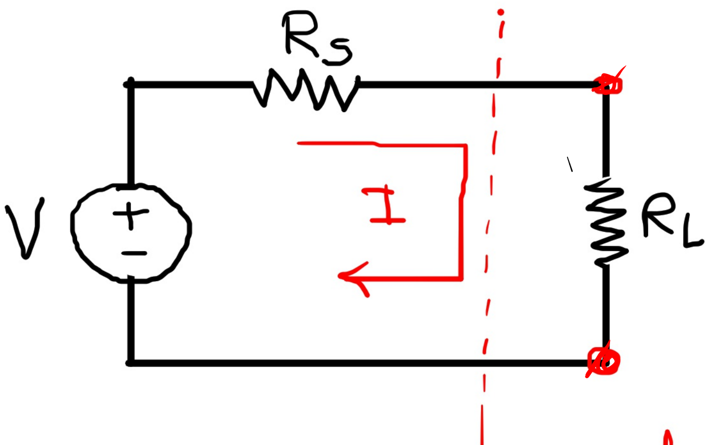
Maximum Power transfer Theorem

Statement:

The maximum power is delivered from a source to a load when load resistance is equal to source resistance.



$$I = \frac{V}{R_S + R_L} \quad \text{-----} \quad (1)$$

$R_L = R_S$

$$I = \left(\frac{V}{R_S + R_L}\right)$$

Power delivered to load

$$P_L = I^2 R_L = \frac{V^2}{(R_S+R_L)^2} \times R_L$$

to determine value of R_L for which maximum power is delivered

$$\frac{dP_L}{dR_L} = 0$$

$$\frac{dP_L}{dR_L} = \frac{d}{dR_L}\left(\frac{V^2 R_L}{(R_S+R_L)^2}\right) = \frac{V^2 (R_S+R_L)^2 - V^2 \cdot R_L \cdot 2(R_S+R_L)}{(R_S+R_L)^4} = 0$$

$$(R_S+R_L)^2 - 2R_L(R_S+R_L) = 0$$

$$R_S^2 + R_L^2 + 2R_L R_S - 2R_L R_S - 2R_L^2 = 0$$

$$R_S^2 - R_L^2 = 0$$

$R_S = R_L$ is condition for maximum power transfer

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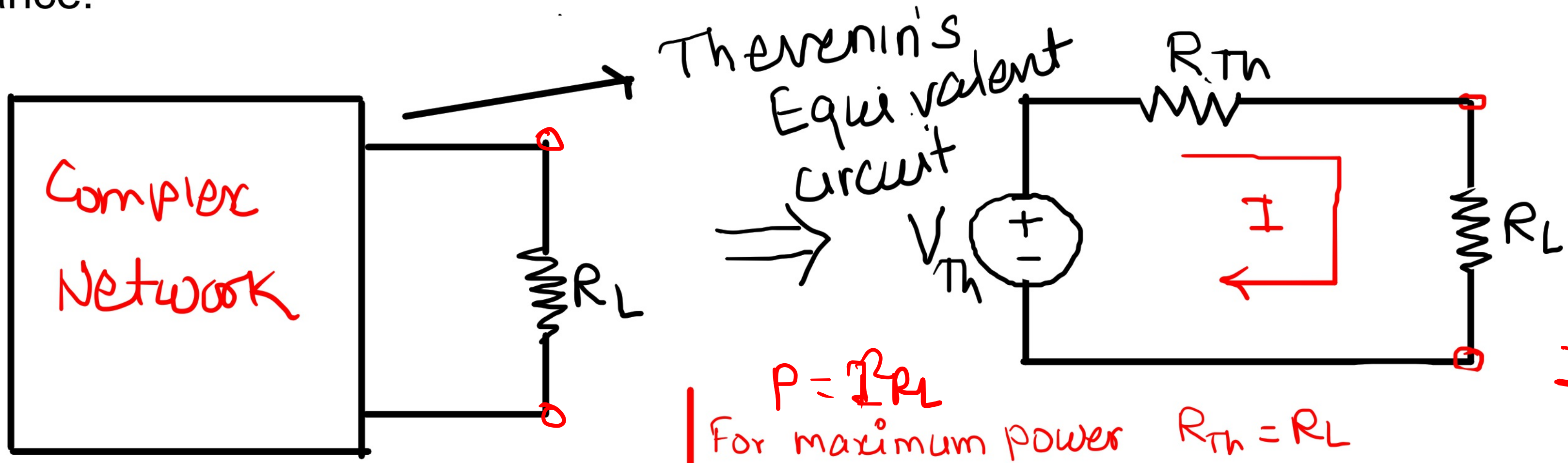


$$V_{RL} = \left(\frac{R_L}{R_L + r_L}\right) V_{th}$$

$$I = \frac{V_{th}}{R_L + R_{Th}}$$

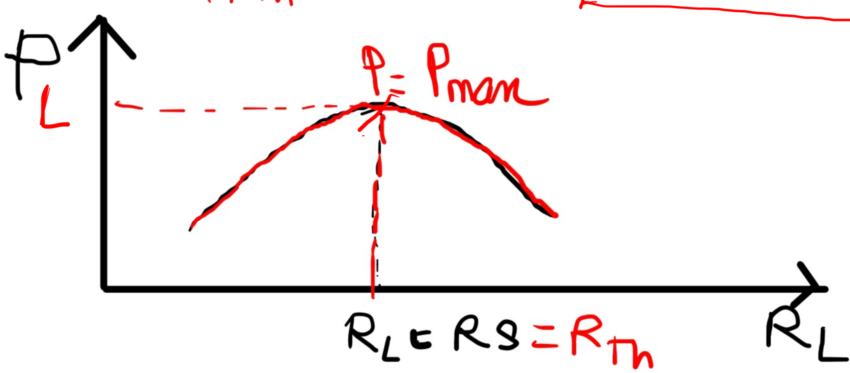
$$P = I^2 R_L$$

For maximum power $R_{Th} = R_L$

$$P_L = \frac{(V_{Th})^2}{(R_L + R_{Th})^2} \times R_L \quad \text{OR} \quad P_{max} = \frac{(V_{Th})^2}{(R_{Th} + R_{Th})^2} \times R_{Th}$$

$$P_{max} = \frac{V_{Th}^2}{4 R_{Th}^2} \times R_{Th}$$

$$\therefore \boxed{P_{max} = \frac{(V_{Th})^2}{4 R_{Th}}}$$

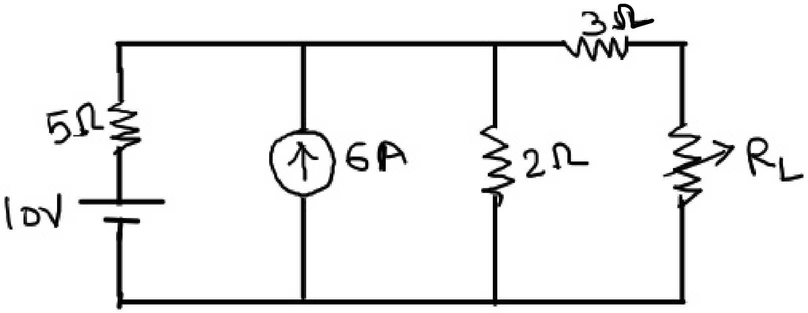


Steps:

1. Remove Load ✓
2. Find Open circuit Voltage Vth ✓
3. Find Rth ✓
4. Find RL for Maximum power transfer (RL=Rth) ✓
5. Find Maximum Power

$$P_{max} = \frac{(V_{th})^2}{4 R_{th}} \checkmark$$

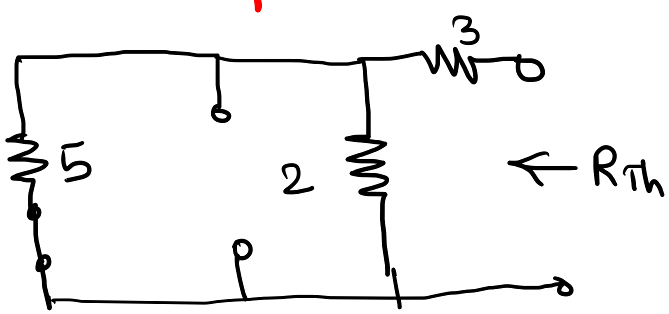
Ex. Find R_L for maximum power transfer & find maximum power.



⇒ Remove Load R_L

⇒ To find R_{Th}

$R_L = R_{Th}$ for maximum power transfer

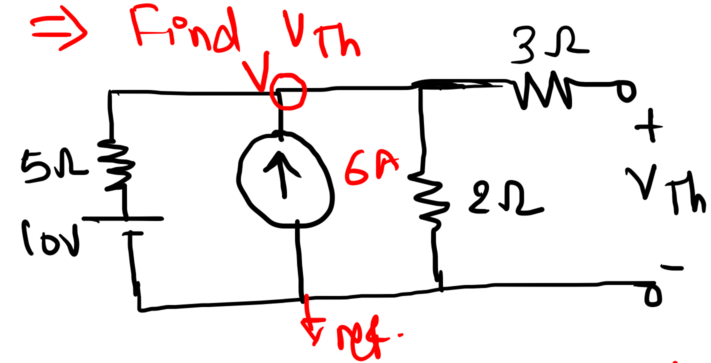


$$R_{Th} = (5 \| 2) + 3 = \frac{10}{7} + 3 =$$

$$R_L = R_{Th} = 4.42\,\Omega$$

$$\Rightarrow P_{L\,max} = \frac{(V_{Th})^2}{4 R_{Th}}$$

⇒ Find V_{Th}



$$V_{Th} = V_{3\Omega} + V_{2\Omega} = V_{2\Omega} = V$$

Using Nodal Analysis

$$\frac{V-10}{5} + \frac{V}{2} = 6$$

$$2V - 20 + 5V = 6 \times 10$$

$$7V = 80$$

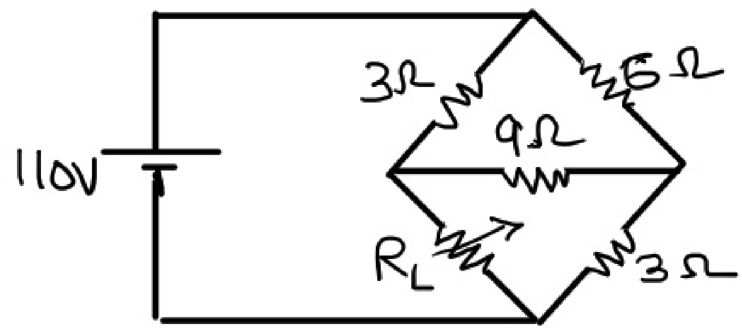
$$V_{Th} = V = \frac{80}{7} = 11.4\,V$$

$$P_{L\,max} = \frac{(11.4)^2}{4 \times 4.42}$$

$$P_{L\,max} = \frac{(11.4)^2}{17.68}$$

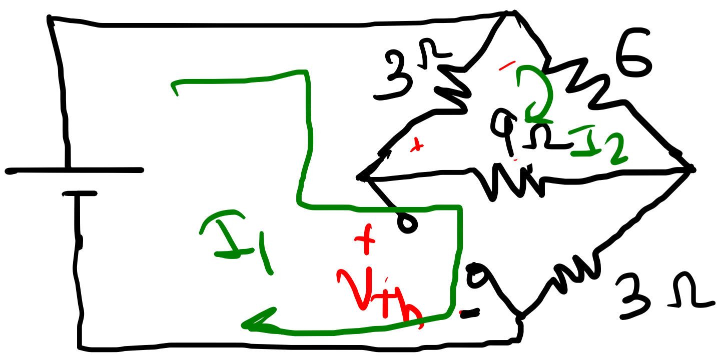
$$P_{L\,max} = 7.35 \text{ Watts}$$

⇒ Find R_L for maximum power transfer. Also find maximum power.



⇒ Remove R_L & find R_{Th} / V_{Th}

⇒ V_{Th}



$V_{th} - V_{9\Omega} - V_{3\Omega} = 0$

$V_{Th} = V_{9\Omega} + V_{3\Omega}$ ✓

OR

$V_{th} - V_{3\Omega} - 110 = 0$

$V_{Th} = V_{3\Omega} + 110$

KVL to mesh (I)

$110 - 3(I_1 - I_2) - 9(I_1 - I_2) - 3I_1 = 0$

$110 - 3I_1 + 3I_2 - 9I_1 + 9I_2 - 3I_1 = 0$

$15I_1 - 12I_2 = 110$ ---- (1)

KVL to mesh (II)

$-3(I_2 - I_1) - 6I_2 - 9(I_2 - I_1) = 0$

$12I_1 - 18I_2 = 0$ ---- (2)

Solving (I) & (II)

$I_1 = 15.71A$, $I_2 = 10.47A$

$V_{9\Omega} = 9(I_1 - I_2) = 9(15.76 - 10.47)$

$V_{9\Omega} = 47.61V$

$V_{3\Omega} = 3 \times I_1 = 3 \times 15.75 = 47.25V$

$V_{Th} = 47.61 + 47.25 = 94.86V$

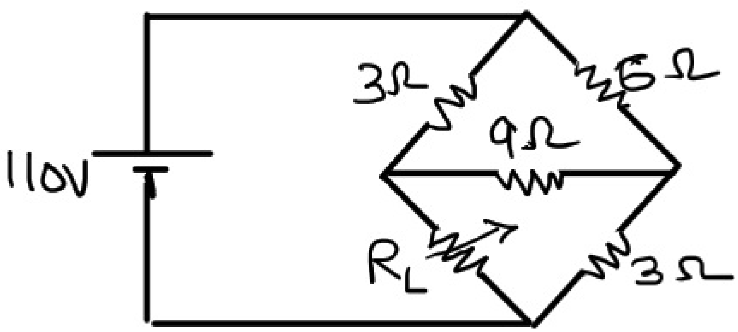
OR

$V_{Th} = 3(I_2 - I_1) + 110$

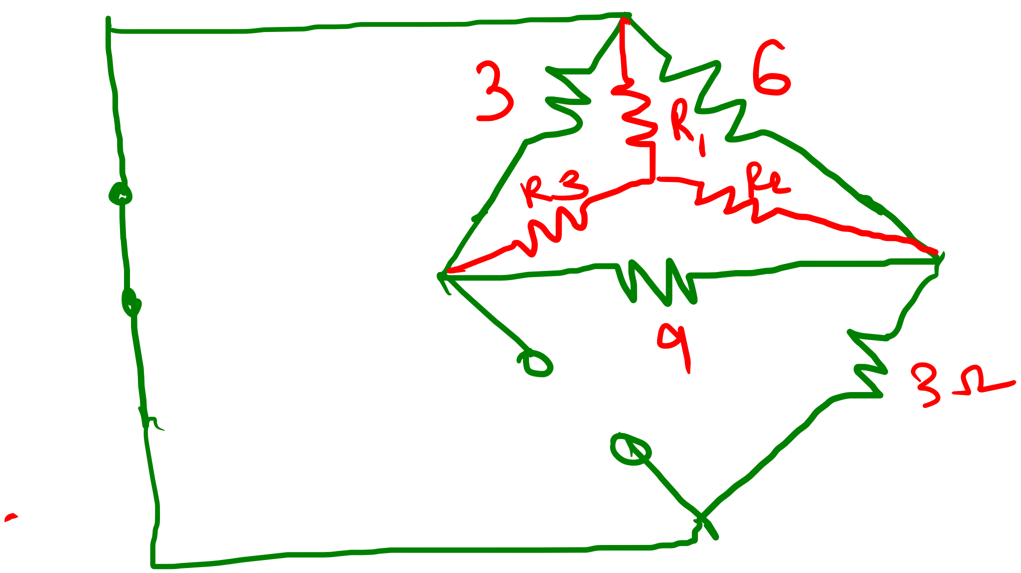
$= 3(10.47 - 15.71) + 110$

$V_{Th} = 94.28V$

⇒ Find R_L for maximum power transfer. Also find maximum power.



⇒ Remove R_L & find R_{Th}

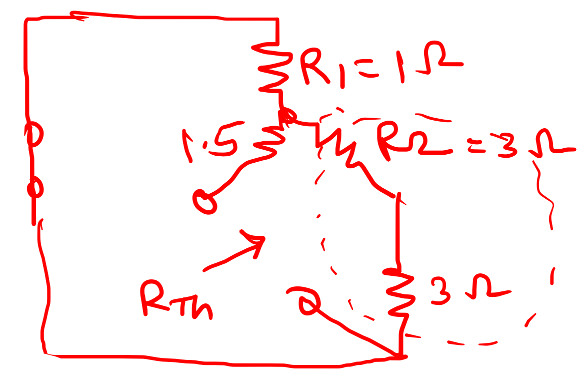


Delta - Star

$$R_1 = \frac{3 \times 6}{18} = \frac{18}{18} = 1\Omega$$

$$R_2 = \frac{9 \times 6}{18} = 3\Omega$$

$$R_3 = \frac{3 \times 9}{18} = \frac{27}{18} = \frac{3}{2} = 1.5\Omega$$



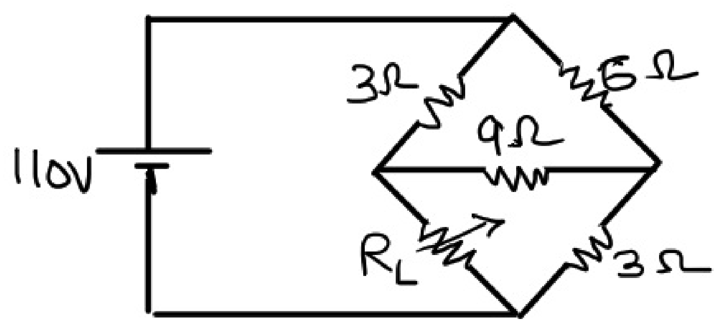
$$R_{Th} = 1.5 + (6 \parallel 1)$$

$$R_{Th} = 1.5 + \frac{6}{7}$$

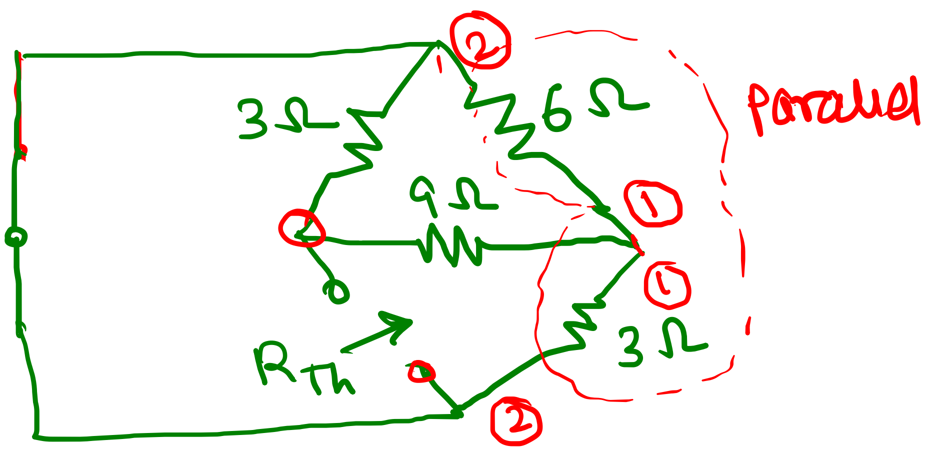
$$R_{Th} = 1.5 + 0.85$$

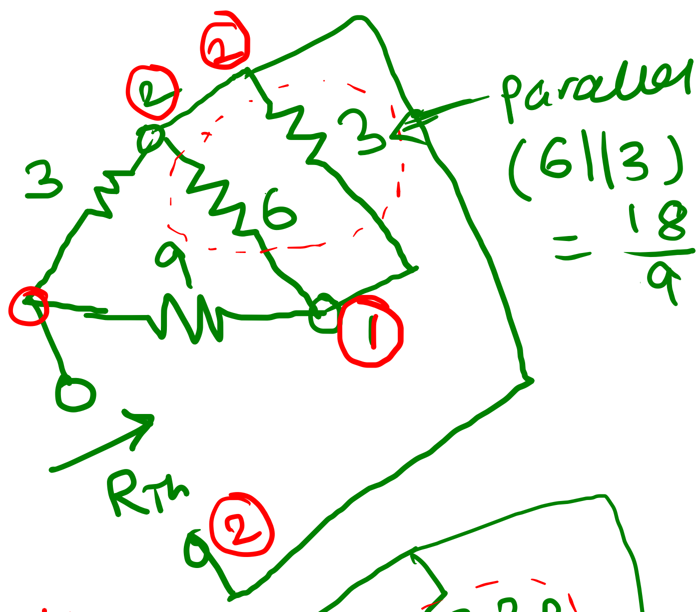
$$R_{Th} = 2.35\Omega$$

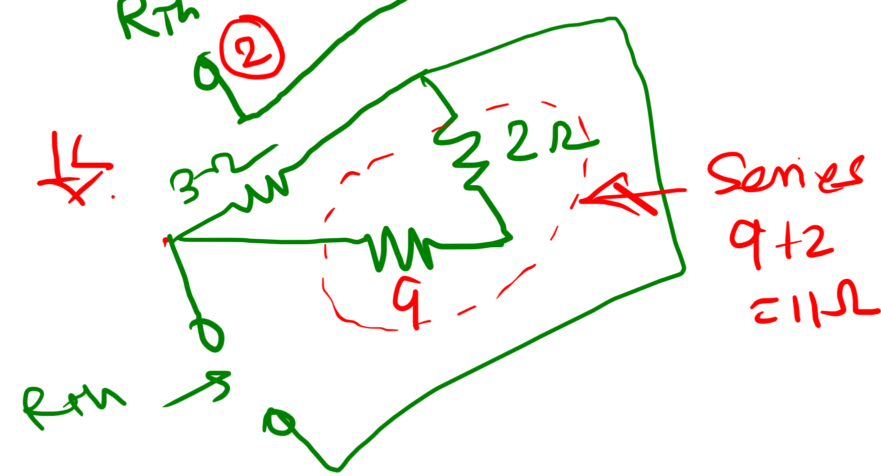
⇒ Find R_L for maximum power transfer. Also find maximum power.



⇒ Remove R_L & find R_{Th}







$R_L = R_{Th} = \frac{33}{14} = 2.35\,\Omega$

$P_{L\,max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(94.28)^2}{4 \times 2.35} = 945$ Watts

$R_{Th} = 3 \,||\, (9+2)$

$= 3 \,||\, 11 = \frac{33}{14}\,\Omega$