

# Engineering Mechanics

## Module 1.1 – Resultant of forces

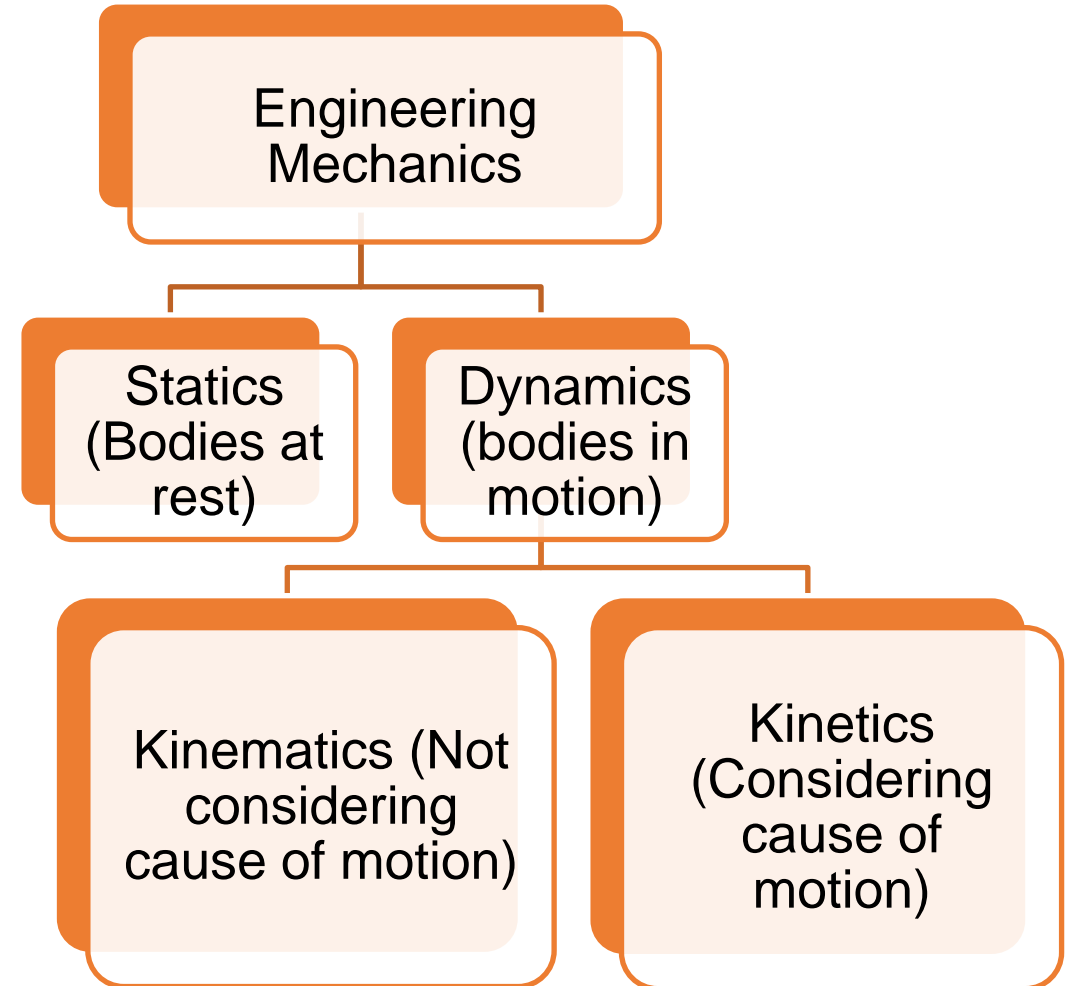
Presented by: Prof. Abhishek P. S. Bhadauria



# Engineering Mechanics

## What is mechanics?

It is the branch of engineering science which deals with effect and analysis of forces acting on the body, which may be at rest or in the motion



# Brief Contents of module 1.1

Types of different force system acting on a body

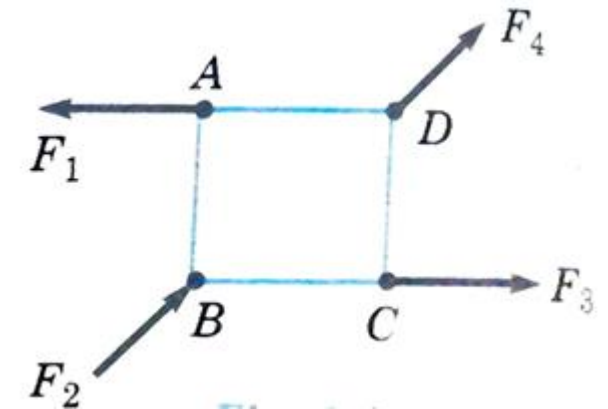
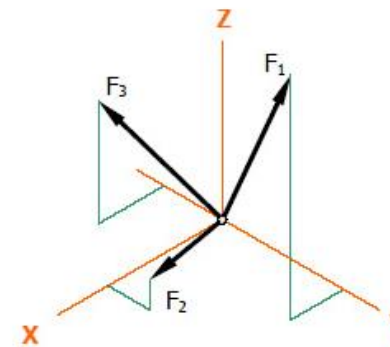
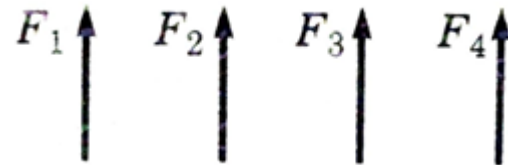
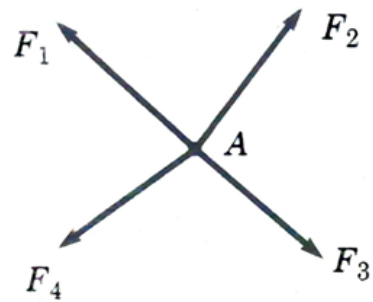
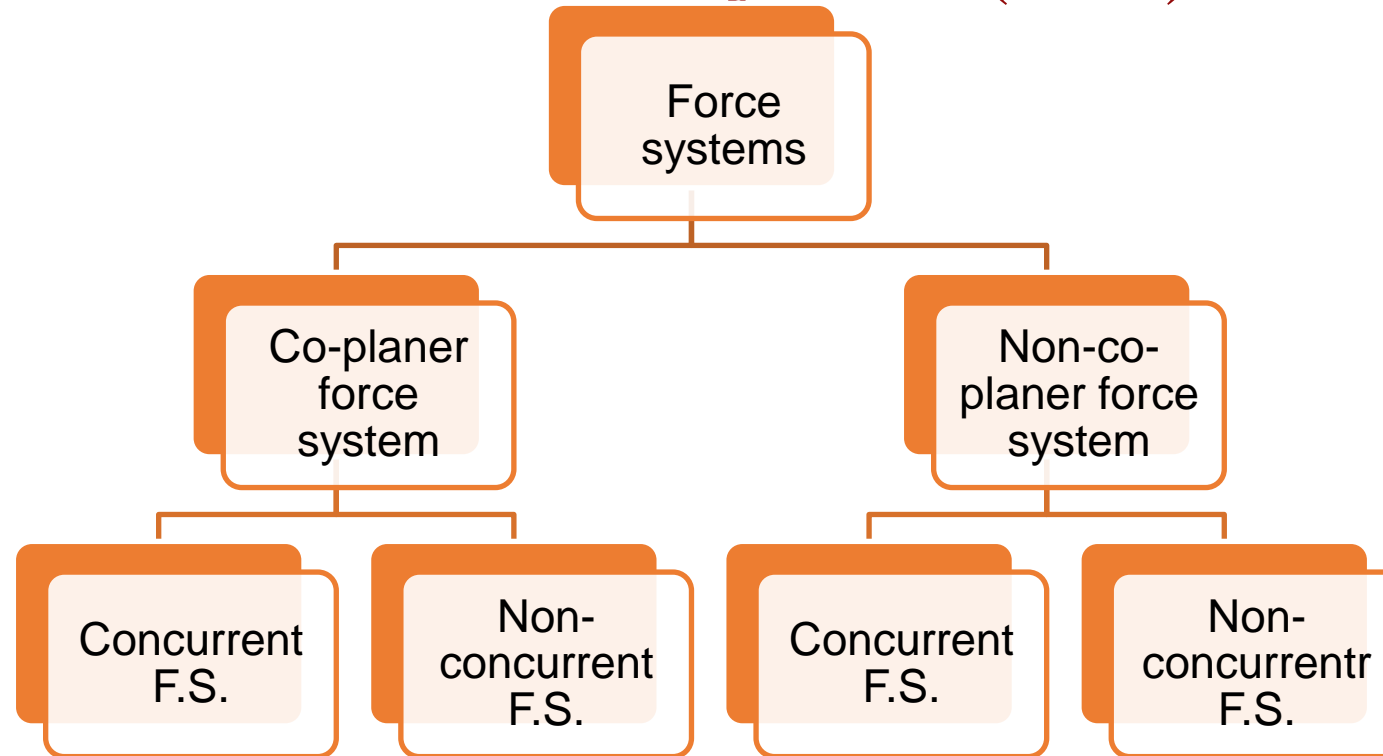
Resultant of a force system and its importance

Resultant of concurrent force system

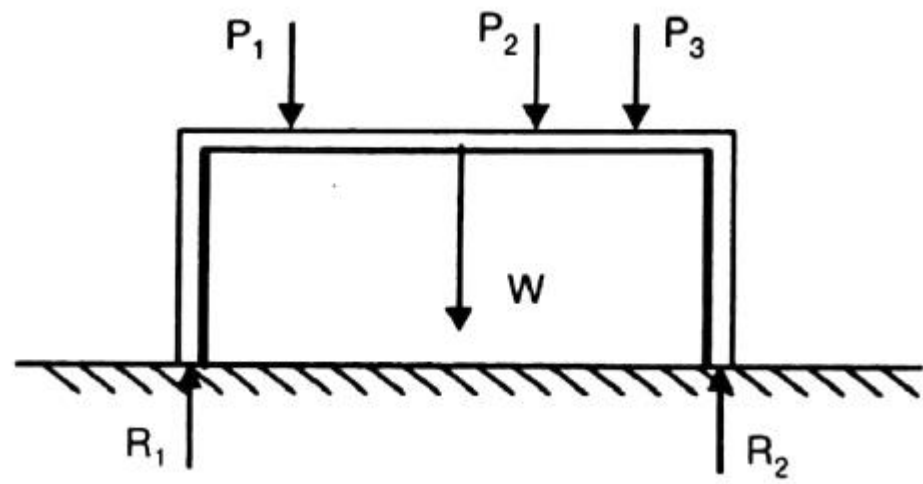
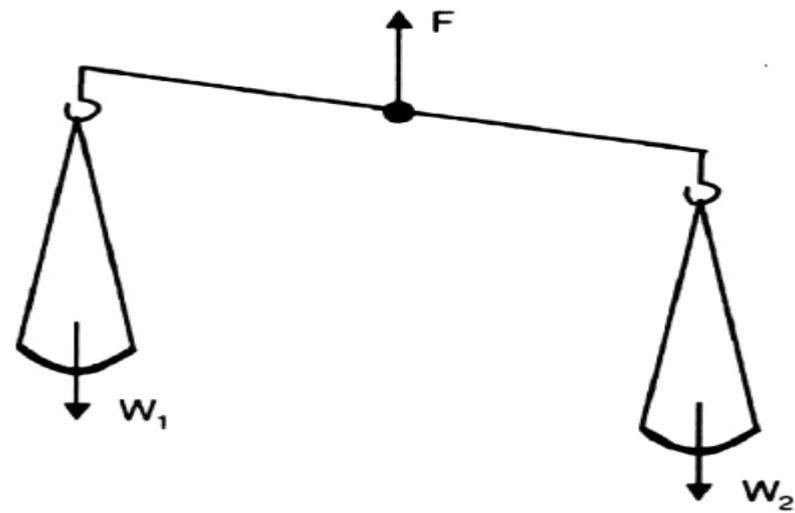
Resultant of Non-Concurrent force system

Resultant of General force system

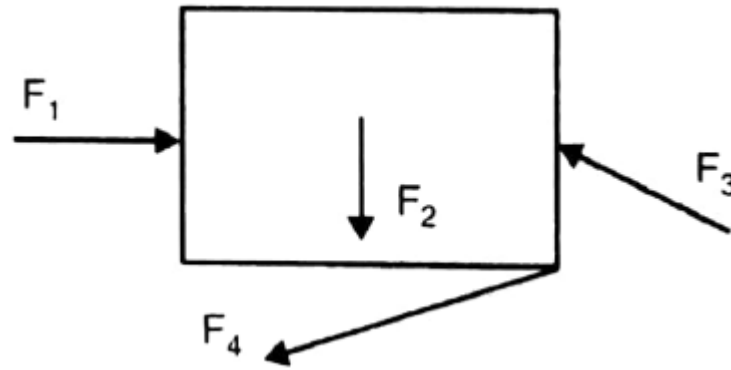
# Types of different force system (F.S.) acting on a body



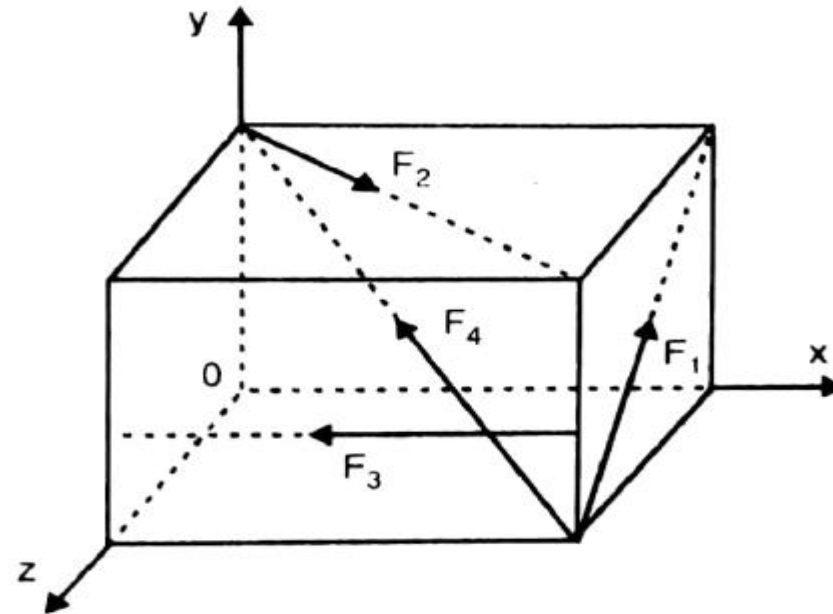
Parallel system :



*General system :*



*Non-coplanar System*



# Resultant of a force system and its importance

## Basic terminologies: Force and moment

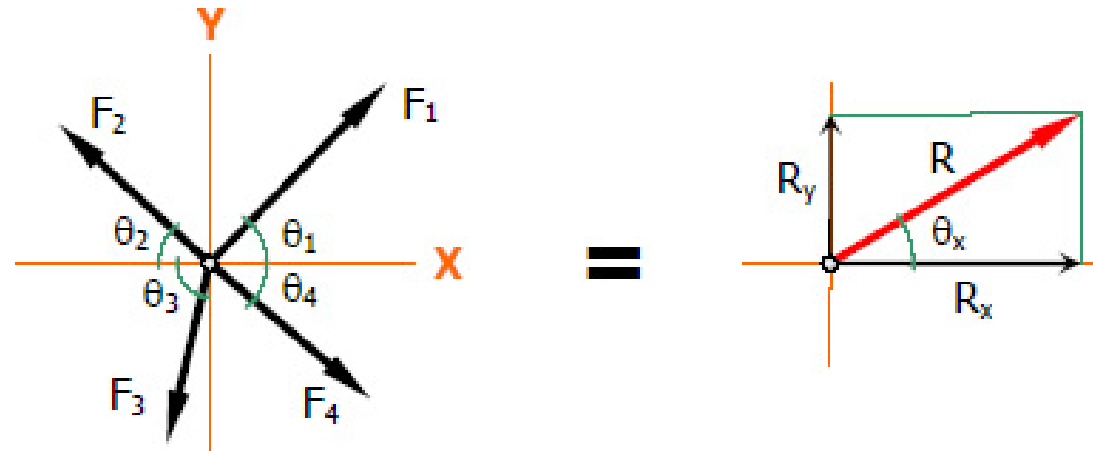
Force: It is an external agency acting on the body which will cause the motion of the body from one location to another

Moment: The turning tendency of the body due to application of a force on it about a particular point is called moment. Moment magnitude is given by force \* perpendicular distance

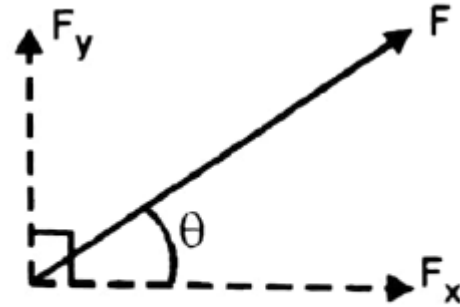
What is resultant?

It is a single equivalent force acting on a body producing the same effect as that of multiple forces producing on it.

$$R_x = \Sigma F_x$$
$$R_y = \Sigma F_y$$
$$R = \sqrt{R_x^2 + R_y^2}$$
$$\tan \theta_x = \frac{R_y}{R_x}$$



## Resolution of a Force



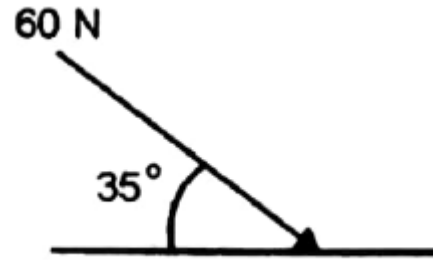
Force  $F$  resolved into components  $F_x$  and  $F_y$

$$F_x = F \cos \theta \rightarrow$$

$$F_y = F \sin \theta \uparrow$$



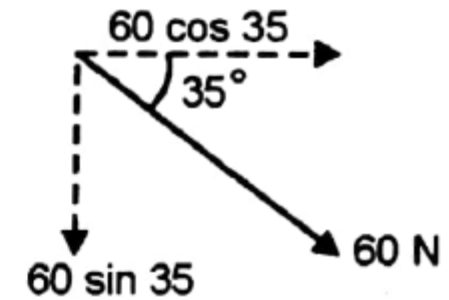
## Resolve the force

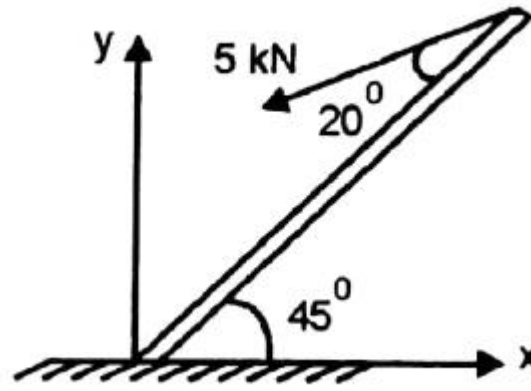


### Solution:

$$F_x = 60 \cos 35 = 49.15 \text{ N} \rightarrow$$

$$F_y = 60 \sin 35 = 34.41 \text{ N} \downarrow$$

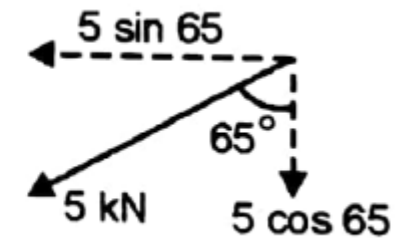


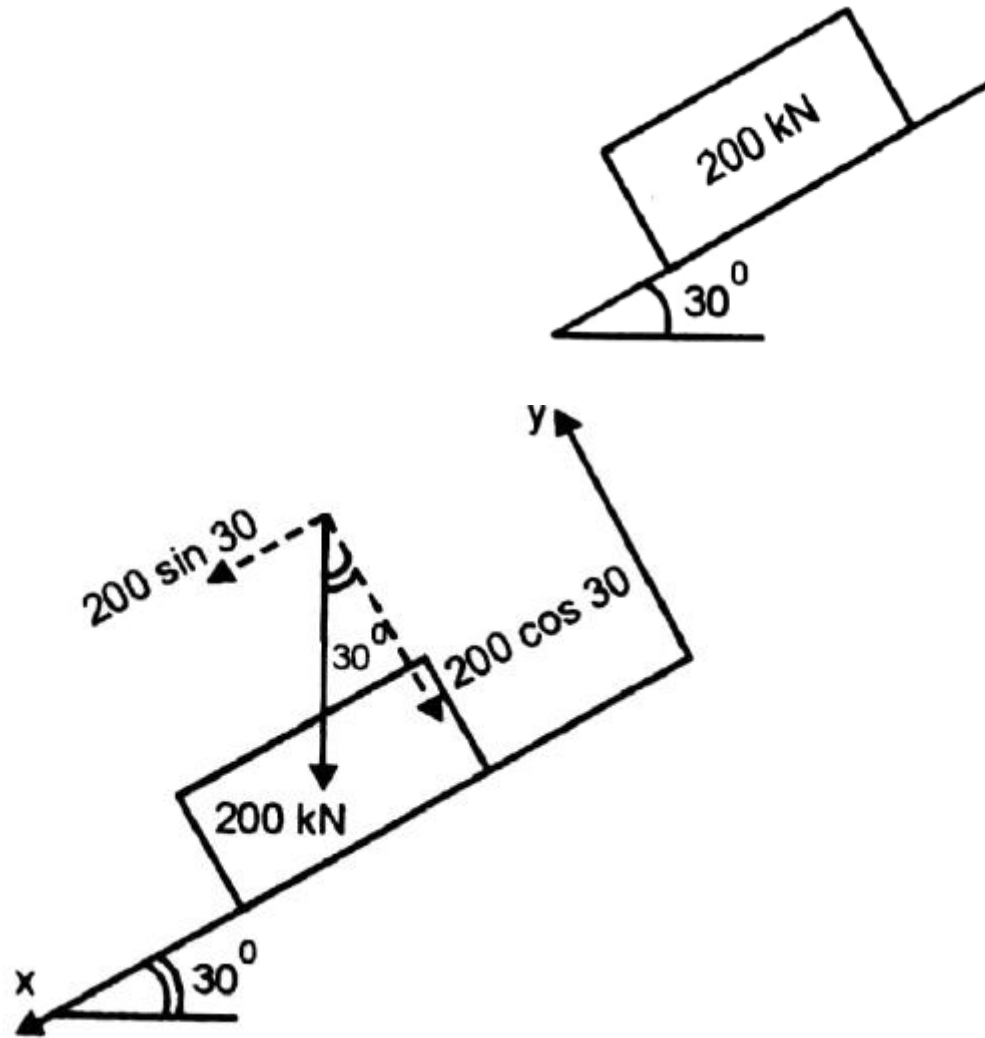


**Solution:** Total angle made by  $5 \text{ kN}$  force with the vertical is  $65^\circ$

$$\therefore F_x = 5 \sin 65 = 4.53 \text{ kN} \leftarrow$$

$$F_y = 5 \cos 65 = 2.11 \text{ kN} \downarrow$$





Resolving we get

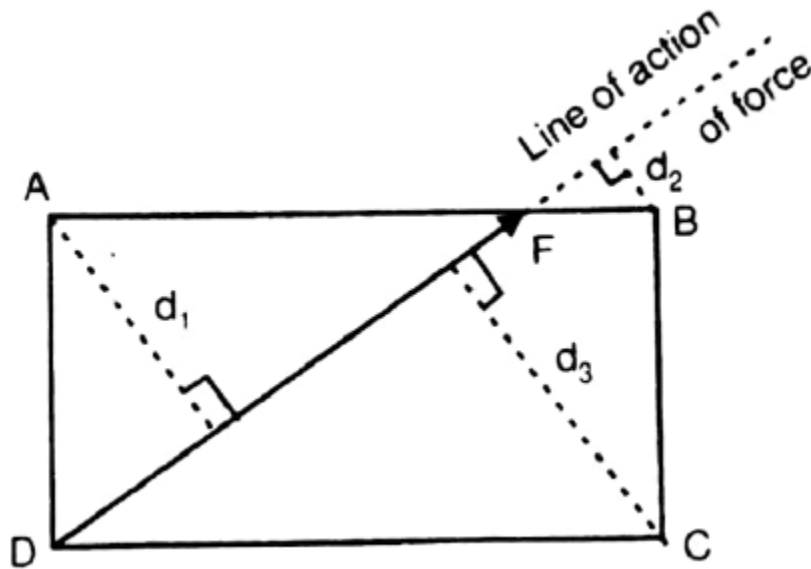
$$F_x = 200 \sin 30 = 100 \text{ N}$$

$$F_y = 200 \cos 30 = 173.2 \text{ N}$$

## Moment of a Force

The rotational effect or moment is measured as the product of the force and the perpendicular distance from the moment centre to the force. This perpendicular distance is known as the *moment arm* 'd'.

$$\therefore \mathbf{M} = \mathbf{F} \times \mathbf{d}$$



The moment of  $F$

$$\text{about A} = F \times d_1 \text{ (anti-clockwise)} = +(F \times d_1)$$

$$\text{about B} = F \times d_2 \text{ (clockwise)} = -(F \times d_2)$$

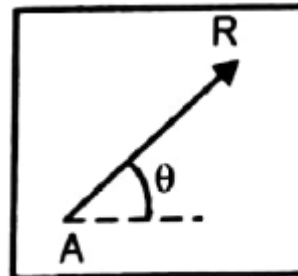
$$\text{about C} = F \times d_3 \text{ (clockwise)} = -(F \times d_3)$$

$$\text{about D} = 0$$

# Composition of Forces (Resultant of forces)

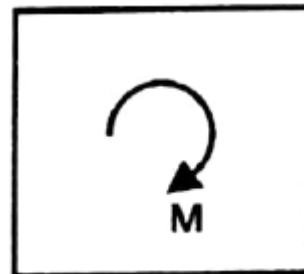
## Types of Resultant

### 1. Resultant- Force



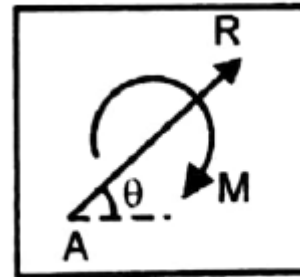
Resultant-Force

### 2. Resultant- Couple



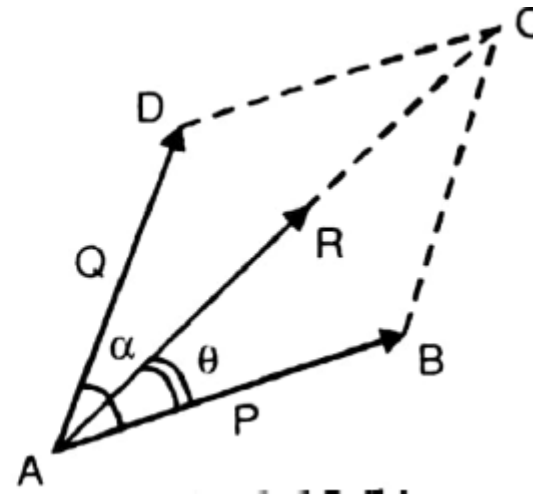
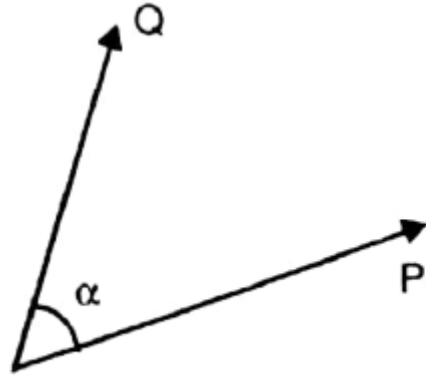
Resultant-Couple

### 3. Resultant-Force Couple



Resultant-Force Couple

## Parallelogram Law of Forces :



Mathematically

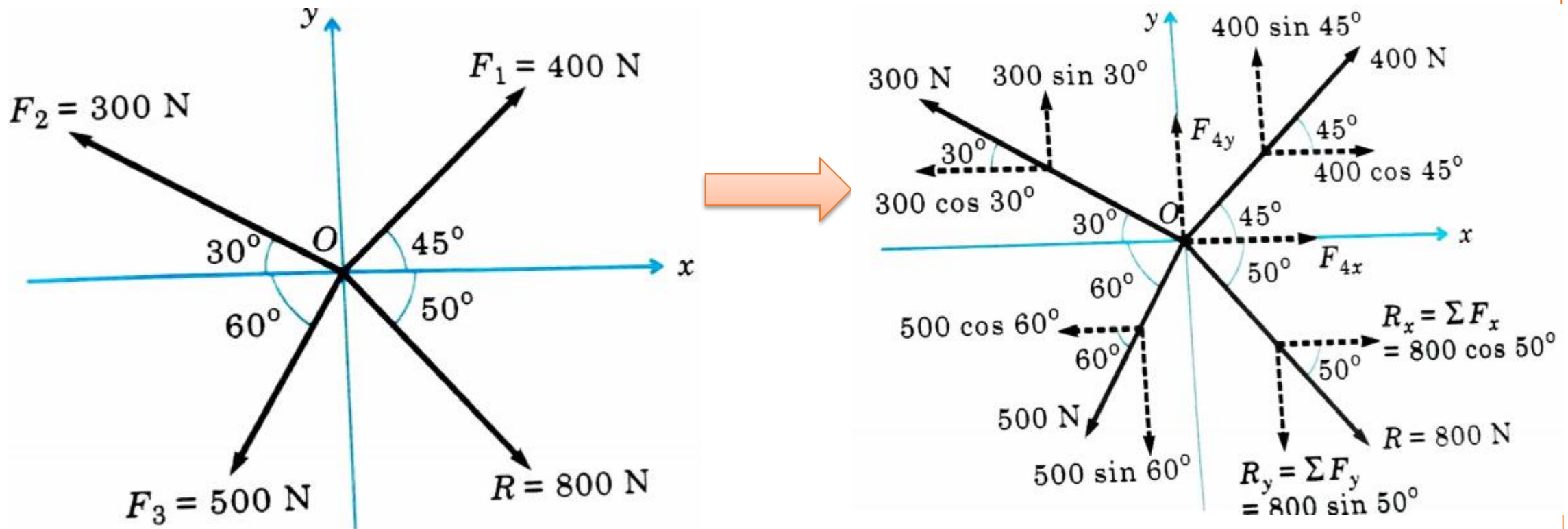
$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \alpha}$$

$$\tan \theta = \frac{Q \sin \alpha}{P + Q \cos \alpha}$$

Where  $\theta$  is the angle made by resultant R with the force P

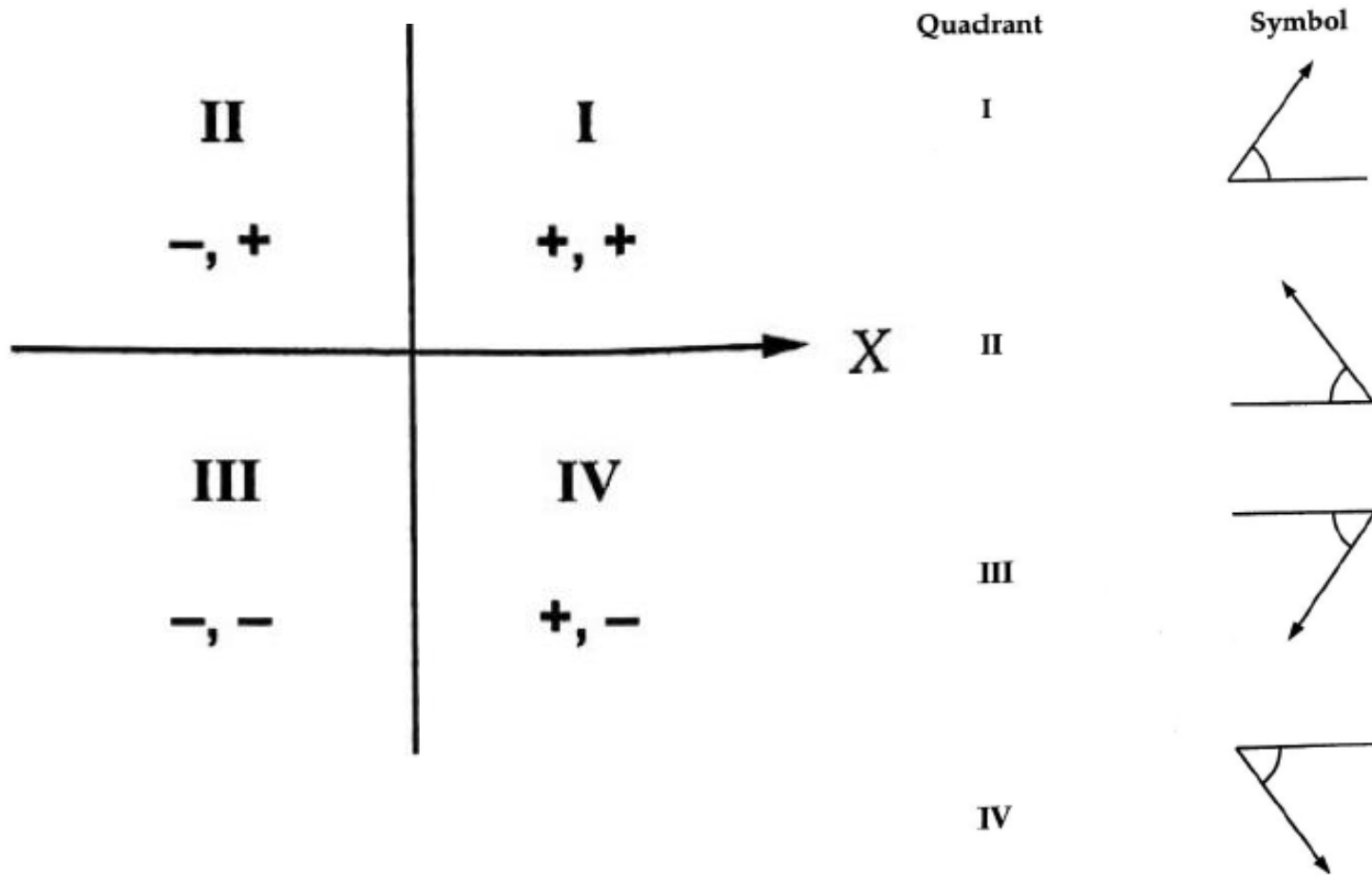
# Resolution of force

Resolution of any particular force is nothing but representing a single force into its equivalent rectangular components

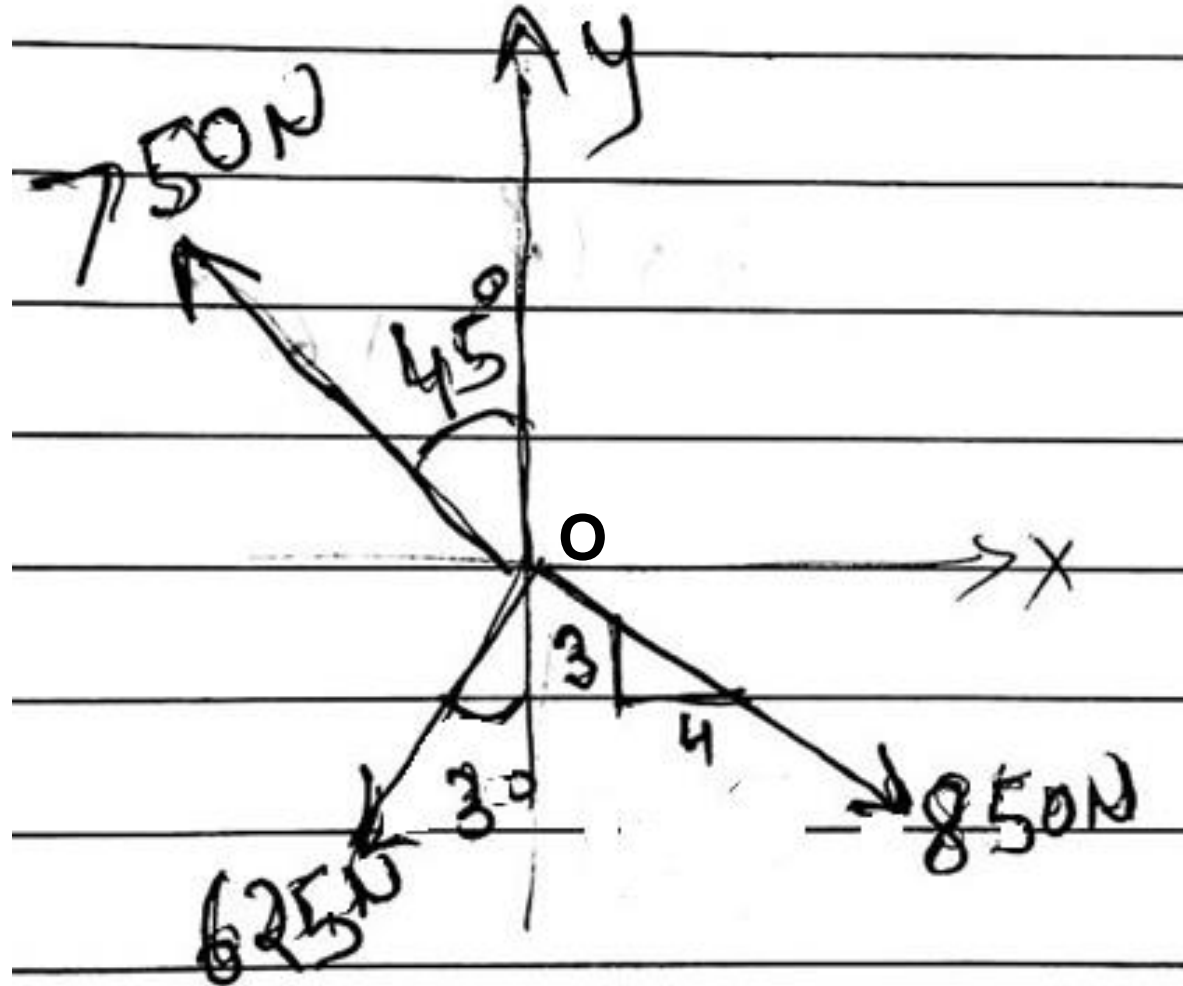




# Sign convention and symbolic representation of Resultant



Determine the resultant of given concurrent force system in magnitude and direction and locate it.



# Solution

$$\begin{aligned}\Sigma F_x &= -750 \sin 45^\circ - 625 \sin 30^\circ \\ &+ 850 \sin 53.13^\circ \\ &= -162.83 \text{ N (←)}\end{aligned}$$

$$\begin{aligned}\Sigma F_y &= 750 \cos 45^\circ - 625 \cos 30^\circ \\ &- 850 \cos 53.13^\circ \\ &= -520.93 \text{ N}\end{aligned}$$

$$\tan \theta = \frac{520.93}{162.83} \quad \theta = 72.51^\circ$$

$$R = 545.78 \text{ N}$$

$$R_x = \Sigma F_x$$

$$R_y = \Sigma F_y$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\tan \theta_x = \frac{R_y}{R_x}$$

Resultant lies in third quadrant

Location of resultant

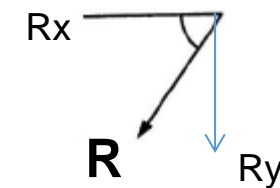
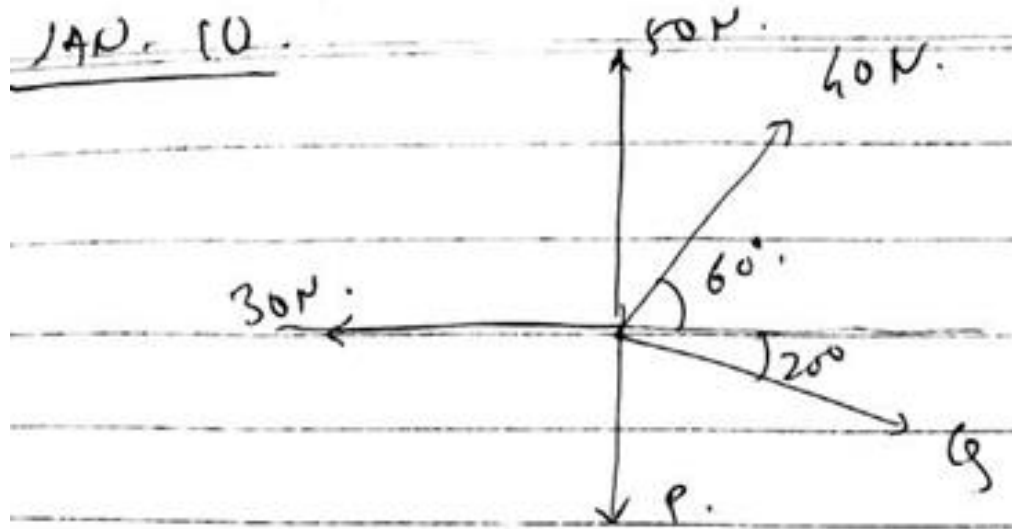


Figure shows coplanar concurrent force system. Find P & Q if the resultant of force system is zero.



As result is zero.

$$\sum F_x = 0 \quad \sum F_y = 0$$

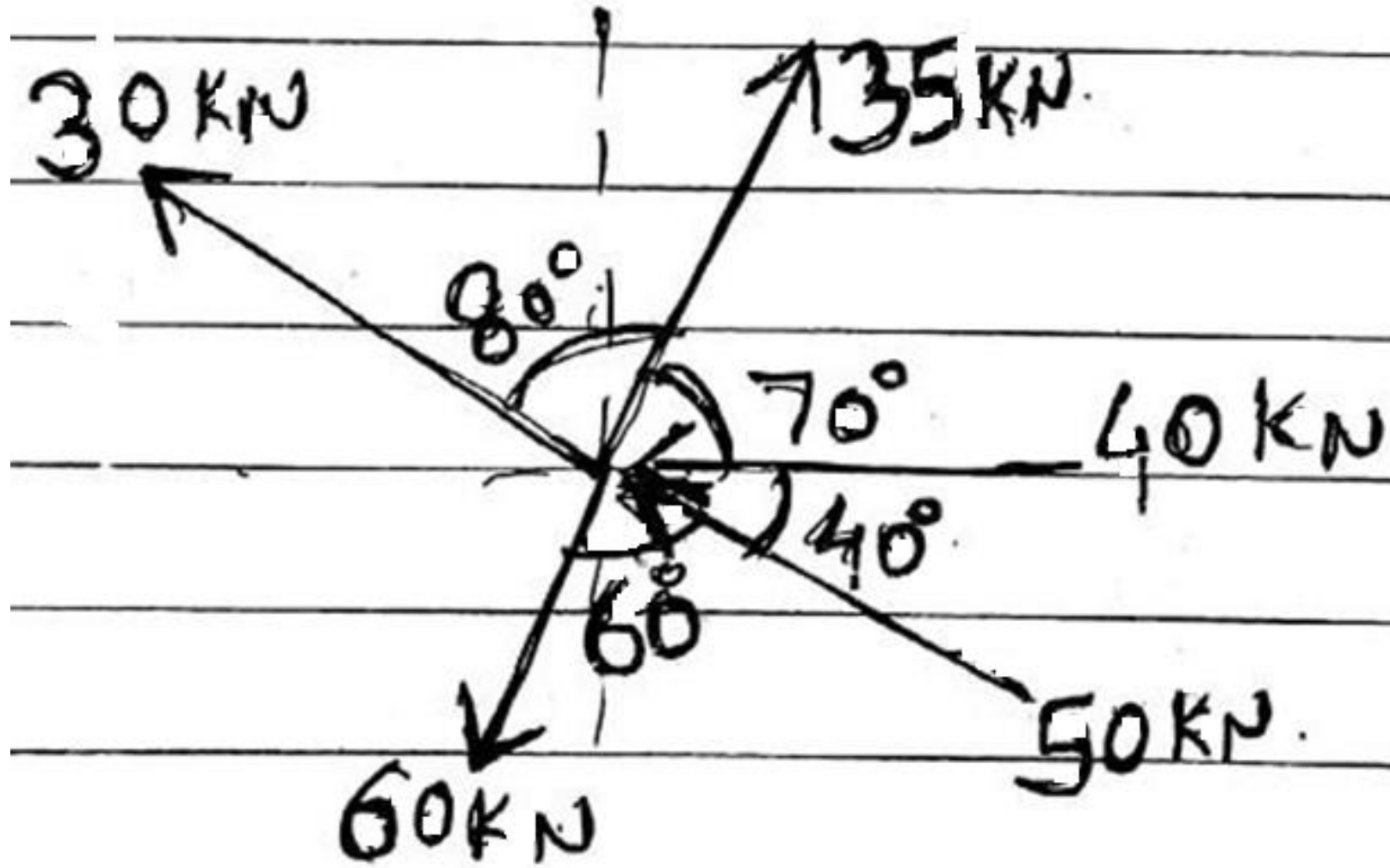
$$\sum F_x = 40 \cos 60 - 30 + Q \cos 20 = 0$$

$$Q = 10.64 \text{ N}$$

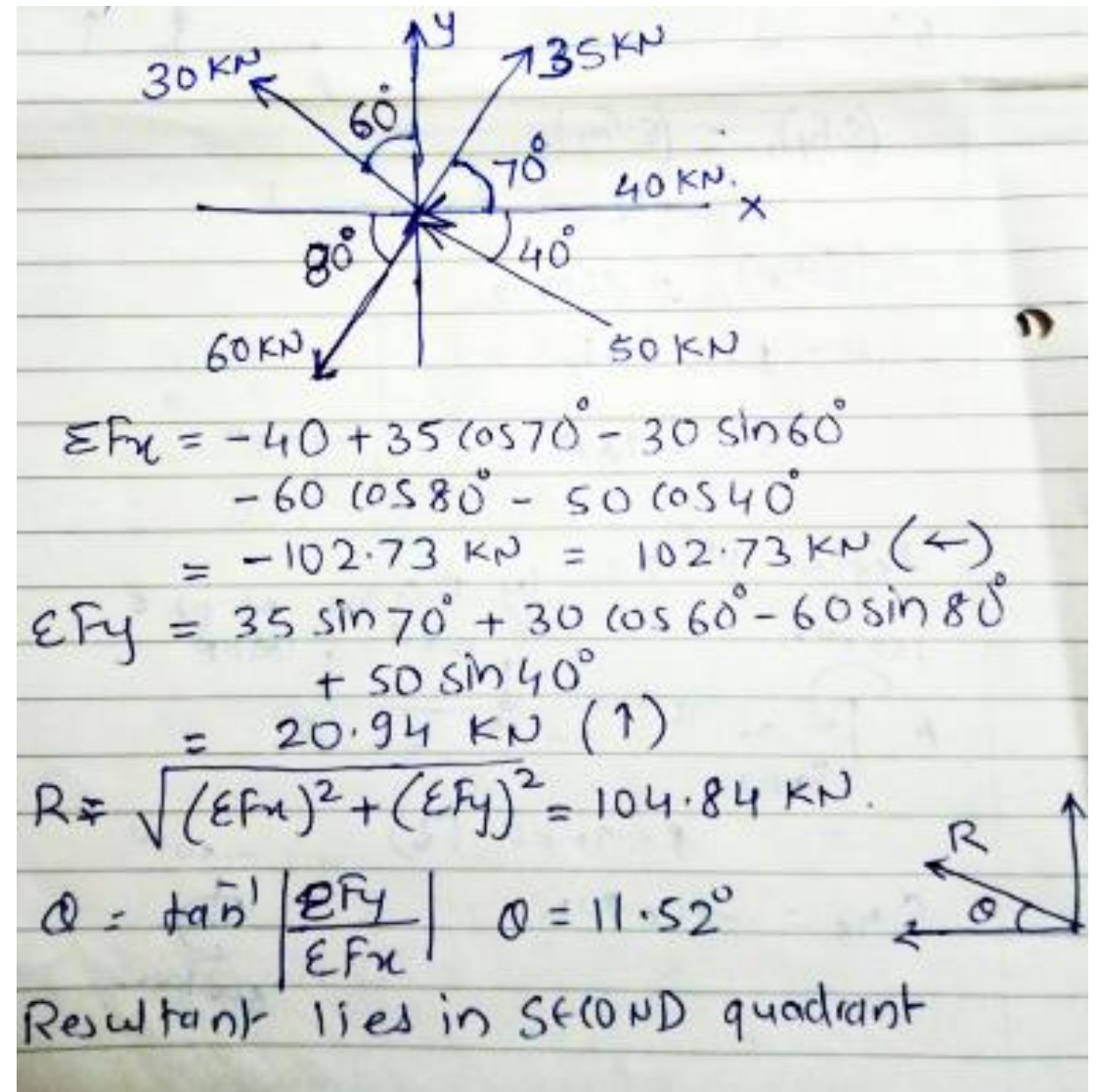
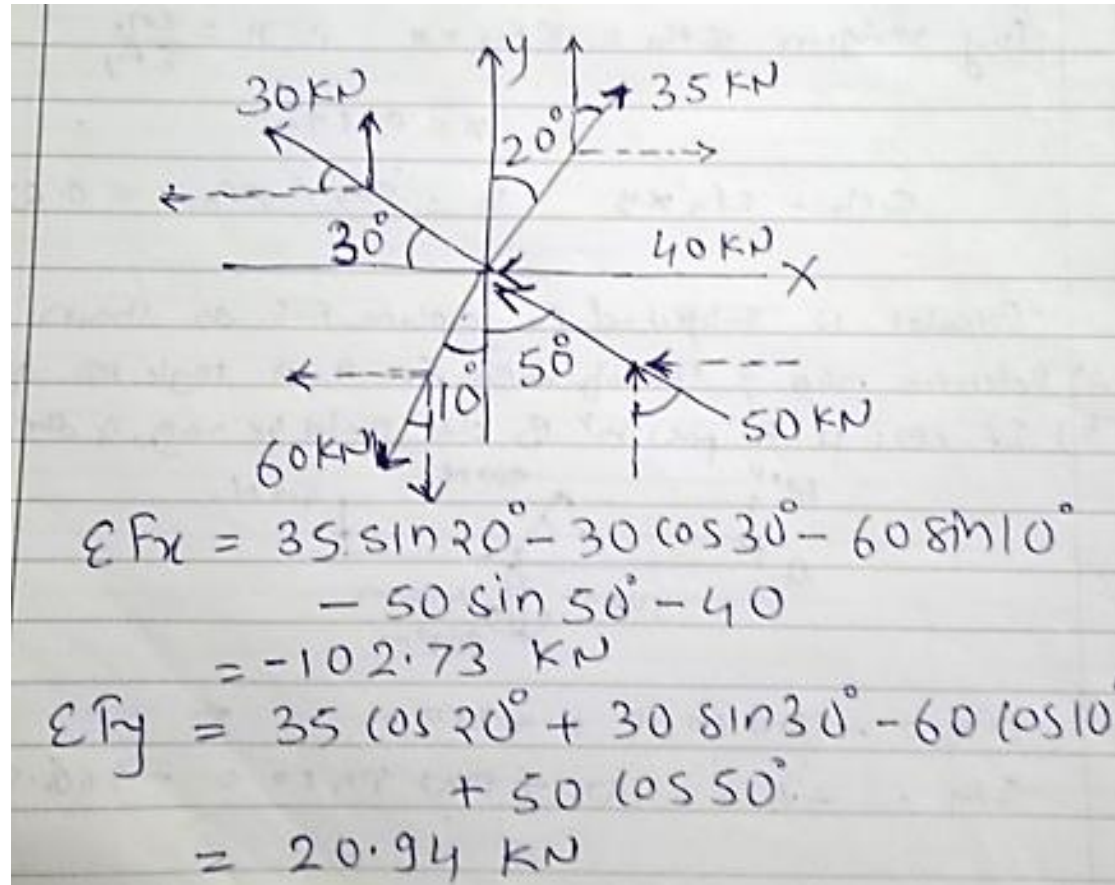
$$\sum F_y = 50 - P + 40 \sin 60 - Q \sin 20$$

$$P = 81 \text{ N}$$

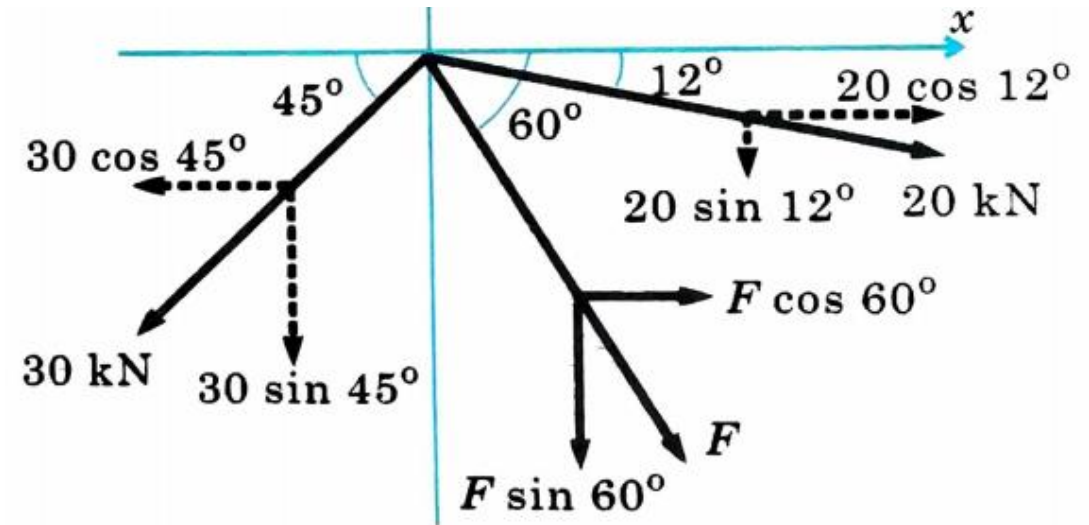
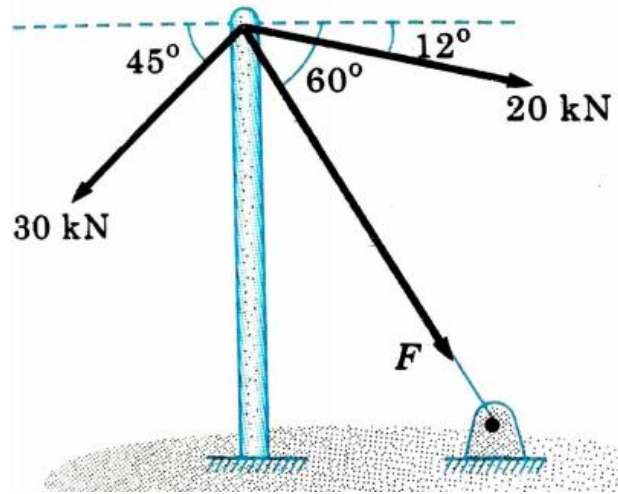
Determine the resultant of given concurrent force system in magnitude and direction and locate it.



# Solution



For the force system shown in fig. determine the value of force  $F$  so that the resultant of the system is vertical



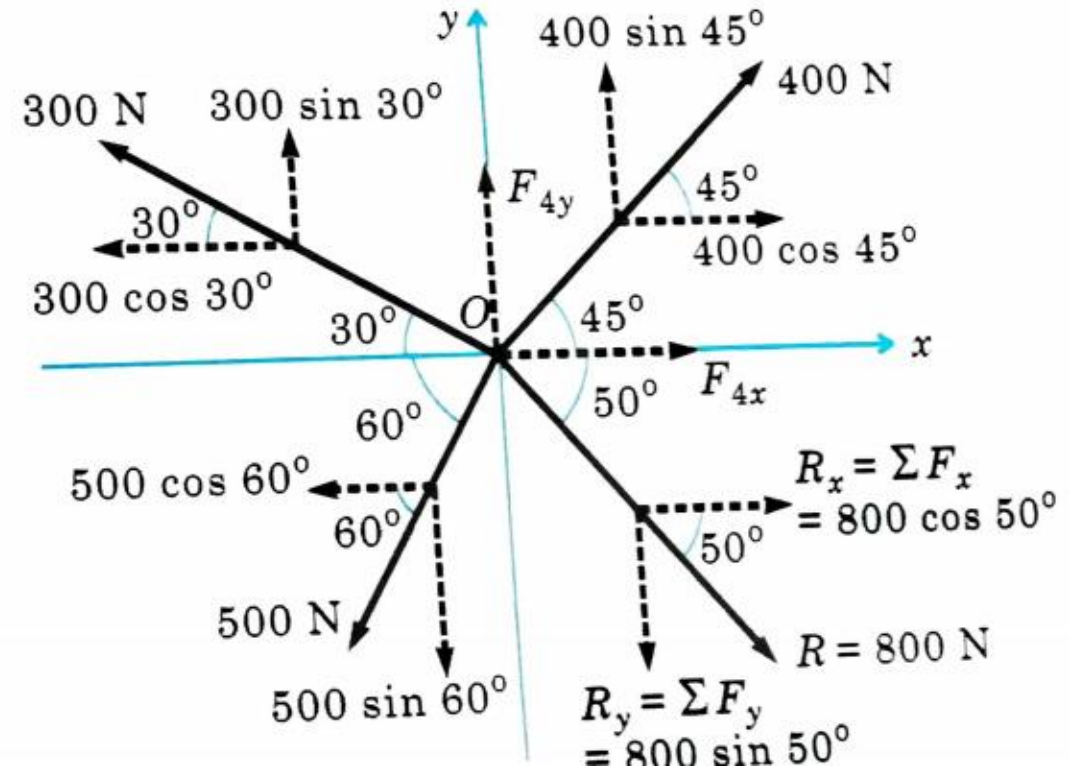
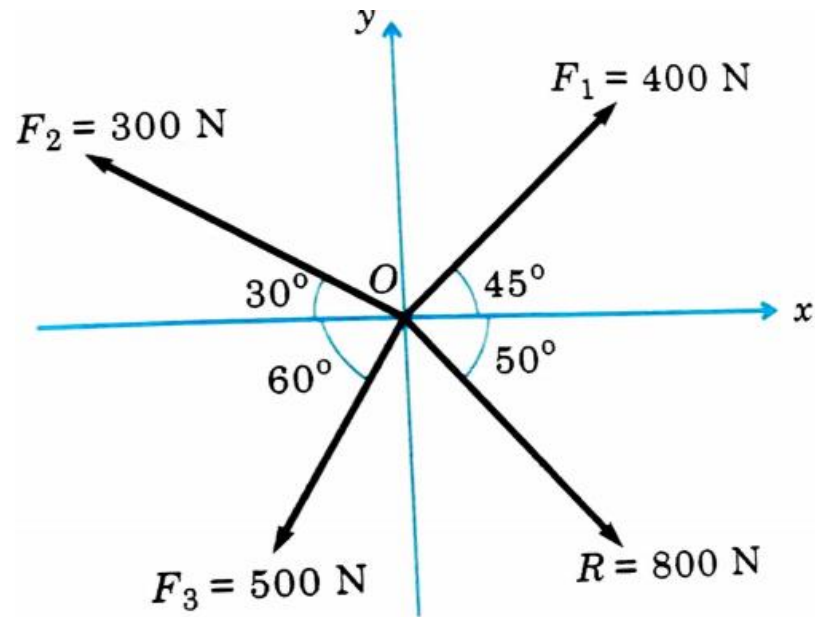
$$\Sigma F_x = 20 \cos 12^\circ + F \cos 60^\circ - 30 \cos 45^\circ = 0$$

$$\therefore F = 3.3 \text{ kN}$$

$$\begin{aligned} R = \Sigma F_y &= -30 \sin 45^\circ - F \sin 60^\circ - 20 \sin 12^\circ \\ &= -30 \sin 45^\circ - 3.3 \sin 60^\circ - 20 \sin 12^\circ \\ &= -28.23 \text{ kN} = 28.23 \text{ kN} (\downarrow) \end{aligned}$$

# Resultant of coplanar concurrent force system

Practice problem : Find the force  $F_4$  completely so that the resultant of the force system is as shown in fig.





**Solution:** This is a concurrent system of four forces.

Let  $(F_4)_x$  and  $(F_4)_y$  be the perpendicular components of the fourth force

Since it is given  $R = 800 \text{ N}$  at  $\theta = 50^\circ$

$$\therefore \sum F_x = 800 \cos 50 \rightarrow$$

$$\text{and } \sum F_y = 800 \sin 50 \downarrow$$

$$\sum F_x \rightarrow +ve$$

$$800 \cos 50 = 400 \cos 45 - 300 \cos 30 - 500 \cos 60 + (F_4)_x$$

$$\therefore (F_4)_x = 741.2 \text{ N} \rightarrow$$

$$\sum F_y \uparrow +ve$$

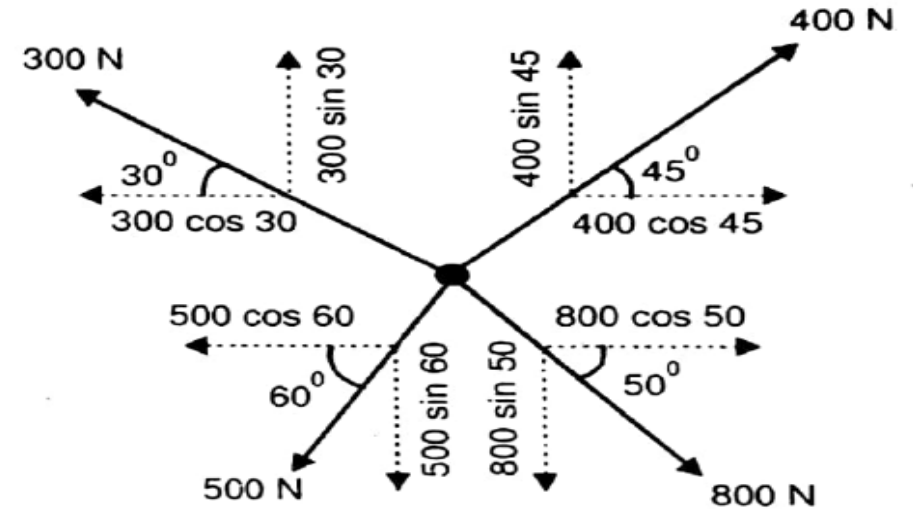
$$-800 \sin 50 = 400 \sin 45 + 300 \sin 30 - 500 \sin 60 + (F_4)_y$$

$$\therefore (F_4)_y = -612.6 \text{ N} \\ = 612.6 \text{ N} \downarrow$$

$$\text{Now } F_4 = \sqrt{(F_4)_x^2 + (F_4)_y^2} = \sqrt{741.2^2 + 612.6^2} = 961.6 \text{ N}$$

$$\text{also } \tan \theta = \frac{(F_4)_y}{(F_4)_x} = \frac{612.6}{741.2} \quad \therefore \theta = 39.6^\circ$$

The fourth force  $F_4 = 961.6 \text{ N}$  at  $\theta = 39.6^\circ$  .....**Ans.**

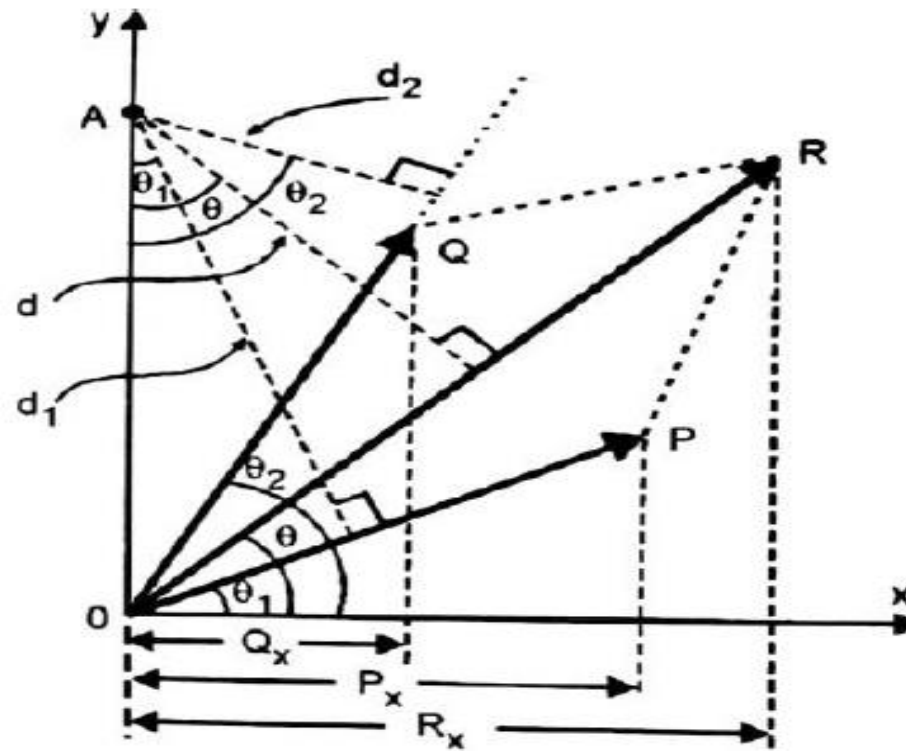


# Varignon's Theorem:

*"the algebraic sum of the*

*moments of a system of coplanar forces about any point in the plane is equal to the moment of the resultant force of the system about the same point".*

$$\Sigma M_A^F = M_A^R$$



Let the x component of forces P, Q and R be  $P_x$ ,  $Q_x$  and  $R_x$  respectively

$$M_A^P = P \times d_1 \quad \text{-----(1)}$$

$$M_A^Q = Q \times d_2 \quad \text{-----(2)}$$

$$\begin{aligned} M_A^R &= R \times d \\ &= R ( OA \cos \theta ) \\ &= OA ( R_x ) \quad \text{-----(3)} \end{aligned}$$

Adding equations (1) and (2) we have

$$M_A^P + M_A^Q = P d_1 + Q d_2$$

$$\begin{aligned} \Sigma M_A^F &= + ( P \times OA \cos \theta_1 ) + ( Q \times OA \cos \theta_2 ) \\ &= OA \cdot P_x + OA \cdot Q_x \quad \begin{array}{l} \text{since } P_x = P \cos \theta_1 \\ \text{and } Q_x = Q \cos \theta_2 \end{array} \end{aligned}$$

$$= OA ( P_x + Q_x )$$

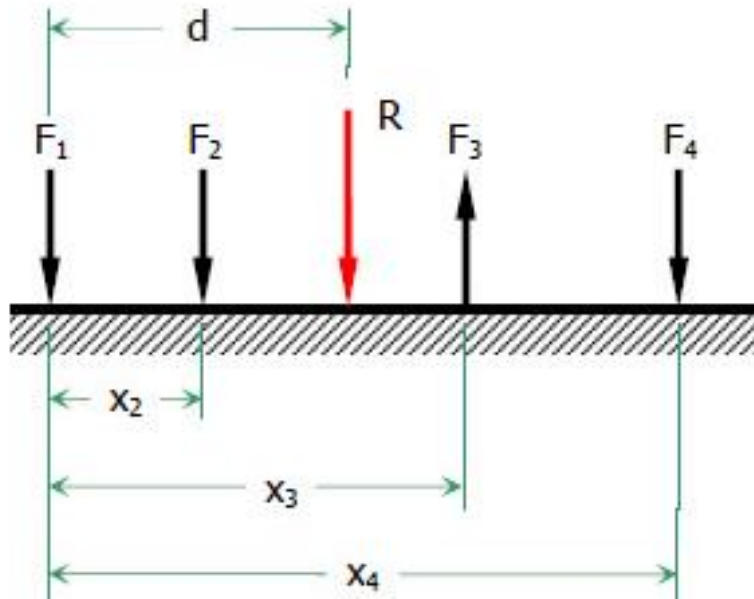
$$\Sigma M_A^F = OA ( R_x ) \text{ -----(4)}$$

Comparing equation (4) with (3)

$$\Sigma M_A^F = M_A^R$$

# Varignon's Theorem and Resultant of parallel forces

It states, "If a number of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point."



Coplanar parallel force system

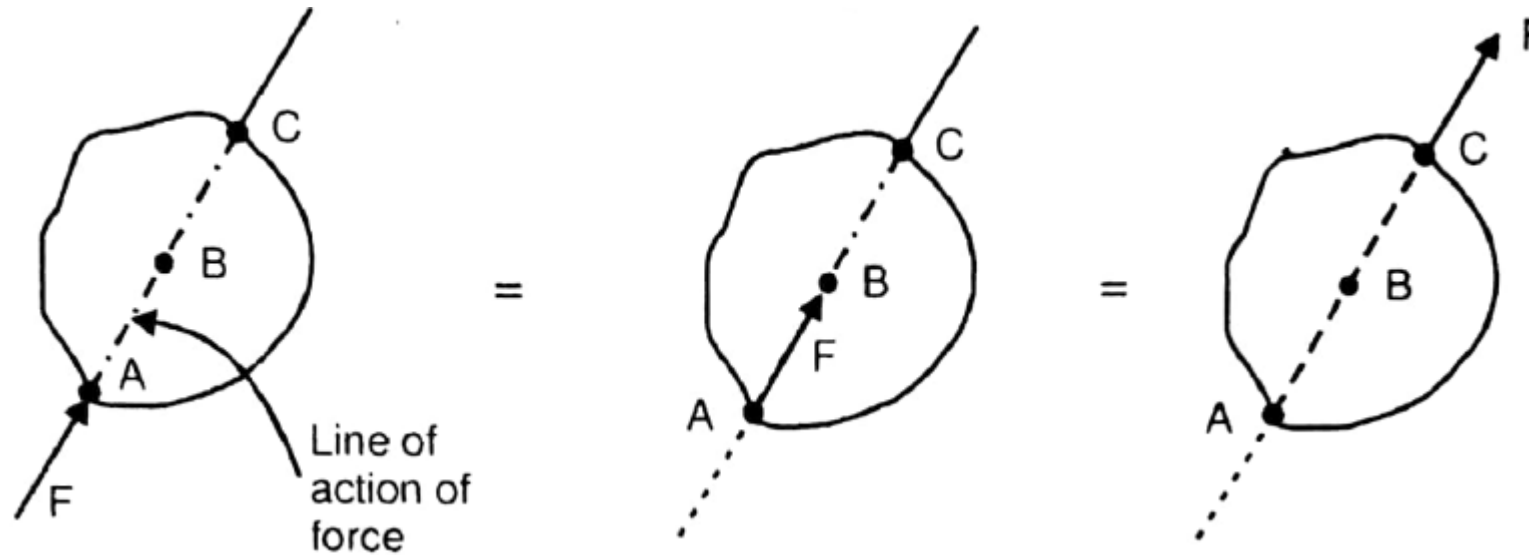
$$R = \Sigma F = F_1 + F_2 + F_3 + \dots$$

By applying Varignon's theorem,

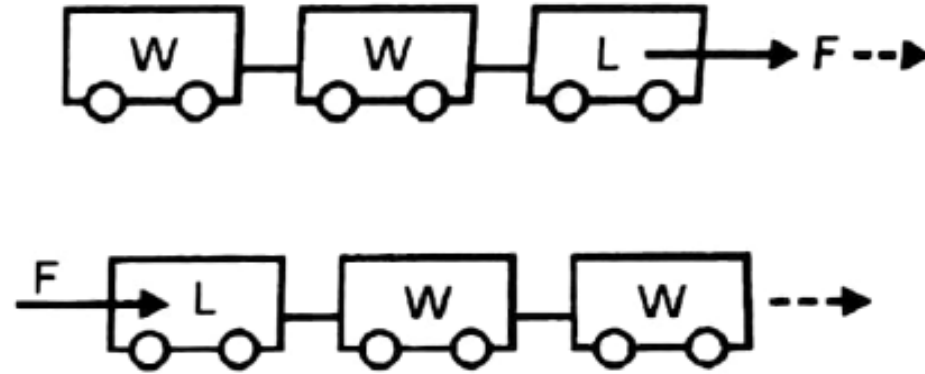
$$Rd = \Sigma Fx = F_1x_1 + F_2x_2 + F_3x_3 + \dots$$

# Principle of Transmissibility of Force:

It states “A force being a sliding vector continues to act along its line of action and therefore makes no change if it acts from a different point on its line of action on a rigid body”.

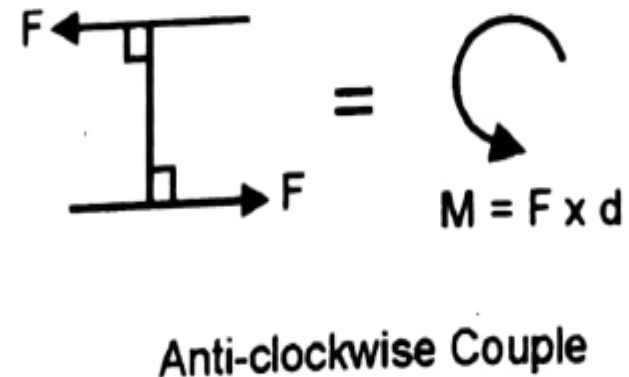
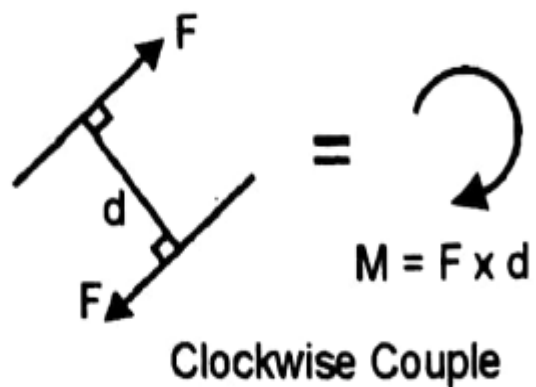


Example:



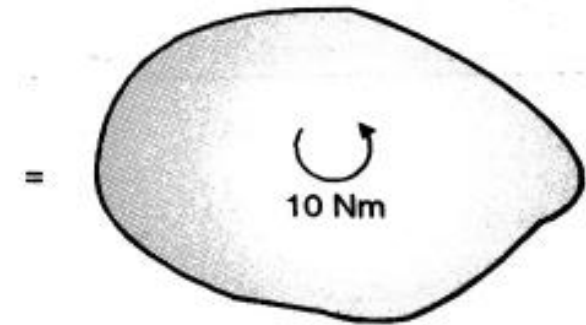
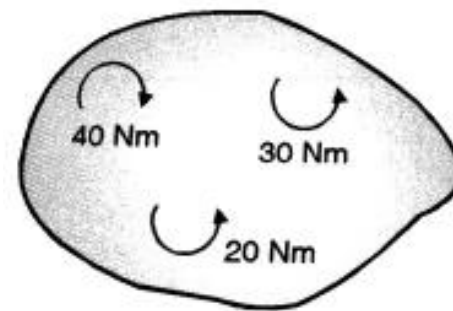
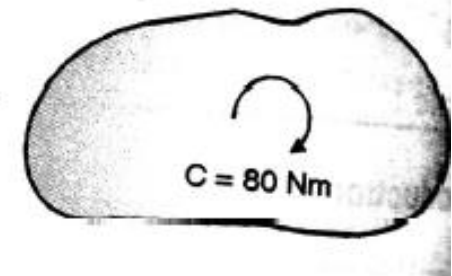
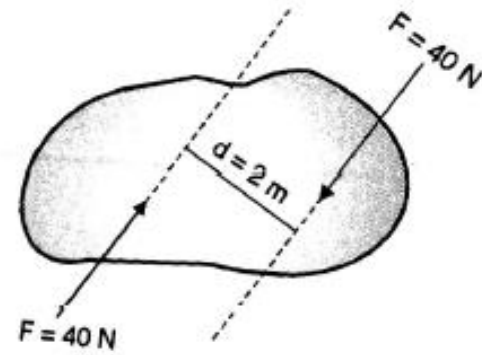
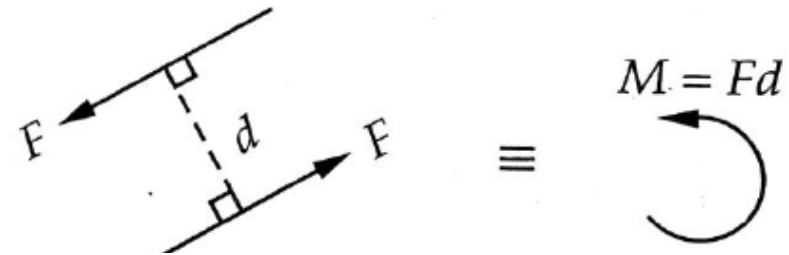
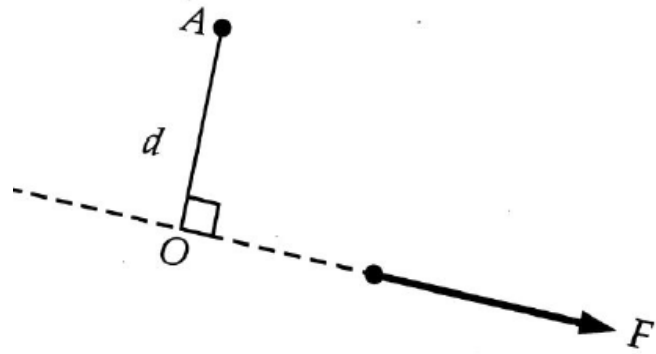
## Couple

Couple is a special case of parallel forces. Two parallel forces of equal magnitude and opposite sense form a couple. The effect of a couple is to rotate the body on which it acts. Fig. — shows a couple formed by two forces of same magnitude  $F$ , separated by a  $\perp$  distance  $d$  known as the arm of the couple.

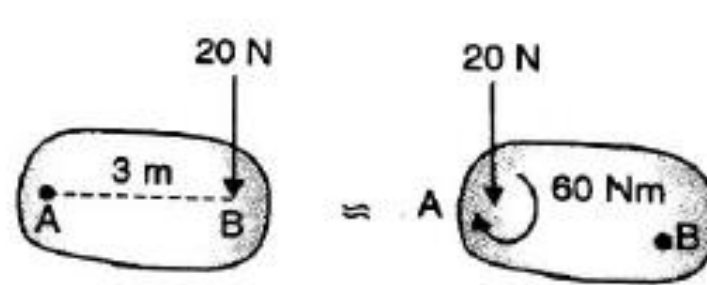




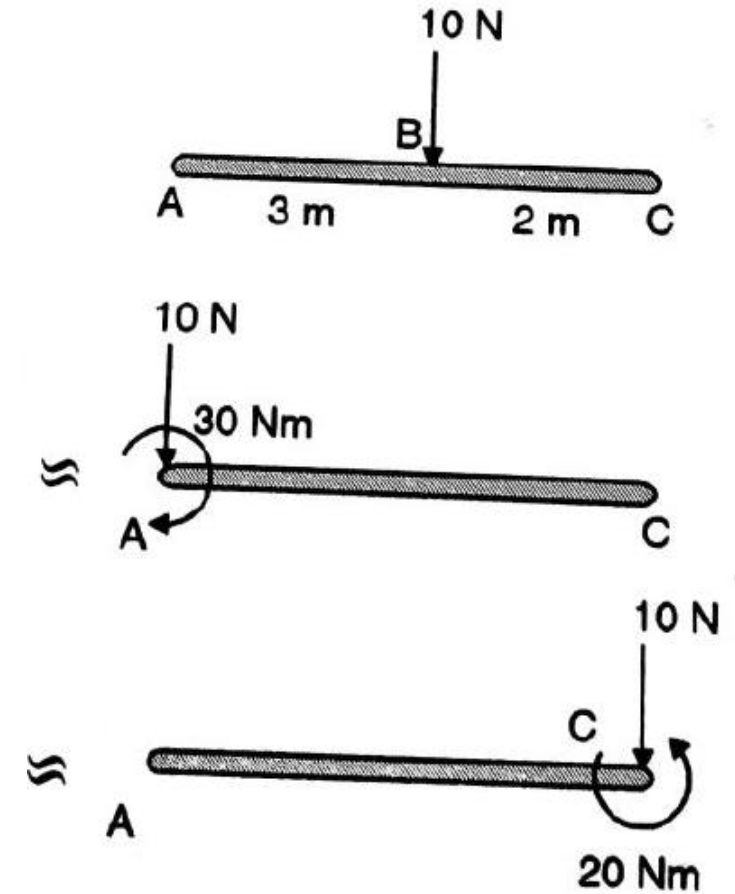
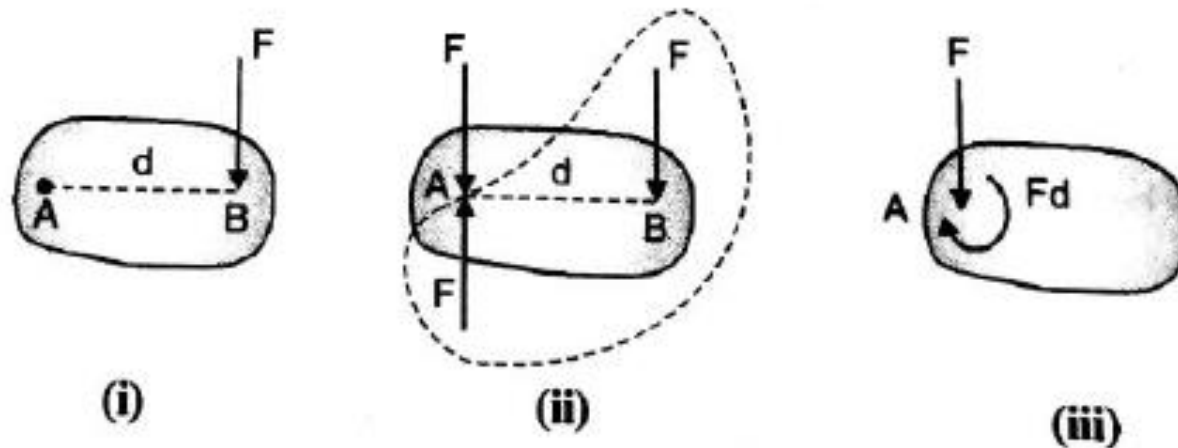
# Moment of a force and Couple Moment



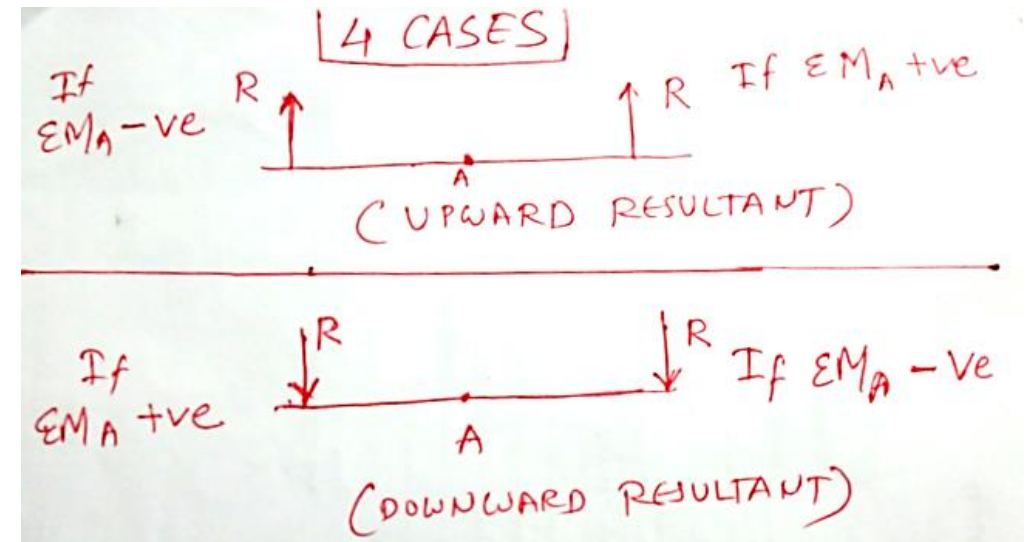
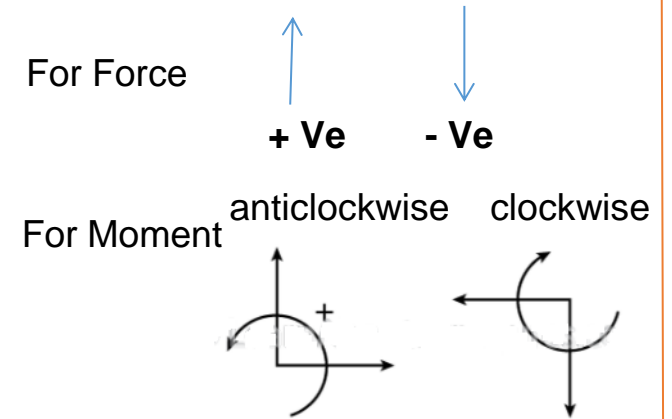
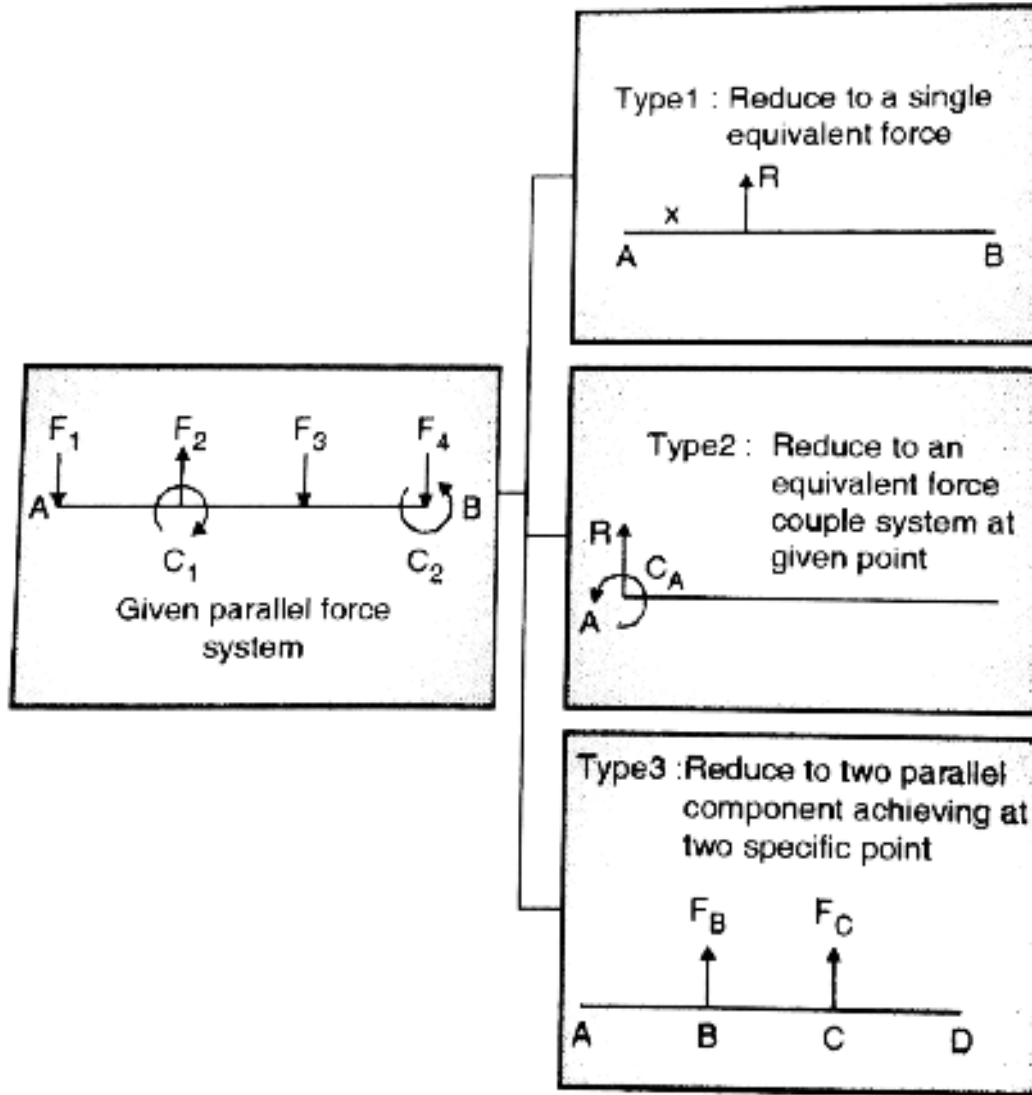
# Conversion of single force to a force couple system



**Explanation :** Consider a force of magnitude  $F$  acting at point B as shown in figure (i).

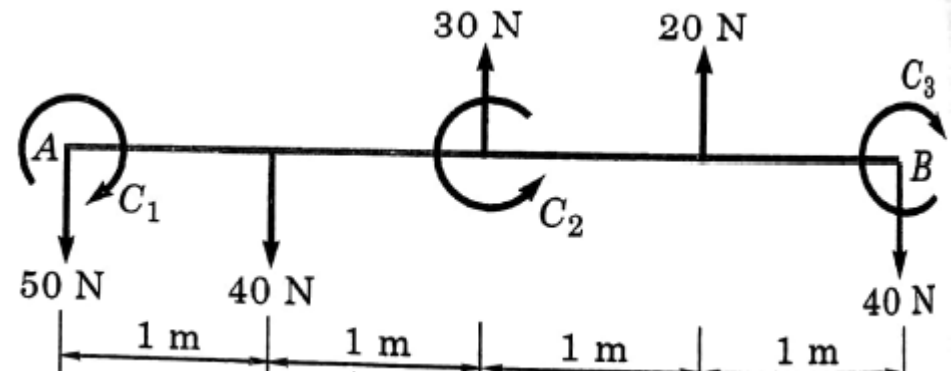


# Resultant of parallel force system



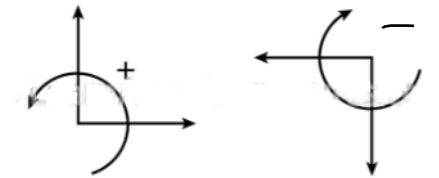
## Problem on parallel force system

Replace the force system shown in figure by a single force. Take  $C_1 = 85 \text{ N.m}$ ,  $C_2 = 65 \text{ N.m}$  and  $C_3 = 90 \text{ N.m}$ .



$$R = (+\uparrow) \Sigma F_y = -50 - 40 + 30 + 20 - 40 = -80 \text{ N} = 80 \text{ N} (\downarrow) \dots \text{Ans.}$$

To find perpendicular distance of resultant from point A, we use Varignon's theorem as



Now,

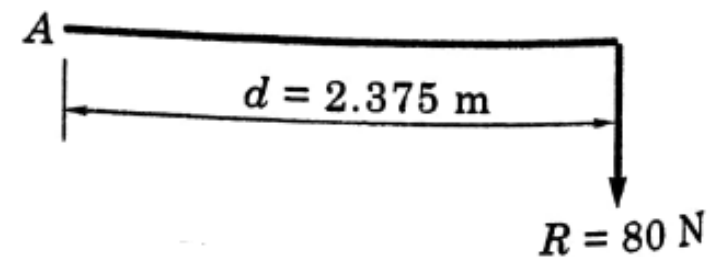
$$|\Sigma M_A| = |R \times d| \dots (I)$$

$$\begin{aligned} (+\odot) \Sigma M_A &= -40 \times 1 + 30 \times 2 + 20 \times 3 - 40 \times 4 - 85 + 65 - 90 \\ &= -190 \text{ N.m} = 190 \text{ N.m} (\odot) \end{aligned}$$

Using equation (I)

$$190 = 80 \times d \quad \therefore d = 2.375 \text{ m from point A} \dots \text{Ans.}$$

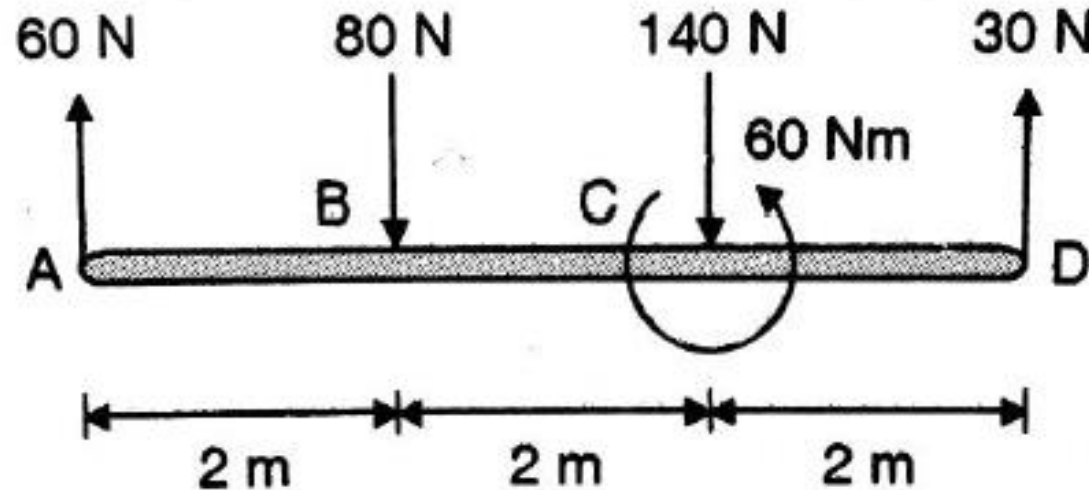
Resultant is represented as shown



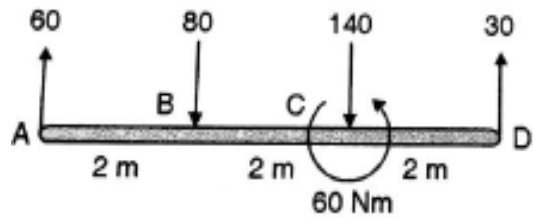
# Problem on parallel force system

Consider the parallel force system acting on rod AD.

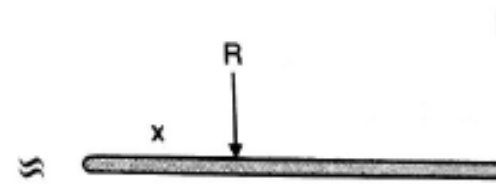
- (i) Find the resultant of the given parallel force system.
- (ii) Convert the given force system into a force couple system at point A
- (iii) Replace the given force system into two parallel components at B and C respectively.



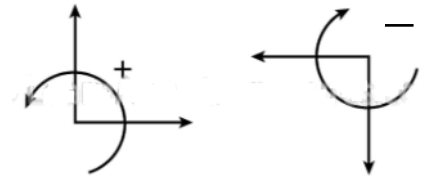
# Solution



Given system



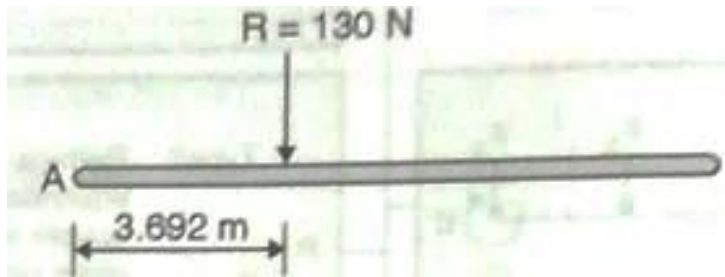
Resultant force



$$R = \sum F_y$$

$$\therefore R = 60 - 80 - 140 + 30$$

$$\therefore R = 130 \text{ N } (\downarrow)$$

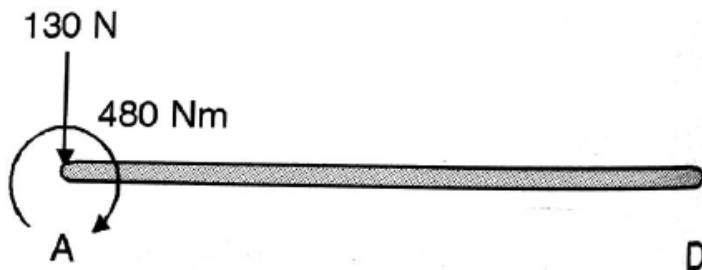


Using Varignon's principle

$$\sum M_A = M_A^R$$

$$60(0) - 80(2) - 140(4) + 30(6) + 60 = -130(x)$$

$$M = -480 \text{ Nm} \quad x = 3.692 \text{ m}$$



The required couple at point A is given by  $M = \sum M_A$

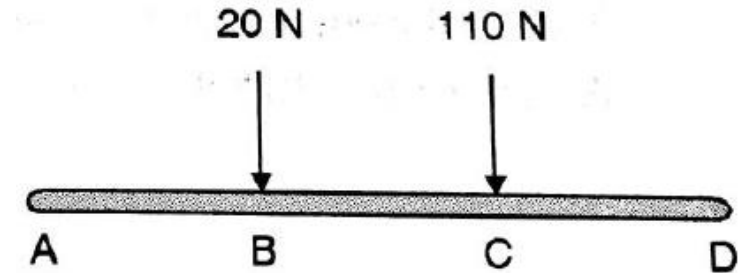
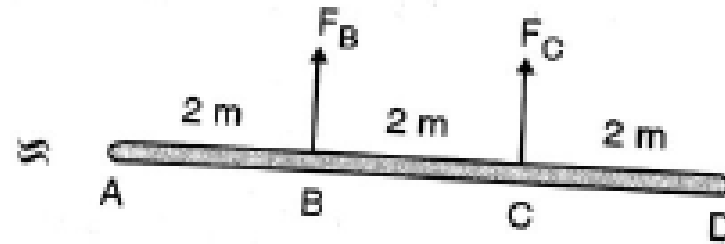
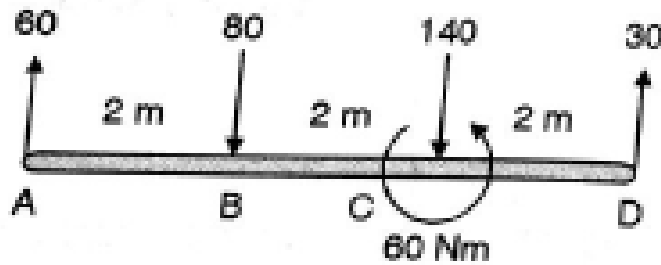
$$\therefore M = 60(0) - 80(2) - 140(4) + 30(6) + 60$$

$$\therefore M = -480 \text{ Nm}$$

The negative sign indicates that M is clockwise,

$$\therefore M = 480 \text{ Nm } (\curvearrowright)$$

## Solution contd...



$$\therefore 60 - 80 - 140 + 30 = F_B + F_C$$

$$\therefore F_B + F_C = -130 \quad \dots(1)$$

$$(\sum M_A)_1 = (\sum M_A)_2$$

$$\therefore 60(0) - 80(2) - 140(4) + 30(6) + 60 = F_B(2) + F_C(4)$$

$$\therefore 2F_B + 4F_C = -480 \quad \dots(2)$$

From Equations (1) and (2)

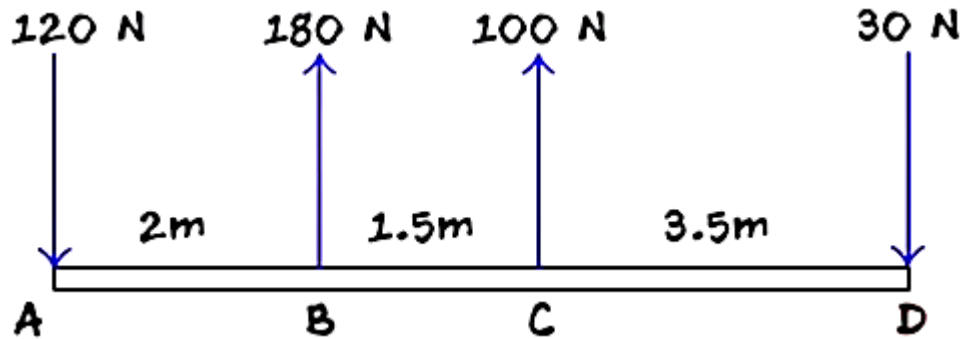
$$F_B = -20 \text{ N,}$$

$$F_C = -110 \text{ N}$$

Negative sign indicates, direction assumed is wrong for both forces

# Problem on parallel force system

- Determine the resultant of the system and its location from A.
- Replace the system by a single force and couple acting at point B.
- Replace the system by a single force and couple acting at point D.



$$R = \sum F (\uparrow +ve)$$

$$R = -120 + 180 + 100 - 30$$

$$R = 130 \text{ N} \dots (\uparrow)$$

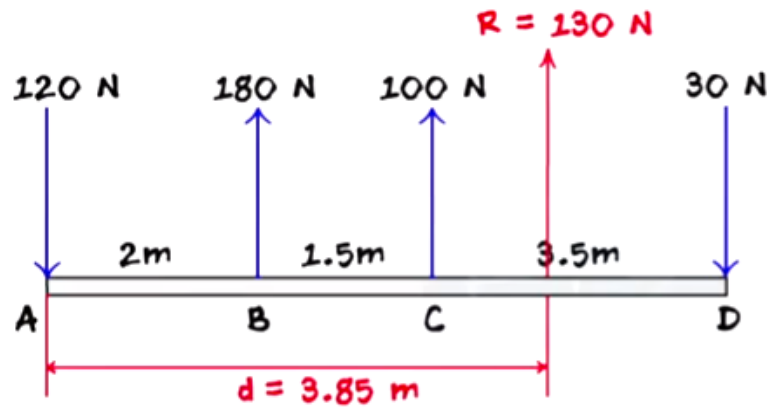
Location of resultant force from A

Using Varignon's Theorem,

$$M_A^R = \sum M_A^F \dots \curvearrowright +ve$$

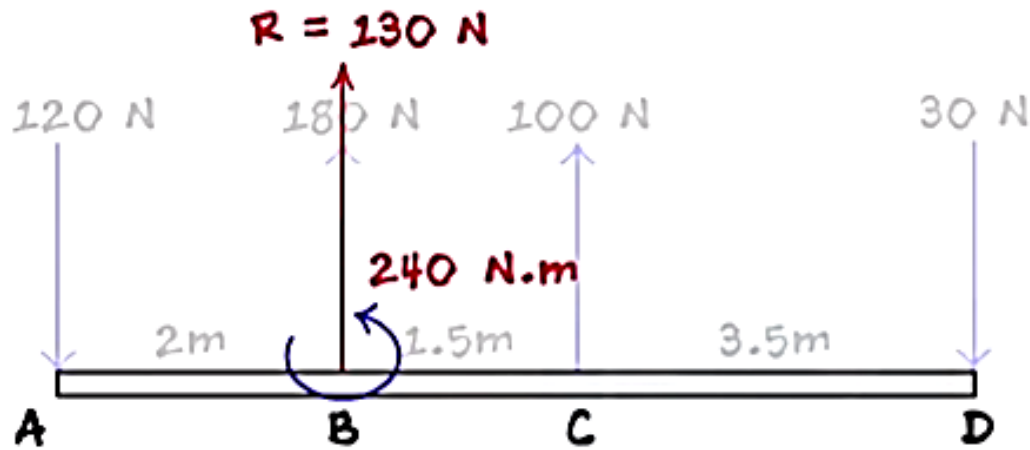
$$130(d) = 180(2) + 100(3.5) - 30(7)$$

$$d = 3.85 \text{ m}$$





## Solution contd...

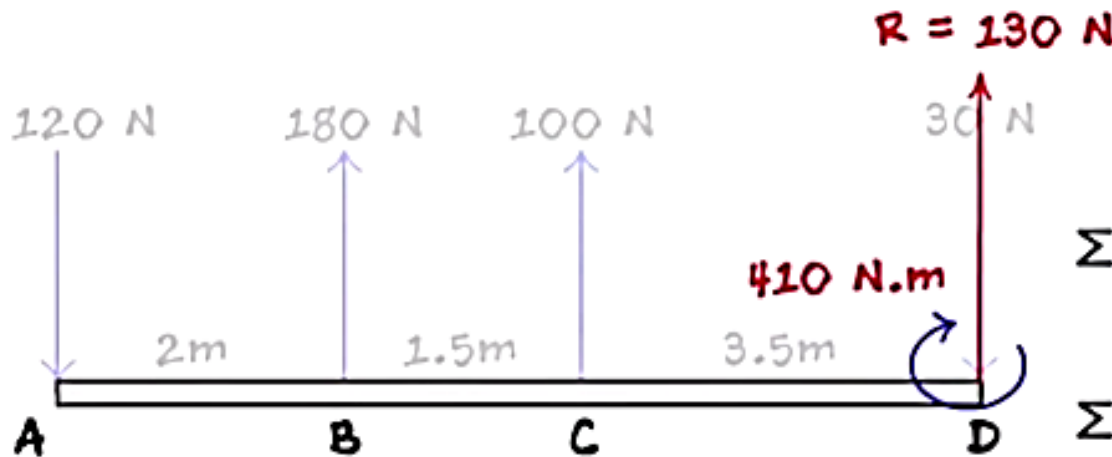


Force-Couple System at point B

To find the couple at B,  $\curvearrowright$  +ve

$$\sum M_B^F = 120(2) + 100(1.5) - 30(5)$$

$$\sum M_B^F = 240 \text{ N.m} \dots\dots \curvearrowright$$



Force-Couple System at point D

To find the couple at D,  $\curvearrowright$  +ve

$$\sum M_D^F = 120(7) - 180(5) - 100(3.5)$$

$$= -410 \text{ N.m}$$

$$\sum M_D^F = 410 \text{ N.m} \dots\dots \curvearrowleft$$

# Resultant of Non concurrent force system

To find the resultant of a general force system

To find the magnitude of resultant force R,

$$R_X = \sum F_X \quad , \quad R_Y = \sum F_Y \quad R = \sqrt{R_X^2 + R_Y^2}$$

To find the direction of resultant force R,

$$\theta = \tan^{-1} \left| \frac{R_Y}{R_X} \right| ,$$

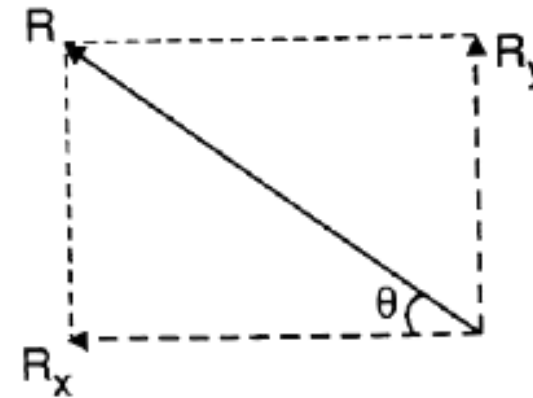
$\theta$  gives the angle made by R with the X-axis.

The quadrant towards which R acts depends on the direction of  $R_X$  and  $R_Y$

e.g. If  $\sum F_X$  is negative, implies that  $R_X (\leftarrow)$

$\sum F_Y$  is positive implies that  $R_Y (\uparrow)$

Then R would act towards the II<sup>nd</sup> quadrant



To find the line of action of the resultant force  $R$ , we use Varignon's principle,

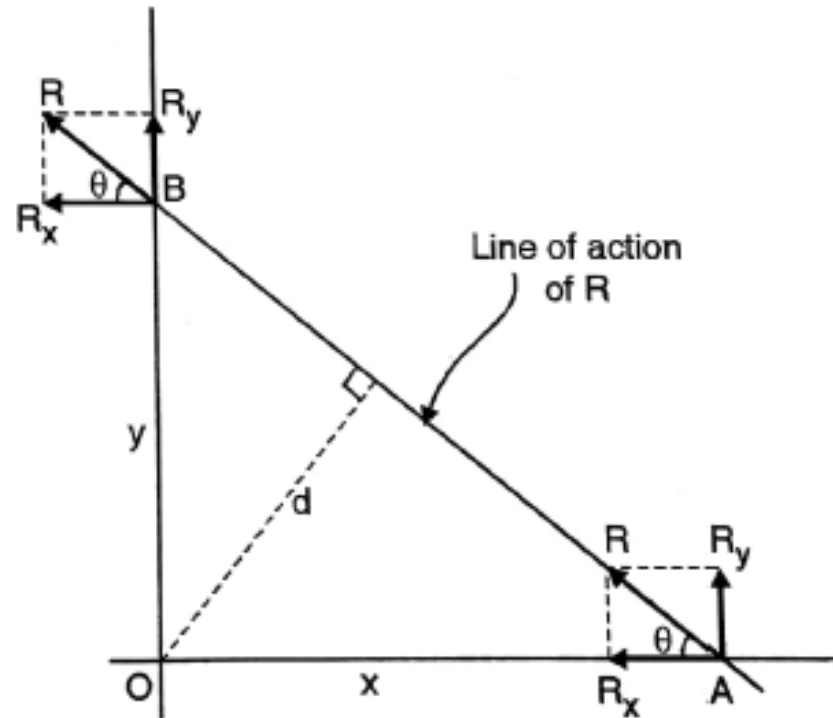
$$\sum M_O = M_O^R \quad \dots(1)$$

where  $O$  is any point

In Equation (1),  $\sum M_O$  is calculated by adding the moment of all forces and couples about point  $O$ .

To find moment of a force  $R$  about origin  $O$  ( $M_O^R$ ),

As per principle of transmissibility, we can assume  $R$  to be acting at any point along its line of action



$$\begin{aligned} \therefore M_O^R &= R \times d, && \dots(2), \text{ considering perpendicular distance} \\ M_O^R &= R_Y \times x && \dots(3), \text{ Considering A as point of application} \\ M_O^R &= R_X \times y && \dots(4), \text{ considering B as point of application} \end{aligned}$$

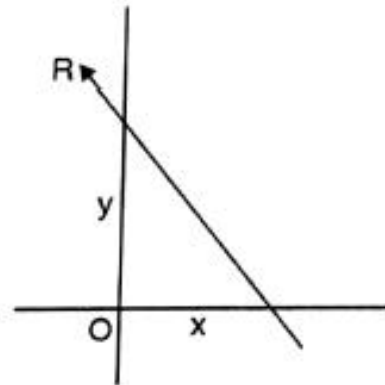
where,  $x$  and  $y$  are the  $x$  and  $y$  intercepts of the line of action of resultant force  $R$

Perpendicular distance,  $d = \frac{\sum M_O}{R}$ , from (1) and (2)

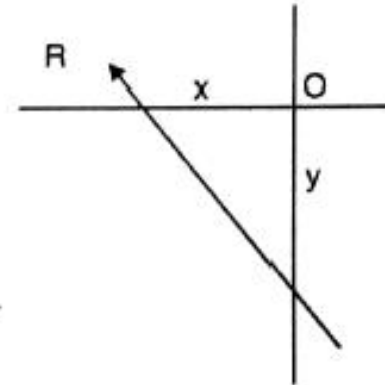
X-intercept,  $x = \frac{\sum M_O}{R_Y}$ , from (1) and (3)

Y-intercept,  $y = \frac{\sum M_O}{R_X}$ , from (1) and (4)

The exact position of  $R$ , with respect to the origins  $O$ . depends on the sense of  $\sum M_O$  and the quadrant towards which  $R$  acts. For instance,



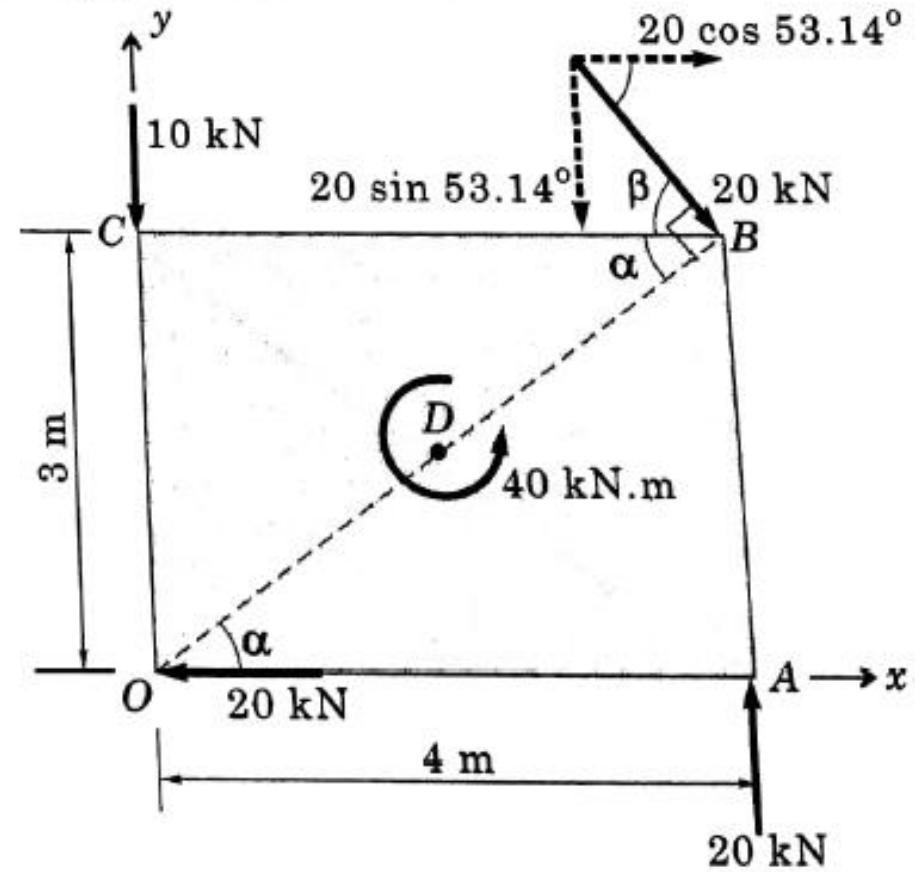
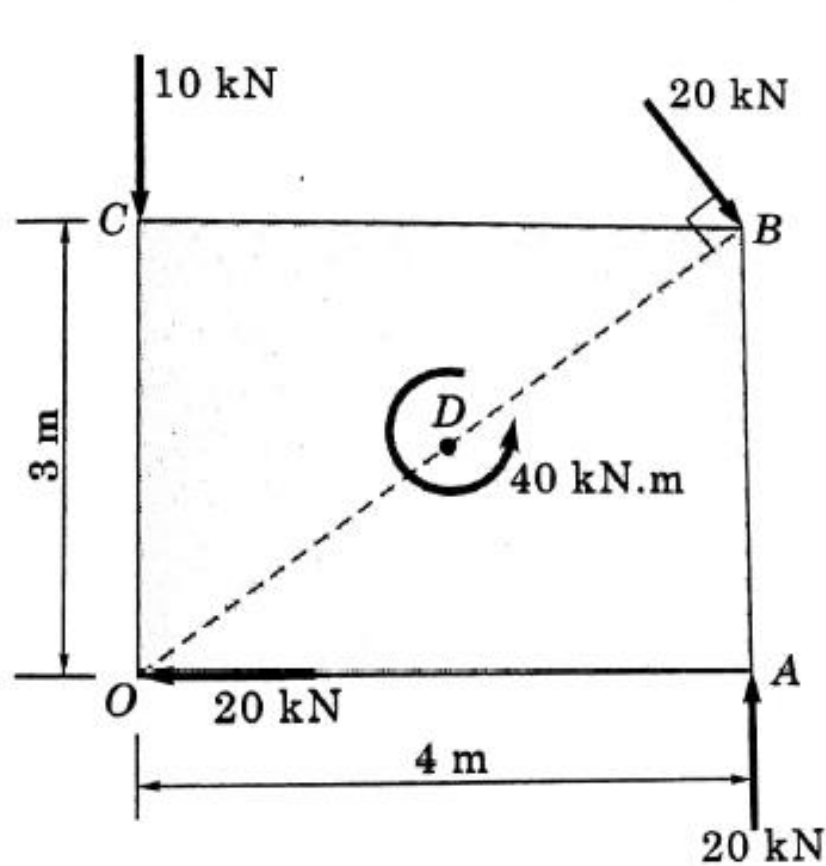
(a) If  $R$  acts towards II<sup>nd</sup> quadrant and  $\sum M_O (\cup)$



(b) If  $R$  acts towards II<sup>nd</sup> quadrant and  $\sum M_O (\cap)$

# Resultant of Non concurrent force system

Find the resultant of the force system acting on a body  $OABC$  as shown in the figure. Also find the point where the resultant will cut  $x$  and  $y$ -axis. What is the distance of resultant from point  $O$ ?



## Solution

Now,

$$\tan \alpha = \frac{3}{4} = 36.86^\circ \quad \text{and} \quad \beta = (90 - \alpha) = 53.14^\circ$$

$$(+\rightarrow) \Sigma F_x = -20 + 20 \cos 53.14^\circ = -8 \text{ kN} = 8 \text{ kN} (\leftarrow)$$

$$(+\uparrow) \Sigma F_y = -10 + 20 - 20 \sin 53.14^\circ = -6 \text{ kN} = 6 \text{ kN} (\downarrow)$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(-8)^2 + (-6)^2} = 10 \text{ kN} \dots \text{Ans.}$$

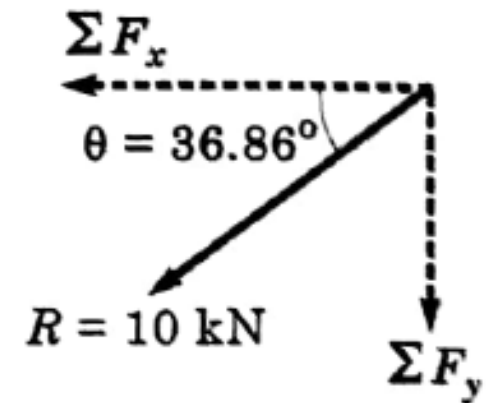
$$\theta = \tan^{-1} \left| \frac{\Sigma F_y}{\Sigma F_x} \right| = \tan^{-1} \left| \frac{6}{8} \right| = 36.86^\circ \dots \text{Ans.}$$

$R$  lies in third quadrant as  $\Sigma F_x$  and  $\Sigma F_y$  are negative.

Resultant is represented as shown in the figure Ex.21(c).

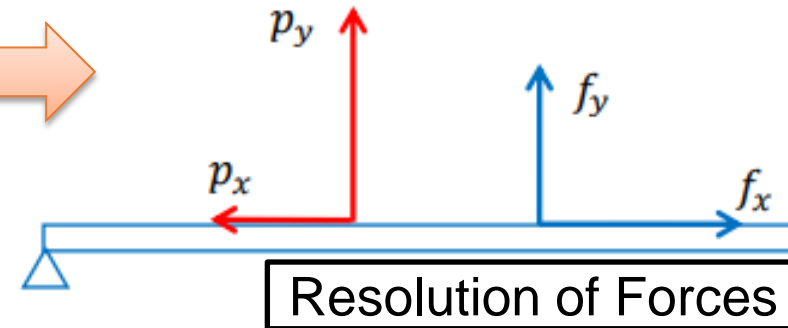
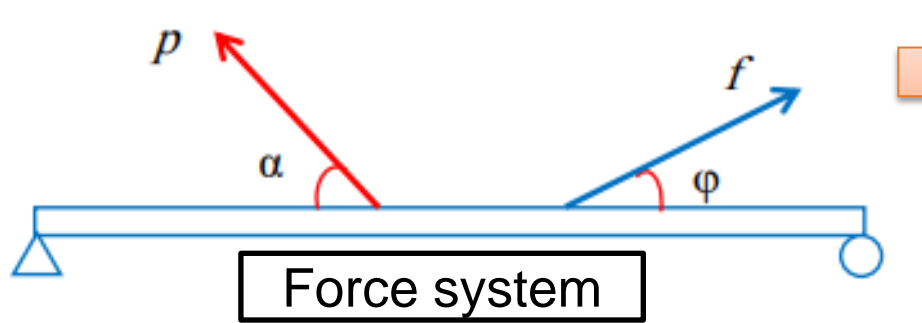
To locate the position of resultant, we use Varignon's theorem as

$$|\Sigma M_O| = |R \times d| = |\Sigma F_x \times y| = |\Sigma F_y \times x| \dots (I)$$





# Resultant of Non concurrent force system



$$f_x = f \times \cos \varphi$$

$$f_y = f \times \sin \varphi$$

$$p_x = p \times \cos \alpha$$

$$p_y = p \times \sin \alpha$$

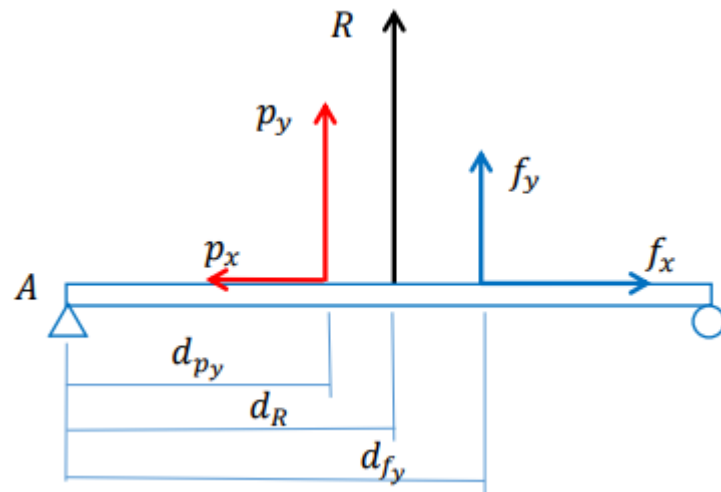


$$R_x = \Sigma F_x$$

$$R_y = \Sigma F_y$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\tan \theta_x = \frac{R_y}{R_x}$$



$$\Sigma M \text{ about } A =$$

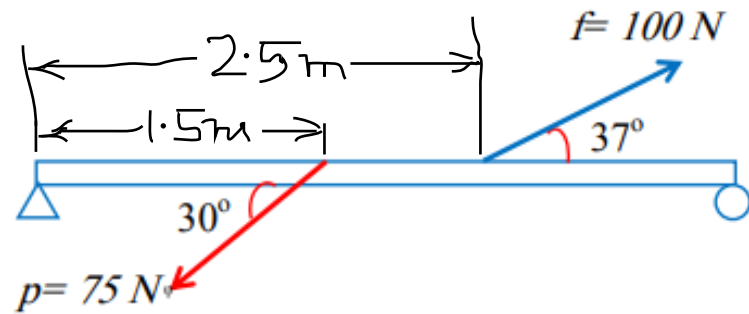
$$R \times d_R = \Sigma (F_y \times d_f) = p_y \times d_{p_y} + f_y \times d_{f_y}$$

Location of the resultant

Calculation of resultant



# Problem- Determine the resultant of the force system and also locate the same

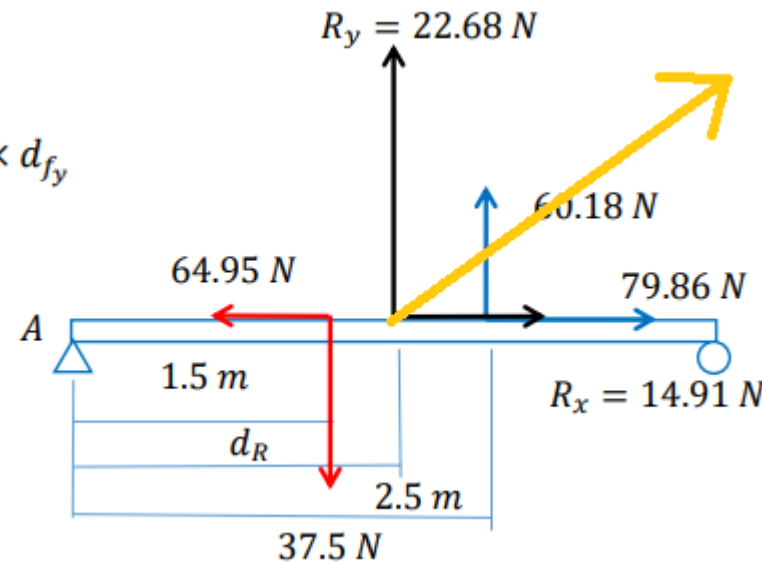


$$\sum M \text{ about } A =$$

$$R_y \times d_R = \sum (F_y \times d_f) = -p_y \times d_{p_y} + f_y \times d_{f_y}$$

$$22.68 \times d_R = -37.5 \times 1.5 + 60.18 \times 2.5$$

$$d_R = 4.15 \text{ m}$$



Location of resultant

$$f_x = 100 \times \cos 37 = 79.86 \text{ N} \rightarrow$$

$$f_y = 100 \times \sin 37 = 60.18 \text{ N} \uparrow$$

$$p_x = 75 \times \cos 30 = -64.95 \text{ N} = 64.95 \text{ N} \leftarrow$$

$$p_y = 75 \times \sin 30 = -37.5 \text{ N} = 37.5 \text{ N} \downarrow$$

## Resultant calculation

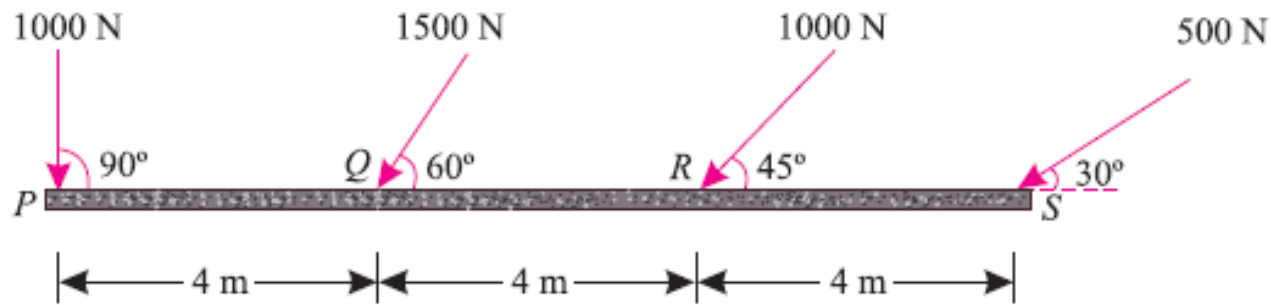
$$R_x = \sum F_x = f_x + p_x = 79.86 - 64.95 = 14.91 \text{ N} \rightarrow$$

$$R_y = \sum F_y = f_y + p_y = 60.18 - 37.5 = 22.68 \text{ N} \uparrow$$

$$R = \sqrt{14.91^2 + 22.68^2} = 27.14 \text{ N} \nearrow$$

$$\text{Slope} = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{22.68}{14.91}\right) = 56.69^\circ$$

# Problem – Determine Resultant and x-intercept of the F. S.



$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(1890)^2 + (3256)^2} = 3765 \text{ N}$$

$$\tan \theta = \frac{\Sigma V}{\Sigma H} = \frac{3256}{1890} = 1.722 \quad \text{or} \quad \theta = 59.8^\circ$$

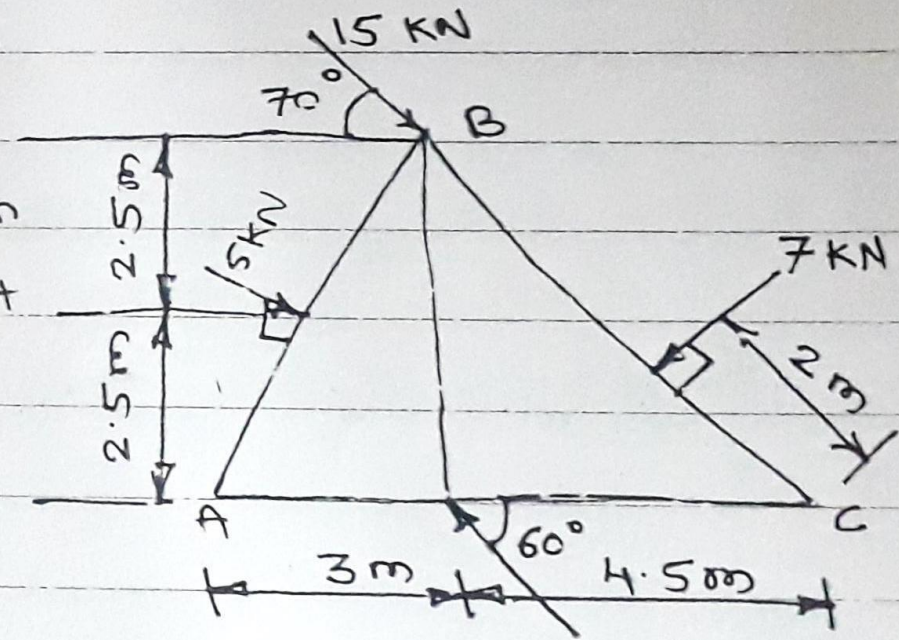
To determine x-intercept, the moments of all the forces are taken about point P

$$3765 x = (1000 \times 0) + (1500 \times 0.866) 4 + (1000 \times 0.707) 8 + (500 \times 0.5) 12$$
$$= 13852$$

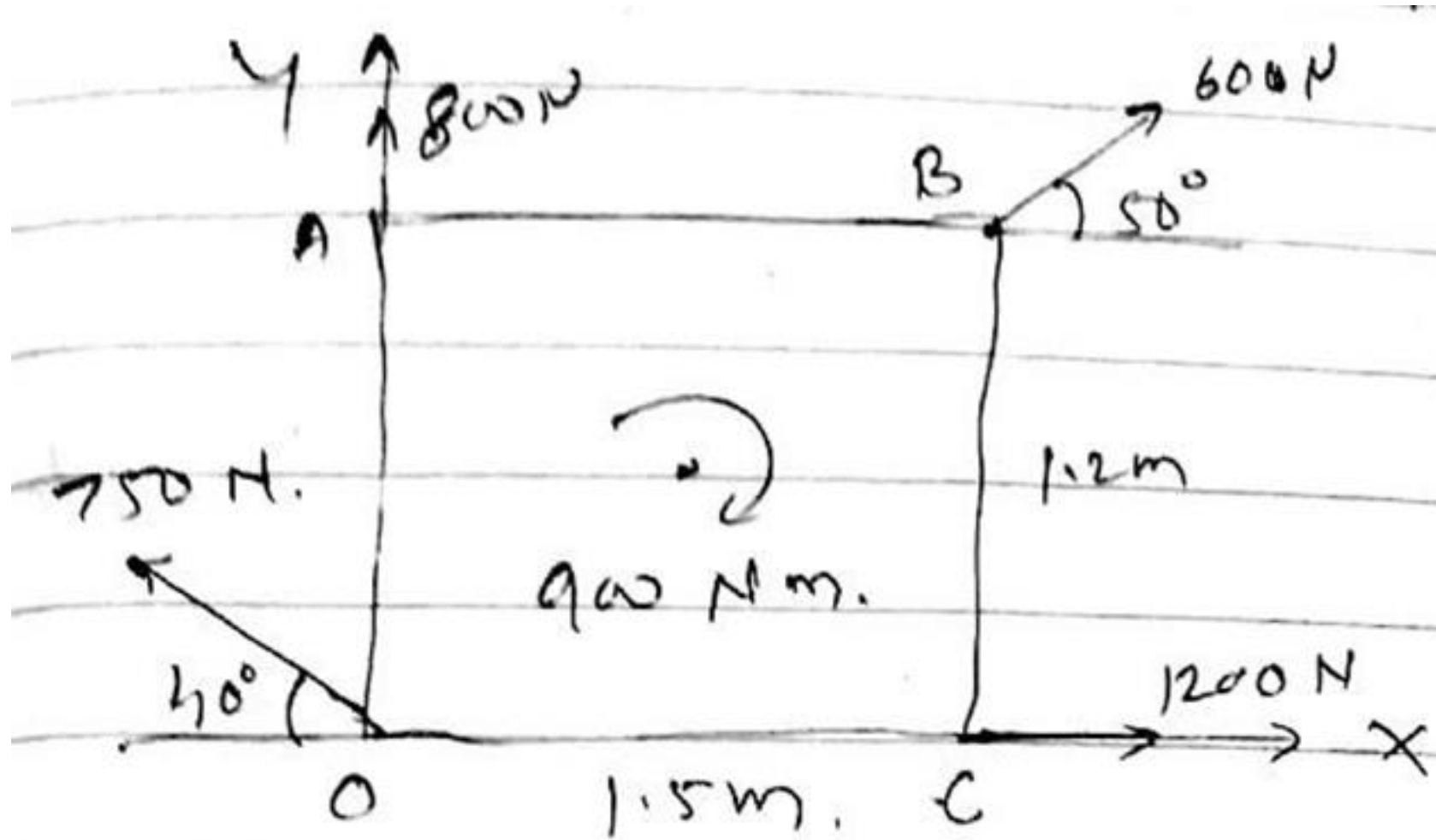
$$x = \frac{13852}{3765} = 3.679 \text{ m}$$

# Problem for Practice:

(H) A triangular plate ABC is subjected to four coplanar forces as shown in figure. Find the resultant completely and locate its position with respect to point A.



Determine resultant of given force system and locate it with respect to point 'O' Also find x and y intercept of resultant



## Solution

$$\begin{aligned}\Sigma F_x &= 600 \cos 50^\circ + 1200 - 750 \cos 40^\circ \\ &= 1011.14 \text{ N. } (\rightarrow)\end{aligned}$$

$$\begin{aligned}\Sigma F_y &= 800 + 750 \sin 40^\circ + 600 \sin 50^\circ \\ &= 1741.72 \text{ N } (\uparrow)\end{aligned}$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = 2013.95 \text{ N}$$

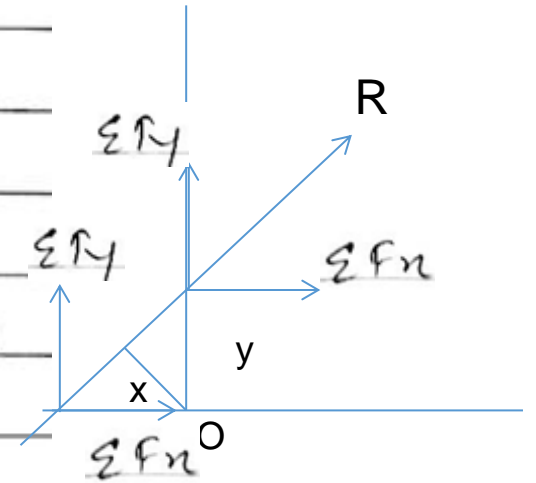
$$\theta = \tan^{-1} \left( \frac{\Sigma F_y}{\Sigma F_x} \right) = 59.86^\circ$$

$$\begin{aligned}\Sigma M_o &= -600 \cos 50^\circ \times 1.2 + 600 \sin 50^\circ \times 1.5 - 900 \\ &= -673.37 \text{ N.m} \\ &= 673.37 \text{ N.m } \curvearrowright\end{aligned}$$

By varignon  $\Sigma M_o = \Sigma F_y \times x \quad \therefore x = \frac{\Sigma M_o}{\Sigma F_y}$

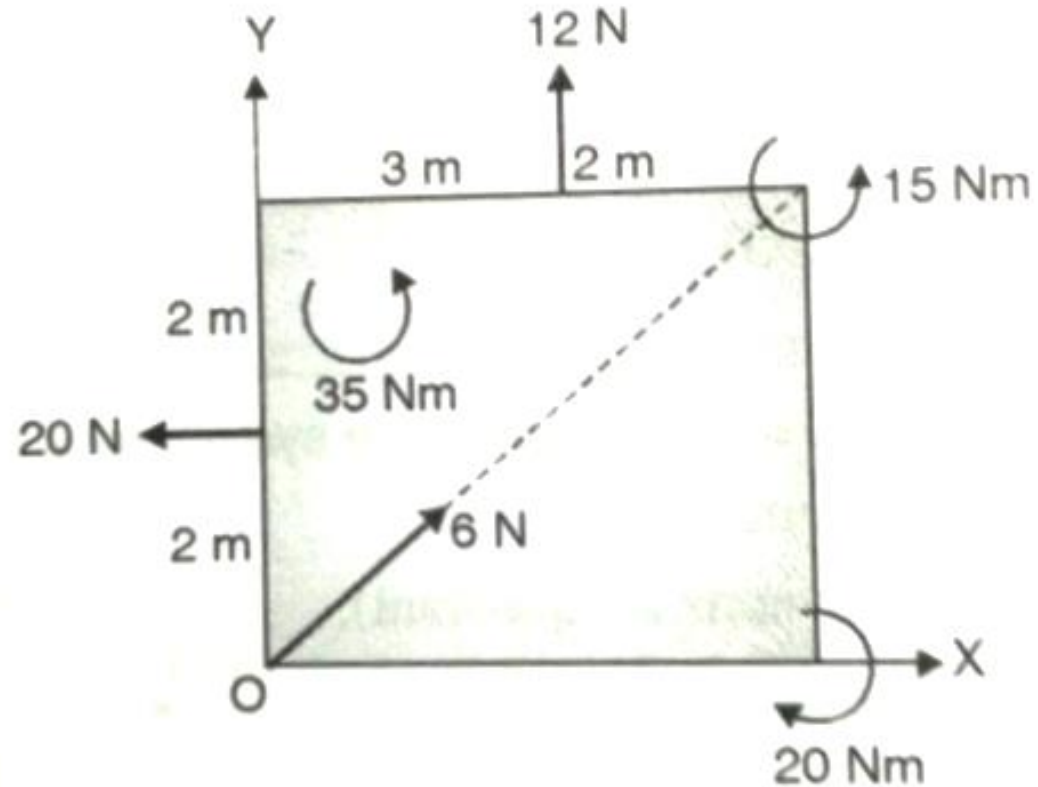
$$x = 0.39 \text{ m.}$$

$$\Sigma M_o = \Sigma F_x \times y \quad y = \Sigma M_o / \Sigma F_x = 0.67$$



# Problems for Practice:

Determine resultant of given force system and locate it with respect to point 'O'  
Also find x and y intercept of resultant.



Determine resultant of given force system and locate it with respect to point 'O'  
Also find x and y intercept of resultant.

