EVALUATION OVER GIVEN REGION

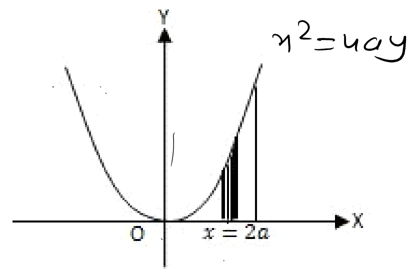
Monday, April 26, 2021 8:46 PM

Evaluate the following integrals

1. $\iint xy\,dx\,dy$ over the region bounded by the x-axis, ordinate at $x = 2a$ and the parabola $x^2 = 4ay$.

Sol^n :- In the region y varies from 0 to $\frac{x^2}{4a}$

and x varies from 0 to $2a$



$$\therefore I = \int_0^{2a}\int_0^{x^2/4a} xy\,dy\,dx$$

$$= \int_0^{2a} x\cdot\left(\frac{y^2}{2}\right)_0^{x^2/4a} dx = \int_0^{2a} \frac{x}{2}\left(\frac{x^4}{16a^2}\right) dx$$

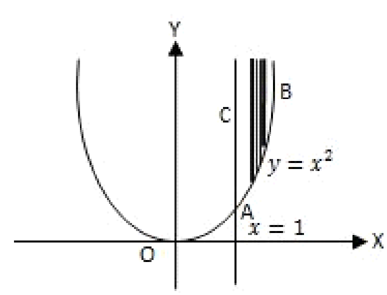
$$= \frac{1}{32a^2}\int_0^{2a} x^5\,dx = \frac{1}{32a^2}\left(\frac{x^6}{6}\right)_0^{2a} = \frac{1}{32a^2}\left(\frac{2^6a^6}{6}\right)$$

$$I = \frac{a^4}{3}.$$

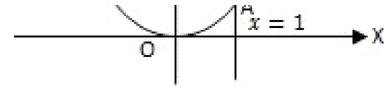
2. $\iint_R \frac{1}{x^4+y^2}\,dx\,dy$ where R is the region $x \geq 1, y \geq x^2$

The boundaries of the region are $y=x^2$ a parabola with vertex at origin and opening upwards.

The line $x=1$ is the line parallel to y-axis.



y-axis.



The region of integration is betn the line $x=1$ and the branch of parabola in the first quadrant.

In this region, consider a vertical strip

On this strip, y varies from $y=x^2$ to $y=\infty$

Then x varies from $x=1$ to $x=\infty$

$$I = \int_1^\infty \int_{x^2}^\infty \frac{dy}{y^2+(x^2)^2}\, dx$$

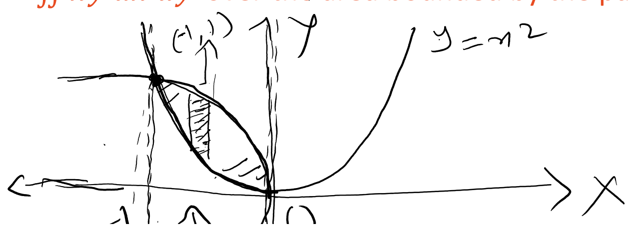
$$= \int_1^\infty \left[\frac{1}{x^2}\tan^{-1}\left(\frac{y}{x^2}\right)\right]_{x^2}^\infty dx$$

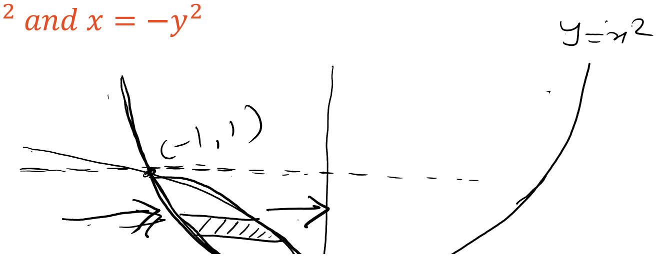
$$= \int_1^\infty \frac{1}{x^2}\left[\tan^{-1}(\infty) - \tan^{-1}(1)\right] dx$$

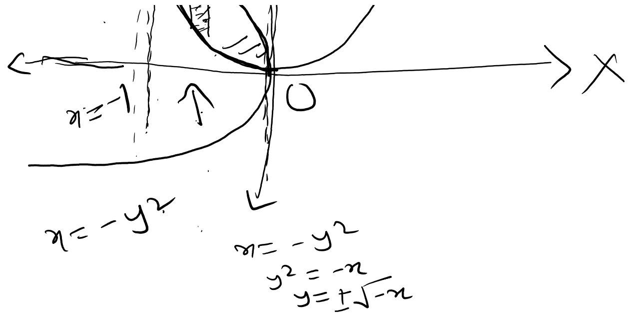
$$= \int_1^\infty \frac{1}{x^2}\left(\frac{\pi}{2}-\frac{\pi}{4}\right) dx = \frac{\pi}{4}\int_1^\infty \frac{1}{x^2}\,dx = \frac{\pi}{4}\left(\frac{-1}{x}\right)_1^\infty$$

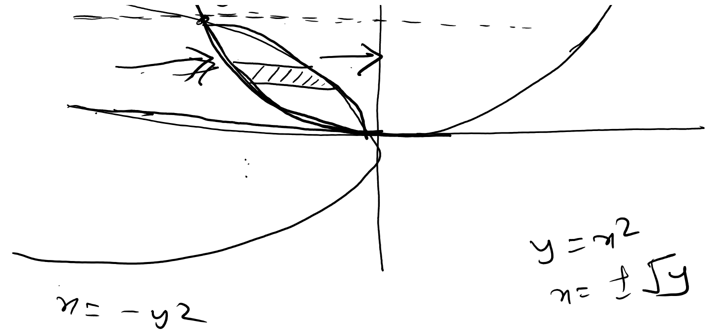
$$= \frac{\pi}{4}\left[0-(-1)\right] = \frac{\pi}{4}$$

3. $\iint xy\, dx\, dy$ over the area bounded by the parabolas $y = x^2$ and $x = -y^2$









Vertical strip

$y \to x^2$ to $\sqrt{-x}$

$x \to -1$ to 0

$$\int_{-1}^{0}\int_{x^2}^{\sqrt{-x}} xy \, dy \, dx$$

or

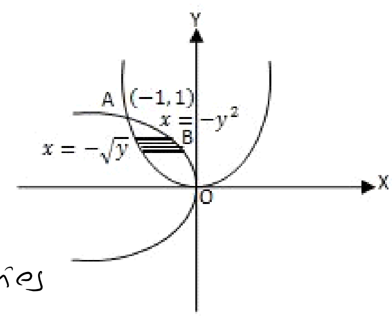
Horizontal strip

$x \to -\sqrt{y}$ to $-y^2$

$y \to 0$ to 1

$$\int_{0}^{1}\int_{-\sqrt{y}}^{-y^2} xy \, dx \, dy$$

The two parabolas are as shown in the figure. They intersect at $O(0,0)$ and $A(-1,1)$



In the region OAB, consider a strip parallel to x-axis, on this strip x varies from $x=-\sqrt{y}$ to $x=-y^2$ and they y varies from $y=0$ to $y=1$

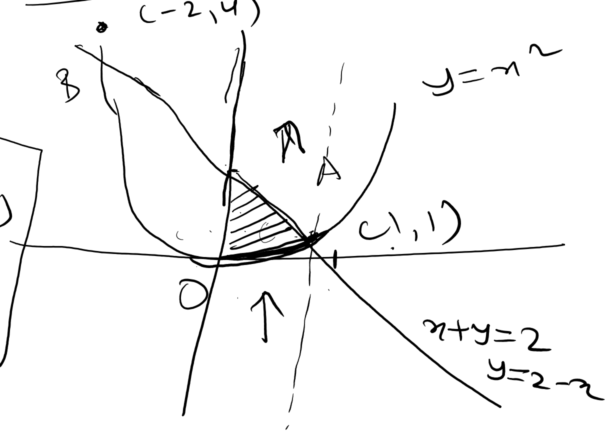
$$\therefore I = \int_{0}^{1}\int_{-\sqrt{y}}^{-y^2} y \cdot x \, dx \, dy = \int_{0}^{1} y\left[\frac{x^2}{2}\right]_{-\sqrt{y}}^{-y^2} dy$$

$$= \frac{1}{2}\int_{0}^{1} y\left(y^4 - y\right)dy = \frac{1}{2}\int_{0}^{1}\left(y^5 - y^2\right)dy$$

$$= \frac{1}{2}\left[\frac{y^6}{6} - \frac{y^3}{3}\right]_0^1 = \frac{1}{2}\left(\frac{1}{6} - \frac{1}{3}\right) = -\frac{1}{12}$$

$$= \frac{1}{2}\left[\frac{y^6}{6} - \frac{y^3}{3}\right]_0^1 = \frac{1}{2}\left[\frac{1}{6} - \frac{1}{3}\right] = -\frac{1}{12}$$

4. $\iint y\, dx\, dy$ over the area bounded by $x = 0, y = x^2, x + y = 2$ in the first quadrant



$x + y = 2$

$x = 0,\ y = 2\ (0,2)$

$x = 2,\ y = 0\ (2,0)$

for point of intersection

$y = x^2,\ x + y = 2$

$y = 2 - x$

$2 - x = x^2$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2,\ x = 1$

$y = 4,\ y = 1$

$(1,1)$

$(-2,4)$

Vertical strip

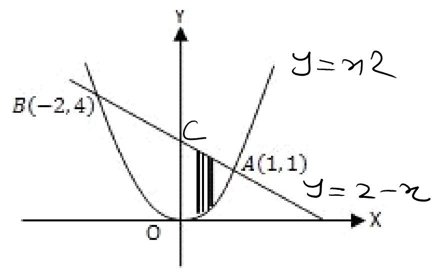
$y \to x^2$ to $2 - x$

$x \to 0$ to 1.

we ~~take~~ a vertical strip in the region OAC.

y varies from x^2 to $2-x$

and then x varies from 0 to 1



$$\therefore I = \int_0^1 \int_{x^2}^{2-x} y\, dy\, dx$$

$$= \int_0^1 \left[\frac{y^2}{2}\right]_{x^2}^{2-x} dx = \frac{1}{2}\int_0^1 (2-x)^2 - (x^2)^2\, dx$$

$$= \frac{1}{2}\int_0^1 (4 - 4x + x^2 - x^4)\, dx = \frac{1}{2}\left[4x - 2x^2 + \frac{x^3}{3} - \frac{x^5}{5}\right]_0^1$$

$$\therefore I = \frac{1}{2}\left[4 - 2 + \frac{1}{3} - \frac{1}{5}\right] = \frac{16}{15}$$

5. $\iint_R \frac{y}{(a-x)\sqrt{ax-y^2}}\,dx\,dy$ where R is the region bounded by $y^2 = ax$ and $y = x$.

point of intersection

$y = x$

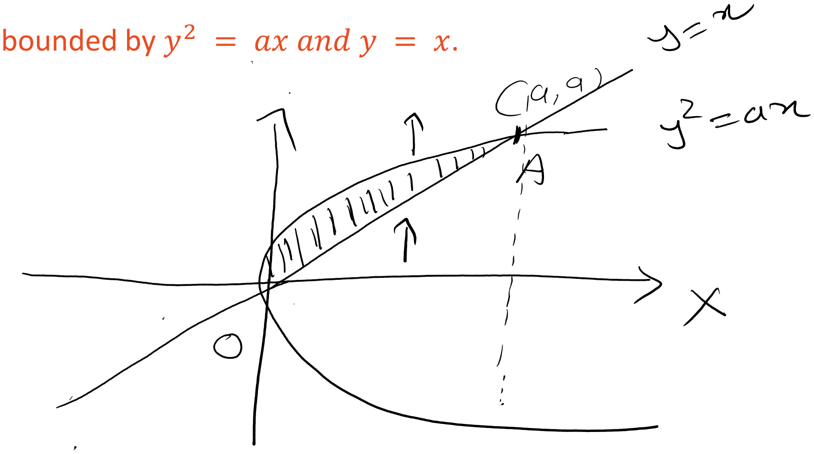
$y^2 = ax$

$y^2 = ay \quad (y^2 - ay) = 0$

$y(y-a) = 0$

$y = 0 \ , \ y = a$

$x = 0 \qquad x = a$



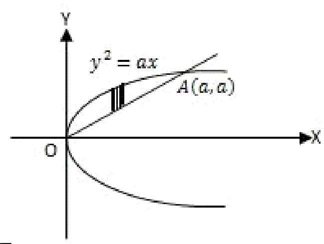
taking a vertical strip

$y \to x$ to $\sqrt{ax}$

$x \to 0$ to a

we will take a vertical strip i.e. integrate wrt y first and then wrt x

both the curves intersect at $(0,0)$ at (a,a)



In this region, y varies from x to $\sqrt{ax}$ and then x varies from $x=0$ to $x=a$

$$\therefore I = \int_0^a \left(\frac{1}{a-x}\right)\left[\int_x^{\sqrt{ax}} \frac{y}{\sqrt{ax-y^2}}\,dy\right] dx$$

put $ax - y^2 = t \qquad -2y\,dy = dt$

when $y = x$, $t = ax - x^2$ | $y = \sqrt{ax}$, $t = 0$

$$I = \int_0^a \left(\frac{1}{a-x}\right)\left[\int_{ax-x^2}^{0} \frac{-dt/2}{\sqrt{t}}\right] dx$$

$a \qquad ax-x^2$

$$= \int_0^a \frac{1}{a-x} \cdot \frac{1}{2} \int_0^{ax-x^2} t^{-1/2}\, dt \cdot dx$$

$$= \int_0^a \frac{1}{a-x} \cdot \frac{1}{2} \Big[2\sqrt{t} \Big]_0^{ax-x^2} dx$$

$$= \int_0^a \frac{\sqrt{ax-x^2}}{a-x}\, dx = \int_0^a \frac{\sqrt{x}\,\sqrt{a-x}}{a-x}\, dx$$

$$= \int_0^a \frac{\sqrt{x}}{\sqrt{a-x}}\, dx \qquad \text{or } x = at$$

put $x = a \sin^2\theta$

$dx = 2a \sin\theta \cos\theta \, d\theta$

$x = 0 \quad \theta = 0 \qquad x = a \quad \theta = \pi/2$

$$I = \int_0^{\pi/2} \frac{\sqrt{a}\sin\theta}{\sqrt{a - a\sin^2\theta}} \cdot 2a\sin\theta\cos\theta\, d\theta$$

$$= \int_0^{\pi/2} 2a\sin^2\theta\, d\theta = 2a \int_0^{\pi/2} \sin^2\theta\, d\theta$$

$$= 2a \cdot \frac{1}{2} B\left(\frac{2+1}{2}, \frac{0+1}{2}\right) = a\, B\left(\frac{3}{2}, \frac{1}{2}\right)$$

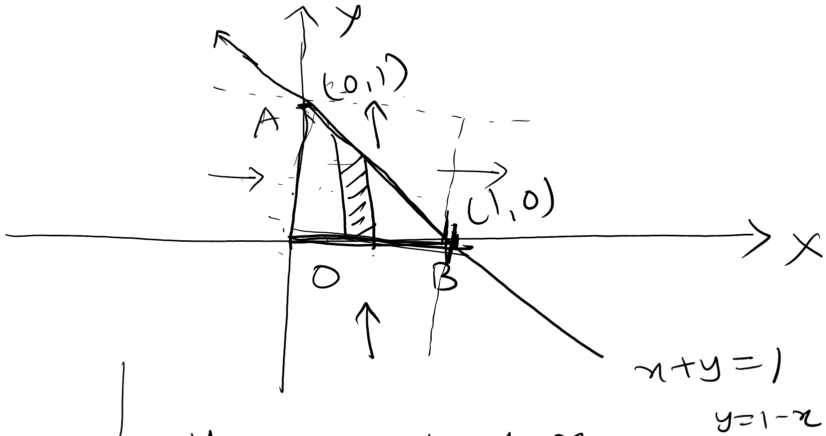
$$\therefore I = \frac{\pi a}{2}$$

6. $\iint \sqrt{xy(1-x-y)}\, dxdy$ over the area bounded by $x = 0, y = 0$ and $x + y = 1$

$x + y = 1$

$x = 0, \; y = 1 \quad (0,1)$

$x = 1, \; y = 0 \quad (1,0)$



Horizontal strip

$x \to 0$ to $1-y$

$y \to 0$ to 1

$y \to 0$ to $1-x$

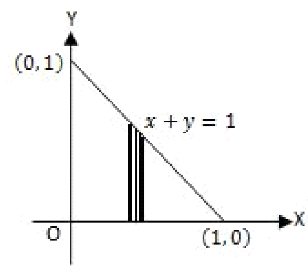
$x \to 0$ to 1

$$I = \iint \sqrt{xy(1-x-y)}\, dx\, dy$$

we shall integrate wrt y first

y varies from 0 to $1-x$

then x varies from 0 to 1



$$I = \int_0^1 \sqrt{x} \left[\int_0^{1-x} \sqrt{y(1-x-y)}\, dy \right] dx \quad \text{—} \quad (1)$$

Let $I_1 = \int_0^{1-x} \sqrt{y(1-x-y)}\, dy$

put $1-x = a$

$$= \int_0^a \sqrt{y(a-y)}\, dy$$

put $y = at \qquad dy = a\,dt$

$y = 0, \; t = 0 \;|\; y = a, \; t = 1$

$$I_1 = \int_0^1 \sqrt{at}\, \sqrt{a-at}\; a\,dt = a^2 \int t^{1/2} (1-t)^{1/2}\, dt$$

$$I_1 = \int_0^1 \sqrt{at}\,\sqrt{a-at}\; a\,dt = a^2 \int_0^1 t^{1/2}(1-t)^{1/2}\,dt$$

$$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

$$\therefore I_1 = a^2 \cdot B\left(\frac{3}{2},\frac{3}{2}\right) = \frac{\pi a^2}{8}$$

$$\therefore I = \int_0^1 \sqrt{x}\,\frac{\pi a^2}{8}\,dx = \frac{\pi}{8}\int_0^1 \sqrt{x}\,(1-x)^2\,dx$$

This is beta function

$$I = \frac{\pi}{8}\cdot B\left(\frac{3}{2},3\right) = \frac{\pi}{8}\,\frac{\Gamma\frac{3}{2}\,\Gamma 3}{\Gamma\left(\frac{3}{2}+3\right)} = \frac{2\pi}{105}$$

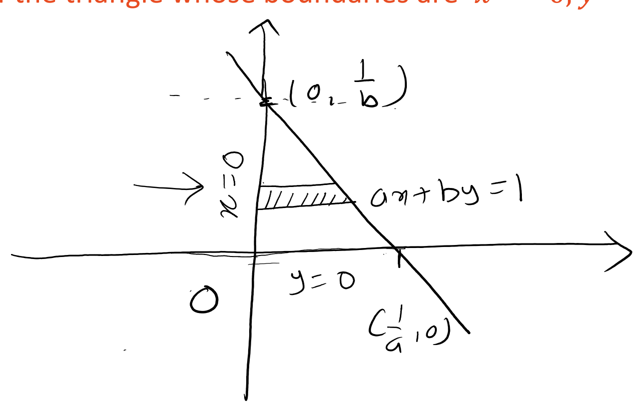
10th May 2021 11:29 AM

7) Prove that $\iint_R e^{ax+by}\,dxdy = 2R$ where R is area of the triangle whose boundaries are $x = 0, y = 0$ and $ax + by = 1$

$ax+by=1$

$x=0,\quad y=\frac{1}{b}$

$x=\frac{1}{a}\quad y=0$

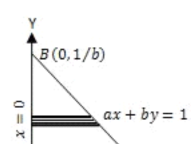


Taking a horizontal strip

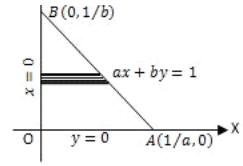
$x \to 0$ to $x = \frac{1}{a}(1-by)$

$y \to 0$ to $\frac{1}{b}$

$\frac{1}{b}\quad \frac{1}{a}(1-by)$



$$\therefore I = \int_0^{\frac{1}{b}} \int_0^{\frac{1}{a}(1-by)} e^{by}\left[e^{ax}\right] dx\, dy$$



$$= \int_0^{1/b} e^{by}\left[\frac{e^{ax}}{a}\right]_0^{\frac{1}{a}(1-by)} = \int_0^{1/b} \frac{1}{a} e^{by}\left[e^{1-by} - 1\right] dy$$

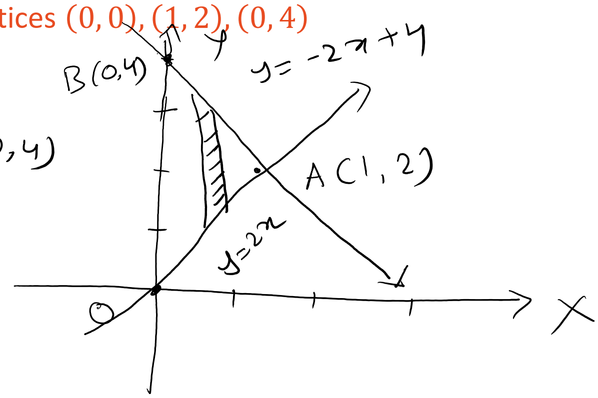
$$= \frac{1}{a}\int_0^{1/b} \left(e - e^{by}\right) dy = \frac{1}{a}\left[ey - \frac{e^{by}}{b}\right]_0^{1/b}$$

$$= \frac{1}{a}\left[e\left(\frac{1}{b}\right) - \frac{e}{b} + \frac{1}{b}\right] = \boxed{\frac{1}{ab}}$$

If R is the area of the triangle then $R = \frac{1}{2} \times \frac{1}{a} \times \frac{1}{b}$

$$\therefore I = \frac{1}{ab} = 2R.$$

8. $\iint_R x(x-y)\,dx\,dy$ where R is the triangle with vertices $(0,0)$, $(1,2)$, $(0,4)$



Soln:- Let $O(0,0)$, $A(1,2)$ and $B(0,4)$ be the vertices of the triangle OAB.

Eqn of OA is

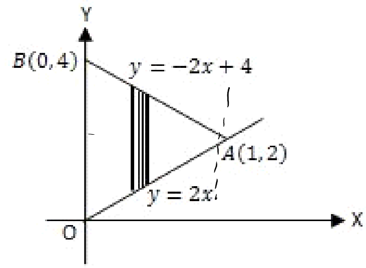
$$\frac{y-0}{x-0} = \frac{0-2}{0-1} \Rightarrow \frac{y}{x} = 2 \Rightarrow y = 2x$$

Eqn of AB

$$y-2 \quad 2-4 \quad \Rightarrow \quad \frac{y-2}{\quad} = -2 \Rightarrow y = -2x+4$$

$\overline{1-0}$ $\qquad x-1$

Taking a vertical strip



$y \rightarrow \quad y = 2x \quad$ to $\quad y = -2x+4$

$x \rightarrow \quad x = 0 \quad$ to $\quad x = 1$

$$I = \int_0^1 \int_{2x}^{-2x+4} (x^2 - xy)\, dy\, dx$$

$$= \int_0^1 \left[x^2 y - \frac{xy^2}{2} \right]_{2x}^{-2x+4} dx$$

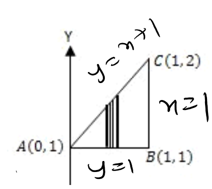
$$= \int_0^1 \left[x^2(-2x+4) - \frac{x}{2}(-2x+4)^2 - x^2(2x) + \frac{x}{2}(2x)^2 \right] dx$$

$$= \int_0^1 (-4x^3 + 12x^2 - 8x)\, dx$$

$$= \left(-x^4 + 4x^3 - 4x^2\right)_0^1$$

$$I = -1$$

9. $\iint (x^2 + y^2)\, dx\, dy$ over the area of the triangle whose vertices are $(0,1), (1,1), (1,2)$

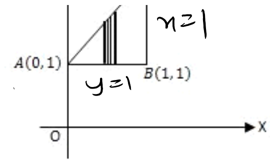


eq^n of AB : $y = 1$

eq^n of BC : $x = 1$

eq^n of AC : $\frac{y-2}{\ } = \frac{1-2}{\ }$

eq^n of BC



eq^n of AC: $\dfrac{y-2}{x-1} = \dfrac{1-2}{0-1}$

$y-2 = x-1 \Rightarrow y = x+1$

$$I = \int_0^1 \int_1^{x+1} (x^2+y^2)\, dy\, dx$$

$$= \int_0^1 \left(x^2y + \frac{y^3}{3}\right)_1^{x+1} dx$$

$$= \int_0^1 \left[x^2(x+1) + \frac{1}{3}(x+1)^3 - x^2(1) - \frac{1}{3}\right] dx$$

$$= \frac{1}{3}\int_0^1 (4x^3 + 3x^2 + 3x)\, dx$$

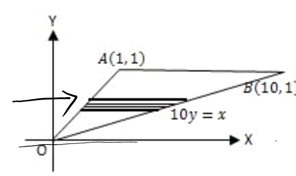
$$= \frac{1}{3}\left(x^4 + x^3 + \frac{3x^2}{2}\right)_0^1$$

$$\therefore I = \frac{7}{6}$$

10. $\iint_R \sqrt{xy - y^2}\, dx\, dy$ where R is a triangle whose vertices are $(0,0)$, $(10,1)$ and $(1,1)$

Eq^n of AB: $y = 1$

Eq^n of OA: $\dfrac{y-0}{x-0} = \dfrac{0-1}{x-1}$



$\therefore y = x$

Eq^n of OB: $\dfrac{y-0}{x-0} = \dfrac{0-1}{0-10} \Rightarrow x = 10y$

Taking a Horizontal strip 1 10y

Taking a Horizontal Strip

$x \rightarrow \quad x = y \quad$ to $\quad x = 10y$

$y \rightarrow \quad y = 0$ to $\quad y = 1$

$$\therefore I = \int_0^1 \int_y^{10y} \sqrt{xy - y^2}\, dx\, dy$$

$$I = \int_0^1 \left[\frac{(xy - y^2)^{3/2}}{\left(\frac{3}{2}\right) y} \right]_y^{10y}$$

$$= \int_0^1 \left[\frac{(9y^2)^{3/2}}{\left(3/2\, y\right)} - 0 \right] dy \qquad = \frac{2}{3} \int_0^1 27 y^2\, dy$$

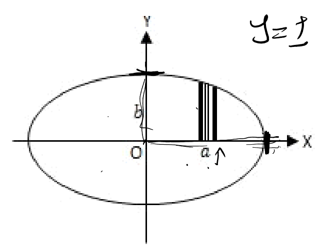
$$= 18 \left(\frac{y^3}{3} \right)_0^1$$

$$\therefore I = 6$$

11. $\iint x^{m-1} y^{n-1} dx\, dy$ over the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$y = \pm b \sqrt{1 - \frac{x^2}{a^2}}$$



Soln: To evaluate the integral, consider a strip parallel to the y-axis

on this strip,

y varies from $y = 0$ to $\frac{b}{a}\sqrt{a^2 - x^2}$

x varies from $x = 0$ to $x = a$

$a \quad \frac{b}{a}\sqrt{a^2 - x^2}$

$$I = \int_0^a \int_0^{\frac{b}{a}\sqrt{a^2-x^2}} x^{m-1} y^{n-1} \, dy \, dx$$

$$= \int_0^a x^{m-1} \left[\frac{y^n}{n}\right]_0^{\frac{b}{a}\sqrt{a^2-x^2}} dx$$

$$= \int_0^a x^{m-1} \cdot \frac{1}{n} \cdot \frac{b^n}{a^n} (a^2-x^2)^{n/2} \, dx$$

$$I = \frac{b^n}{na^n} \int_0^a x^{m-1} (a^2-x^2)^{n/2} \, dx$$

put $x = a\sin\theta \quad \therefore dx = a\cos\theta \, d\theta$

x	0	a
θ	0	$\pi/2$

$$I = \frac{b^n}{na^n} \int_0^{\pi/2} (a\sin\theta)^{m-1} (a^2 - a^2\sin^2\theta)^{n/2} a\cos\theta \, d\theta$$

$$= \frac{b^n}{na^n} \int_0^{\pi/2} a^{m+n} \sin\theta^{m-1} \cos^{n+1}\theta \, d\theta$$

$$= \frac{a^m b^n}{n} \int_0^{\pi/2} \sin^{m-1}\theta \cos^{n+1}\theta \, d\theta$$

$$= \frac{a^m b^n}{n} \cdot \frac{1}{2} B\left(\frac{m-1+1}{2}, \frac{n+1+1}{2}\right)$$

$$I = \frac{a^m b^n}{2n} \cdot B\left(\frac{m}{2}, \frac{n}{2}+1\right)$$

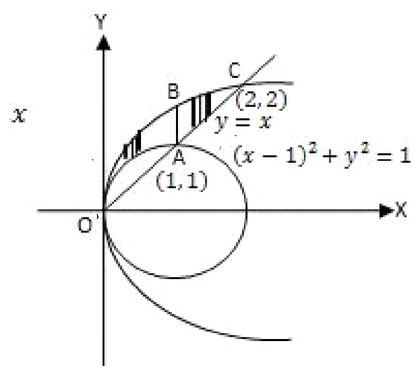
12. $\iint_R xy\, dx\, dy$ over the region R given by $x^2 + y^2 - 2x = 0, y^2 = 2x, y = x$

Soln. The given region is bounded by the line $y = x$, the parabola $y^2 = 2x$ and the circle

$$x^2 + y^2 - 2x = 0$$

$(x-1)^2 + y^2 = 1$ → circle with center $(1,0)$ and radius 1.

A is the point of intersection of circle and line



$y = x$ and $x^2 + y^2 - 2x = 0$

$\Rightarrow x^2 + x^2 - 2x = 0$

$2x^2 - 2x = 0$

$2x(x-1) = 0$

$x = 0$ or $x = 1$ $\Rightarrow y = 0$ or $y = 1$ $\therefore A(1,1)$

C is the point of intersection of line and parabola.

$y = x$, $y^2 = 2x$

$y^2 = 2y \Rightarrow y^2 - 2y = 0 \Rightarrow y(y-2) = 0 \Rightarrow y = 0$ or $y = 2$

$\Rightarrow x = 0$ or $x = 2$

$\therefore$ C is $(2,2)$

For evaluating the integral, we see that the region is divided into two parts, OAB and ABC.

In the region OAB, consider a vertical strip

on this strip y varies from $\sqrt{2x-x^2}$ to $\sqrt{2x}$ and x varies from $x=0$ to $x=1$

In the region ABC, again consider a vertical strip

on this strip, y varies from $y=x$ to $y=\sqrt{2x}$

and x varies from $x=1$ to $x=2$

$$\therefore J = \int_0^1 \int_{\sqrt{2x-x^2}}^{\sqrt{2x}} xy\,dy\,dx + \int_1^2 \int_x^{\sqrt{2x}} xy\,dy\,dx$$

$$= \int_0^1 x\left(\frac{y^2}{2}\right)_{\sqrt{2x-x^2}}^{\sqrt{2x}} dx + \int_1^2 x\left(\frac{y^2}{2}\right)_x^{\sqrt{2x}} dx$$

$$= \int_0^1 \frac{x}{2}\left(2x-2x+x^2\right)dx + \int_1^2 \frac{x}{2}\left(2x-x^2\right)dx$$

$$= \frac{1}{2}\int_0^1 x^3\,dx + \frac{1}{2}\int_1^2 \left(2x^2-x^3\right)dx$$

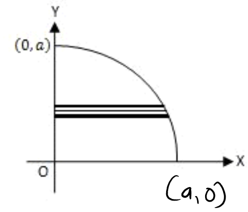
$$= \frac{1}{2}\left(\frac{x^4}{4}\right)_0^1 + \frac{1}{2}\left(\frac{2x^3}{3}-\frac{x^4}{4}\right)_1^2$$

$$= \frac{1}{8} + \frac{1}{2}\left[\frac{16}{3}-\frac{16}{4}-\frac{2}{3}+\frac{1}{4}\right] = \frac{7}{12}$$

13. Evaluate $\iint (x^2-y^2)\,x\,dx\,dy$ over the positive quadrant of the circle $x^2+y^2=a^2$

Soln.: $x^2+y^2=a^2$ is circle with centre at origin and radius a.

We consider only the $\frac{1}{4}$th part of circle in the first quadrant.



Taking a horizontal strip in this region

x varies from 0 to $\sqrt{a^2-y^2}$ and y varies from 0 to a

$$\therefore I = \int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2-y^2)\,x\,dx\,dy$$

$$= \int_0^a \left[\frac{x^4}{4} - \frac{x^2}{2}y^2\right]_0^{\sqrt{a^2-y^2}} dy$$

$$= \int_0^a \left[\frac{(a^2-y^2)^2}{4} - \frac{(a^2-y^2)y^2}{2}\right] dy$$

$$= \int_0^a \left[\frac{1}{4}(a^4-2a^2y^2+y^4) - \frac{1}{2}(a^2y^2-y^4)\right] dy$$

$$= \frac{1}{4}\left[a^4y - 2a^2\frac{y^3}{3} + \frac{y^5}{5}\right]_0^a - \frac{1}{2}\left(a^2\frac{y^3}{3} - \frac{y^5}{5}\right)_0^a$$

$$= \frac{1}{4}\left[a^5 - \frac{2a^5}{3} + \frac{a^5}{5}\right] - \frac{1}{2}\left(\frac{a^5}{3} - \frac{a^5}{5}\right)$$

$$= 2\,a^5 \quad 1\,a^5$$

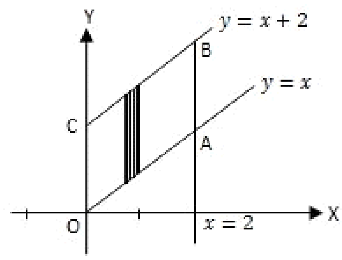
$$= \frac{2}{15}a^5 - \frac{1}{15}a^5$$

$$\therefore I = \frac{a^5}{15}$$

14. Evaluate $\iint_R (x+y)\,dxdy$ where R is the region bounded by $x=0, x=2, y=x, y=x+2$

Soln: The region of integration is bounded by $x=0$ ie the y-axis

$x=2$ ie the line parallel to the y-axis



$y=x$ ie the line through the origin

$y=x+2$ ie the line parallel to $y=x$ and making intercept 2 on the y-axis

Thus the region of integration is OABC

Take a strip parallel to y-axis

On this strip, y varies from $y=x$ to $y=x+2$ and then x varies from $x=0$ to $x=2$

$$\therefore \iint_R (x+y)\,dx\,dy = \int_0^2 \int_x^{x+2} (x+y)\cdot dy\,dx$$

$$= \int_0^2 \left[xy + \frac{y^2}{2}\right]_x^{x+2} dx$$

$$= \int^2 \left[x(x+2) + \frac{(x+2)^2}{2} - x^2 - \frac{x^2}{2}\right] dx$$

$$= \int_0 \left[x(x+2) + \frac{(x+2)^2}{2} - x^2 - \frac{x^2}{2} \right] dx$$

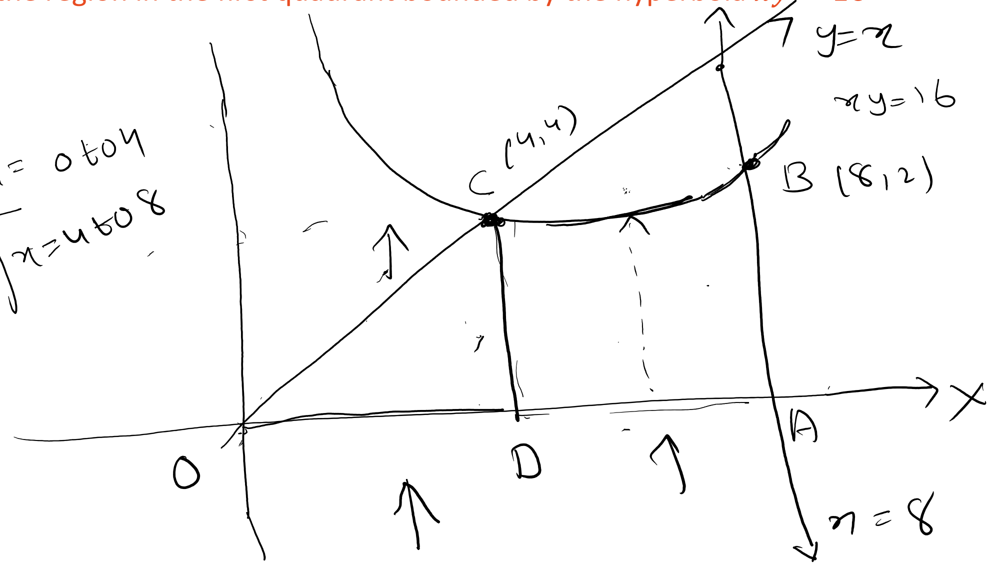
$$= \int_0^2 (4x+2)\, dx$$

$$= 2(x^2+x)_0^2$$

$$\therefore I = 12$$

15. Evaluate $\iint_R x^2\, dA$ where R is the region in the first quadrant bounded by the hyperbola $xy = 16$ and the lines $y = x, y = 0$ and $x = 8$

$y=0$	$y=x$	$x = 0$ to 4
$y=0$	$y=\frac{16}{x}$	$x=4$ to 8



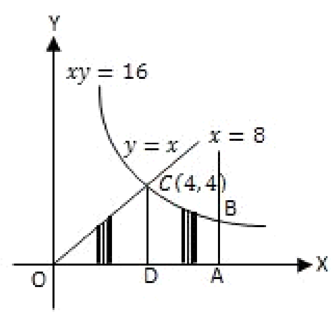
The point C, is point of intersection of $y=x$ and hyperbola $xy=16$.

$\therefore$ C is $(4,4)$

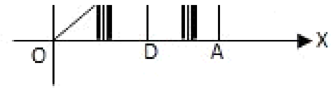
point B is point of intersection of $x=8$ and $xy=16$

$\therefore B(8,2)$

The region of integration is OABC



we have to split it into 2 regions



we have to split it into 2 regions ODC and DABC

consider a vertical strip in the region ODC

On this strip, y varies from $y=0$ to $y=x$ and then x varies from $x=0$ to $x=4$

consider a vertical strip in the region CDAB

On this strip y varies from $y=0$ to $y=\frac{16}{x}$ and then x varies from $x=4$ to $x=8$

$$\iint_R x^2\,dA = \int_0^4\int_0^x x^2\,dy\,dx + \int_4^8\int_0^{16/x} x^2\,dy\,dx$$

$$= \int_0^4 x^2 (y)_0^x\,dx + \int_4^8 x^2 (y)_0^{16/x}\,dx$$

$$= \int_0^4 x^2(x-0)\,dx + \int_4^8 x^2\left(\frac{16}{x}-0\right)dx$$

$$= \int_0^4 x^3\,dx + \int_4^8 16x\,dx$$

$$= \left(\frac{x^4}{4}\right)_0^4 + \left(8x^2\right)_4^8$$

$$I = 64 + 8(64 - 16)$$

$$I = 448$$