vectors

$$\bar{M}_1 = \bar{r}_1 \times \bar{F}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 0 & 0.6 & -0.8 \end{vmatrix} \qquad \therefore \bar{M}_1 = -1.2\,\mathbf{i}$$

$$\bar{M}_2 = \bar{r}_2 \times \bar{F}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1.5 & 2 \\ -2.34 & 0 & 1.87 \end{vmatrix} \qquad \therefore \bar{M}_2 = 2.8\,\mathbf{i} - 4.68\,\mathbf{j} + 3.51\,\mathbf{k}$$

$$\bar{M}_3 = \bar{r}_3 \times \bar{F}_3 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.5 & 0 & 0 \\ 0 & 1.2 & 1.6 \end{vmatrix} \qquad \therefore \bar{M}_3 = -4\,\mathbf{j} + 3\,\mathbf{k}$$

$$\overline{M}_4 = \overline{r}_4 \times \overline{F}_4 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 3.9 & 0 & -3.12 \end{vmatrix} \qquad \therefore \ \overline{M}_4 = 7.8\,\mathbf{j}$$

$$\overline{M}_5 = \overline{r}_5 \times \overline{F}_5 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.5 & 0 & 0 \\ 0 & -2 & 0 \end{vmatrix} \qquad \therefore \ \overline{M}_5 = -5\,\mathbf{k}$$

(v) Resultant force vector

$$\overline{R} = \overline{F}_1 + \overline{F}_2 + \overline{F}_3 + \overline{F}_4 + \overline{F}_5$$

$$\overline{R} = 1.56\,\mathbf{i} - 0.2\,\mathbf{j} - 0.45\,\mathbf{k} \quad \textit{Ans.}$$

(vi) Resultant moment vector

$$\Sigma\,\overline{M} = \overline{M}_1 + \overline{M}_2 + \overline{M}_3 + \overline{M}_4 + \overline{M}_5$$

$$\Sigma\,\overline{M} = 1.6\,\mathbf{i} - 0.88\,\mathbf{j} + 1.51\,\mathbf{k} \quad \textit{Ans.}$$

Practice Problem:

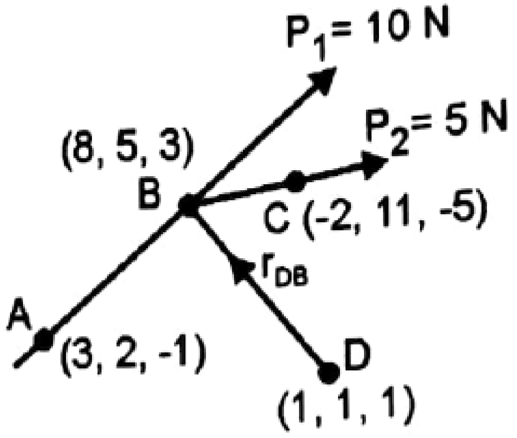
P1. A force P_1 = 10 N in magnitude acts along direction AB whose co-ordinates of points A and B are (3, 2, –1) and (8, 5, 3) respectively. Another force P_2 = 5 N in magnitude acts along BC where C has co-ordinates (–2, 11, –5). Determine a) The resultant of P_1 and P_2. b) The moment of the resultant about a point D (1, 1, 1).

Solution: a) This is a concurrent space force system consisting of two forces P_1 and P_2 meeting at B.

$$\therefore \quad \overline{P}_1 = P_1 \cdot \hat{e}_{AB}$$

$$= 10\left[\frac{5\mathbf{i}+3\mathbf{j}+4\mathbf{k}}{\sqrt{5^2+3^2+4^2}}\right]$$

$$\therefore \quad \overline{P}_1 = 7.07\,\mathbf{i}+4.24\,\mathbf{j}+5.65\,\mathbf{k}\ \text{N}$$



$$\overline{P}_2 = P_2 \cdot \hat{e}_{BC}$$

$$= 5\left[\frac{-10\,\mathbf{i} + 6\,\mathbf{j} - 8\,\mathbf{k}}{\sqrt{10^2 + 6^2 + 8^2}}\right]$$

$$\therefore \quad \overline{P}_2 = -3.535\,\mathbf{i} + 2.12\,\mathbf{j} - 2.828\,\mathbf{k}\ \text{N}$$

The resultant force $\overline{R} = \overline{P}_1 + \overline{P}_2$

$$= (7.07\,\mathbf{i} + 4.24\,\mathbf{j} + 5.65\,\mathbf{k}) + (-3.535\,\mathbf{i} + 2.12\,\mathbf{j} - 2.828\,\mathbf{k})$$

$$\overline{R} = 3.535\,\mathbf{i} + 6.36\,\mathbf{j} + 2.822\,\mathbf{k}\ \text{N}$$ .... **Ans.**

b) Moment of the force R about the origin D.

$$\overline{M}_D^R = \overline{r}_{DB} \times \overline{R}$$

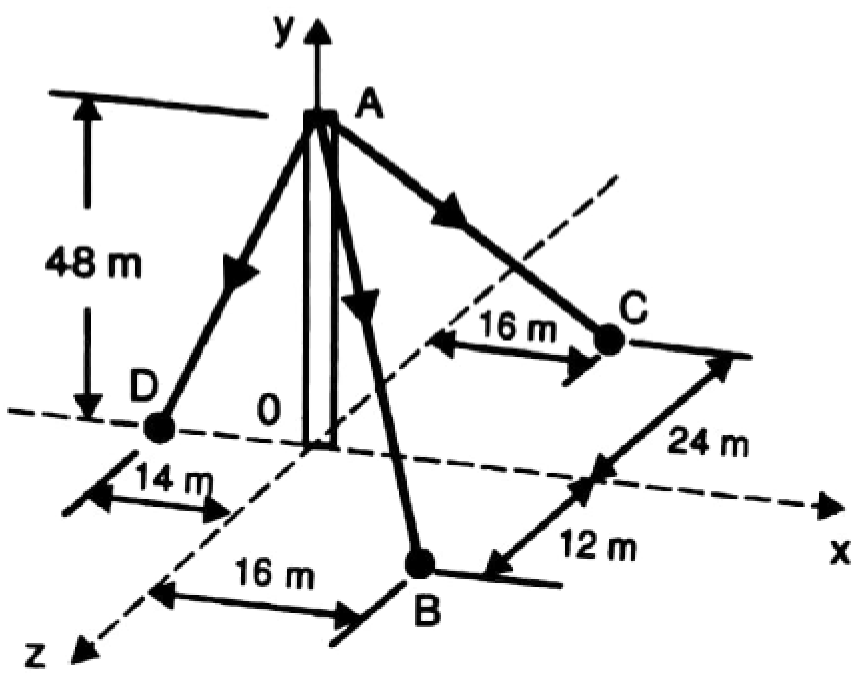
$$= (7\,\mathbf{i} + 4\,\mathbf{j} + 2\,\mathbf{k}) \times (3.535\,\mathbf{i} + 6.36\,\mathbf{j} + 2.822\,\mathbf{k})$$

$$\overline{M}_D^R = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & 4 & 2 \\ 3.535 & 6.36 & 2.822 \end{vmatrix}$$

$$\therefore \quad \overline{M}_D^R = -1.432\,\mathbf{i} - 12.68\,\mathbf{j} + 30.38\,\mathbf{k}\ \text{Nm}$$ .... **Ans.**

Practice Problem:

Knowing that the tension in AC = 20 kN, determine the required values of T_{AB} and T_{AD} so that the resultant of the three forces applied at A is vertical. Also find the resultant.



Solution: This is a concurrent space force system consisting of three forces T_{AC}, T_{AB} and T_{AD} meeting at A. Coordinates of different points are A (0, 48, 0) m, B (16, 0, 12) m, C (16, 0, – 24) m and D (– 14, 0, 0) m

$$\bar{T}_{AC} = T_{AC} \cdot \hat{e}_{AC}$$

$$= 20\left[\frac{16\mathbf{i} - 48\mathbf{j} - 24\mathbf{k}}{\sqrt{16^2 + 48^2 + 24^2}}\right]$$

$$\therefore \bar{T}_{AC} = 5.714\,\mathbf{i} - 17.143\,\mathbf{j} - 8.57\,\mathbf{k} \text{ kN}$$

$$\bar{T}_{AB} = T_{AB} \cdot \hat{e}_{AB}$$

$$= \bar{T}_{AB}\left[\frac{16\mathbf{i} - 48\mathbf{j} + 12\mathbf{k}}{\sqrt{16^2 + 48^2 + 12^2}}\right]$$

$$\therefore \bar{T}_{AB} = \bar{T}_{AB}\,(0.3077\,\mathbf{i} - 0.923\,\mathbf{j} + 0.2307\,\mathbf{k}) \text{ kN}$$

$$\bar{T}_{AD} = T_{AD} \cdot \hat{e}_{AD}$$

$$= \bar{T}_{AD}\left[\frac{-14\mathbf{i} - 48\mathbf{j}}{\sqrt{14^2 + 48^2}}\right]$$

$$\therefore \bar{T}_{AD} = \bar{T}_{AD}\,(-0.28\,\mathbf{i} - 0.96\,\mathbf{j}) \text{ kN}$$

The resultant force $\bar{R} = \bar{T}_{AC} + \bar{T}_{AB} + \bar{T}_{AD}$

Also since the resultant force is vertical, i.e. along y axis, implies that $\sum F_y = R$, $\sum F_x =$ and $\sum F_z = 0$,

Using $\sum F_x = 0$

$$5.714 + 0.3077\, T_{AB} - 0.28\, T_{AD} = 0 \qquad \ldots\ldots\ldots (1)$$

Using $\sum F_z = 0$

$$-8.57 + 0.2307\, T_{AB} = 0 \qquad \ldots\ldots\ldots (2)$$

Solving equations (1) and (2), we get

$T_{AB} = 37.15$ kN and $T_{AD} = 61.23$ kN .... **Ans.**

Using $\sum F_y = R$

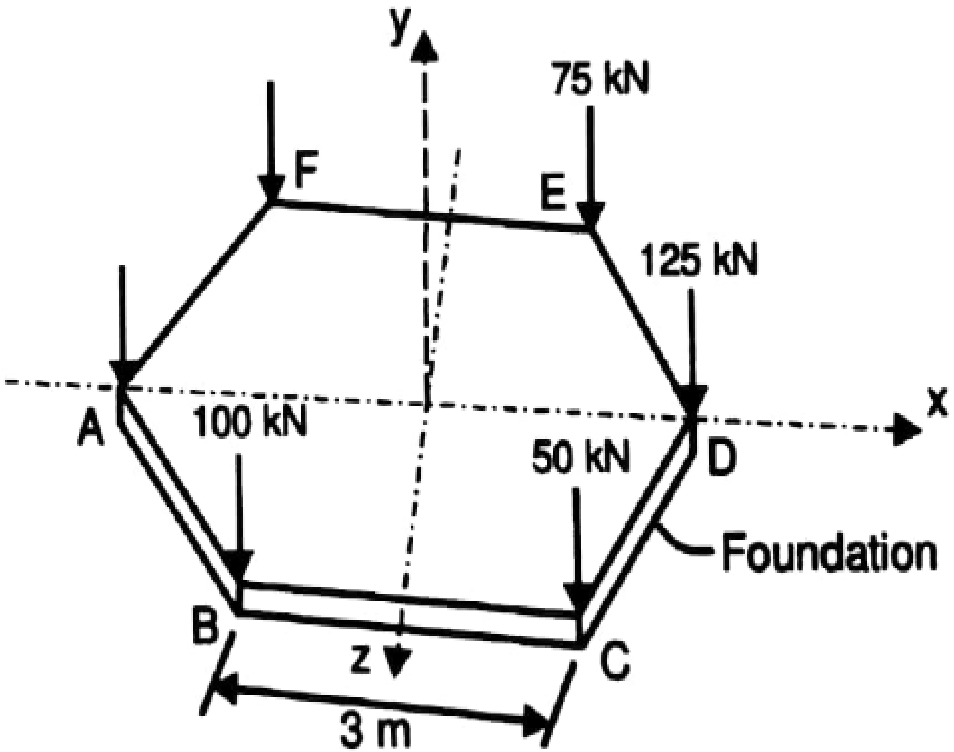
$$-17.143 - 0.923\, T_{AB} - 0.96\, T_{AD} = R$$

$\therefore$ $R = -17.143 - 0.923(37.15) - 0.96(61.23)$

$\therefore$ $R = -110.2$ kN Or $\overline{R} = -110.2\,\mathbf{j}$ kN .... **Ans.**

Practice Problem:

Determine the loads to be applied at A and F, if the resultant of all six loads is to pass through the centre of the foundation of hexagonal shape of side 3 m.



Solution: This is a parallel space force system consisting of six forces.

$F_1 = 75$ kN, $F_2 = 125$ kN, $F_3 = 50$ kN,

$F_4 = 100$ kN, F_5 and F_6.

$\bar{F}_1 = -75\,\mathbf{j}$ kN

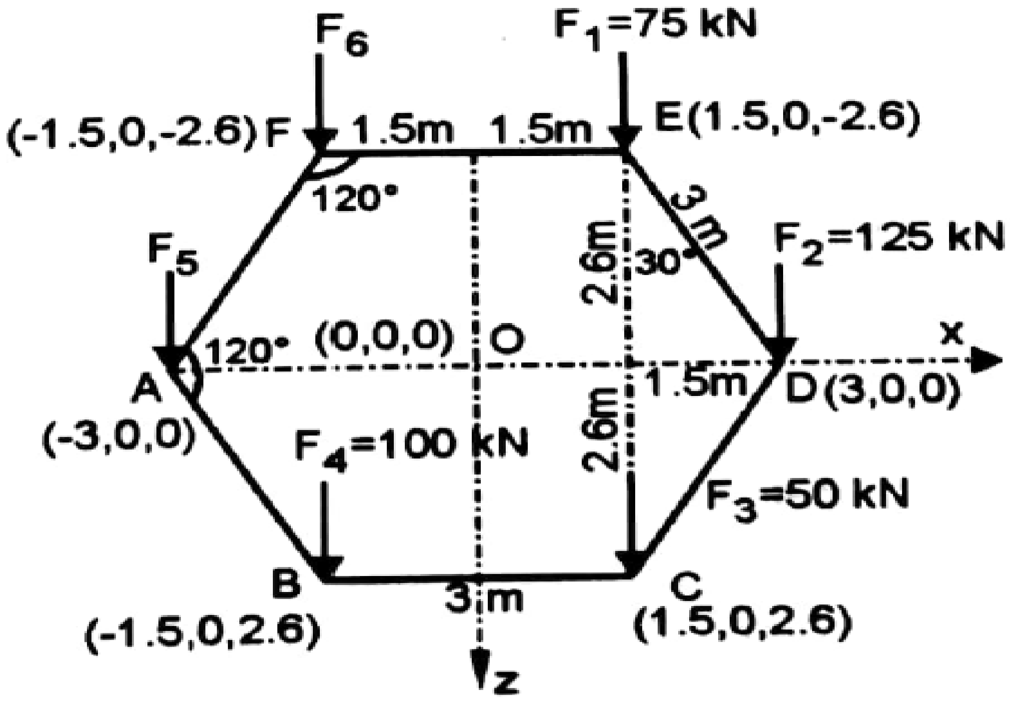
$\bar{F}_2 = -125\,\mathbf{j}$ kN

$\bar{F}_3 = -50\,\mathbf{j}$ kN

$\bar{F}_4 = -100\,\mathbf{j}$ kN

$\bar{F}_5 = -F_5\,\mathbf{j}$ kN

$\bar{F}_6 = -F_6\,\mathbf{j}$ kN



Taking moments of all the forces about the origin.

$$\overline{M}_O^{F_1} = \overline{r}_{OE} \times \overline{F}_1$$

$$= (1.5\,\mathbf{i} - 2.6\,\mathbf{k}) \times (-75\,\mathbf{j})$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & 0 & -2.6 \\ 0 & -75 & 0 \end{vmatrix}$$

$$\therefore \quad \overline{M}_O^{F_1} = -195\,\mathbf{i} - 112.5\,\mathbf{j} \text{ kNm}$$

$$\overline{M}_O^{F_2} = \overline{r}_{OD} \times \overline{F}_2$$

$$= (3\,\mathbf{i}) \times (-125\,\mathbf{j})$$

$$\therefore \quad \overline{M}_O^{F_2} = -375\,\mathbf{k} \text{ kNm}$$

$$\overline{M}_O^{F_3} = \overline{r}_{OC} \times \overline{F}_3$$

$$= (1.5\,\mathbf{i} + 2.6\,\mathbf{k}) \times (-50\,\mathbf{j})$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & 0 & 2.6 \\ 0 & -50 & 0 \end{vmatrix}$$

$$\therefore \quad \overline{M}_O^{F_3} = 130\,\mathbf{i} - 75\,\mathbf{k} \ \text{kNm}$$

$$\overline{M}_O^{F_5} = \overline{r}_{OA} \times \overline{F}_5$$

$$= (-3\,\mathbf{i}) \times (-F_5\,\mathbf{j})$$

$$\therefore \quad \overline{M}_O^{F_5} = 3F_5\,\mathbf{k} \ \text{kNm}$$

$$\overline{M}_O^{F_4} = \overline{r}_{OB} \times \overline{F}_4$$

$$= (-1.5\,\mathbf{i} + 2.6\,\mathbf{k}) \times (-100\,\mathbf{j})$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1.5 & 0 & 2.6 \\ 0 & -100 & 0 \end{vmatrix}$$

$$\therefore \quad \overline{M}_O^{F_4} = 260\,\mathbf{i} + 150\,\mathbf{k} \ \text{kNm}$$

$$\overline{M}_O^{F_6} = \overline{r}_{OF} \times \overline{F}_6$$

$$= (-1.5\,\mathbf{i} - 2.6\,\mathbf{k}) \times (-F_6\,\mathbf{j})$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1.5 & 0 & -2.6 \\ 0 & -F_6 & 0 \end{vmatrix}$$

$$\therefore \quad \overline{M}_O^{F_6} = (-2.6F_6)\,\mathbf{i} + (1.5F_6\,\mathbf{k}) \ \text{kNm}$$

$\overline{M}_O^R = 0$ .... Since the resultant passing through the origin.

Using Varinon's Theorem

$$\sum \overline{M}_O^F = \sum \overline{M}_O^R$$

$$\overline{M}_O^{F_1} + \overline{M}_O^{F_2} + \overline{M}_O^{F_3} + \overline{M}_O^{F_4} + \overline{M}_O^{F_5} + \overline{M}_O^{F_6} = \overline{M}_O^R$$

$$(-195\,\mathbf{i} - 112.5\,\mathbf{j}) + (-375\,\mathbf{k}) + (130\,\mathbf{i} - 75\,\mathbf{k}) + (260\,\mathbf{i} + 150\,\mathbf{k}) + (3F_5\,\mathbf{k})$$

$$+ ((-2.6F_6)\,\mathbf{i} + (1.5F_6\,\mathbf{k})) = 0$$

$$(195 - 2.6F_6)\mathbf{i} + (-412.5 + 3F_5 + 1.5F_6)\mathbf{k} = 0$$

Equating the coefficients we get

$$195 - 2.6F_6 = 0$$

$\therefore$ $F_6 = 75$ kN .... **Ans.**

$$-412.5 + 3F_5 + 1.5F_6 = 0$$

$$-412.5 + 3F_5 + 1.5 \times (75) = 0$$

$\therefore$ $F_5 = 100$ kN .... **Ans.**

Equilibrium of Space Forces:

When the resultant of a system is zero, the system is said to be in equilibrium. For the resultant to be zero, the resultant force $\bar{F}$ and the resultant moment $\bar{M}$ should be zero.

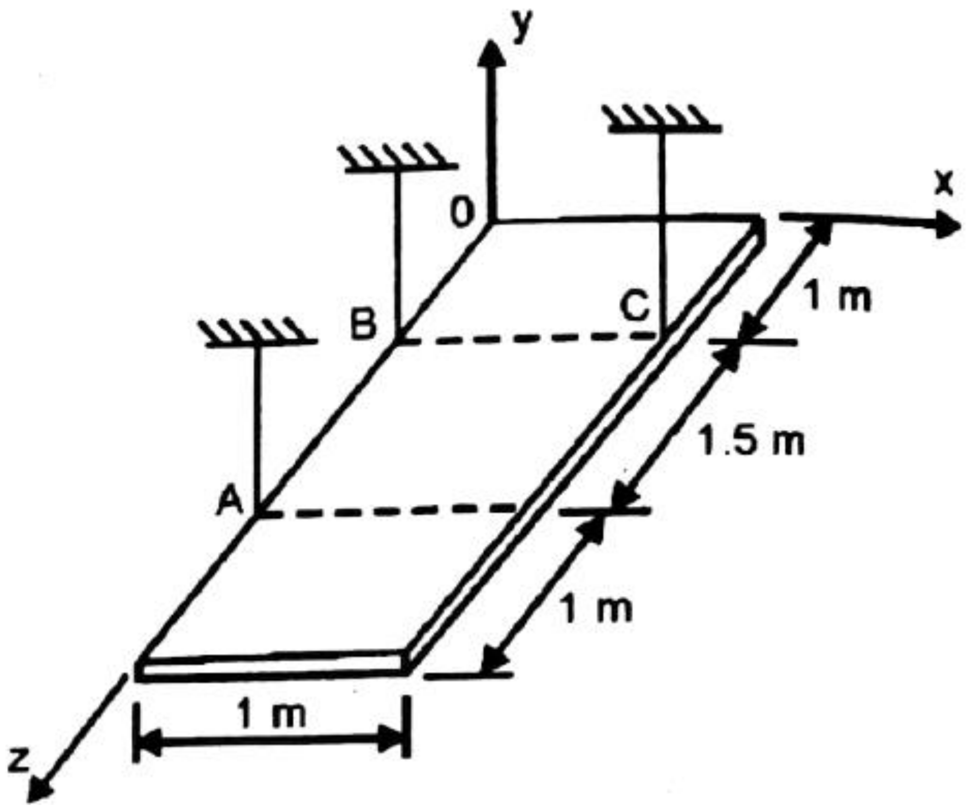
$$\Sigma F_x = 0 \qquad \Sigma M_x = 0$$

$$\Sigma F_y = 0 \qquad \Sigma M_y = 0$$

$$\Sigma F_z = 0 \qquad \Sigma M_z = 0$$

Problem:

Find the tension in each of the cable supporting the rectangular plate. The plate weighs 500 N.



Solution:

Applying COE

Equating moments @ xx axis to zero.

$$\Sigma M_{xx} = 0$$

$$-T_A \times 2.5 - T_B \times 1 - T_C \times 1 + 500 \times 1.75 = 0$$

$$2.5\,T_A + T_B + T_C = 875 \quad \text{.........} (1)$$

Equating moments @ zz axis to zero.

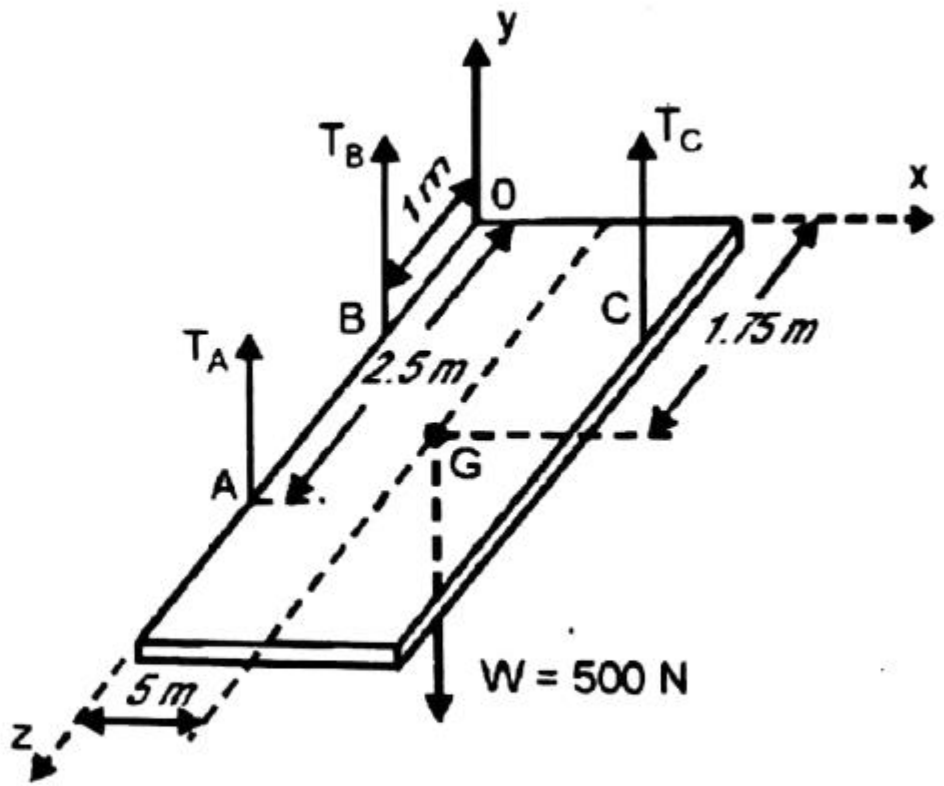
$$\Sigma M_{zz} = 0$$

$$-T_C \times 1 - 500 \times 0.5 = 0$$

$$\therefore \quad T_C = 250 \text{ N}$$

$$\Sigma F_y = 0$$

$$T_A + T_B + T_C - 500 = 0 \quad \text{.........} (2)$$



Subsituting value of T_C in equation (2)

$$T_A + T_B + 250 - 500 = 0$$

$$T_A + T_B = 250 \qquad \text{.........} \ (3)$$

Subsituting value of T_C in equation (2)

$$2.5\ T_A + T_B + 250 = 875$$

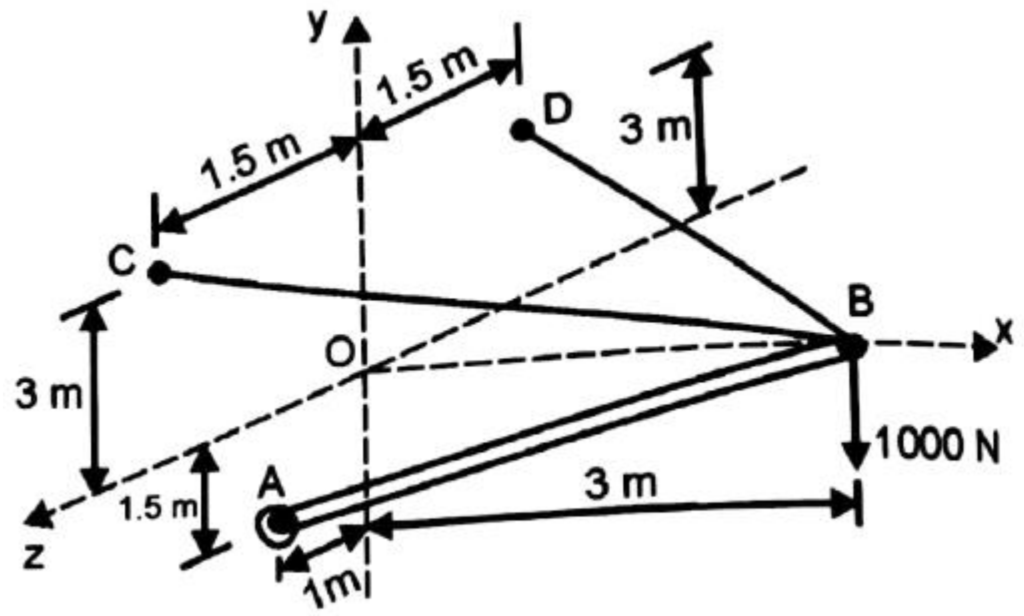
$$\therefore \qquad 2.5\ T_A + T_B = 625 \qquad \text{.........} \ (4)$$

Solving equations (3) and (4), we get

$T_A = 250$ N and $T_B = 0$ .... **Ans.**

Problem:

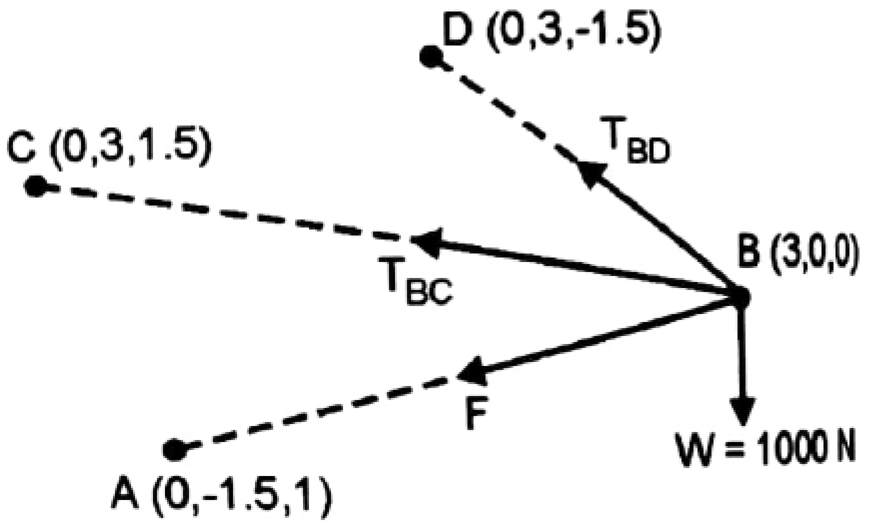
A rod AB supports a load of 1000 N as shown. Neglect weight of the rod. Determine tension in each cable and force in the rod AB.



Solution:

Let T_{BD}, T_{BC} be the tension in the cables BD and BC respectively.

F ---> Force in rod AB



$$\overline{T}_{BD} = T_{BD} \cdot \hat{e}_{BD}$$

$$= T_{BD}\left[\frac{-3\,\mathbf{i} + 3\,\mathbf{j} - 1.5\,\mathbf{k}}{\sqrt{3^2 + 3^2 + 1.5^2}}\right]$$

$$\overline{T}_{BD} = T_{BD}\,(-0.6667\,\mathbf{i} + 0.6667\,\mathbf{j} - 0.3333\,\mathbf{k})\ \text{N}$$

$$\overline{T}_{BC} = T_{BC} \cdot \hat{e}_{BC}$$

$$= T_{BC} \left[\frac{-3\,\mathbf{i} + 3\,\mathbf{j} + 1.5\,\mathbf{k}}{\sqrt{3^2 + 3^2 + 1.5^2}} \right]$$

$$\therefore \quad \overline{T}_{BC} = T_{BC} \left(-0.6667\,\mathbf{i} + 0.6667\,\mathbf{j} + 0.3333\,\mathbf{k} \right) \text{ N}$$

$$\overline{F} = F \cdot \hat{e}_{BA}$$

$$= F \left[\frac{-3\,\mathbf{i} - 1.5\,\mathbf{j} + \mathbf{k}}{\sqrt{3^2 + 1.5^2 + 1^2}} \right]$$

$$\therefore \quad \overline{F} = F \left(-0.8571\,\mathbf{i} - 0.4286\,\mathbf{j} + 0.2857\,\mathbf{k} \right) \text{ N}$$

$$\overline{W} = -1000\,\mathbf{j} \text{ N}$$

Applying COE

$\sum F_X = 0$

$$-0.6667\,T_{BD} - 0.6667\,T_{BC} - 0.8571\,F = 0 \qquad \text{.........} \;(1)$$

$\sum F_Y = 0$

$$0.6667\,T_{BD} + 0.6667\,T_{BC} - 0.4286\,F - 1000 = 0 \qquad \text{.........} \;(2)$$

$\sum F_Z = 0$

$$-0.3333\,T_{BD} + 0.3333\,T_{BC} + 0.2857\,F = 0 \qquad \text{.........} \;(3)$$

Solving equation (1) , (2) and (3) we get

$T_{BD} = 166.7$ N, $T_{BC} = 833.4$ N and $F = -777.8$ N