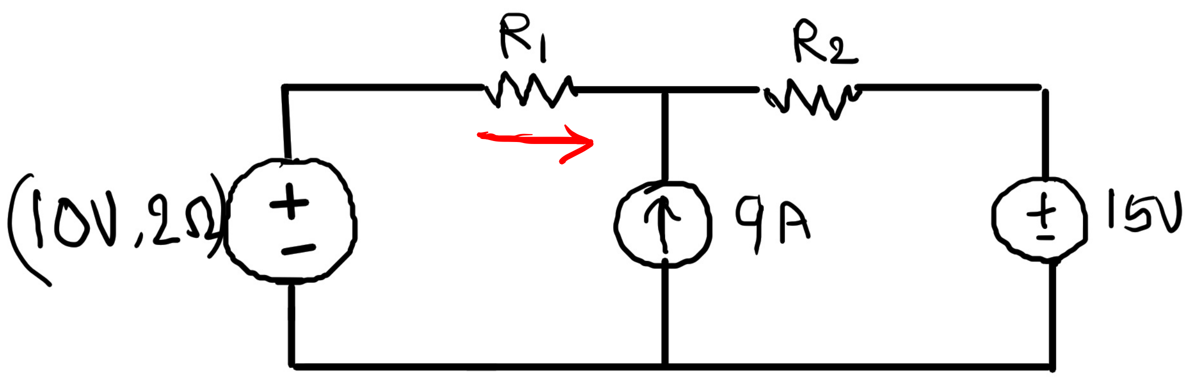
Superposition Theorem

Statement

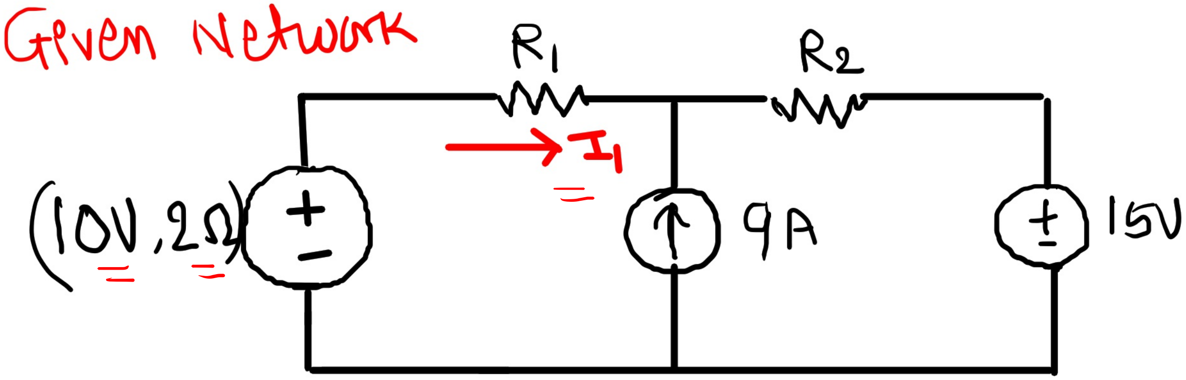
In a network of linear element containing more than one source(Current/Voltage) of energy , the current flowing/Voltage across any element is the sum of all the currents/Voltages which would result if each source is considered separately and all the other sources replaced for the time being by their internal resistances.

e.g. If the current through R1 is to be found in the network below

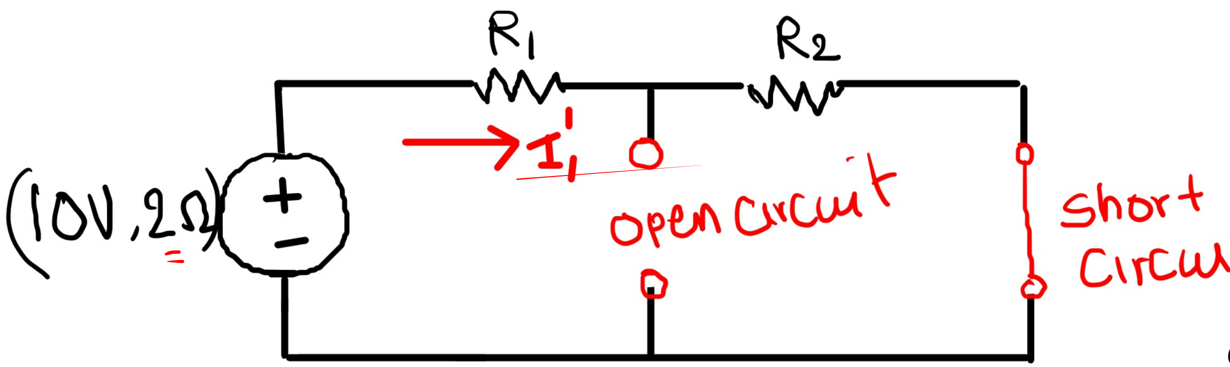


Superposition Theorem

Given Network



1. Consider 10V source in the circuit and replace 9A and 15V by their internal resistances
   Ideal current source has infinite internal resistance (Parallel to it) so It is replaced by open circuit and Ideal Voltage source has zero series resistance so it is replaced by short circuit.

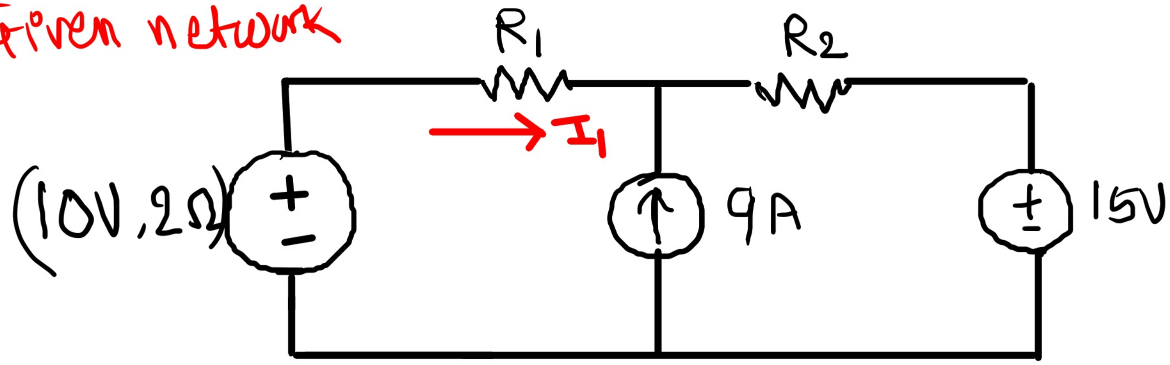


Find I_1' by any suitable analysis method.

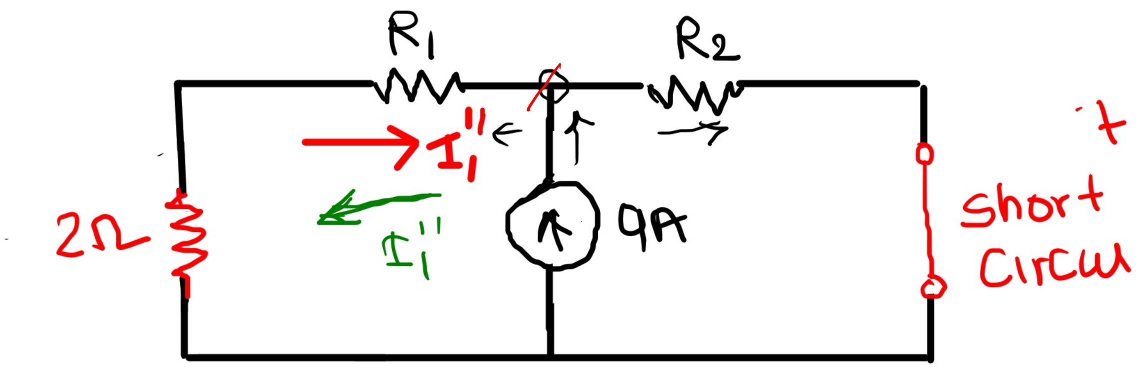
e.g. $I_1' = \left(\dfrac{10}{2 + R_1 + R_2} \right) (\rightarrow)$

Superposition Theorem

Given network



2. Consider 9A source in the circuit and replace 10V and 15V by their internal resistances
   10V source has internal resistance 2 Ohm so it is replaced by 2Ohm resistance.
   and Ideal Voltage source has zero series resistance so it is replaced by short circuit.



Find I_1'' by any suitable analysis method.

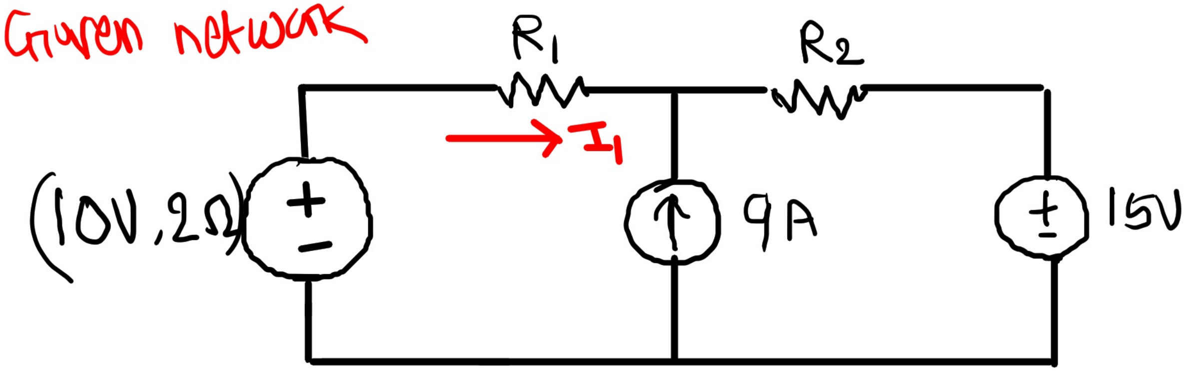
$$I_1'' = -\left(\frac{R_2 \times 9}{R_1 + R_2 + 2}\right) (\rightarrow)$$

Using Current division formula

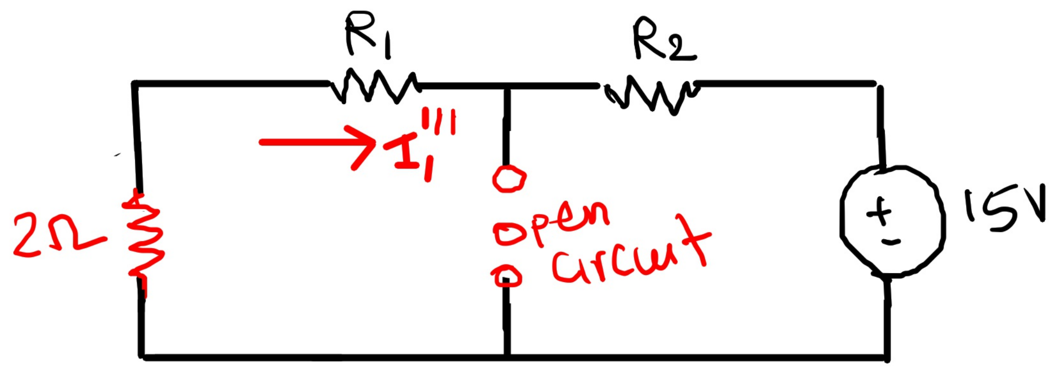
$$I_1'' = \frac{R_2 \times 9}{R_1 + R_2 + 2} (\leftarrow)$$

Superposition Theorem

Given network



3. Consider 15 V source in the circuit and replace 10V and 9A by their internal resistances
   10V source has internal resistance 2 Ohm so it is replaced by 2Ohm resistance.
   and Ideal 9A current source is replaced by open circuit.



$-2I_1''' - R_1 I_1''' - R_2 I_1''' - 15 = 0$

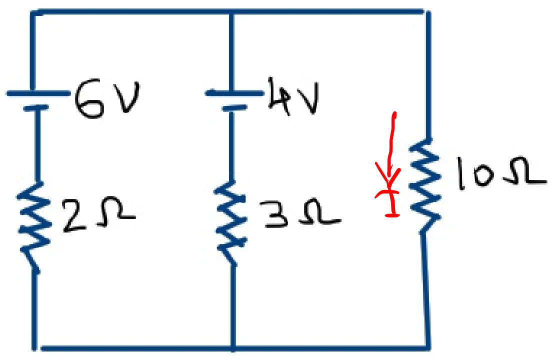
Find I_1''' by any suitable analysis method.

$I_1''' = -\left(\frac{15}{R_1 + R_2 + 2}\right) (\rightarrow)$

4. The current flowing through R1 is $I_1 = I_1' + I_1'' + I_1'''$

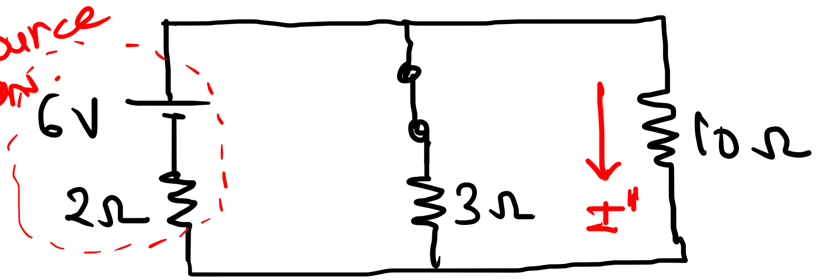
Ex ① Find current flowing through 10Ω resistance using SPT.

0.464

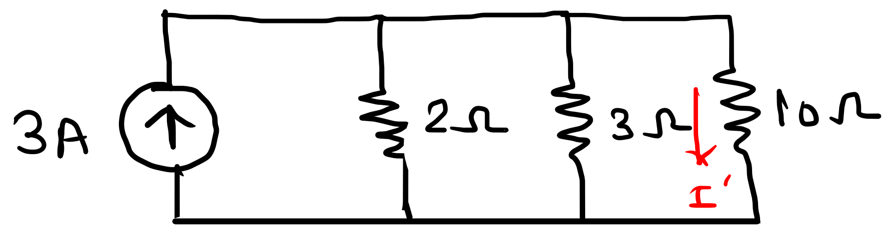


⇒ Consider 6V source in the circuit & replace 4V source by its internal resistance

Source tran:



Using source transformation



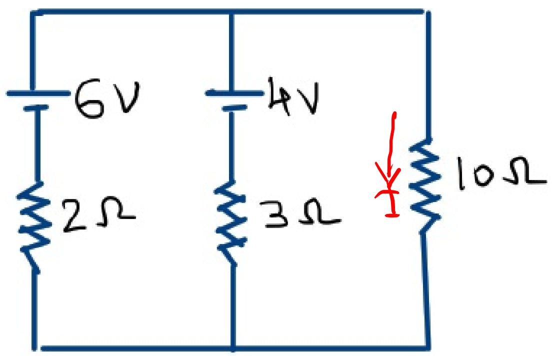
Using current division formula

$$I' = \frac{\frac{1}{10} \times 3}{\frac{1}{2} + \frac{1}{3} + \frac{1}{10}}$$

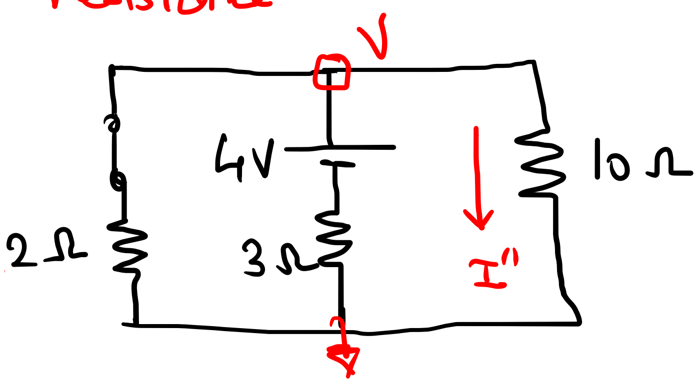
$$I' = \frac{0.3}{0.5 + 0.33 + 0.1}$$

$$I' = \frac{0.3}{0.93} = 0.32 (\downarrow)$$

Ex① Find current flowing through 10Ω resistance using SPT. 0·464



$\Rightarrow$ Consider 4V source in the circuit & replace 6V source by its internal resistance



Using Nodal Analysis

KCL at node

$$\frac{V}{2} + \frac{V-4}{3} + \frac{V}{10} = 0$$

$$\frac{15V + 10V - 40 + 3V}{30} = 0$$

$$28V = 40$$

$$V = \frac{40}{28} =$$

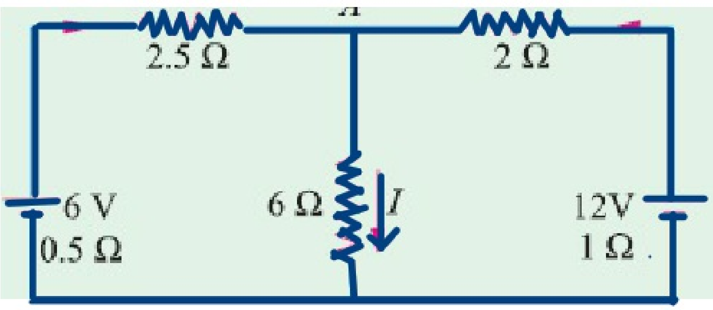
$$I'' = \frac{40}{28} \times \frac{1}{10} = \frac{4}{28} = \frac{1}{7} = 0.14(\downarrow)$$

$$I_{10\Omega} = I' + I'' = (0.32 + 0.14)$$

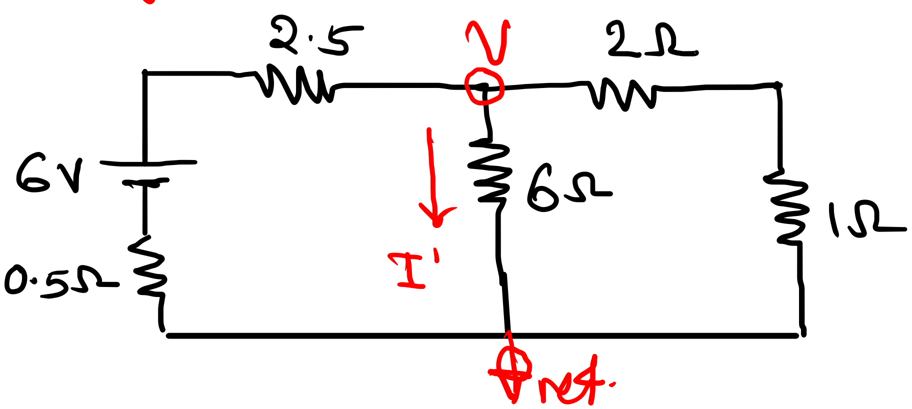
$$\boxed{I_{10\Omega} = 0.46(\downarrow)}$$

Ex ② Find current I using superposition principles.

1.2A↓



⇒ Consider 6V source in the circuit & replace 12V source by its internal resistance.



Using Nodal Analysis
KCL at node

$$\frac{V-6}{3} + \frac{V}{6} + \frac{V}{3} = 0$$

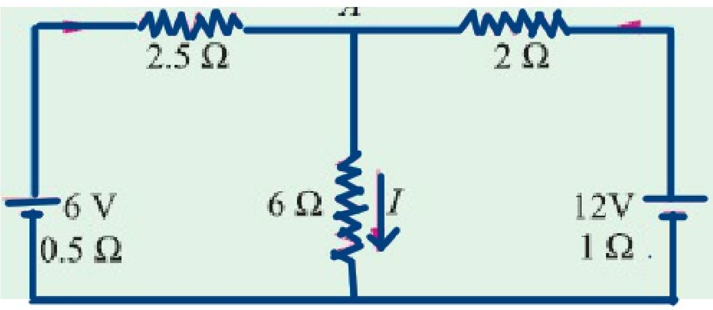
$$\frac{2V-12+V+2V}{6} = 0$$

$$5V = 12 \qquad \boxed{V = 2.4V}$$

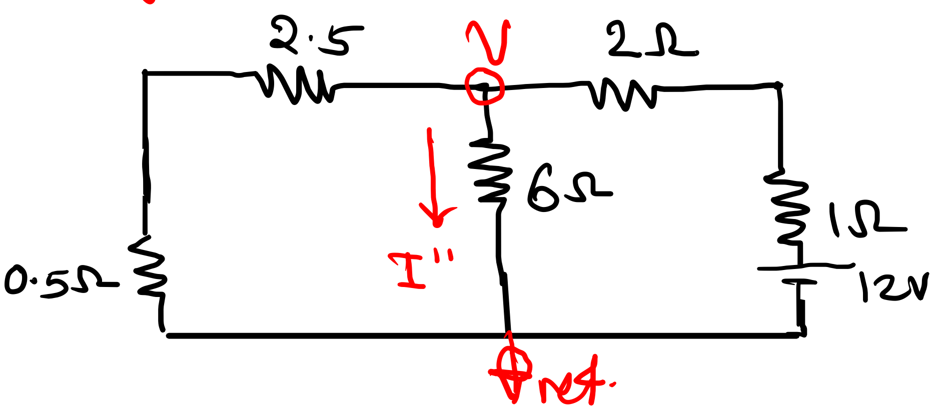
$$I' = \frac{V}{6} = \frac{2.4}{6} = 0.4(\downarrow)$$

Ex ② Find current I using superposition principles.

1.2A ↓



⇒ Consider 12V source in the circuit & replace 6V source by its internal resistance.



Using Nodal Analysis
KCL at node

$$\frac{V}{3} + \frac{V}{6} + \frac{V-12}{3} = 0$$

$$\frac{2V + V + 2V - 24}{6} = 0$$

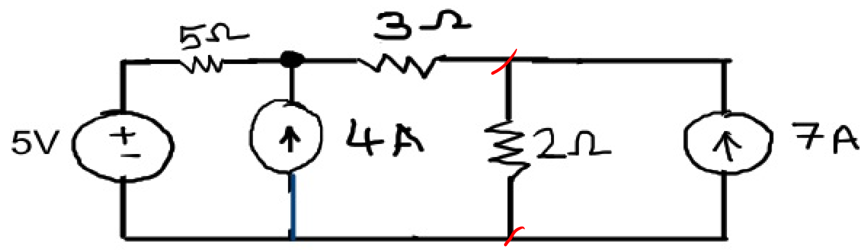
$$5V = 24 \quad \therefore V = 4.8\ V$$

$$I'' = \frac{V}{6} = \frac{4.8}{6} = 0.8\,A(\downarrow)$$

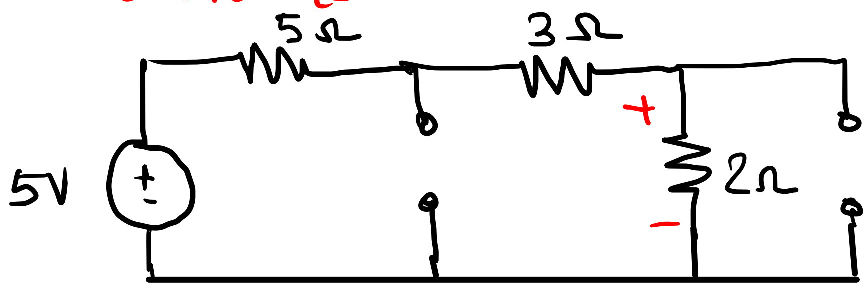
$$I = I' + I''$$

$$I = (0.4 + 0.8) = 1.2\,A(\downarrow)$$

Ex ③ Find voltage across 2Ω resistor using SPT. 162V



⇒ Consider 5V source & replace 4A & 7A by their internal resistances.



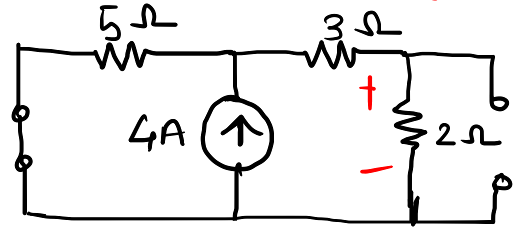
Using voltage division formula

$$V'_{2\Omega} = \frac{2\times 5}{5+3+2} = 1V$$

$V_{2\Omega} = V'_{2\Omega} + V''_{2\Omega} + V'''_{2\Omega} =$

$$\boxed{V_{2\Omega} = 1+4+11.2 = 16.2V}$$

⇒ 4A source in the circuit & replace 5V & 7A by their internal resistances.

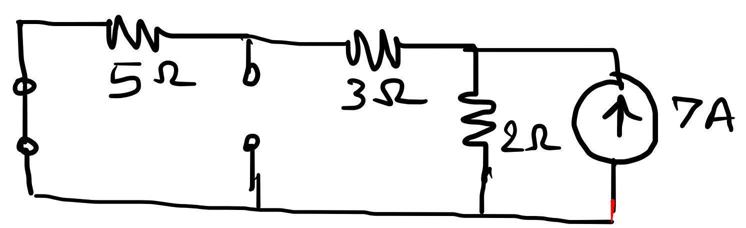


Using Current division

$$I_{2\Omega} = \frac{5\times 4}{5+3+2} = 2A$$

$$\boxed{V''_{2\Omega} = 2\times 2 = 4V}$$

⇒ Consider 7A & replace 5V & 4A

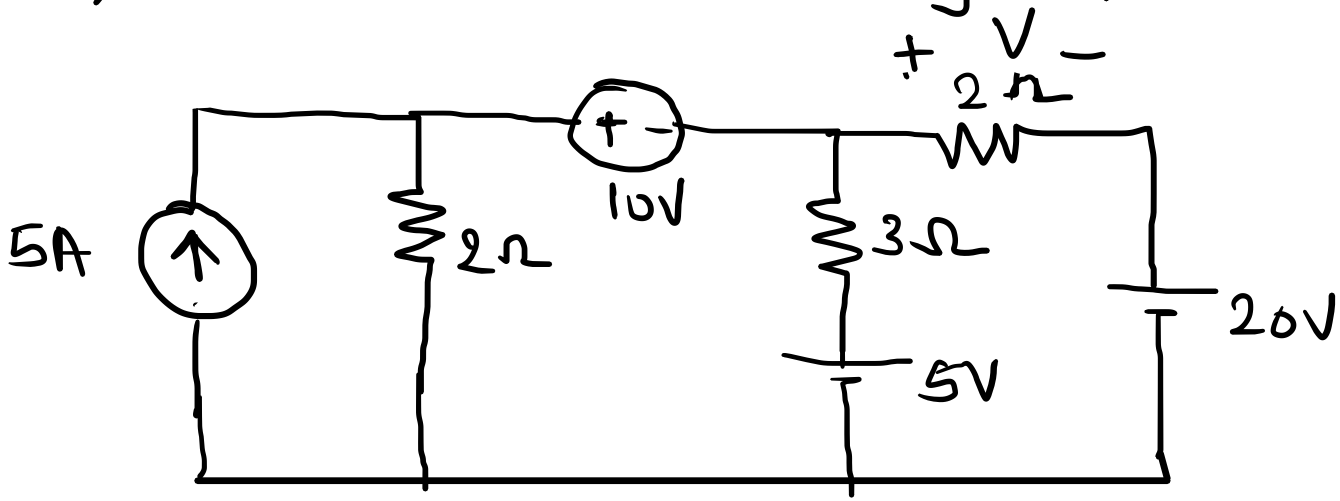


Using current division

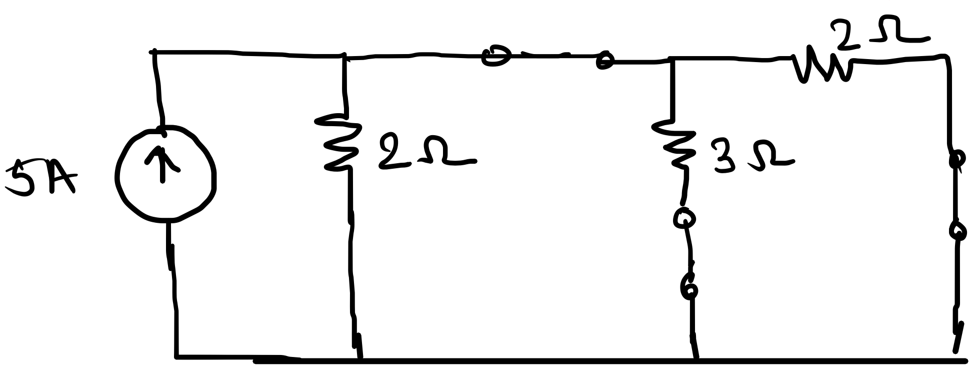
$$I_{2\Omega} = \frac{8\times 7}{5+3+2} = \frac{56}{10} = 5.6A$$

$$V'''_{2\Omega} = 5.6\times 2 = 11.2V$$

⇒ Find voltage 'V' using SPT.



⇒ Consider 5A & replace 10V, 5V & 20V



Using Current division

$$I_{2\Omega} = \frac{\frac{1}{2}\times 5}{\frac{1}{2}+\frac{1}{3}+\frac{1}{2}} = \frac{2.5}{0.5+0.33+0.5} = \frac{2.5}{1.33}$$

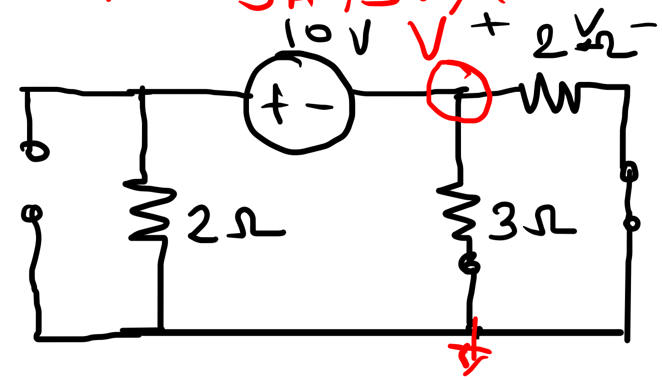
$I_{2\Omega} = 1.87\,A$

$V' = I_{2\Omega}\times 2$

$= 1.87\times 2$

$\boxed{V' = 3.74\,V}$

⇒ Consider 10V & replace 5A, 5V, & 20V



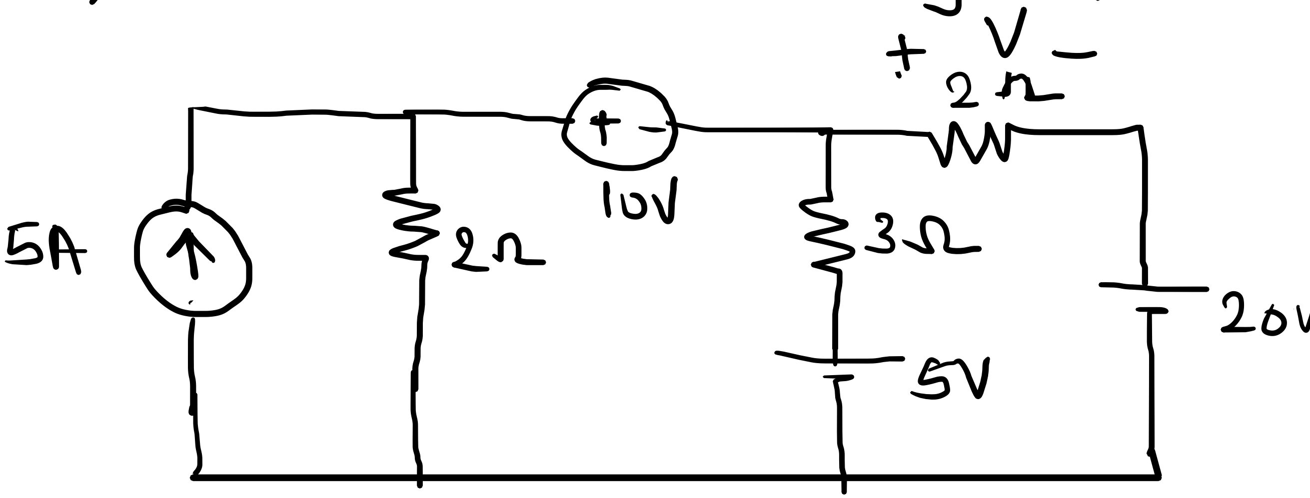
Using Nodal / KCL

$$\frac{V+10}{2}+\frac{V}{3}+\frac{V}{2}=0$$

$$\frac{3V+30+2V+3V}{6}=0$$

$8V = -30$ | $V''_{2\Omega} = -3.75V$

⇒ Find voltage 'V' using SPT.

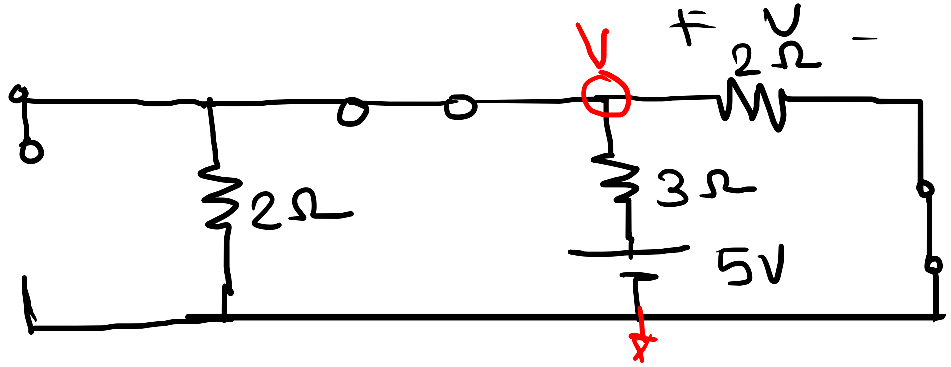


$8V = 10$

$V = \frac{10}{8}$

$V'''_{2\Omega} = V = 1.25V$

⇒ Consider 5V & replace 10V, 5A & 20V

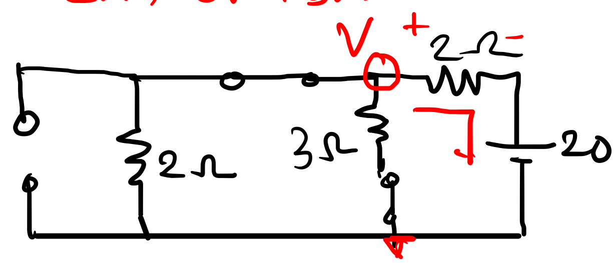


Using Nodal KCL.

$$\frac{V}{2} + \frac{V-5}{3} + \frac{V}{2} = 0$$

$$\frac{3V + 2V - 10 + 3V}{6} = 0$$

⇒ Consider 20V & replace 5A, 10V & 5V.



Using Nodal

$$\frac{V}{2} + \frac{V}{3} + \frac{V-20}{2} = 0$$

$$\frac{3V + 2V + 3V - 60}{6} = 0$$

$8V = 60 \mid V = \frac{60}{8} = 7.5V.$

$$I_{2\Omega} = \frac{V-20}{2} = \frac{7.5-20}{2} = -6.25A$$

$$V''_{2\Omega} = 2 \times I_{2\Omega} = -6.25 \times 2 = -12.5V$$

$$V_{2\Omega} = 3.74 - 3.75 + 1.25 - 12.5$$

$$\boxed{V_{2\Omega} = -11.26V}$$