

### PRACTICE PROBLEMS ON HIGHER ORDER DIFFERENTIAL EQUATIONS

#### TYPE-1

Solve the following equations.

1.  $9 \frac{d^2y}{dx^2} + 18 \frac{dy}{dx} - 16y = 0$
2.  $\frac{d^3y}{dx^3} - 6 \frac{d^2y}{dx^2} + 11 \frac{dy}{dx} - 6y = 0$
3.  $6 \frac{d^3x}{dt^3} + 23 \frac{d^2x}{dt^2} + 29 \frac{dx}{dt} + 12x = 0$
4.  $\frac{d^4y}{dx^4} - 5 \frac{d^3y}{dx^3} + 5 \frac{d^2y}{dx^2} + 5 \frac{dy}{dx} - 6y = 0$
5.  $\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 4y = 0$
6.  $\frac{d^3y}{dx^3} - 3 \frac{d^2y}{dx^2} + 3 \frac{dy}{dx} - y = 0$
7.  $\frac{d^3y}{dx^3} - 5 \frac{d^2y}{dx^2} + 8 \frac{dy}{dx} - 4y = 0$
8.  $\frac{d^4y}{dx^4} - 18 \frac{d^2y}{dx^2} + 81y = 0$
9.  $\frac{d^3y}{dx^3} + 8y = 0$
10.  $\frac{d^4y}{dx^4} - m^4y = 0$
11.  $\frac{d^4y}{dx^4} + 4 \frac{d^3y}{dx^3} + 8 \frac{d^2y}{dx^2} + 8 \frac{dy}{dx} + 4y = 0$
12.  $\frac{d^4y}{dx^4} + k^4y = 0$
13.  $\frac{d^6y}{dx^6} - 64y = 0$
14.  $(D^3 - D^2 + D - 1)^2y = 0$
15.  $\{(D - 1)^4(D^2 + 2D + 2)^2\}y = 0$
16.  $\{(D^2 + 1)^3(D^2 + D + 1)^2\}y = 0$
17.  $(D^4 + 8D^2 + 16)y = 0$

#### TYPE-2

Solve the following equations.

1.  $\frac{d^2y}{dx^2} + 3 \frac{dy}{dx} + 2y = e^{-x}$
2.  $(D^3 - 2D^2 - 5D + 6)y = e^{3x} + 8$
3.  $(D^3 - 2D^2 - 5D + 6)y = (e^{2x} + 3)^2$
4.  $6 \frac{d^2y}{dx^2} + 17 \frac{dy}{dx} + 12y = e^{-3x/2} + 2^x$
5.  $(D^2 - 2D + 1)y = e^x + 1$
6.  $(D^2 - (a + b)D + ab)y = e^{ax} + e^{bx}$
7.  $(D^2 + 6D + 9)y = \sin h 3x$
8.  $(D^2 + 4D + 4)y = \cos h 2x$
9.  $(D^3 - 4D)y = 2 \cosh 2x$
10.  $(D^3 - a^2D)y = 2 \cos h ax$
11.  $(D^4 + 1)y = \cos h 4x \sin h 3x$
12.  $\frac{d^3y}{dx^3} - 4 \frac{dy}{dx} = 2 \cos h^2 2x$
13.  $\frac{d^3y}{dx^3} - y = (1 + e^x)^2$

#### TYPE-3

Solve the following equations.

1.  $(D^2 + 4)y = \cos 2x$
2.  $\frac{d^3y}{dx^3} - 3 \frac{d^2y}{dx^2} + 9 \frac{dy}{dx} - 27y = \cos 3x$
3.  $\frac{d^4y}{dx^4} - a^4y = \sin ax$
4.  $(D^4 + 10D^2 + 9)y = \cos (2x + 3)$

5.  $(D^2 - 4)y = \sin^2 x$
7.  $(D^4 + 8D^2 + 16)y = \sin^2 x$
9.  $(D^2 + D + 1)y = (1 + \sin x)^2$
11.  $(D^2 - 2D + 1)y = e^x + \sin(\sqrt{3}x)$
13.  $(D - 1)^2(D^2 + 1)y = e^x + \sin^2(x/2)$
15.  $\frac{d^2y}{dx^2} + y = \sin x \sin 2x + 2^x$
6.  $(D^2 + 1)y = \sin x \sin 2x$
8.  $(D^3 + D^2 + D + 1)y = \sin^2 x$
10.  $\frac{d^2y}{dx^2} + 9y = e^x - \cos 2x$
12.  $(D^4 - 1)y = e^x + \cos x \cos 3x$
14.  $(D - 1)^2(D^2 + 1)^2y = \sin^2 \frac{x}{2} + e^x$

**TYPE-4**

Solve the following equations.

1.  $(D^4 - 2D^3 + D^2)y = x^3$
2.  $\frac{d^3y}{dx^3} - 2\frac{dy}{dx} + 4y = 3x^2 - 5x + 2$
4.  $(D^2 + 2D + 2)y = x^2 + 1$
6.  $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2 + \cos 2x + e^x$
8.  $(D^2 + D - 2)y = x + e^{-2x}$
9.  $(D^2 - 4D + 4)y = 8(x^2 + \sin 2x + e^{2x})$
10.  $(D^2 - 4D + 4)y = e^{2x} + x^3 + \cos 2x$
12.  $(D^4 + 4)y = x^2 + \sin 2x$
3.  $(D^3 - 2D^2 + D)y = x^2 + x$
5.  $(D^3 - D^2 - 6D)y = x^2 + 1$
7.  $\frac{d^3y}{dt^3} + \frac{dy}{dt} = \cos t + t^2 + 3$
11.  $(D^3 - D)y = 2e^x + 2x + 1 - 4\cos x$

**TYPE-5**

Solve the following equations.

1.  $(D^2 - 4)y = x^2 e^{3x}$
2.  $(D^2 - 3D + 2)y = x^2 e^{2x}$
4.  $(D^2 - 6D + 9)y = \frac{e^{3x}}{x^2}$
6.  $(D^2 + 6D + 9)y = \frac{1}{x^3} e^{-3x} + 2^x$
8.  $(D^3 - 7D - 6)y = e^{2x}(x + 1)$
10.  $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = (x^2 e^x)^2$
12.  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2e^x \cos \frac{x}{2}$
14.  $(D^2 + D - 6)y = e^{2x} \sin 3x$
16.  $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 5y = e^x \cos 3x$
18.  $(D^2 + 1)y = \sin x \sin hx$
20.  $(D^2 - 1)y = \cos hx \cos x$
3.  $(D^2 - 8D + 16)y = \frac{e^{4x}}{x^2}$
5.  $(D^2 + 4D + 4)y = \frac{e^{-2x}}{x^5}$
7.  $(D^2 - 4D + 4)y = \frac{e^{2x}}{1+x^2}$
9.  $(D^3 - 7D - 6)y = (1 + x^2)e^{2x}$
11.  $(D^2 - 1)y = x \sin hx$
13.  $(D^2 - 3D + 2)y = 2e^x \sin \left(\frac{x}{2}\right)$
15.  $(D^3 + 1)y = e^{x/2} \sin \left(\frac{\sqrt{3}}{2} x\right)$
17.  $(D^2 - 4D + 3)y = e^x \cos 2x + \cos 3x$
19.  $(D^4 - 1)y = \cos x \cos hx$
21.  $(D^3 - 7D - 6)y = \cos hx \cos x$

22.  $\frac{d^2y}{dx^2} + 2y = x^2e^{3x} + e^x - \cos 2x$

23.  $(D^2 + 2)y = e^x \cos x + x^2e^{3x}$

24.  $\frac{d^2y}{dx^2} + 2y = x^2e^{3x} + e^x \cos 3x$

25.  $(D^2 - 2D + 2)y = e^x(x + \sin x)$

**TYPE-6**

1.  $(D^2 + a^2)y = \sec ax$

2.  $(D^2 + a^2)y = \operatorname{cosec} ax$

4.  $(D^2 + 5D + 6)y = e^{-2x} \sec^2 x (1 + 2 \tan x)$

3.  $(D^2 + 3D + 2)y = \sin(e^x)$

5.  $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$

6.  $(D^2 + D)y = \frac{1}{1+e^x}$

7.  $(D^2 - D - 2)y = 2 \log x + \frac{1}{x} + \frac{1}{x^2}$

8.  $(D^2 + a^2)y = 2a \tan ax$

9.  $(D^2 + 2D + 1)y = 4e^{-x} \log x$

10.  $(D^2 - 1)y = e^{-x} \sin(e^{-x}) + \cos(e^{-x})$

11.  $(D^2 - 1)y = \frac{2}{1+e^x}$

**ANSWERS****TYPE-1**

1.  $y = c_1 e^{(2/3)x} + c_2 e^{(-8/3)x}$

2.  $y = c_1 e^x + c_2 e^{2x} + c_3 e^{3x}$

3.  $x = c_1 e^{-t} + c_2 e^{(-3/2)t} + c_3 e^{(-4/3)t}$

4.  $y = c_1 e^{-x} + c_2 e^{2x} + c_3 e^{3x} + c_4 e^x$

5.  $y = (c_1 + c_2 x) e^{-2x}$

6.  $y = (c_1 + c_2 x + c_3 x^2) e^{3x}$

7.  $y = (c_1 + c_2 x) e^{-2x}$

8.  $y = (c_1 + c_2 x) e^{3x} + (c_3 + c_4 x) e^{-3x}$

9.  $y = c_1 e^{-2x} + e^x (c_2 \cos(\sqrt{3}x) + c_3 \sin(\sqrt{3}x))$

10.  $y = c_1 e^{mx} + c_2 e^{-mx} + c_3 \cos mx + c_4 \sin mx$

11.  $y = e^{-x} [(c_1 + c_2 x) \cos x + (c_3 + c_4 x) \sin x]$

12.  $y = e^{(k/\sqrt{2})x} (c_1 \cos(k/\sqrt{2})x + c_2 \sin(k/\sqrt{2})x) + e^{(-k/\sqrt{2})x} (c_3 \cos(k/\sqrt{2})x + c_4 \sin(k/\sqrt{2})x)$

13.  $y = c_1 e^{2x} + c_2 e^{-2x} + e^{-x} (c_3 \cos(\sqrt{3}x) + c_4 \sin(\sqrt{3}x)) + e^x (c_5 \cos(\sqrt{3}x) + c_6 \sin(\sqrt{3}x))$

14.  $y = (c_1 + c_2 x) e^x + (c_3 + c_4 x) \cos x + (c_5 + c_6 x) \sin x$

15.  $y = (c_1 + c_2 x + c_3 x^2 + c_4 x^3) e^x + e^{-x} \{ (c_5 + c_6 x) \cos x + (c_7 + c_8 x) \sin x \}$

16.  $y = [(c_1 + c_2 x + c_3 x^2) \cos x + (c_4 + c_5 x + c_6 x^2) \sin x] + e^{-x/2} [(c_7 + c_8 x) \cos(\sqrt{3}/2)x + (c_9 + c_{10} x) \sin(\sqrt{3}/2)x]$

17.  $y = (c_1 + c_2 x) \cos 2x + (c_3 + c_4 x) \sin 2x$

**TYPE-2**

1.  $y = c_1 e^{-x} + c_2 e^{-2x} + x e^{-x}$

2.  $y = c_1 e^x + c_2 e^{-2x} + c_3 e^{3x} + \frac{x}{10} e^{3x} + \frac{4}{3}$

3.  $y = c_1 e^x + c_2 e^{-2x} + c_3 e^{3x} + \frac{1}{18} e^{4x} - \frac{3}{2} e^{2x} + \frac{3}{2}$

4.  $y = c_1 e^{-4x/3} + c_2 e^{-3x/2} - x e^{-3x/2} + \frac{2^x}{6(\log 2)^2 + 17 \log 2 + 12}$

5.  $y = (c_1 + c_2 x) e^x + \frac{x^2}{2} e^x + 1$

6.  $y = c_1 e^{ax} + c_2 e^{bx} + \frac{x}{a-b} [e^{ax} - e^{bx}]$
7.  $y = (c_1 + c_2 x) e^{-3x} + \frac{1}{2} \left[ \frac{e^{3x}}{36} + \frac{x^2}{2} e^{-3x} \right]$
8.  $y = (c_1 + c_2 x) e^{-2x} + \frac{1}{32} e^{2x} + \frac{x^2}{4} e^{-2x}$
9.  $y = c_1 + c_2 e^{2x} + c_3 e^{-2x} + \frac{x}{8} (e^{2x} + e^{-2x})$
10.  $y = c_1 + c_2 e^{ax} + c_3 e^{-ax} + \frac{x}{2a^2} (e^{ax} + e^{-ax})$
11.  $y = e^{x/\sqrt{2}} \left\{ c_1 \cos \left( \frac{x}{\sqrt{2}} \right) + c_2 \sin \left( \frac{x}{\sqrt{2}} \right) \right\} + e^{-x/\sqrt{2}} \left\{ c_3 \cos \left( \frac{x}{\sqrt{2}} \right) + c_4 \sin \left( \frac{x}{\sqrt{2}} \right) \right\}$   
 $+ \frac{1}{9608} (e^{7x} - e^{-7x}) - \frac{1}{8} (e^x - e^{-x})$
12.  $y = c_1 + c_2 e^{2x} + c_3 e^{-2x} - \frac{x}{4} + \frac{1}{48} \sin h 4x$
13.  $y = c_1 e^x + e^{-x/2} [c_2 \cos(\sqrt{3}/2)x + c_3 \sin(\sqrt{3}/2)x] - 1 + \frac{2}{3} x e^x + \frac{1}{7} e^{2x}$

**TYPE-3**

1.  $y = c_1 \cos 2x + c_2 \sin 2x + \frac{x}{4} \sin 2x$
2.  $y = c_1 e^{3x} + (c_2 \cos 3x + c_3 \sin 3x) - \frac{x}{36} (\cos 3x + \sin 3x)$
3.  $y = c_1 e^{ax} + c_2 e^{-ax} + (c_3 \cos ax + c_4 \sin ax) + \frac{1}{4a^3} x \cos ax$
4.  $y = c_1 \cos x + c_2 \sin x + c_3 \cos 3x + c_4 \sin 3x - \frac{1}{15} \cos (2x + 3)$
5.  $y = C_1 e^{2x} + C_2 e^{-2x} - \frac{1}{8} + \frac{1}{16} \cos 2x$
6.  $y = c_1 \cos x + c_2 \sin x + \frac{1}{4} x \sin x + \frac{1}{16} \cos 3x$
7.  $y = (c_1 + c_2 x) \cos 2x + (c_3 + c_4 x) \sin 2x + \frac{1}{32} + \frac{x^2}{64} \cos 2x$
8.  $y = c_1 \cos x + c_2 \sin x + c_3 e^{-x} + \frac{1}{2} + \frac{1}{30} (2 \sin 2x + \cos 2x)$
9.  $y = e^{-x/2} \left( c_1 \cos \left( \frac{\sqrt{3}}{2} x \right) + c_2 \sin \left( \frac{\sqrt{3}}{2} x \right) \right) + \frac{3}{2} - 2 \cos x - \frac{1}{26} (2 \sin 2x - 3 \cos 2x)$
10.  $y = c_1 \cos 3x + c_2 \sin 3x + \frac{1}{10} e^x - \frac{1}{5} \cos 2x$
11.  $y = (c_1 + c_2 x) e^x + \frac{1}{8} (\sqrt{3} \cos(\sqrt{3}x) - \sin(\sqrt{3}x)) + \frac{x^2}{2} e^x$
12.  $y = c_1 e^x + c_2 e^{-x} + (c_3 \cos x + c_4 \sin x) + \frac{x}{4} e^x + \frac{1}{510} \cos 4x + \frac{1}{30} \cos 2x$
13.  $y = (c_1 + c_2 x) e^x + (c_3 \cos x + c_4 \sin x) + \frac{x^2}{4} e^x + \frac{1}{2} - \frac{x}{8} \cos x$
14.  $y = (c_1 + c_2 x) e^x + ((c_3 + c_4 x) \cos x + (c_5 + c_6 x) \sin x) + \frac{1}{2} - \frac{1}{32} x^2 \sin x + \frac{1}{8} x^2 e^x$
15.  $y = c_1 \cos x + c_2 \sin x + \frac{1}{16} \cos 3x + \frac{x}{4} \sin x + \frac{1}{(\log 2)^2 + 1} \cdot 2^x$

**TYPE-4**

1.  $y = (c_1 + c_2 x) + (c_3 + c_4 x) e^x + \frac{x^5}{20} + \frac{x^4}{2} + 3x^3 + 12x^2$
2.  $y = c_1 e^{-2x} + e^x (c_2 \cos x + c_3 \sin x) + \frac{1}{4} [3x^2 - 2x + 1]$

3.  $y = c_1 + (c_2 + c_3x)e^x + \frac{x^3}{3} + \frac{5x^2}{2} + 8x$
4.  $y = (c_1 \cos x + c_2 \sin x)e^{-x} + \frac{1}{2}(x^2 - 2x + 2)$
5.  $y = c_1 + c_2e^{-2x} + c_3e^{3x} - \frac{1}{6}\left[\frac{x^3}{3} - \frac{x^2}{6} + \frac{25x}{18}\right]$
6.  $y = (c_1 + c_2x)e^{2x} + \frac{1}{4}\left[x^2 + 2x + \frac{3}{2}\right] + e^x - \frac{1}{8}\sin 2x$
7.  $y = c_1 + c_2 \cos t + c_3 \sin t - \frac{t}{2} \cos t + \frac{t^3}{3} + t$
8.  $y = c_1e^x + c_2e^{-2x} - \frac{1}{2}\left(x + \frac{1}{2}\right) - \frac{x}{3}e^{-2x}$
9.  $y = (c_1 + c_2x)e^{2x} + 2x^2 + 4x + 3 + \cos 2x + 4x^2e^{2x}$
10.  $y = (c_1 + c_2x)e^{2x} + \frac{x^2}{2}e^x + \frac{1}{4}\left[x^3 + 3x^2 + \frac{9x}{2} + 3\right] - \frac{\sin 2x}{8}$
11.  $y = c_1 + c_2e^x + c_3e^{-x} - x^2 - x + 2 \sin x + xe^x$
12.  $y = c_1 \cos 2x + c_2 \sin 2x - \frac{x}{4} \cos 2x + \frac{1}{4}\left(x^2 - \frac{1}{2}\right)$

**TYPE-5**

1.  $y = c_1e^{2x} + c_2e^{-2x} + \frac{e^{3x}}{5}\left(x^2 - \frac{12x}{5} + \frac{62}{25}\right)$
2.  $y = c_1e^x + c_2e^{2x} + e^{2x}\left(\frac{x^3}{3} - x^2 + 2x\right)$
3.  $y = (c_1 + c_2x)e^{4x} - e^{4x} \log x$
4.  $y = (c_1 + c_2x)e^{3x} - e^{3x} \log x \quad y$
5.  $y = (c_1 + c_2x)e^{-2x} + \frac{e^{-2x}}{12x^3}$
6.  $y = (c_1 + c_2x)e^{-3x} + \frac{1}{2x}e^{-3x} + \frac{1}{(3+\log 2)^2}2^x$
7.  $y = (c_1 + c_2x)e^{2x} + e^{2x}\left[x \tan^{-1}x - \frac{1}{2} \log(1+x^2)\right]$
8.  $y = c_1e^{-x} + c_2e^{-2x} + c_3e^{3x} - e^{2x}\frac{1}{12}\left(x + \frac{17}{12}\right)$
9.  $y = c_1e^{-x} + c_2e^{-2x} - e^{2x}\frac{1}{12}\left(x^2 + \frac{5}{6}x + \frac{169}{72}\right)$
10.  $y = c_1e^x + c_2e^{3x} - e^{2x}(x^4 + 12x^2 + 24)$
11.  $y = c_1e^x + c_2e^{-x} + \frac{x^2}{4} \cos hx - \frac{x}{4} \sin hx$
12.  $y = c_1e^x + c_2e^{2x} - \frac{8}{5}e^x\left[2 \sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right]$
13.  $y = c_1e^x + c_2e^{2x} - \frac{8}{5}e^x\left(\sin \frac{x}{2} - 2 \cos \frac{x}{2}\right)$
14.  $y = c_1e^{2x} + c_2e^{-3x} - \frac{e^{2x}}{102}(5 \cos 3x + 3 \sin 3x)$
15.  $y = c_1e^{-x} + e^{x/2}\left(c_2 \cos \frac{\sqrt{3}}{2}x + c_3 \sin \frac{\sqrt{3}}{2}x\right) - \frac{xe^{x/2}}{6}\left[\sqrt{3} \cos(\sqrt{3}/2)x + \sin(\sqrt{3}/2)x\right]$
16.  $y = c_1e^{-x} + e^x(c_2 \cos 2x + c_3 \sin 2x) - \frac{e^x}{65}(3 \sin 3x + 2 \cos 3x)$
17.  $y = c_1e^{3x} + c_2e^x - \frac{1}{30}(2 \sin 3x + \cos 3x) - e^x \frac{1}{8}(\sin 2x + \cos 3x)$
18.  $y = c_1 \cos x + c_2 \sin x + \frac{1}{5}[-2 \cos x \cos hx + \sin x \sin hx]$

19.  $y = c_1 e^x + c_2 e^{-x} + c_3 \cos x + c_4 \sin x - \frac{1}{5} \cos x \cos hx$
20.  $y = c_1 e^x + c_2 e^{-x} + \frac{1}{5} [2 \sin x \sin hx - \cos x \cos hx]$
21.  $y = c_1 e^{-x} + c_2 e^{-2x} + c_3 e^{3x} - \frac{1}{100} \cdot e^x (\sin x + 3 \cos x) + \frac{1}{68} \cdot e^{-3x} (3 \cos x - 5 \sin x)$
22.  $y = (c_1 \cos(\sqrt{2}x) + c_2 \sin(\sqrt{2}x)) + \frac{e^{3x}}{11} \left[ x^2 - \frac{12x}{11} + \frac{50}{121} \right] + \frac{1}{3} e^x + \frac{1}{2} \cos 2x$
23.  $y = (c_1 \cos(\sqrt{2}x) + c_2 \sin(\sqrt{2}x)) + e^x \frac{1}{4} (\sin x + \cos x) + \frac{e^{3x}}{11} \left( x^2 - \frac{12x}{11} + \frac{50}{121} \right)$
24.  $y = (c_1 \cos(\sqrt{2}x) + c_2 \sin(\sqrt{2}x)) + \frac{e^{3x}}{11} \left[ x^2 - \frac{12x}{11} + \frac{50}{121} \right] + \frac{e^x}{12} (\sin 3x - \cos 3x)$
25.  $y = e^x (c_1 \cos x + c_2 \sin x) + x e^x - \frac{x}{2} e^x \cos x$

**TYPE-6**

1.  $y = c_1 \cos ax + c_2 \sin ax + \frac{x}{a} \sin ax - \frac{1}{a^2} \cos ax \log \cos ax$
2.  $y = c_1 \cos ax + c_2 \sin ax + \frac{1}{a^2} \log(\sin ax) \sin ax - \frac{x}{a} \cos ax$
3.  $y = c_1 e^{-x} + c_2 e^{-2x} - e^{-2x} \sin(e^x)$
4.  $y = c_1 e^{-2x} + c_2 e^{-3x} + e^{-2x} [\tan x]$
5.  $y = c_1 e^{-x} + c_2 e^{-2x} + e^{-2x} e^x$
6.  $y = c_1 + c_2 e^{-x} - e^{-x} [e^x \log(e^{-x} + 1) + \log(1 + e^x)]$
7.  $y = c_1 e^{-x} + c_2 e^{2x} - \log x$
8.  $y = c_1 \cos ax + c_2 \sin ax - \frac{2}{a} \cos ax \log \tan \left( \frac{\pi}{4} + \frac{ax}{2} \right)$
9.  $y = (c_1 + c_2 x) e^{-x} + e^{-x} x^2 (2 \log x - 3)$
10.  $y = c_1 e^{-x} + c_2 e^x - e^x \sin(e^{-x})$
11.  $y = c_1 e^{-x} + c_2 e^x - e^{-x} \log(1 + e^x) - 1 + e^x \log(e^{-x} + 1)$