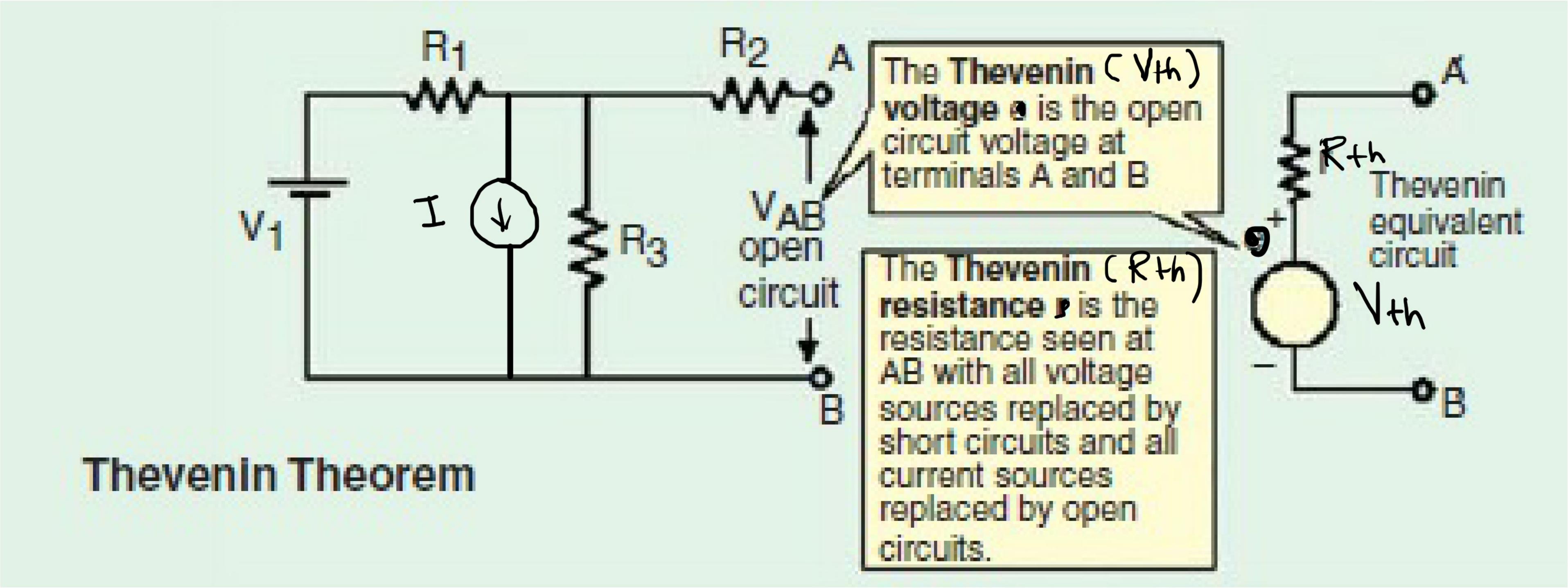
Thevenin's Theorem

Statement:

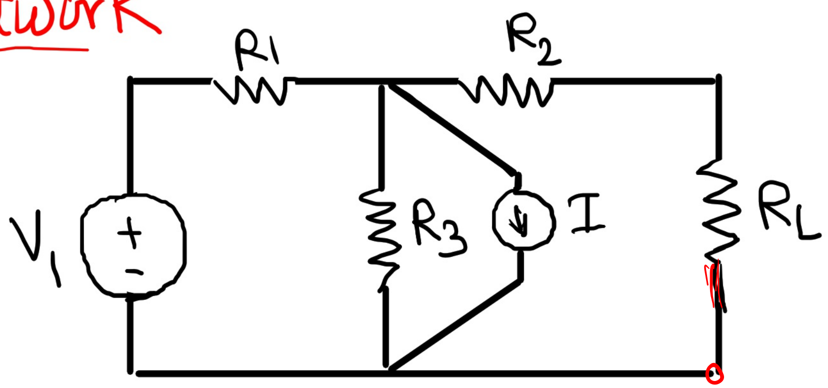
Any linear, active bilateral network can be replaced by a voltage source (Vth) in series with a resistance (Rth) where Vth is the open-circuit voltage (i.e. voltage across the two terminals when RL is removed) and Rth is the internal resistance of the network as viewed back into the open-circuited network from terminals A and B with all energy sources replaced by their internal resistance. (Ideal current sources by infinite resistance and Ideal voltage source by zero resistance.



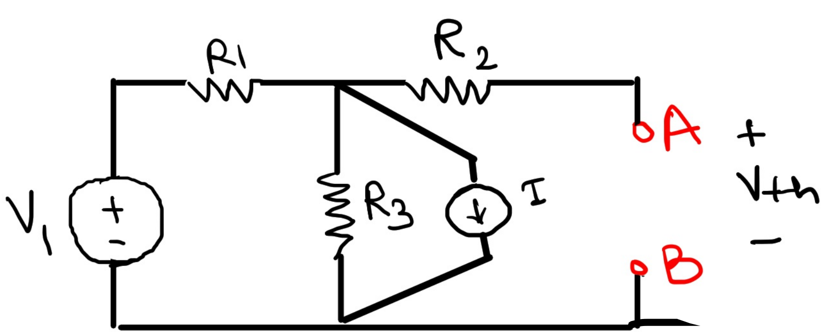
Thevenin's Theorem

Steps to analyse netwrk using Thevenin's Theorem

Given network

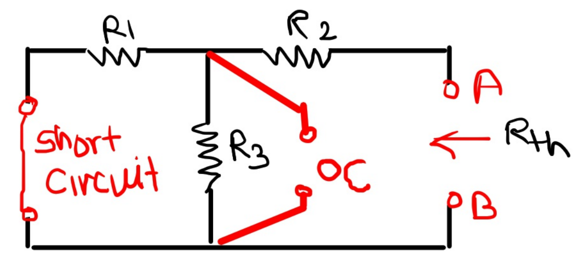


1. Remove load resistance RL from the given network



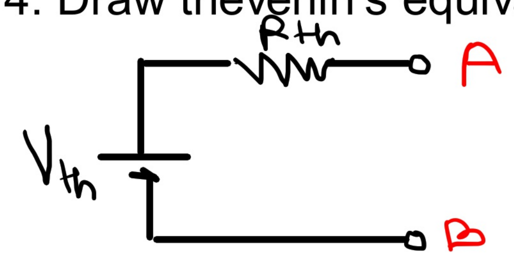
2. Find Vth i.e. the open circuit voltage between the terminals (A-B) from where the load is removed using any suitable method (mesh, nodal, source transformation etc..)

3. Find Rth i.e. resistance looking back into the network from the terminals (A-B) from where the load is removed and energy sources replaced by their internal resistances.

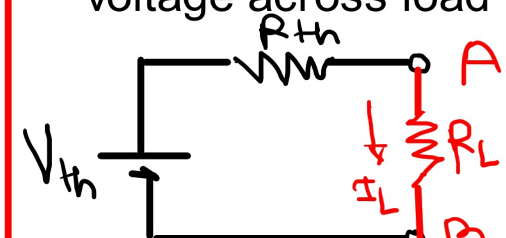


$R_{th} = (R_1 \| R_3) + R_2$

4. Draw thevenin's equivalent circuit



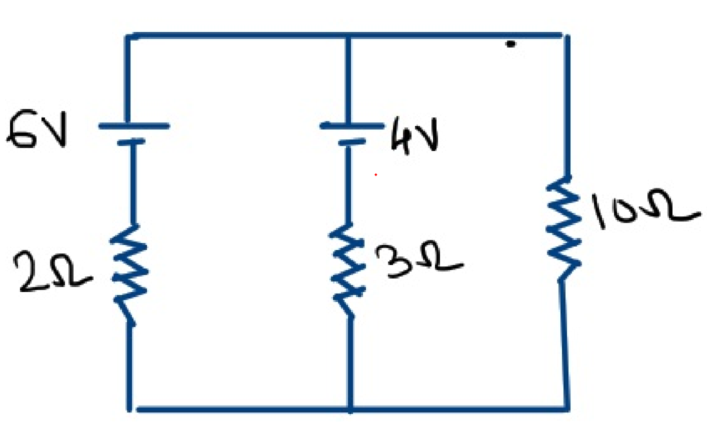
5. Connect the load RL and find current/ voltage across load RL



$I_L = \frac{V_{th}}{R_{th} + R_L}$

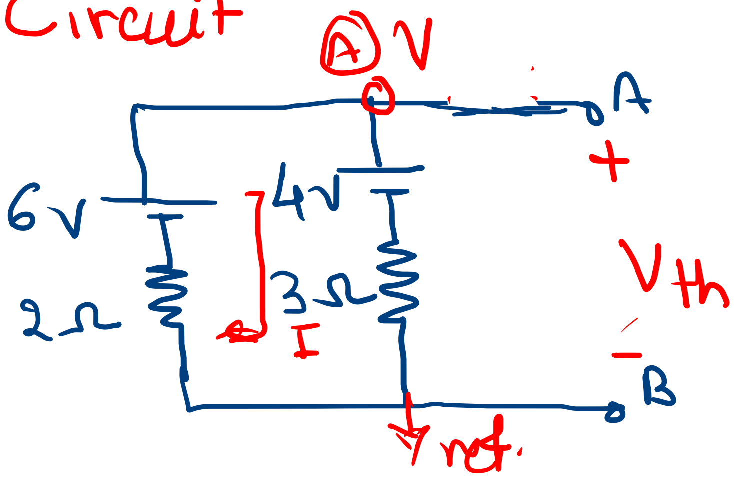
Voltage division $V_L = \frac{V_{th} \times R_L}{R_L + R_{th}}$

Example:1 Find current through 10 Ohm resistor using Thevenin's Theorem



⇒ Solution

① Remove $R_L = 10\Omega$ from the circuit



② Find V_{th} using KVL

$$V_{th} - 4 - 3I = 0$$

KVL to loop to find I

$$6 - 4 - 3I - 2I = 0$$

$$5I = 2$$

$$I = \frac{2}{5} = 0.4A$$

$$V_{th} = 4 + 3I = 4 + 3 \times 0.4$$

$$V_{th} = 5.2V$$

OR

Using Nodal

KCL at node A

$$\boxed{V_{th} = V}$$

$$\frac{V-6}{2} + \frac{V-4}{3} = 0$$

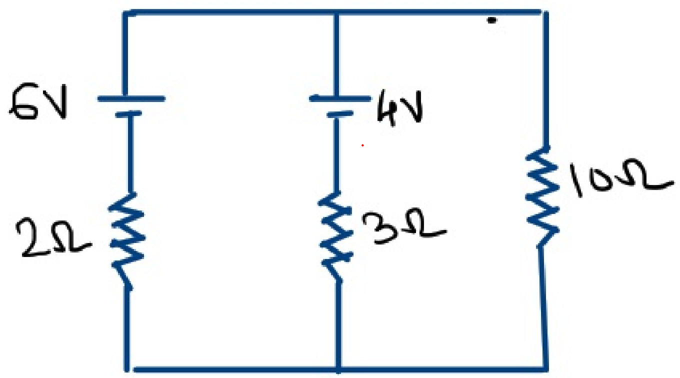
$$\frac{3V - 18 + 2V - 8}{6} = 0$$

$$5V = 26$$

$$V = \frac{26}{5}$$

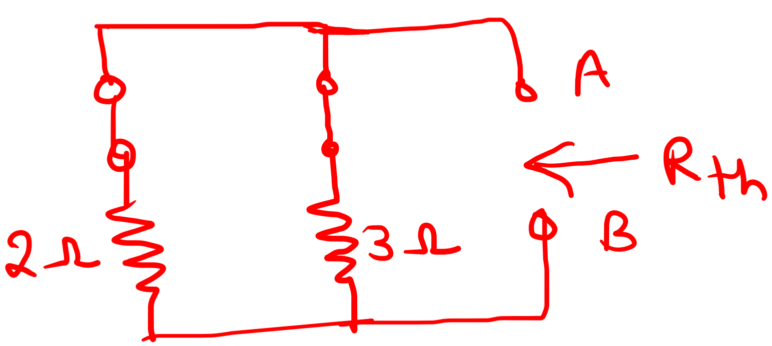
$$\boxed{V = 5.2V}$$

Example:1 Find current through 10 Ohm resistor using Thevenin's Theorem



⇒ Solution

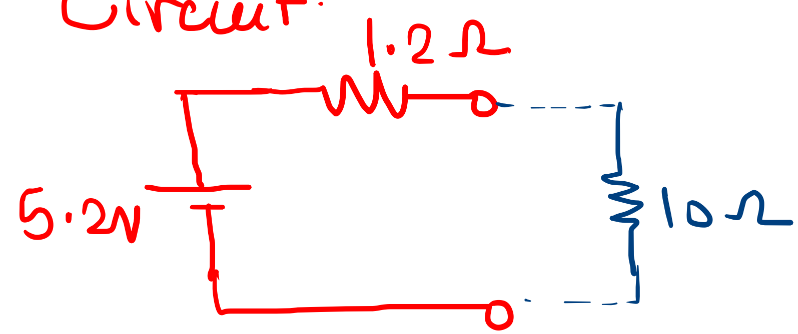
3 find R_{th}



$$R_{th} = (2 \| 3)$$

$$R_{th} = \frac{6}{5} = 1.2\,\Omega$$

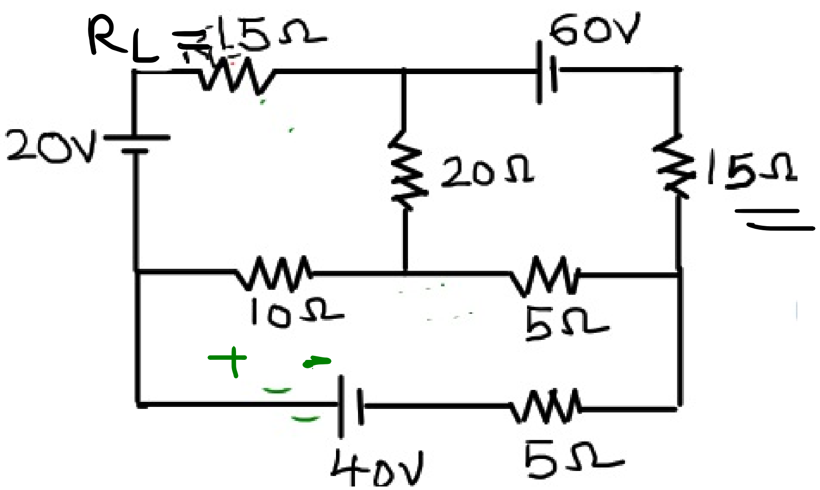
4 Draw Thevenin's Equivalent Circuit.



5 Connect load & find current

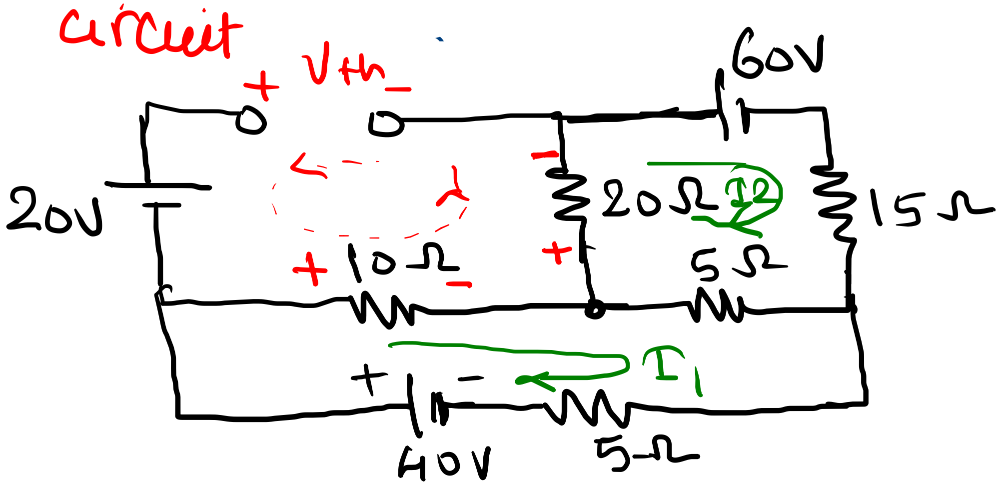
$$I_{10\Omega} = \frac{5.2}{(1.2+10)} = \frac{5.2}{11.2} = 0.46\,A\,(\downarrow)$$

Ex:3 Find current through RL=15 Ohm resistor using Thevenin's Theorem



⇒ Solution

① Remove $R_L=15\Omega$ from the the circuit



② Find Vth

$$V_{th}-20-V_{10\Omega}-V_{20\Omega}=0$$

$$V_{th}=20+V_{10\Omega}+V_{20\Omega}$$

using mesh Analysis

KVL to mesh (I)

$$-10I_1-5(I_1-I_2)-5I_1+40=0$$

$$-20I_1+5I_2=-40$$

$$\boxed{20I_1-5I_2=40 \quad --- (1)}$$

KVL to mesh (II)

$$-60-15I_2-5(I_2-I_1)-20I_2=0$$

$$\boxed{5I_1-40I_2=60 \quad --- (2)}$$

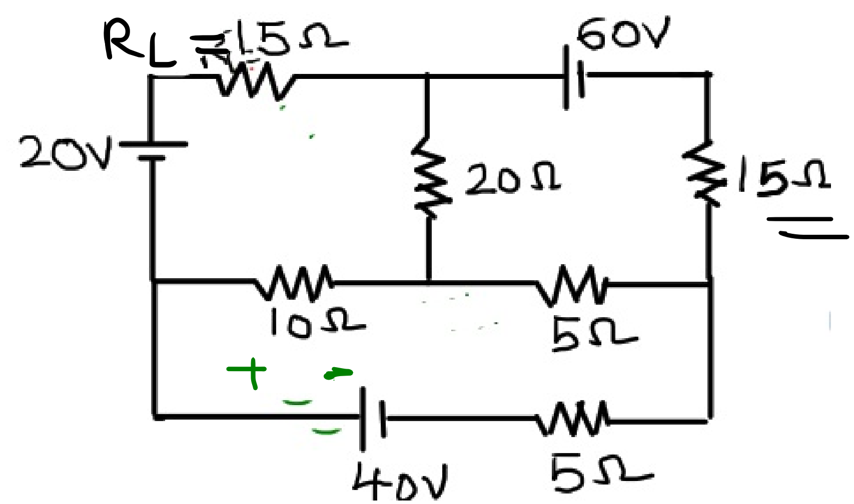
Solving (1) & (2) $I_1=1.68$ A

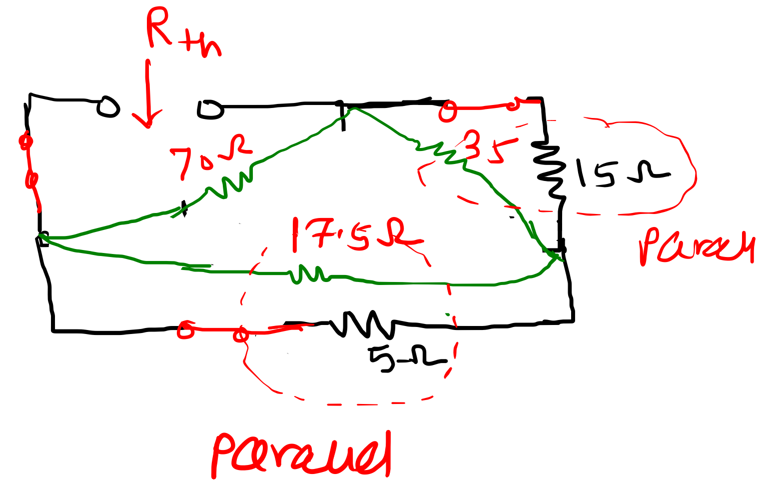
$I_2=-1.29$ A

$$V_{th}=20+(10\times1.68)+(20\times-1.29)$$

$$V_{th}=20+16.8-25.8=11V.$$

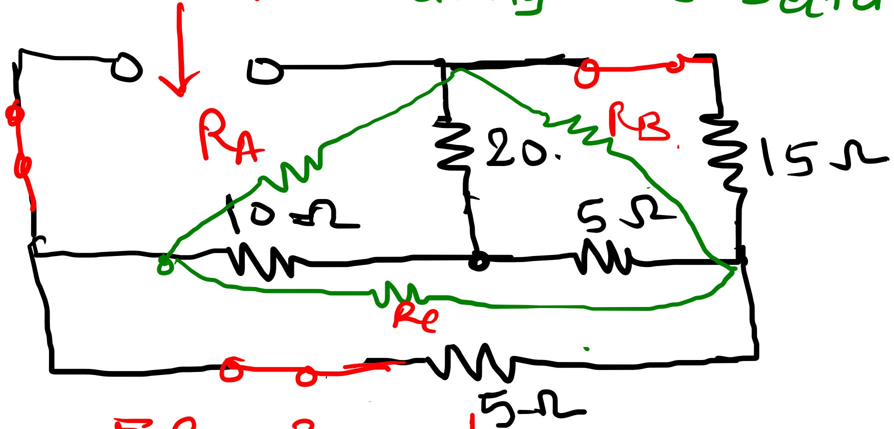
Ex:3 Find current through RL=15 Ohm resistor using Thevenin's Theorem





⇒ (3) Find R...

Rth using star-Delta



$R_A = \frac{\Sigma R}{5} = \frac{350}{5} = 70$ | $\Sigma R = 20\times5 + 5\times10 + 10\times20$

$= 100 + 50 + 200$

$= 350$

$R_B = \frac{\Sigma R}{10} = \frac{350}{10} = 35$

$R_c = \Sigma R / 20 = \frac{350}{20} = 17.5\Omega$

$35 \| 15 = \frac{35\times15}{35+15} = 10.5\Omega$

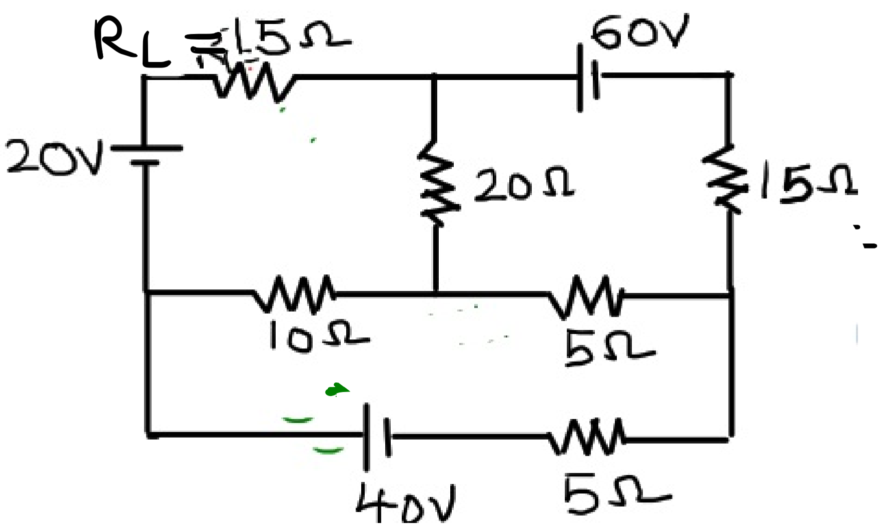
$17.5 \| 5 = \frac{17.5\times5}{17.5+5} = 3.88$

Rth

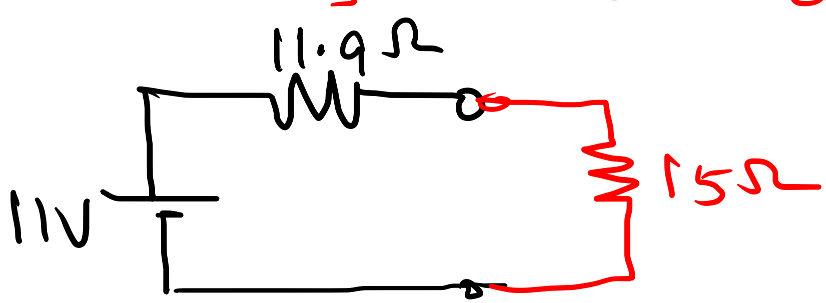
$R_{th} = 70 \| (10.5 + 3.88)$

$R_{th} = 70 \| 14.38 = 11.9\Omega$

Ex:3 Find current through RL=15 Ohm resistor using Thevenin's Theorem

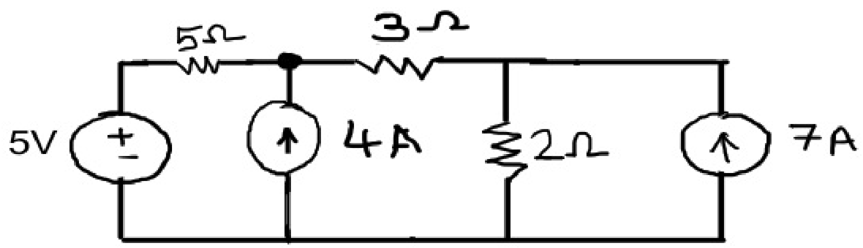


⇒ 4 Draw thevenin's Equivalent Circuit & Connect load.



$$I_{15\Omega} = \frac{11}{11.9+15} = \frac{11}{26.9} = 0.4A \downarrow$$

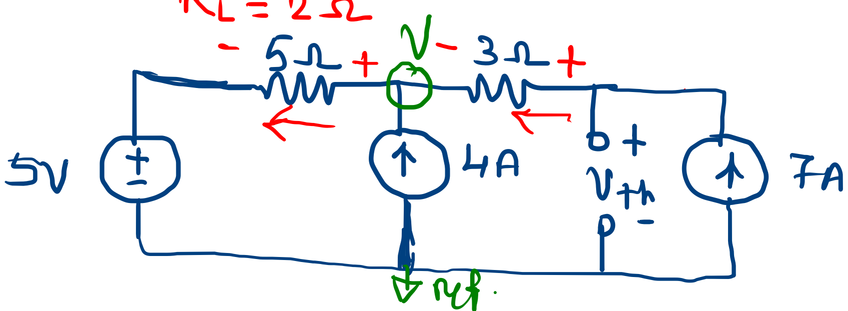
Ex:3 Find voltage across 2 Ohm resistor using Thevenin's Theorem



⇒ Solution:

① Remove load resistance

$R_L = 2\Omega$



V_{th} = voltage across 7A source

OR

V_{th} = voltage across 3Ω & 4A

$= V_{3\Omega} + V_{4A}$

OR

$V_{th} = (V_{3\Omega} + V_{5\Omega} + 5V)$

$V_{3\Omega} = I_{3\Omega} \times 3 = 7 \times 3 = 21V$

$V_{5\Omega} = I_{5\Omega} \times 5 = (4+7) \times 5 = 55V$

$V_{th} = (21 + 55 + 5) = 81V$

OR using Nodal Analysis

KCL at node

$\frac{V-5}{5} = 4+7 = 11$

$V - 5 = 55 \quad \therefore V = 60V$

$V_{th} = V_{3\Omega} + V$

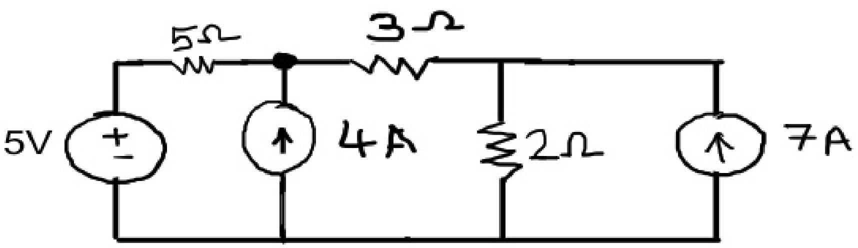
$V_{th} = 21 + 60 = 81V$

81

8

16.7

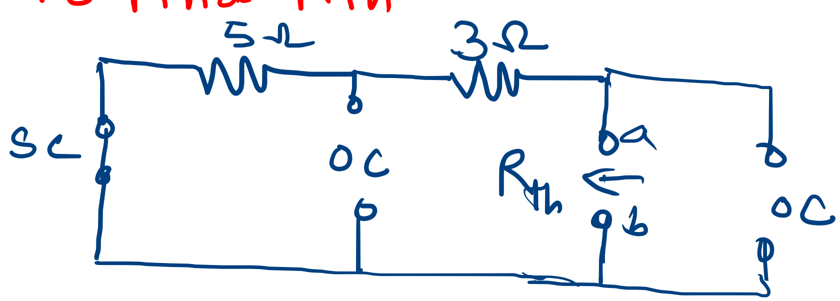
Ex:3 Find voltage across 2 Ohm resistor using Thevenin's Theorem



81
8
16.

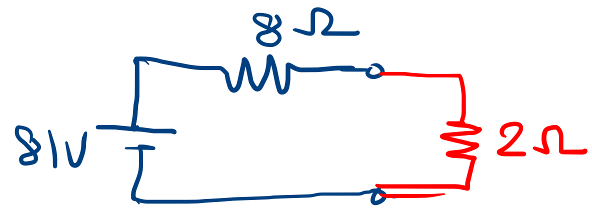
⇒ Solution.

③ To find R_{Th}



$R_{Th} = (5+3) = 8\Omega$

④ Draw equivalent circuit & Connect load.



$$V_{2\Omega} = \frac{2 \times 81}{8+2}$$

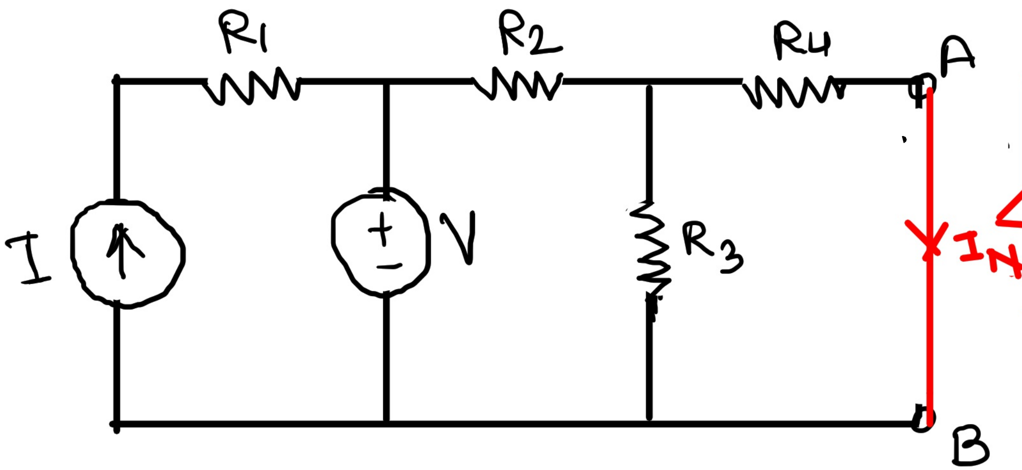
$$V_{2\Omega} = \frac{81}{5}$$

$$\boxed{V_{2\Omega} = 16.2\,V}$$

Norton's Theorem

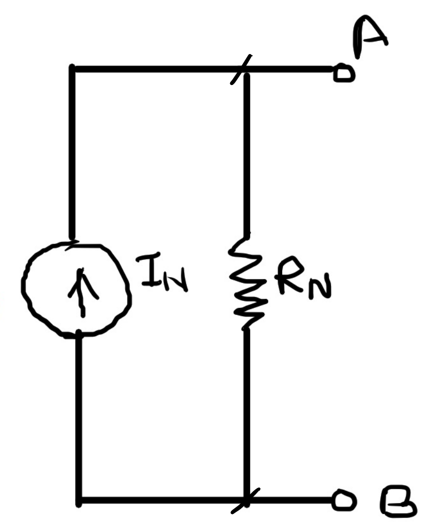
Statement:

Any linear, active bilateral network can be replaced by a current source (I_N) in parallel with a resistance (R_N) where I_N is the short-circuit current (i.e. current through the two terminals when RL is removed) and R_N is the internal resistance of the network as viewed back into the open-circuited network from terminals A and B with all energy sources replaced by their internal resistance (if any) and current sources by infinite resistance.



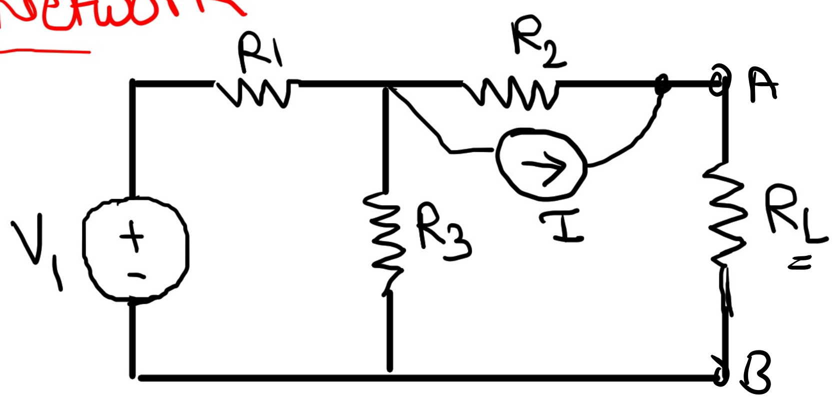
Norton' Current I_N is short circuit current between terminlas A-B

Norton' Resistance R_N is resistance looking back into network from A-B with all energy sources replaced by their internal resistances

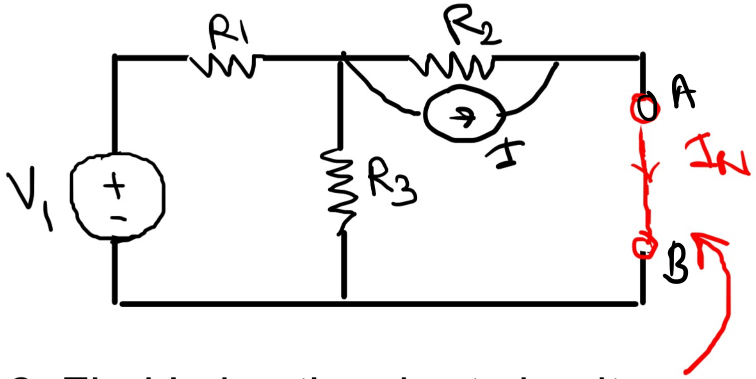


Norton's Theorem

Given Network

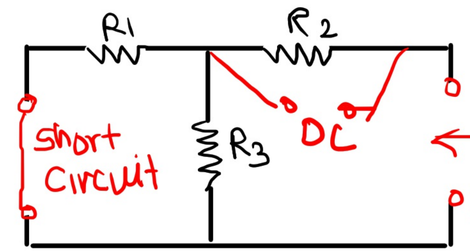


1. Remove the load RL from the circuit



2. Find IN i.e. the short circuit current between the terminals (A-B) from where the load is removed, using any suitable method (mesh, nodal, source transformation etc..)

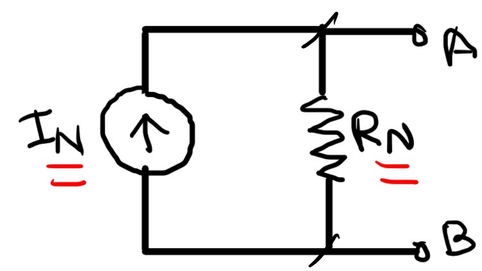
3. Find RN i.e. resistance looking back into the network from the terminals (A-B) from where the load is removed with energy sources replaced by their internal resistances.



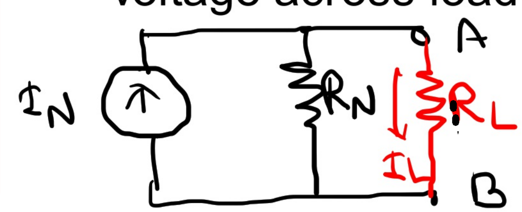
$R_N = (R_1 \| R_3) + R_2$

$R_N = R_{Th}$

4. Draw Norton's equivalent circuit



5. Connect the load RL and find current/ voltage across load RL

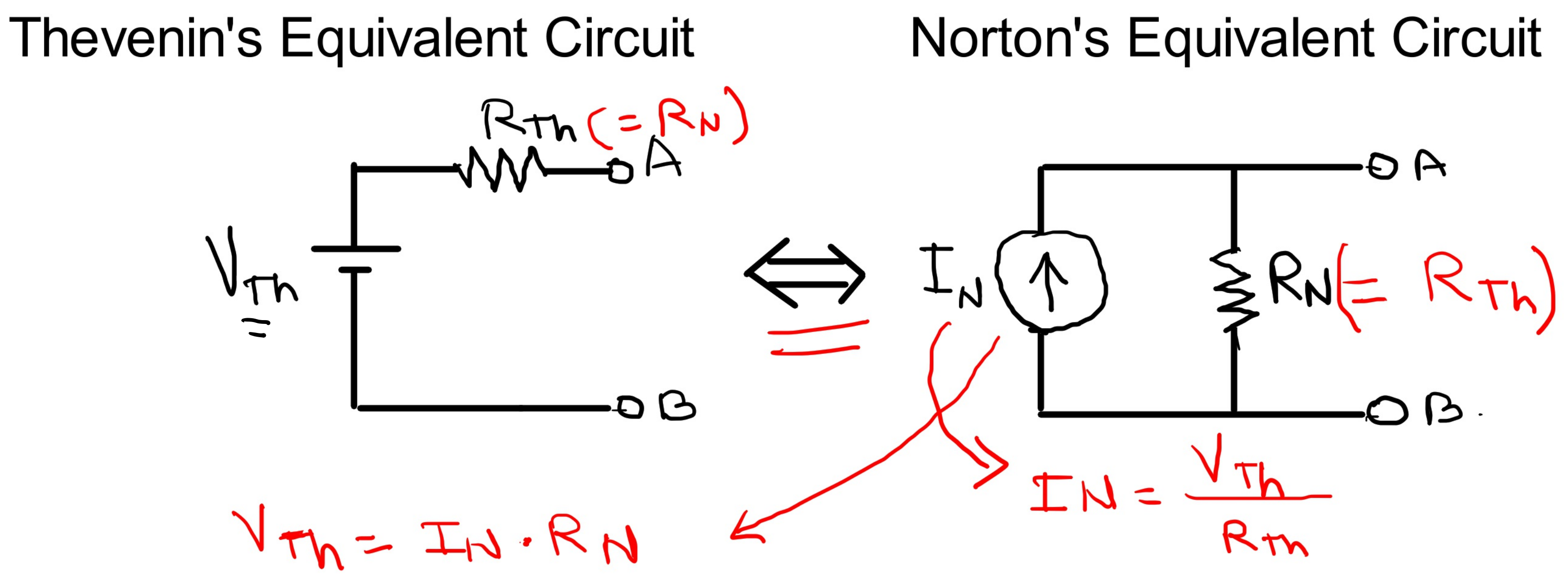


$I_L = \dfrac{R_N \cdot I_N}{R_N + R_L}$

OR

$V_L = I_L \cdot R_L$

Relation between Thevenin and Norton Equivalent Circuit

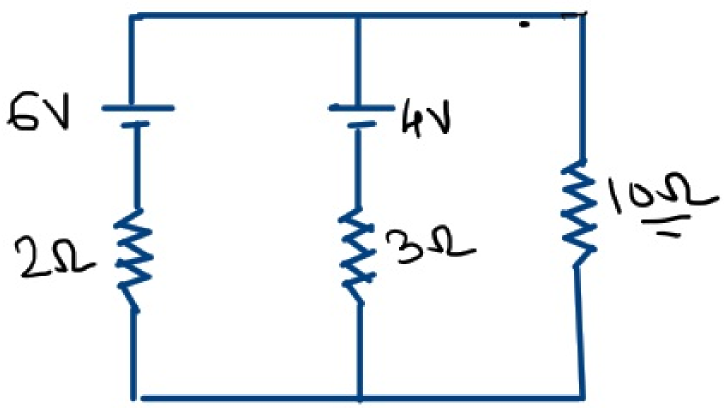


Norton's equivalent circuit can be obtained by applying source transformation to Thevenin's equivalent circuit.

Also

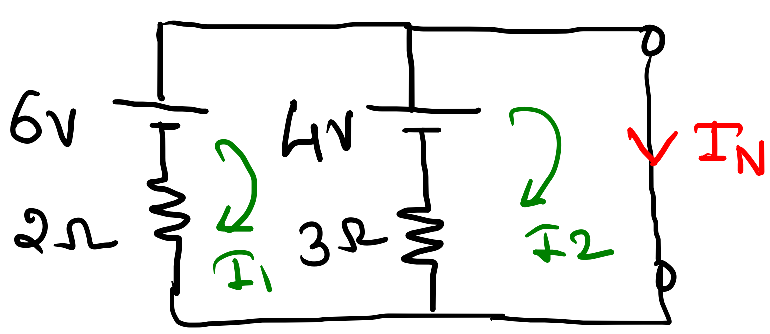
Thevenin's equivalent circuit can be obtained by applying source transformation to Norton's equivalent circuit.

① Find current flowing through 10Ω resistor using Norton's theorem.



$\Rightarrow$ Solution

① Remove load $R_L = 10\Omega$



② Find short circuit current (I_N)

$\Rightarrow$ Using mesh Analysis

KVL to mesh (I)

$$-2I_1 + 6 - 4 - 3(I_1 - I_2) = 0$$

$$-5I_1 + 3I_2 = -2 \quad \text{---(i)}$$

KVL to mesh (II)

$$-3(I_2 - I_1) + 4 = 0$$

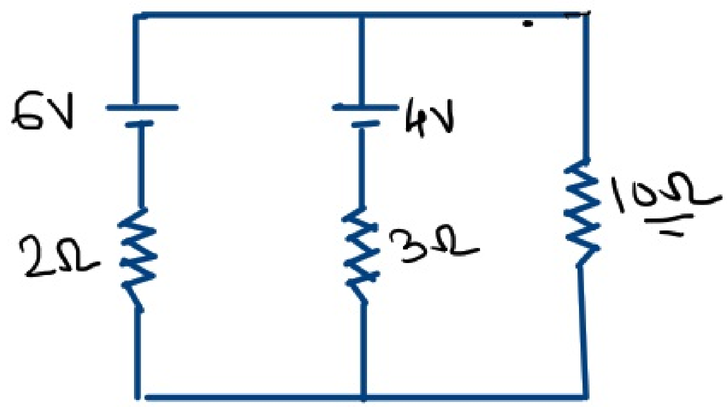
$$3I_1 - 3I_2 = -4 \quad \text{---(2)}$$

solving (i) & (ii)

$I_1 = 3A \quad I_2 = 4.33A$

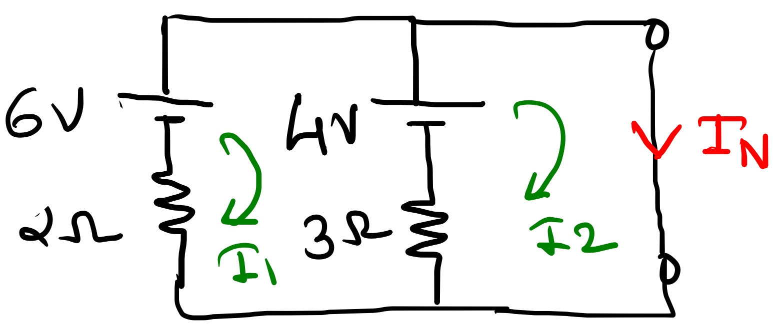
$$\boxed{I_N = I_2 = 4.33A}$$

① Find current flowing through 10Ω resistor using Norton's theorem.



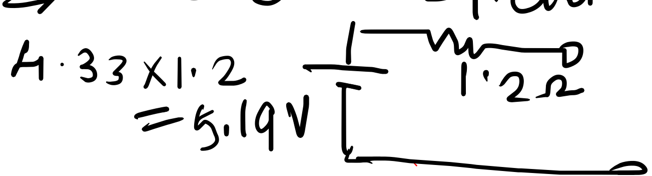
⇒ Solution

① Remove load $R_L = 10\Omega$

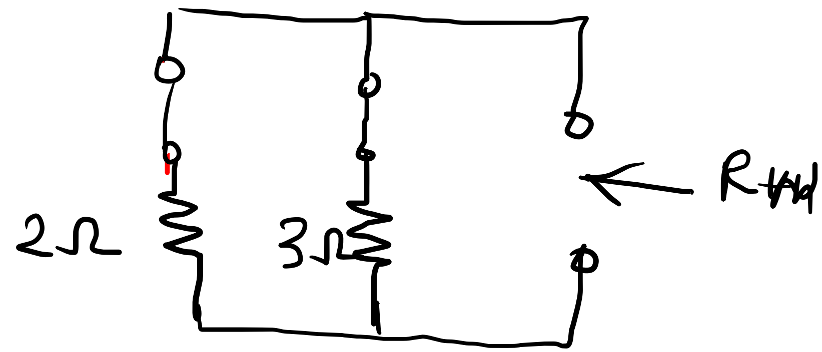


② Find Short circuit current (I_N)

⇒ Thevenins Eq. ckt.

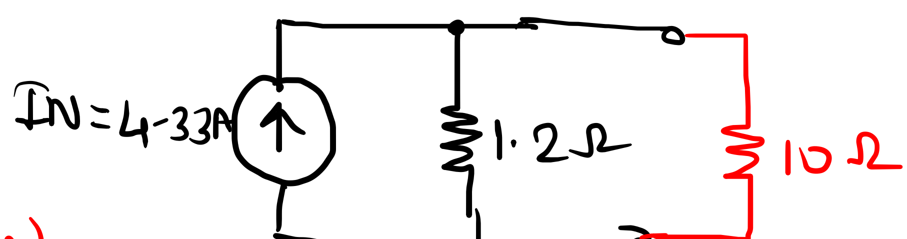


③ to find R_N



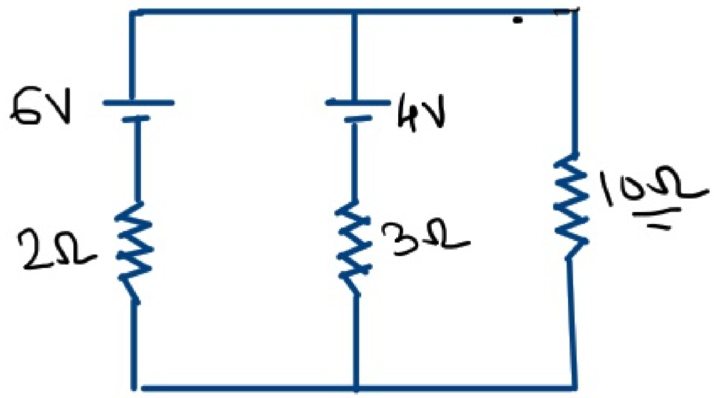
$$R_N = 2 \| 3 = \frac{6}{5} = 1.2\,\Omega$$

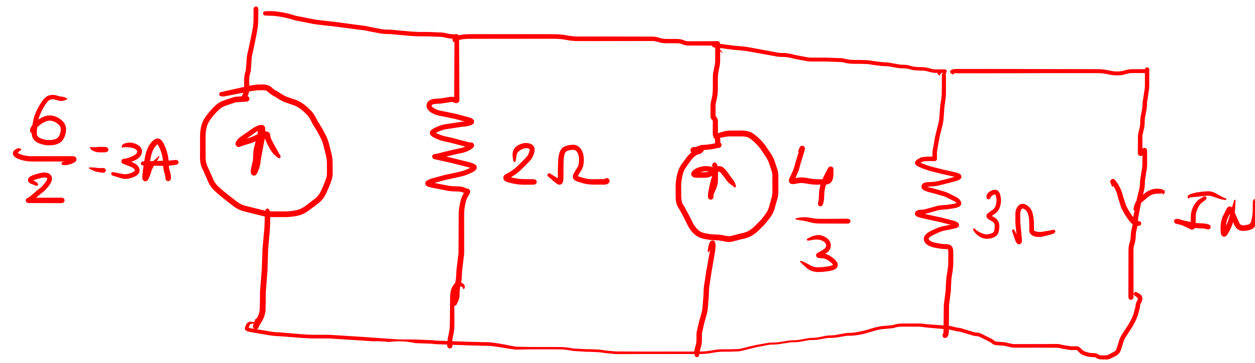
④ Draw Norton's Equivalent circuit



$$I_{10\Omega} = \frac{1.2 \times 4.33}{1.2 + 10} = 0.46\,A$$

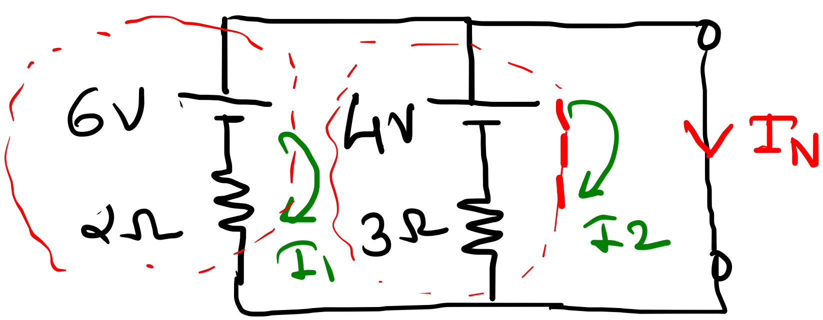
① Find current flowing through 10Ω resistor using Norton's theorem.

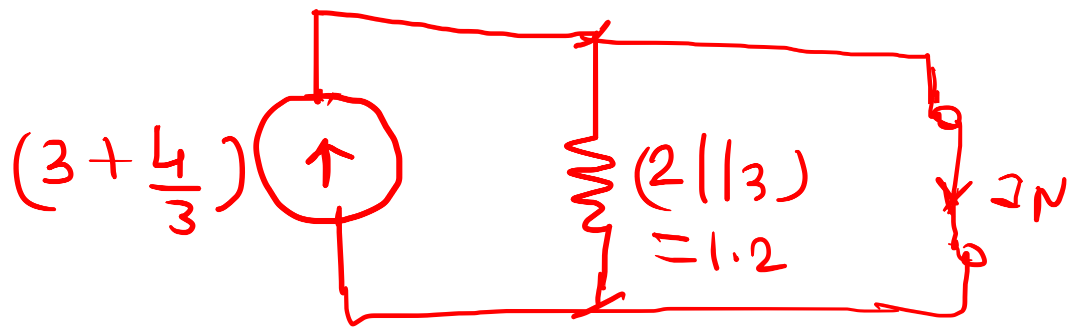


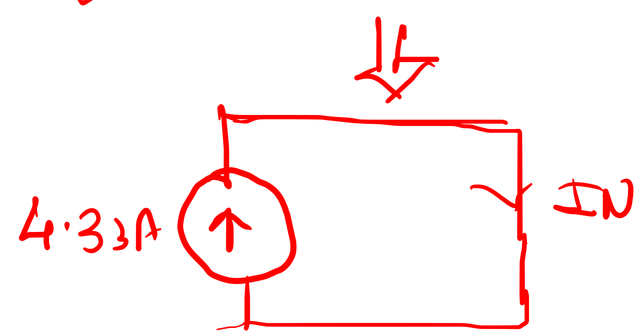


⇒ Solution

① Remove load RL = 10Ω



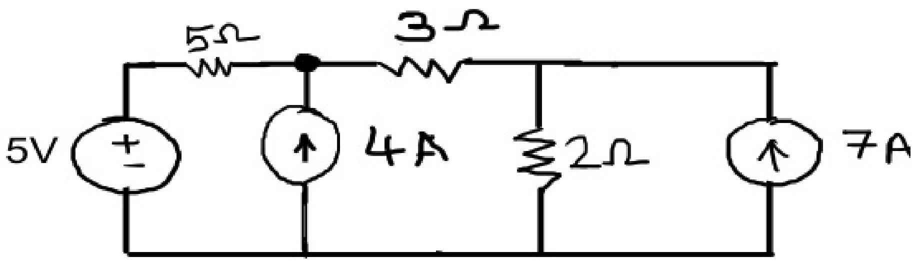




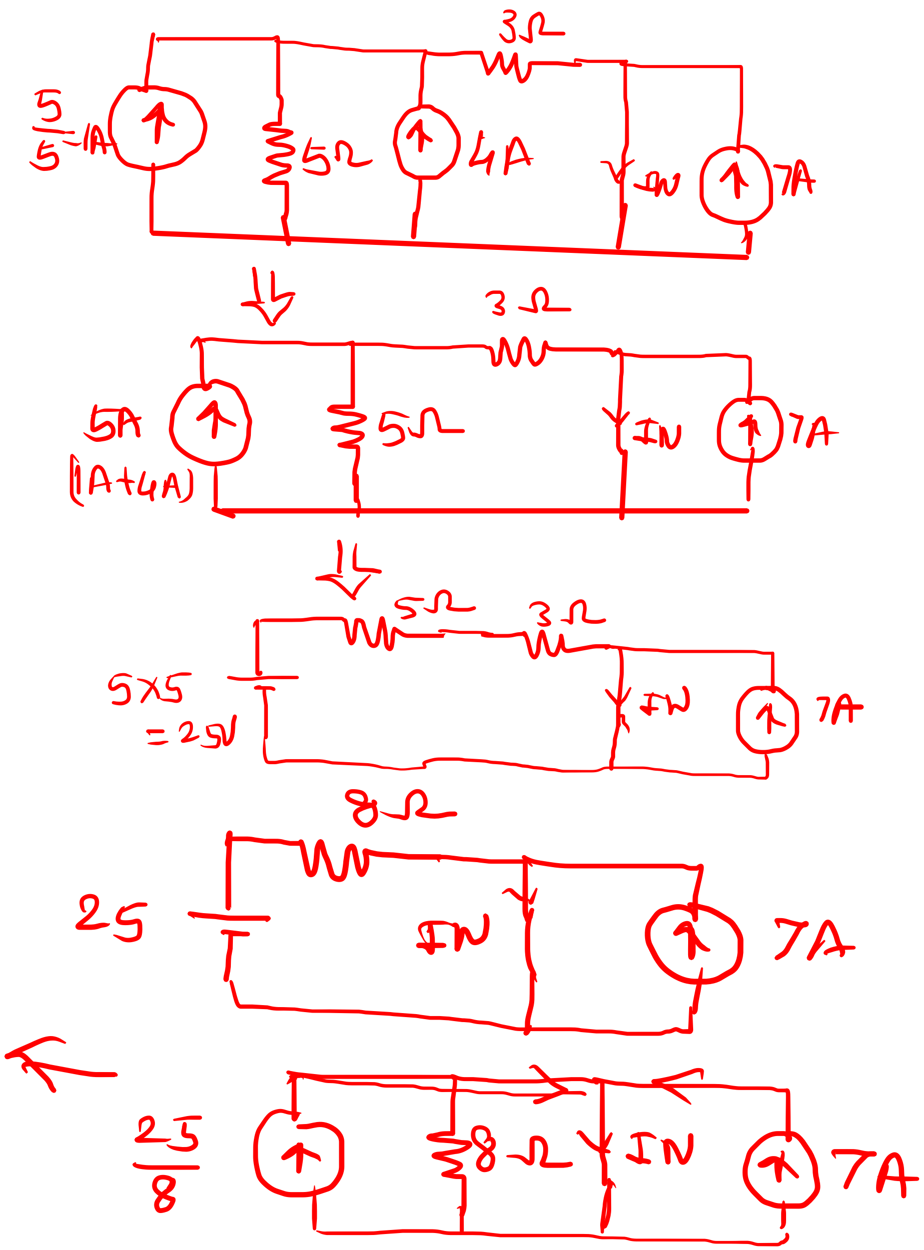
② Find Short circuit current (IN)

$I_N = 4.33A$

Ex. ② Find current in 2Ω resistor using Norton's theorem.



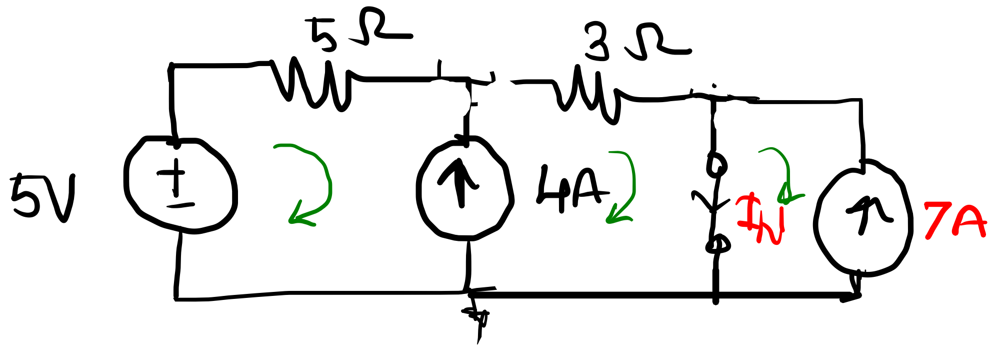
⇒ Source transformation



⇒ Solution

① Remove $R_L = 2\Omega$.

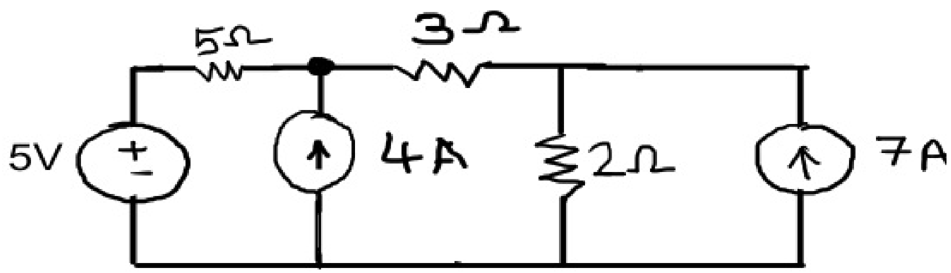
② Short circuit & find I_N.



Using KCL

$$I_N = \frac{25}{8} + 7 = 10.1A$$

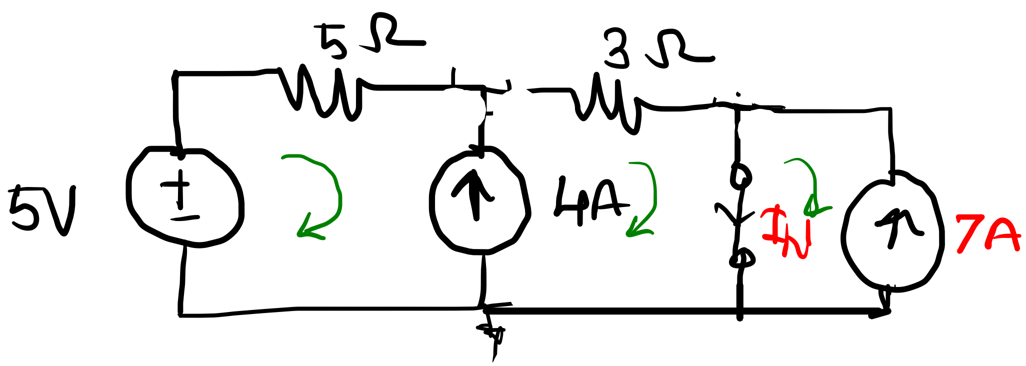
Ex. 2 Find current in 2Ω



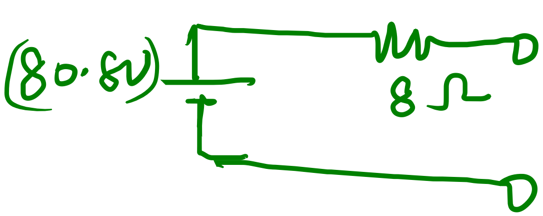
⇒ Solution

1. Remove $R_L = 2\Omega$.

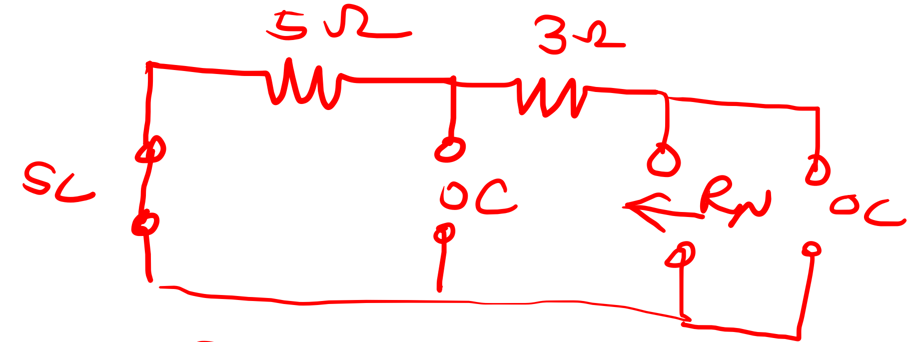
2. Short circuit & find I_N.



⇒ Theverim's Eq. ckt.

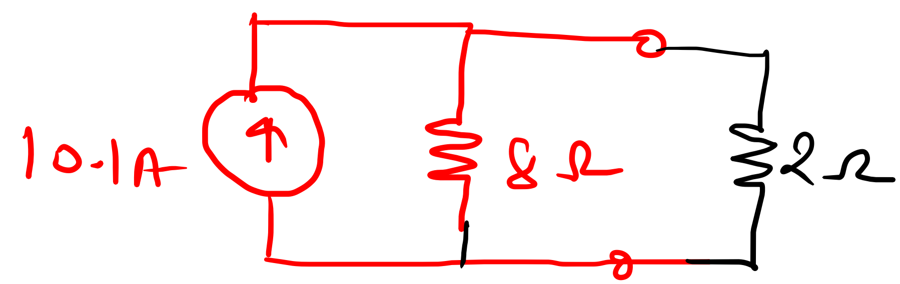


3. find R_N.



$$R_N = 5+3 = 8\Omega$$

4. Norton's Equivalent



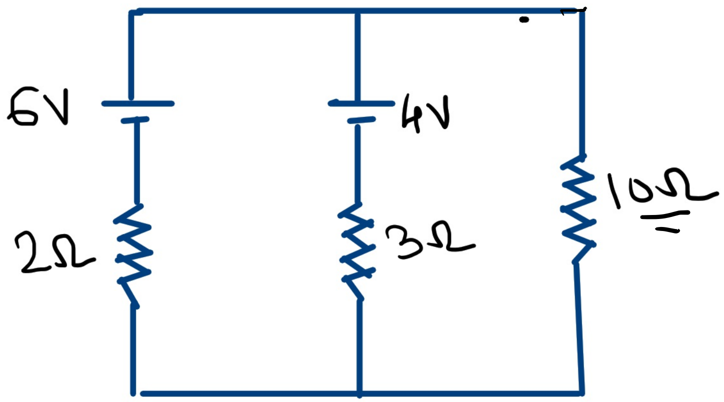
$$I_{2\Omega} = \frac{8 \times 10.1}{8+2} = 8.1\,A$$

$$V_{2\Omega} = 8.1 \times 2 = 16.2V.$$

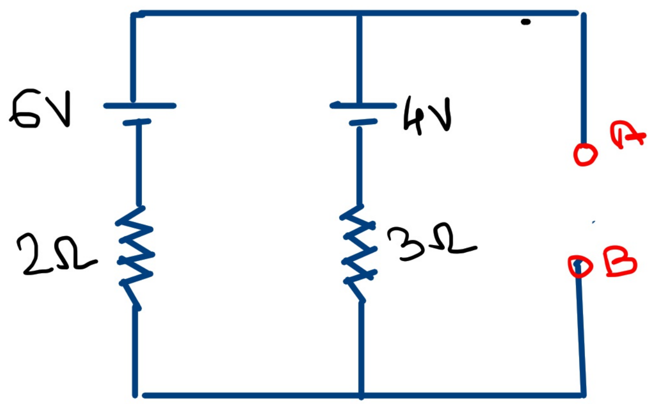
Norton's Theorem

Q.1 Find current flowing through 10 Ohm resistance using Norton's theorem

$I_{10} = 0.46A$

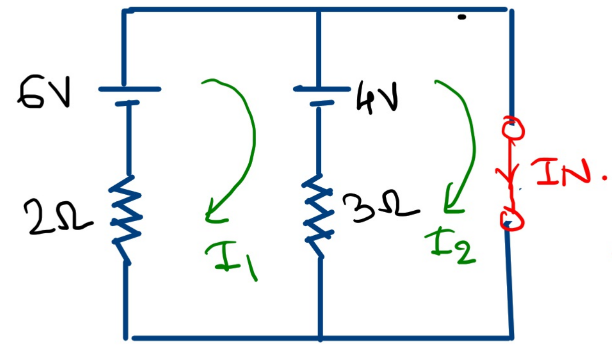


① Remove load $R_L = 10\Omega$



② Short circuit A & B & find

$I_N = I_{sc}$



$I_N = I_2$

→ Using mesh Analysis.

→ Mesh ①

$-2I_1 + 6 - 4 - 3(I_1 - I_2) = 0$

$5I_1 - 3I_2 = 2 \quad \text{---} ①$

→ mesh ②

$-3(I_2 - I_1) + 4 = 0.$

$3I_1 - 3I_2 = -4 \quad \text{---} ②$

Solving ① & ② $I_1 = 3A$ $I_2 = 4.33A$

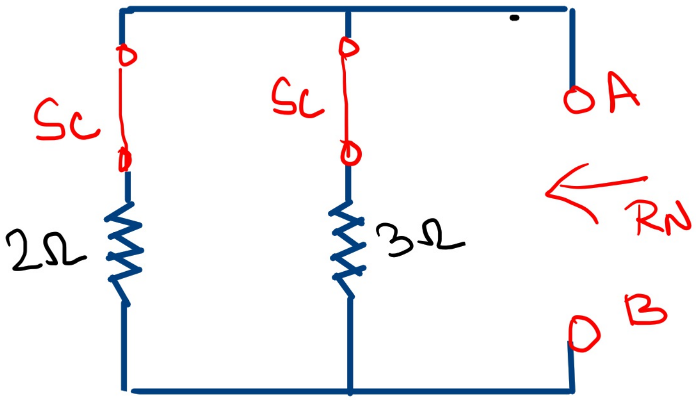
$I_N = I_2 = 4.33\ A$

Norton's Theorem

Q.1 .....

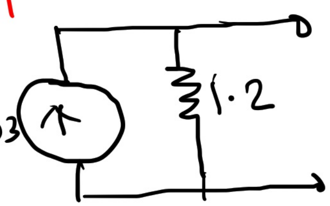
$I_{10} = 0.46A$

(3) Find $R_N = R_{eq}$.



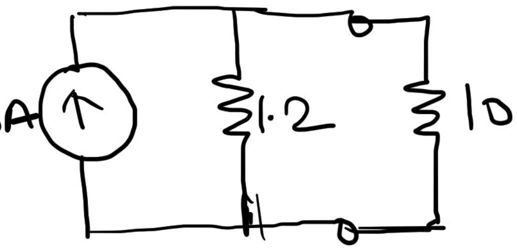
$R_N = 2 \| 3 = \frac{6}{5} = 1.2\,\Omega$

(4) Draw Norton's Equivalent Circuit



$\Rightarrow$

(5) Connect $R_L = 10\,\Omega$



using current division formula

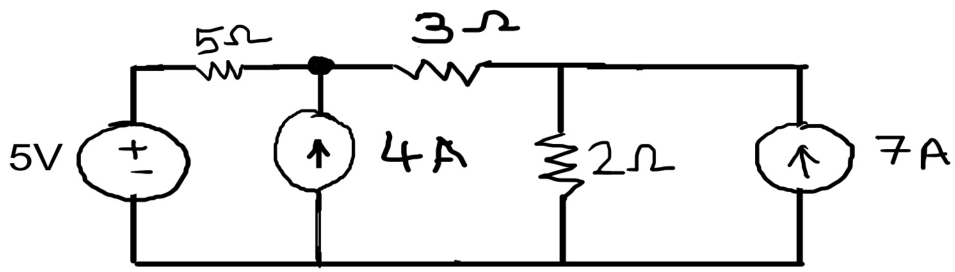
$$I_{10\Omega} = \frac{1.2 \times 4.33}{1.2 + 10}$$

$$\boxed{\underline{I_{10\Omega} = 0.46A}}$$

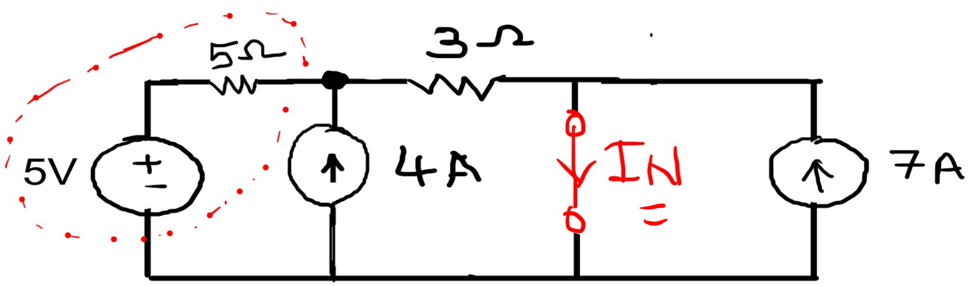
source transformation (Thevenin Equivalent

$V_{Th} = 4.33 \times 1.2$
$= \underline{5.2}$

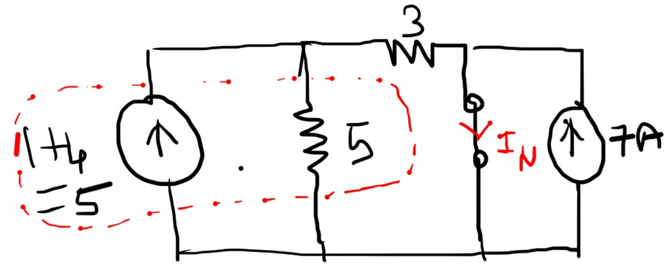
Example:- 2. Find voltage across 2 Ohm resistor using Norton's Theorem

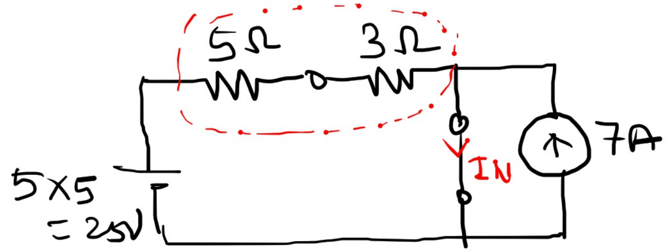


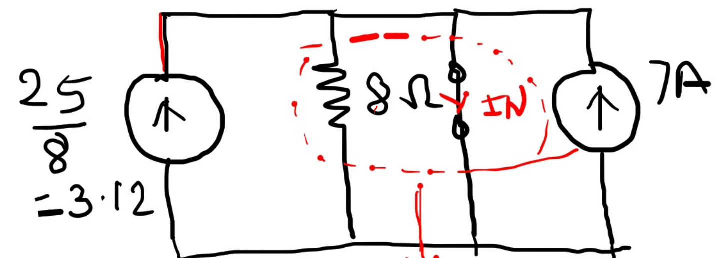
① Remove $R_L = 2\Omega$ & find IN



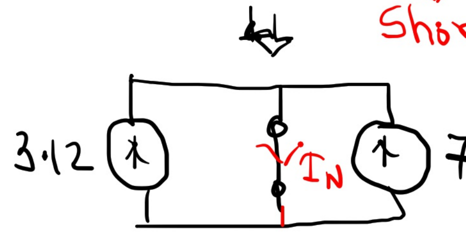
using source transformation







Short circuit (8Ω)



$I_N = 3.12 + 7$

$I_N = 10.12A$

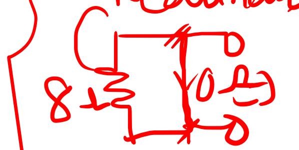
$I_N = \frac{81}{8}$

$R_N = 8\Omega$

$I_{2\Omega} = \frac{81}{10}$

$V_{2\Omega} = 16.2$

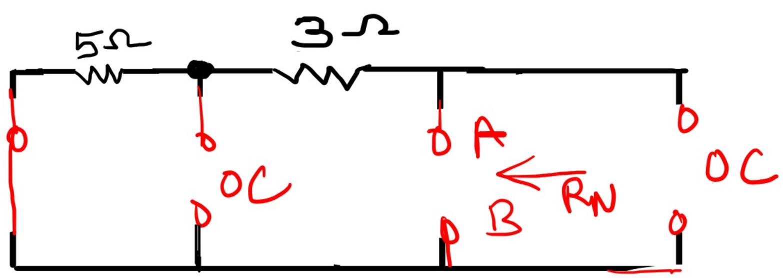
Redundant



$\frac{8 \times 0}{8+0} = 0$

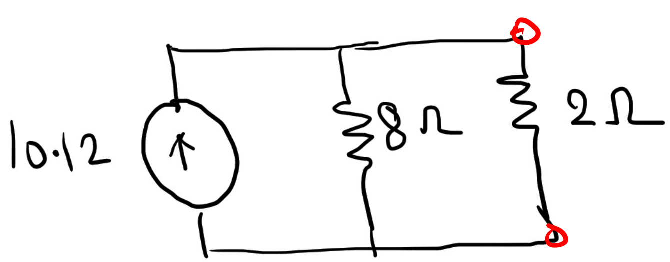
Example:- 2. .....

③ Find RN



$R_N = 5+3 = 8\,\Omega$

④ Draw Norton's Equivalent circuit & Connect load.



Using current division

$$I_{2\Omega} = \frac{10.12 \times 8}{10}$$

$$I_{2\Omega} = 8.09\,A$$

$$V_{2\Omega} = I_{2\Omega} \times 2$$

$$= 8.09 \times 2$$

$$V_{2\Omega} = 16.2\,V$$

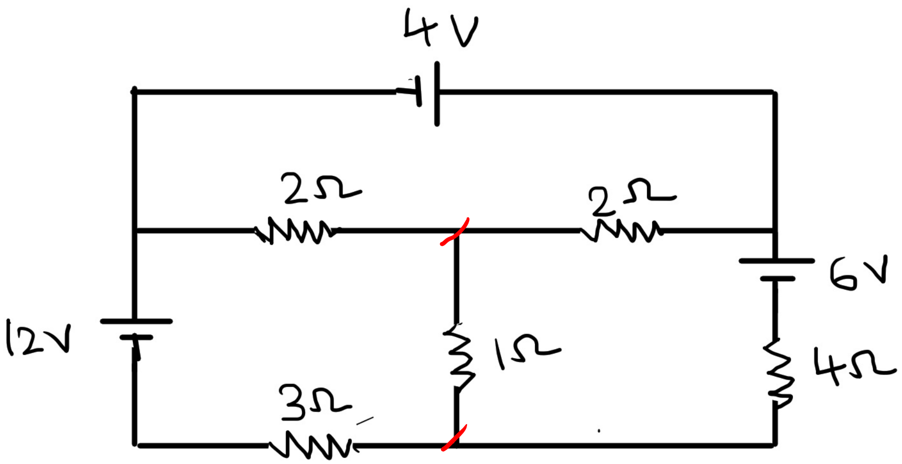
$I_N = \frac{81}{8}$

$R_N = 8\Omega$

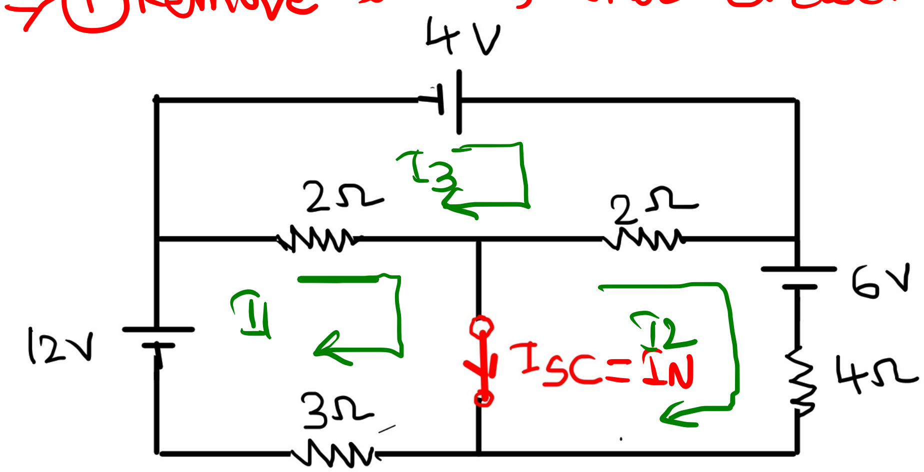
$I_{2\Omega} = \frac{81}{10}$

$V_{2\Omega} = 16.2$

Example:- 3. Find current flowing through 1Ohm resistor, using Norton's Theorem.



⇒ (1) Remove load & short circuit & find IN



Using mesh Analysis

$I_N = (I_1 - I_2) \downarrow$

KVL to mesh (I)

$12 - 2(I_1 - I_3) - 3I_1 = 0$

$5I_1 - 2I_3 = 12$ --- (i)

KVL to mesh (2)

$-2(I_2 - I_3) - 6 - 4(I_2) = 0$

$6I_2 - 2I_3 = -6$ --- (ii)

KVL to mesh (3)

$4 - 2(I_3 - I_2) - 2(I_3 - I_1) = 0$

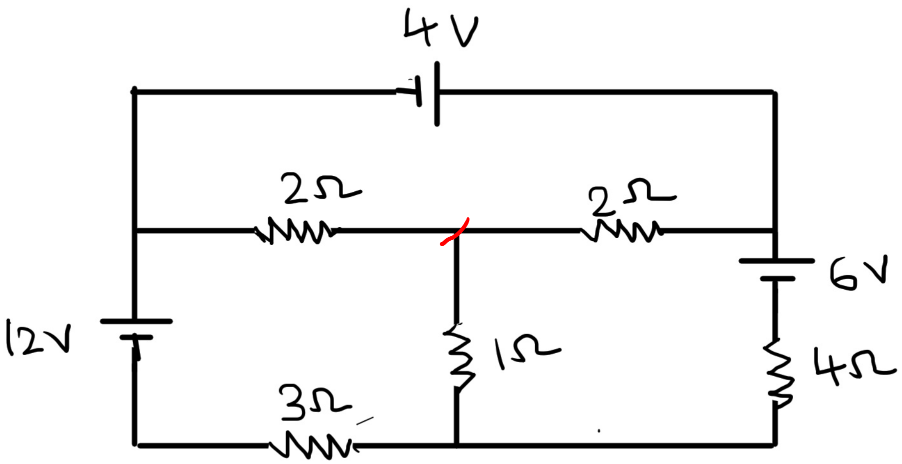
$2I_1 + 2I_2 - 4I_3 = -4$ --- (iii)

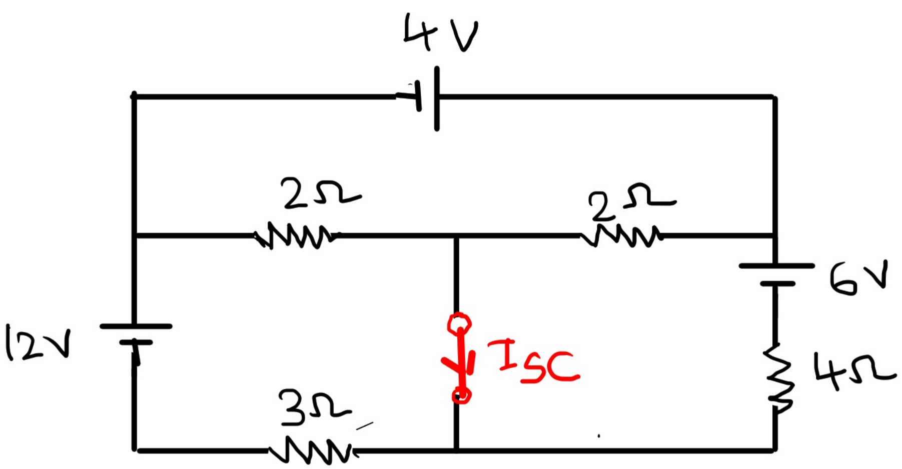
Solving (i), (ii) & (iii)

$I_1 = 3.47A$, $I_2 = -0.1A$, $I_3 = 2.68A$

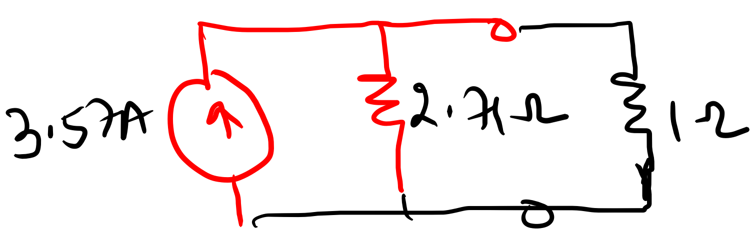
$I_N = I_1 - I_2 = 3.47 - (-0.1) = 3.57A \downarrow$

Example:- 3. Find current flowing through 1Ohm resistor, using Norton's Theorem.

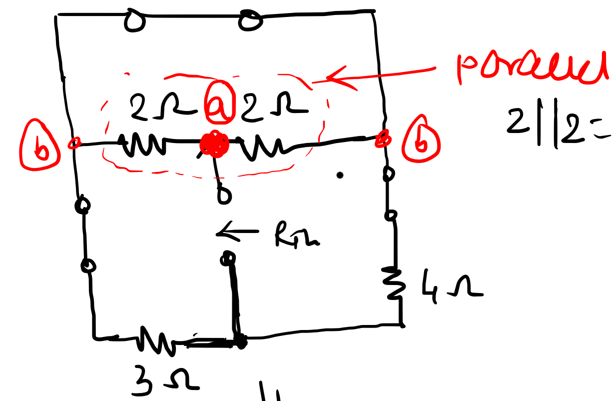




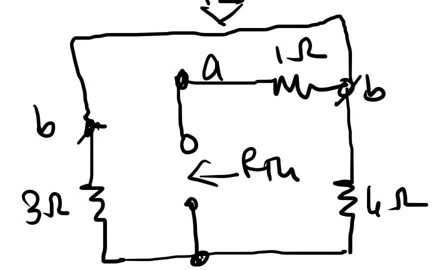
⇒ Norton's Equivalent Circuit



⇒ To find $R_N = R_{Th}$



$2||2 = 1\Omega$



$3||4 = \frac{12}{7}$

$$R_N = \frac{12}{7} + 1 = 2.71\Omega$$

Current division Rule

$$I_{1\Omega} = \frac{2.71 \times 3.57}{2.71 + 1\Omega} = 2.6A$$