PRACTICE PROBLEMS ON FIRST ORDER FIRST DEGREE DIFFERENTIAL EQUATIONS

TYPE- I

Solve the following equations.

1. $(2x^2 + 3y^2 - 7)\,x\,dx + (3x^2 + 2y^2 - 8)\,y\,dy = 0$
2. $\left(y^2e^{xy^2} + 4x^3\right)dx + \left(2xy\,e^{xy^2} - 3y^2\right)dy = 0$
3. $\left[y\left(1+\frac{1}{x}\right) + cos\,y\right]dx + (x + log\,x - x\,sin\,y)dy = 0$
4. $\frac{ydx-xdy}{(x+y)^2} = \frac{dx}{2\sqrt{1-x^2}}$
5. $(x^2 - xtan^2\,y + sec^2\,y)\,dy = (tan\,y - 2xy - y)\,dx$
6. $[y\,sin\,(xy) + xy^2\,cos\,(xy)]dx + [x\,sin\,(xy) + x^2\,y\,cos\,(xy)]\,dy = 0$
7. $x\,dx + y\,dy = \frac{a(x\,dy-y\,dx)}{x^2+y^2}$
8. $(2xy\,cos\,x^2 - 2xy + 1)\,dx + (sin\,x^2 - x^2)\,dy = 0$
9. $\frac{dy}{dx} + \frac{y\,cos\,x+sin\,y+y}{sin\,x+x\,cos\,y+x} = 0$
10. $2\left(1 + x^2\sqrt{y}\right)ydx + \left(x^2\sqrt{y} + 2\right)x\,dy = 0$
11. $\left(x\sqrt{1-x^2y^2} - y\right)dy + \left(x + y\sqrt{1-x^2y^2}\right)dx = 0$
12. $\frac{y}{x^2}cos\left(\frac{y}{x}\right)dx - \frac{1}{x}cos\left(\frac{y}{x}\right)dy + 2xdx = 0$
13. $\left(1 + e^{x/y}\right)dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0, given\ y(0) = 4$
14. $\frac{dy}{dx} = -\frac{4x^3y^2+y\,cos\,(xy)}{2x^4y+x\,cos\,(xy)}$
15. $(2x^2 + 3y^2 - 7)\,x\,dx + (3x^2 + 2y^2 - 8)\,y\,dy = 0$
16. $(\tan y + x)dx + (x\sec^2 y - 3y)dy = 0$
17. $[1 + \log(xy)]dx + \left(1 + \frac{x}{y}\right)dy = 0$

ANSWERS

1. $x^4 + y^4 + 3x^2y^2 - (7x^2 + 8y^2) = c$
2. $e^{xy^2} + x^4 - y^3 = c$
3. $xy + y\,log\,x + x\,cos\,y = c$
4. $\frac{y}{x+y} + \frac{1}{2}sin^{-1}x = c$
5. $x\,tan\,y - x^2y - xy - tan\,y = c$
6. $xy\,sin\,(xy) = c$
7. $\frac{x^2}{2} + \frac{y^2}{2} + a\,tan^{-1}\left(\frac{x}{y}\right) = c$
8. $y\,sin\,x^2 - x^2y + x = c$
9. $y\,sin\,x + x\,sin\,y + xy = c$
10. $2xy + \frac{2}{3}x^3y^{3/2} = c$
11. $x^2 - y^2 + xy\sqrt{1-x^2y^2} + sin^{-1}xy = c$
12. $x^2 - sin\left(\frac{y}{x}\right) = c$
13. $x + ye^{x/y} = 4$
14. $x^4y^2 + sin\,xy = c$
15. $x^4 + 3x^2y^2 - 7x^2 + y^4 - 8y^2 = 0$
16. $2x\tan y + x^2 - 3y^2 = c$
17. $y + x\log(xy) = c$

TYPE – II

Solve the following equations.

1. $\left(y+\frac{1}{3}y^3+\frac{1}{2}x^2\right)dx+\frac{1}{4}(x+xy^2)dy=0$

2. $(x^4e^x - 2m\,xy^2)\,dx + 2mx^2\,y\,dy = 0$

3. $(x^2 + y^2 + 1)dx - 2xy\,dy = 0$

4. $(y - 2x^3)dx - x\,(1 - xy)\,dy = 0$

5. $(2x\log x - xy)dy + 2ydx = 0$

6. $(4xy + 3y^2 - x)dx + x(x + 2y)dy = 0$

7. $\left(xy^2 - e^{1/x^3}\right)dx - x^2ydy = 0$

8. $(x^4 + y^4)\,dx - xy^3\,dy = 0$

9. $(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$

10. $y(xy + e^x)dx - e^xdy = 0$

11. $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$

12. $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$

13. $(2xy^2 - y)dx + xdy = 0$

14. $y(1 + xy)\,dx + x\,(1 - xy)\,dy = 0$

15. $(y - xy^2)dx - (x + x^2y)dy = 0$

16. $y(1 + xy)\,dx + x(1 + xy + x^2y^2)\,dy = 0$

17. $\frac{dy}{dx} = -\frac{x^2y^3+2y}{2x-2x^3y^2}$

18. $(xy\,sin\,xy + cos\,xy)\,y\,dx + (xy\,sin\,xy - cos\,xy)\,x\,dy = 0$

19. $(x^4 + y^4)dx - xy^3\,dy = 0$

20. $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$

21. $y(x + y)dx - x(y - x)dy = 0$

22. $(x^2 + y^2)dx - (x^2 + xy)dy = 0$

23. $(x^2 - xy + y^2)dx - xydy = 0$

24. $\left[2x\,sin\,h\left(\frac{y}{x}\right) + 3ycos\,h\left(\frac{y}{x}\right)\right]dx - 3x.cos\,h\left(\frac{y}{x}\right).dy = 0$

25. $\left(\frac{y}{x}secy - tan\,y\right)dx - (x - secylog\,x)dy = 0$

26. If y^n is an integrating factor of $y(2x^2y + e^x)dx - (e^x + y^2)dy = 0$ find n and solve the equation.

27. If y^n is an integrating factor of $(2xy^2 + e^xy)dx - e^xdy = 0$ then find n and solve the equation.

28. If y^n is an integrating factor of $(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$. Find n and solve the equation.

29. If x^n is an integrating factor of $y\,dx - x\,dy + (1 + x^2)dx + x^2\,sin\,y\,dy = 0$ Find n and solve the equation.

30. If x^n is an integrating factor of $(y - 2x^3)dx - x(1 - xy)dy = 0$ then find n and solve the equation.

31. If x^n is an integrating factor of $(x^4e^x - 2mxy^2)dx + 2mx^2y\,dy = 0$ find n and solve the equation.

32. If $f(x)$ a function of x only is an integrating factor of $(x^4e^x - 2mxy^2)dx + 2mx^2ydy = 0$. Find $f(x)$ and then solve the equation.

33. If $(x+y)^k$ is an integrating factor of $(4x^2 + 2xy + 6y)\,dx + (2x^2 + 9y + 3x)\,dy = 0$. find k and solve the equation.

34. If $(x+y+1)^n$ is an integrating factor of $(2xy - y^2 - y)dx + (2xy - x^2 - x)dy = 0$ find n and solve the equation.

ANSWERS

1. $x^6 + 3x^4y + x^4y^3 = c$

2. $x^2e^x + my^2 = cx^2$

3. $x^2 - y^2 - 1 = cx$

4. $-\frac{y}{x} - x^2 + \frac{y^2}{2} = c$

5. $2ylog\ x - \frac{y^2}{2} = c$

6. $4x^4y + 4x^3y^2 - x^4 = c$

7. $\frac{1}{3}e^{1/x^3} - \frac{y^2}{2x^2} = c$

8. $4x^4\ log\ x - y^4 = cx^4$

9. $x^2e^y + \frac{x^2}{y} + \frac{x}{y^3} = c$

10. $\frac{x^2}{2} + \frac{e^x}{y} = c$

11. $x\left(y + \frac{2}{y^2}\right) + y^2 = c$

12. $3x^2y^4 + 6xy^2 + 2y^6 = c$

13. $x^2y - x = cy$

14. $\frac{1}{xy} - \log\left(\frac{x}{y}\right) = c$

15. $log\frac{x}{y} = c + xy$

16. $\frac{1}{2x^2y^2} + \frac{1}{xy} - log\ y = c$

17. $\frac{1}{3}log\frac{x}{y^2} - \frac{1}{3x^2y^2} = c$

18. $x\ sec\ xy = cy$

19. $4x^4\ log\ x - y^4 = cx^4$

20. $\frac{cy^3}{x^2} = e^{-x/y}$

21. $log\sqrt{xy} - \frac{y}{2x} = c$

22. $\frac{y}{x} = log\frac{x}{(x-y)^2} + c$

23. $\log(x-y) + \frac{y}{x} = c$

24. $x^{-2/3}.sinh\left(\frac{y}{x}\right) = c$

25. $ylog\ x - xsiny = c$

26. $n = -2, \frac{2x^3}{3} + \frac{e^x}{y} - y = c$

27. $n = -2, x^2y + e^x = cy$

28. $n = -4, x^2y^3e^y + x^2y^2 + x = cy^3$

29. $n = -2, y + 1 - x^2 + x\cos y = cx$

30. $n = -2, \frac{y}{x} + x^2 - \frac{y^2}{2} = c$

31. $n = -4, e^x + my^2x^{-2} = c$

32. $f(x) = x^{-4}, e^x + my^2x^{-2} = c$

33. $k = 1,\ x^4 + 2x^3y + 3x^2y + x^2y^2 + 6y^2x + 3y^3 = c$

34. $n = -4, \frac{xy}{(x+y+1)^3} = c$

TYPE – III

Solve the following differential equations.

1. $\frac{dy}{dx} + \frac{4x}{x^2+1}.y = \frac{1}{(x^2+1)^3}$

2. $\frac{dy}{dx} + \left(\frac{1-2x}{x^2}\right)y = 1$

3. $(1-x^2)\frac{dy}{dx} + 2xy = x\sqrt{1-x^2}$

4. $x(x-1)\frac{dy}{dx} - (x-2)y = x^3(2x-1)$

5. $\frac{dy}{dx}coshx = 2cosh^2x.sinhx - ysin\ hx$

6. $x\cos x\frac{dy}{dx} + y(x\sin x + \cos x) = 1$

7. $(1 + x + xy^2)dy + (y + y^3)dx = 0$

8. $(1+y^2)dx = \left(e^{tan^{-1}y} - x\right)dy$

9. $(y + 1)\,dx + [x - (y + 2)e^y]dy = 0$

10. $(1 + \sin y)\frac{dx}{dy} = [2y\cos y - x(\sec y + \tan y)]$

11. $(1 + y^2)dx = (\tan^{-1} y - x)dy$

12. $\frac{dy}{dx} + 2y\,tan\,x = sin\,x \;\; at\; y = 0, x = \frac{\pi}{3}$

13. $\frac{dy}{dx} + \frac{2x}{x^2+1}.y = \frac{1}{(x^2+1)^2}$ given that $y = 0$ when $x = 1$

14. $xlog\,x.\frac{dy}{dx} + y = 2log\,x$

15. $dr + (2r\,cot\,\theta + sin\,2\theta)d\theta = 0$

16. $\frac{dy}{dx} + xsin\,2y = x^3\cos^2 y$

17. $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$

18. $\frac{dy}{dx} + x^3\sin^2 y + x\sin 2y = x^3$

19. $\frac{dy}{dx} + (2xtan^{-1}y - x^3)(1 + y^2) = 0$

20. $y\frac{dy}{dx} + \frac{4x}{3} - \frac{y^2}{3x} = 0$

21. $e^x(x + 1)dx + (y^2e^{2y} - xe^x)dy = 0$

22. $\frac{dx}{dy} = e^{y-x}(e^y - e^x)$

23. $xcos\,y\frac{dy}{dx} + sin\,y = xsin^2 y$

24. $3y^2\frac{dy}{dx} + 2y^3x = 4x^3e^{x^2}$

25. $\frac{dy}{dx} = \frac{e^y}{x^2} - \frac{1}{x}$

26. $\left(xy^2 - e^{1/x^3}\right)dx - x^2y\,dy = 0$

27. $\frac{dy}{dx} = \frac{y^3}{e^{2x}+y^2}$

28. $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$

29. $(x + 1)\frac{dy}{dx} - y = e^x(x + 1)^2$

ANSWERS

1. $y(x^2 + 1)^2 = tan^{-1}x + c$

2. $y = x^2 + ce^{1/x}.x^2$

3. $y = \sqrt{(1 - x^2)} + c(1 - x^2)$

4. $y = x^3 + \frac{cx^2}{x-1}$

5. $y.cos\,hx = \frac{2}{3}cosh^3x + c$

6. $yx\,secx = tan\,x + c$

7. $xy + tan^{-1}y = c$

8. $xe^{tan^{-1}y} = \frac{1}{2}e^{2tan^{-1}y} + c$

9. $(y + 1)(x - e^y) = c$

10. $x.(1 + sin\,y) = y^2\,cos\,y + c\,cos\,y$

11. $x = tan^{-1}\,y - 1 + ce^{-tan^{-1}y}$

12. $y\,sec^2\,x = sec\,x - 2$

13. $y(x^2 + 1) = tan^{-1}x - \frac{\pi}{4}$

14. $y\,log\,x = (log\,x)^2 + c$

15. $2\,r\,sin^2\,\theta + sin^4\,\theta = c$

16. $tan\,y = \frac{1}{2}(x^2 - 1) + ce^{-x^2}$

17. $e^y = e^x - 1 + ce^{-e^x}$

18. $tan\,y.e^{x^2} = \frac{1}{2}e^{x^2}(x^2 - 1) + c$

19. $tan^{-1}y = \frac{(x^2-1)}{2} + ce^{-x^2}$

20. $y^2x^{-2/3} + 2x^{4/3} = c$

21. $xe^{x-y} = -(y^2 - 2y + 2)e^y + c$

22. $e^x = e^y - 1 + ce^{-e^y}$

23. $cosecy + x\,(log\,x + c) = 0$

24. $2y^3e^{x^2} = e^{2x^2}(2x^2 - 1) + c$

25. $2x\,e^{-y} = 2cx^2 + 1$

26. $\frac{y^2}{x^2} = \frac{2}{3}e^{1/x^3} + c$

27. $e^{-2x}y^2 + 2log\,y = c$

28. $ye^{2\sqrt{x}} = 2\sqrt{x} + c$

29. $y = (x + 1)(e^x + c)$