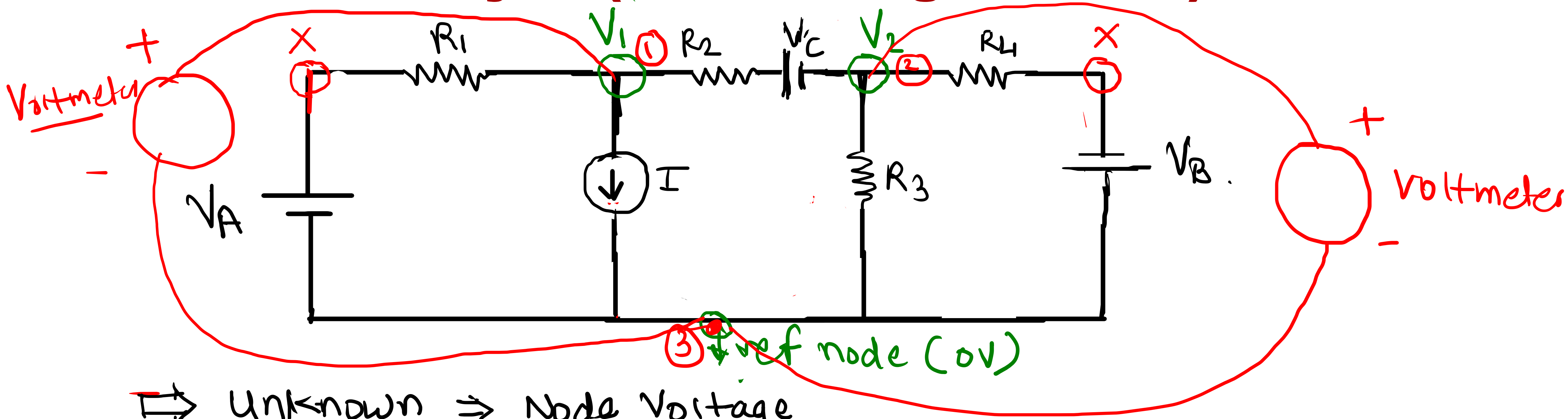
Nodal Analysis (Node Voltage method)



⇒ Unknown ⇒ Node Voltage

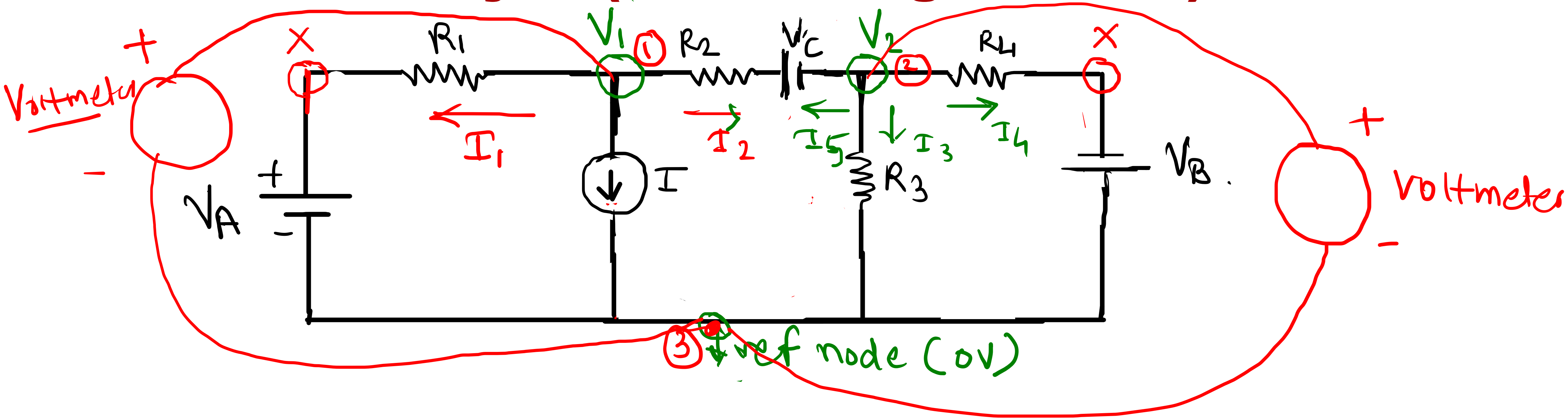
⇒ Identify nodes in given circuit

⇒ One node as reference node (Node 3)

⇒ Other nodes are called non-reference node (node 1 & 2)

⇒ Non-reference node voltages V_1 & V_2 are assumed to be positive w.r.t. ref. node

Nodal Analysis (Node Voltage method)



⇒ Apply KCL at non-reference nodes

KCL at node ① $I_1 + I_2 + I = 0$

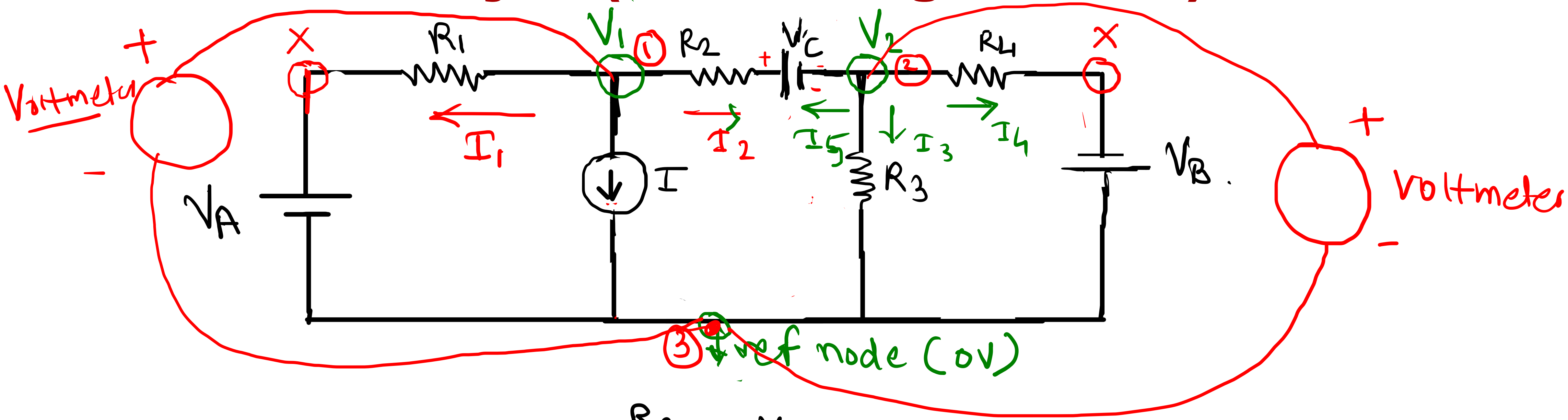
KCL at node ② $I_5 + I_3 + I_4 = 0$

⇒ Represents currents in terms of node voltages.

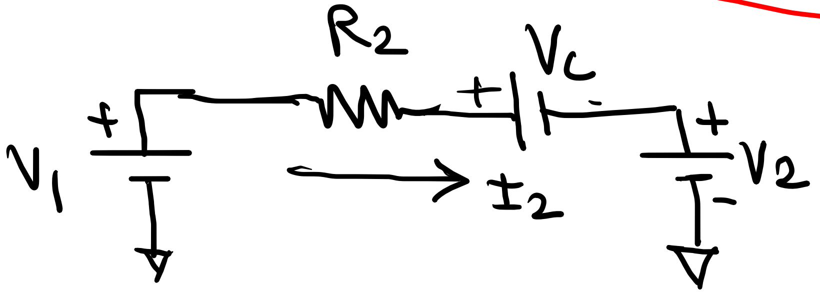
using KVL → $V_1 - I_1 R_1 - V_A = 0$

$$I_1 = \frac{V_1 - V_A}{R_1}$$

Nodal Analysis (Node Voltage method)



$I_2 \Rightarrow$

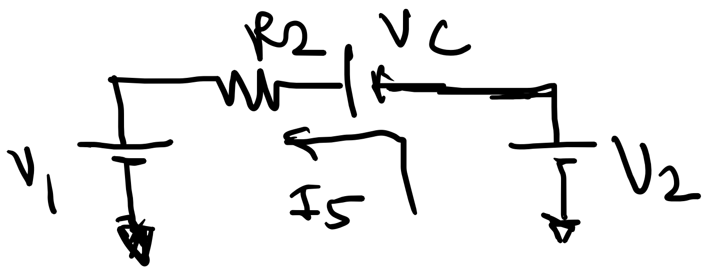


KVL to Loop

$$V_1 - I_2 R_2 - V_c - V_2 = 0$$

$$I_2 = \frac{V_1 - V_c - V_2}{R_2}$$

$I_5 \Rightarrow$

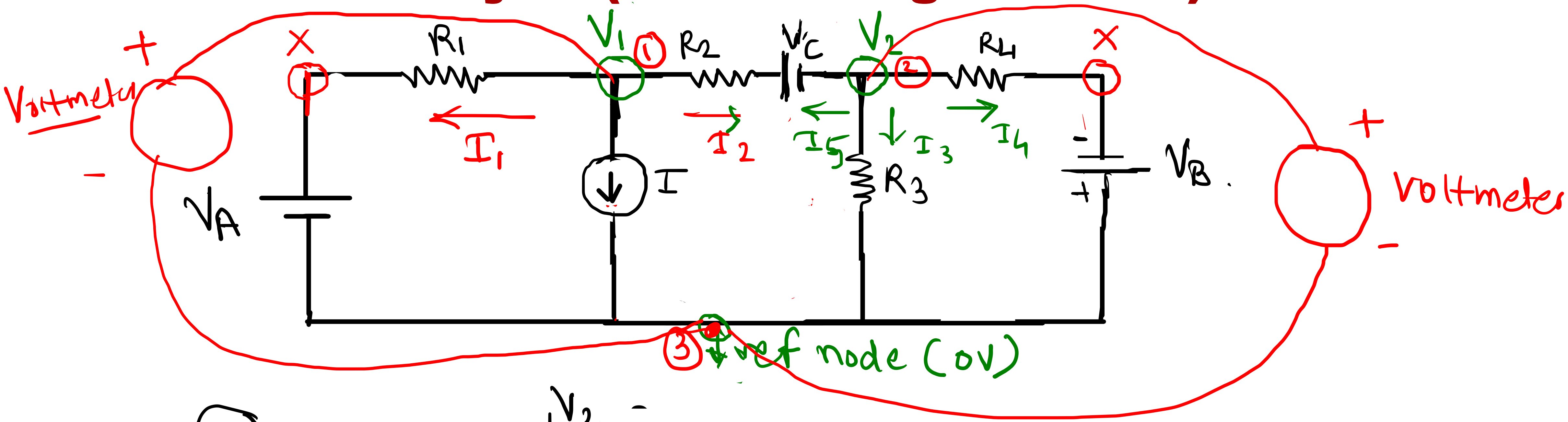


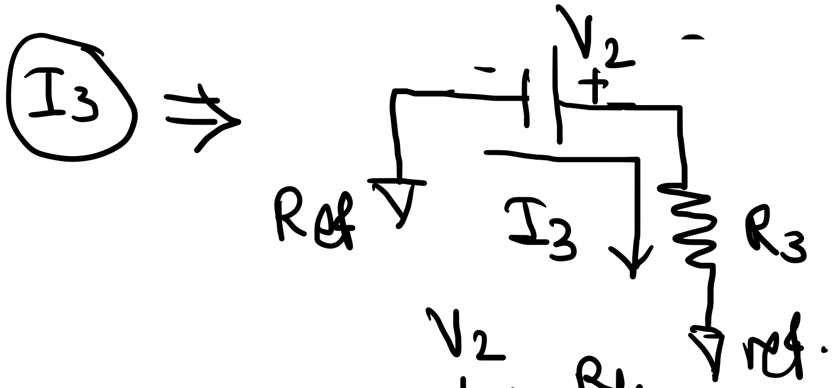
KVL to loop

$$V_2 + V_c - I_5 R_2 - V_1 = 0$$

$$I_5 = \frac{V_2 + V_c - V_1}{R_2}$$

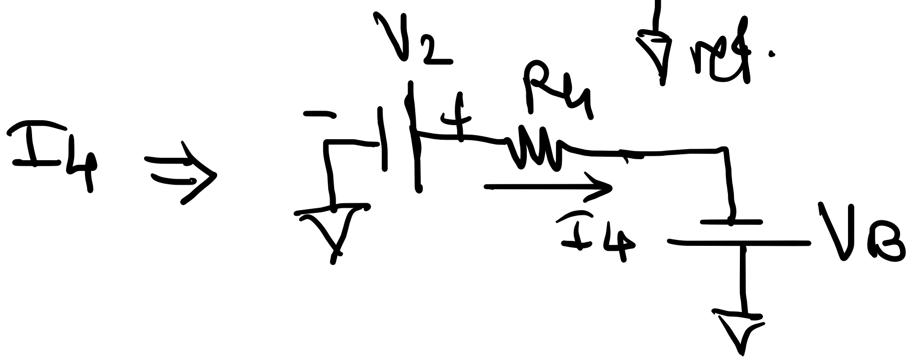
Nodal Analysis (Node Voltage method)





KVL to loop

$$V_2 - I_3 R_3 = 0 \quad \therefore \boxed{I_3 = \frac{V_2}{R_3}}$$

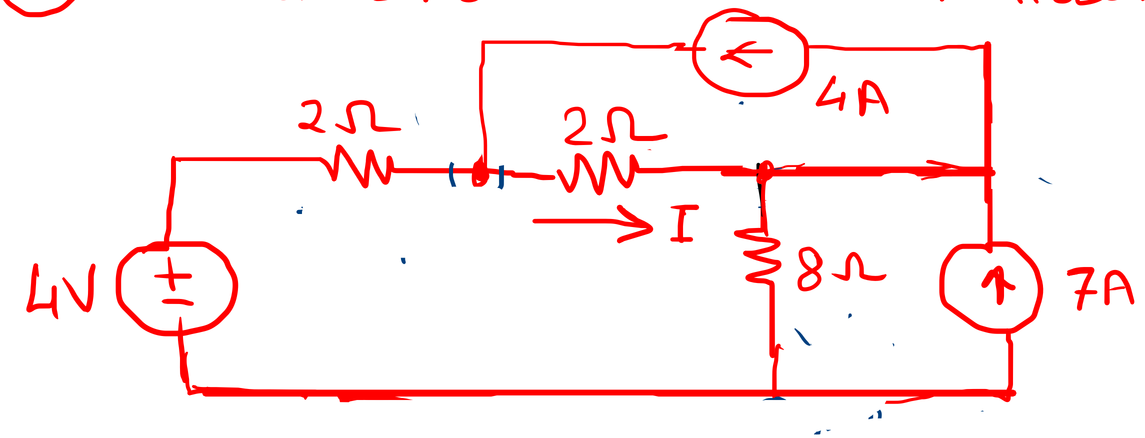


KVL to loop

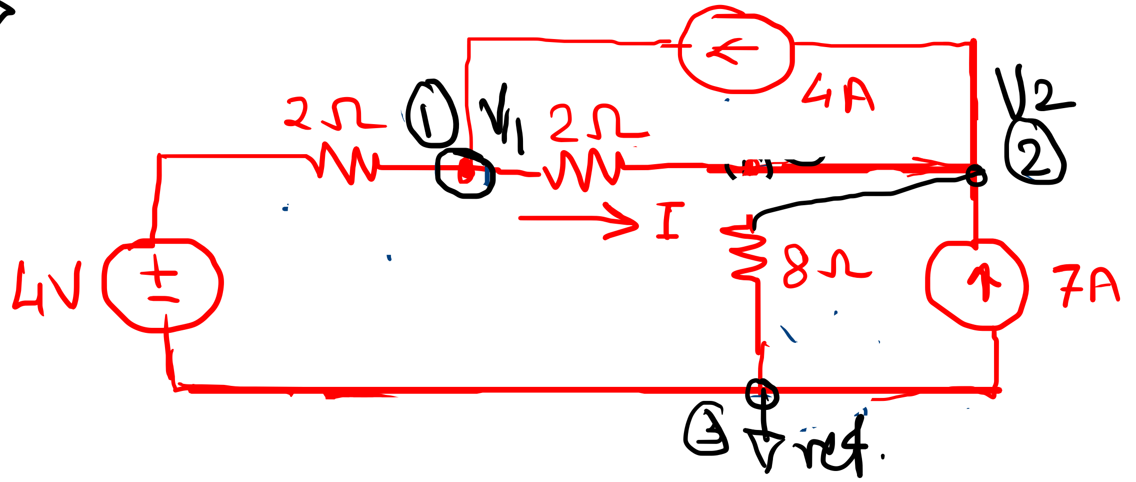
$$V_2 - I_4 R_4 + V_B = 0$$

$$\boxed{I_4 = \frac{V_2 + V_B}{R_4}}$$

(1) Find current I in the following network using Nodal Analysis



⇒



⇒ KCL at node (1)

$$\frac{V_1-4}{2}+\frac{V_1-V_2}{2}=4$$

$$(V_1-4)+(V_1-V_2)=8$$

$$2V_1-V_2=12 \quad \text{---(1)}$$

KCL at node (2)

$$\frac{V_2}{8}+\frac{V_2-V_1}{2}=7-4$$

$$\frac{V_2+4V_2-4V_1}{8}=3$$

$$-4V_1+5V_2=24 \quad \text{---(2)}$$

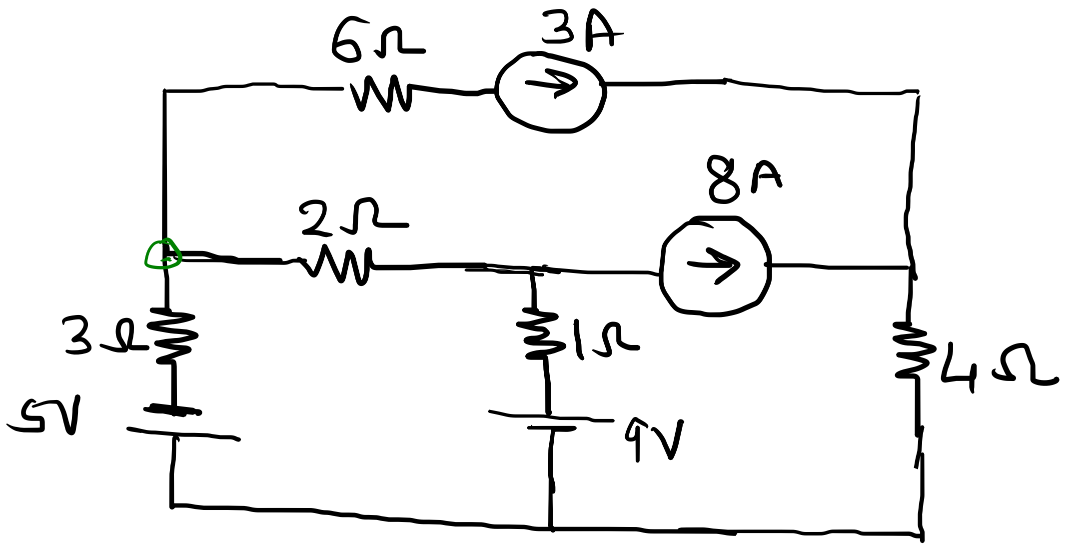
Solving (1) & (2)

$V_1 = 14V$, $V_2 = 16V$.

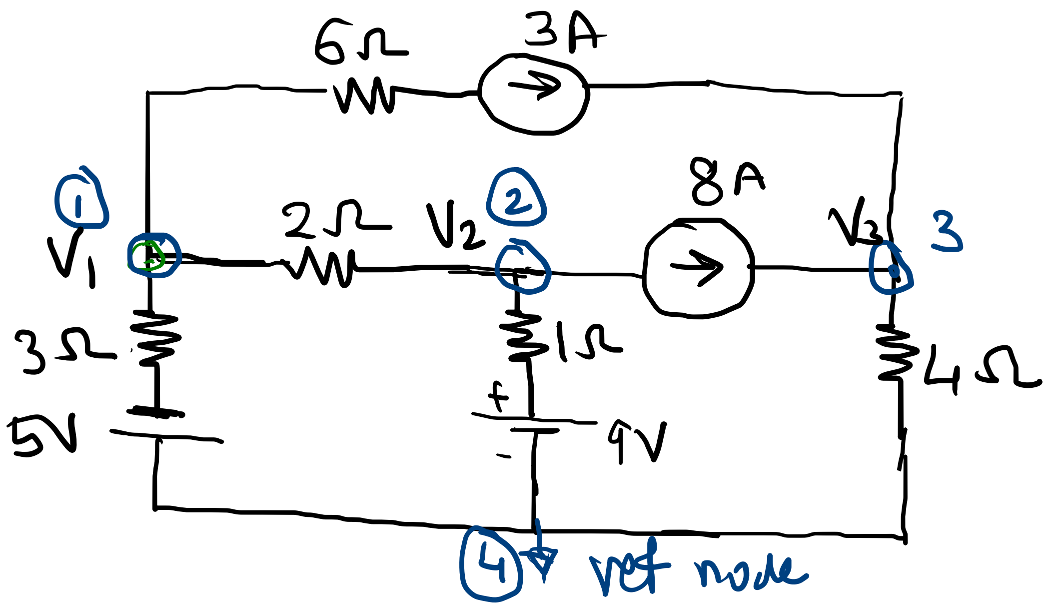
$$I=\frac{V_1-V_2}{2}=\frac{14-16}{2}=-1A$$

$(\rightarrow)$

Example (2) Find current in 3Ω Resistor using Nodal Analysis



⇒ Identify nodes.



⇒ KCL at node (1)

$$\frac{V_1+5}{3} + \frac{V_1-V_2}{2} + 3 = 0$$

$$\frac{2V_1+10+3V_1-3V_2}{6} = -3$$

$$5V_1 - 3V_2 = -28 \quad --- (1)$$

⇒ KCL at node (2)

$$\frac{V_2-V_1}{2} + \frac{V_2-9}{1} + 8 = 0$$

$$V_2 - V_1 + 2V_2 - 18 = -16$$

$$-V_1 + 3V_2 = 2 \quad --- (2)$$

⇒ KCL at node (3)

$$\frac{V_3}{4} = 3+8$$

$$V_3 = 11 \times 4 = 44V$$

$$V_1 = -6.5V$$

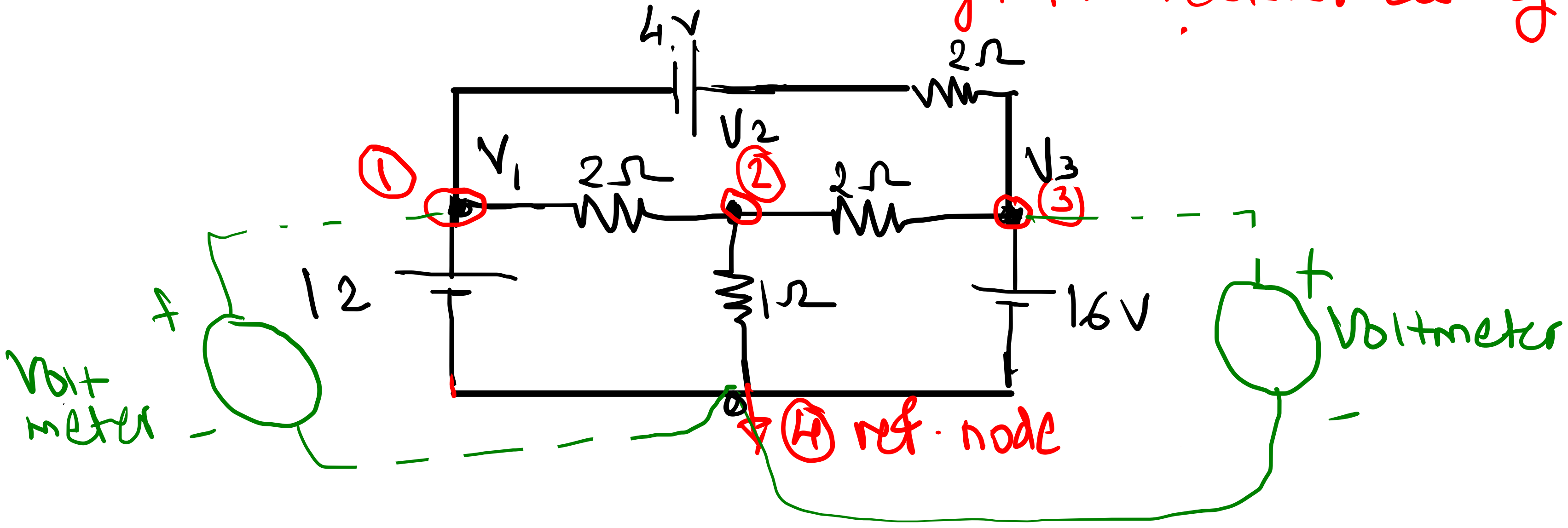
$$V_2 = -1.5V$$

$$I_3 = \frac{V_1+5}{3}$$

$$= \frac{-6.5+5}{3} = -0.5A$$

⇒ Type = 2 ⇒ Voltage Source without series resistance appearing between Non-reference and reference node.

⇒ Example ③ Find current through 1Ω resistor using Nodal Analysis.



⇒ KCL at node-2

$$\frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{2} + \frac{V_2}{1} = 0$$

$$\frac{V_2 - 12}{2} + \frac{V_2 - 16}{2} + \frac{V_2}{1} = 0$$

$$V_2 - 12 + V_2 - 16 + 2V_2 = 0$$

$$4V_2 = 28$$

$$\boxed{V_2 = 7V}$$

$$I_{1\Omega} = \frac{V_2}{1} = 7A$$

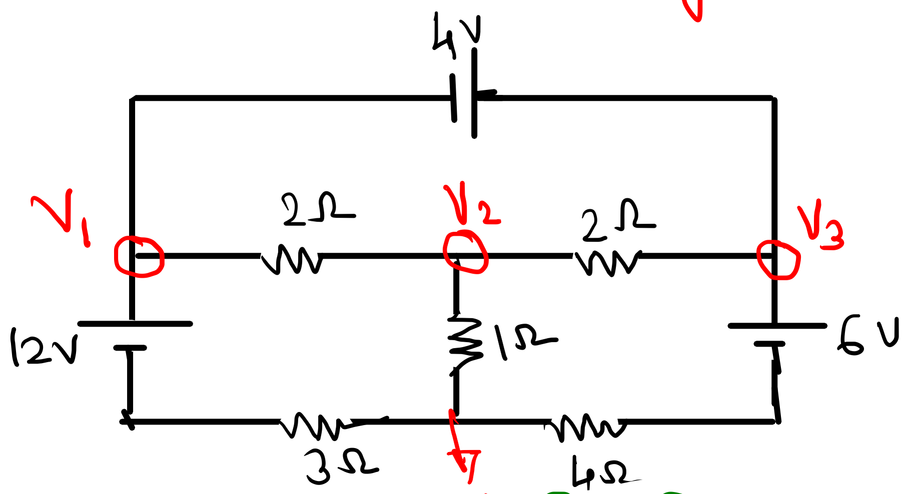
⇒ Identify nodes

⇒ Since 12V & 16V without series resistances appearing between node 1, 3 & ref. node so no need to Apply KCL.

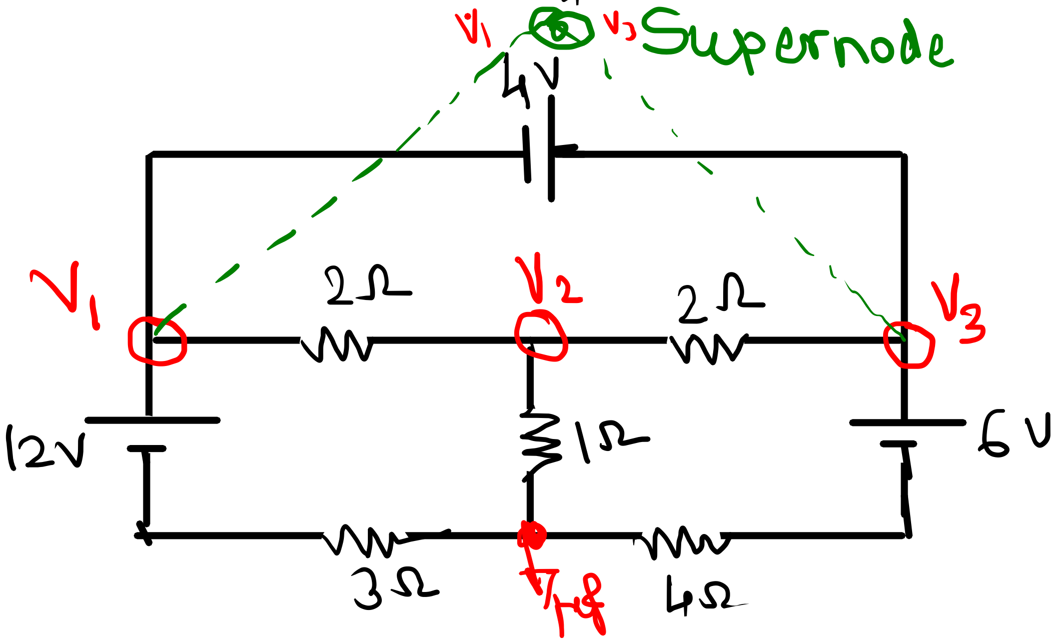
$V_1 = 12V$ & $V_3 = 16V$

⇒ Type-3 : Voltage source without series resistance appearing between two non-reference nodes.

Example : find current through 1Ω resistor



⇒



⇒ 4V source without series resistance between two non-ref. nodes ① & ③

→ Represent 4V source in terms of V_1 & V_3

Using KVL

$V_3 - 4 - V_1 = 0$

$V_1 - V_3 = -4$ --- ①

⇒ Write KCL at supernode keeping V_1 & V_3 intact.

$$\frac{V_1 - V_2}{2} + \frac{V_1 - 12}{3} + \frac{V_3 - V_2}{2} + \frac{V_3 - 6}{4} = 0$$

$$\frac{6V_1 - 6V_2 + 4V_1 - 48 + 6V_3 - 6V_2 + 3V_3 - 18}{12} = 0$$

$10V_1 - 12V_2 + 9V_3 = 66$ --- ②

⇒ KCL to node ②

$$\frac{V_2 - V_1}{2} + \frac{V_2}{1} + \frac{V_2 - V_3}{2} = 0$$

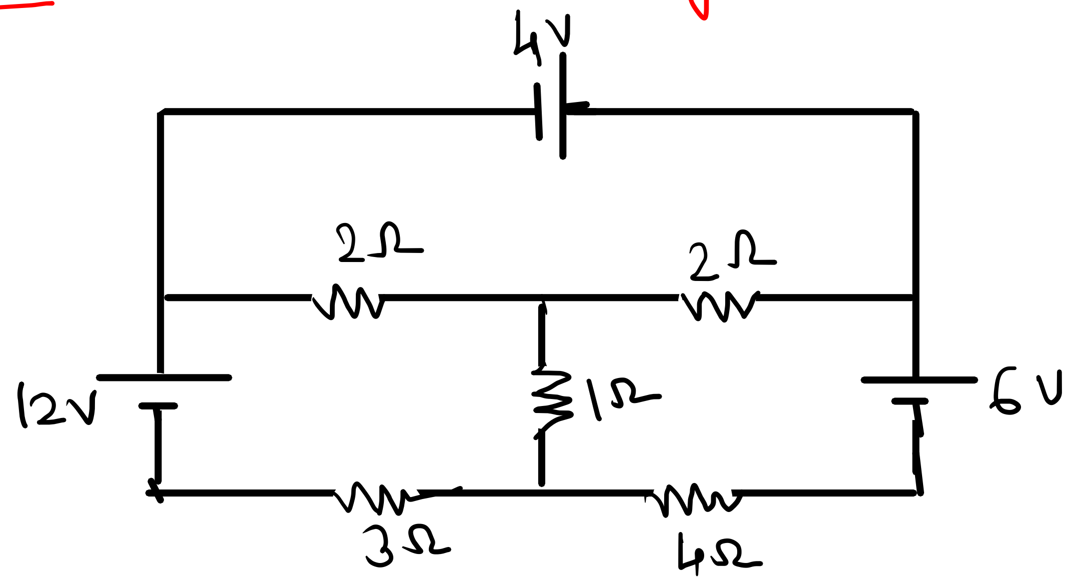
$V_2 - V_1 + 2V_2 + V_2 - V_3 = 0$

$-V_1 + 4V_2 - V_3 = 0$ ---- ③

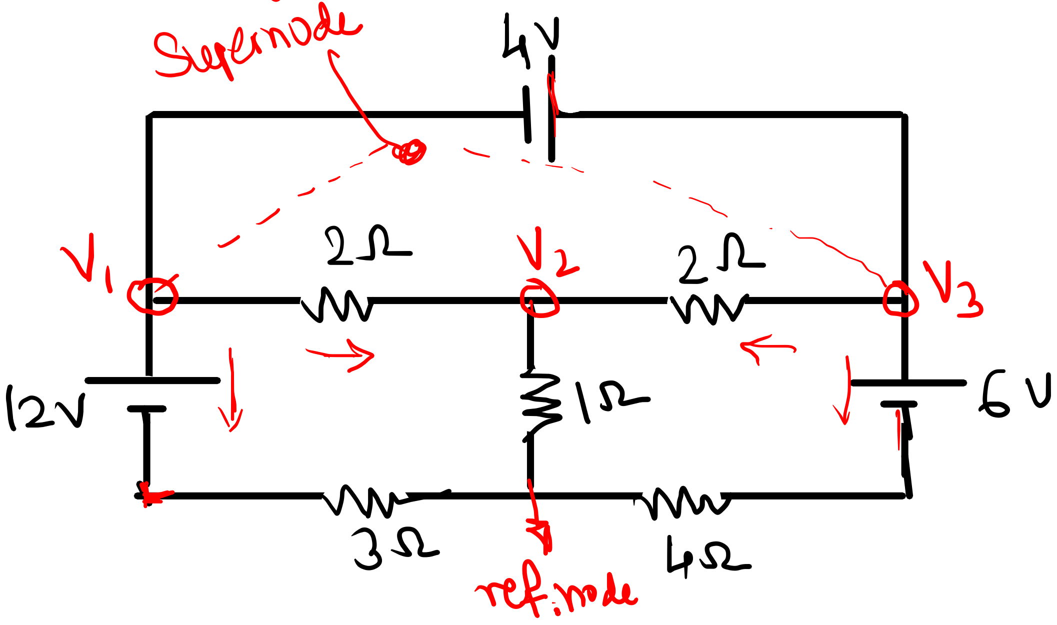
Solving (i) (ii) & (iii) $V_1 = 3.2V$, $V_2 = 2.6V$, $V_3 = 7.2V$

$$I_{1\Omega} = \frac{V_2}{1} = \frac{2.6}{1} = 2.6A$$

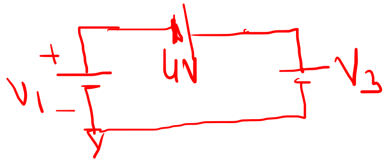
Example : find current through 1Ω resistor



⇒ Identify nodes & mark node voltages.



⇒ 4V source without series resistance betn two non-ref nodes (1 & 3) so its a case of supernode.



$V_1 + 4 - V_3 = 0$

$V_1 - V_3 = -4$ --- (1)

⇒ KCL to supernode

$$\frac{V_1 - V_2}{2} + \frac{V_1 - 12}{3} + \frac{V_3 - V_2}{2} + \frac{V_3 - 6}{4} = 0$$

$$\frac{6V_1 - 6V_2 + 4V_1 - 48 + 6V_3 - 6V_2 + 3V_3 - 18}{12} = 0$$

$10V_1 - 12V_2 + 9V_3 = 66$ --- (2)

⇒ KCL to node (2)

$$\frac{V_2 - V_1}{2} + \frac{V_2}{1} + \frac{V_2 - V_3}{2} = 0$$

$$\frac{V_2 - V_1 + 2V_2 + V_2 - V_3}{2} = 0$$

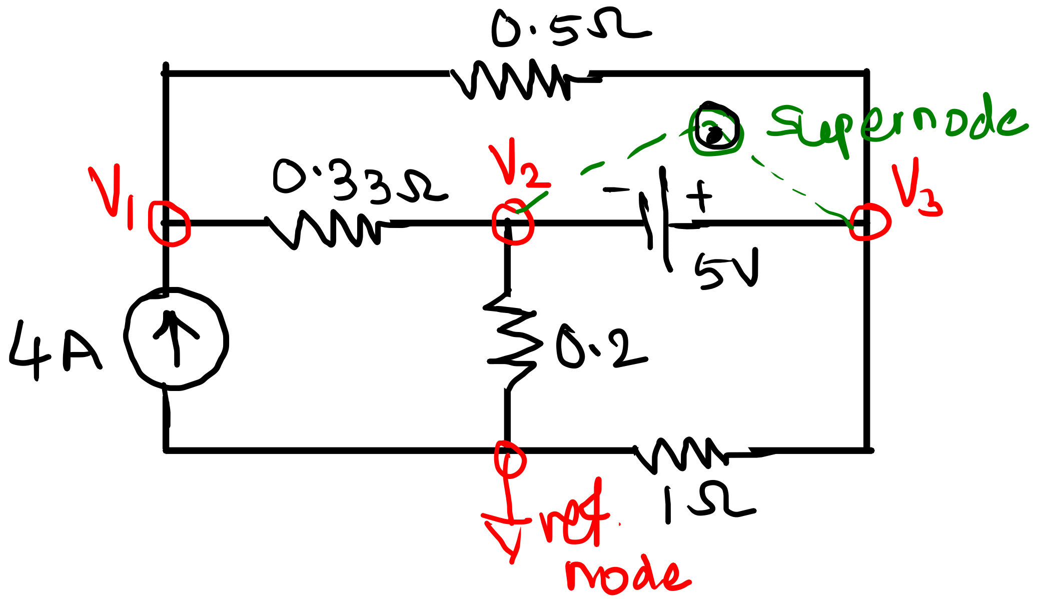
$-V_1 + 4V_2 - V_3 = 0$ ---- (3)

Solving (1), (2) & (3)

$V_1 = 3.2V,\ V_2 = 2.6V,\ V_3 = 7.2V$

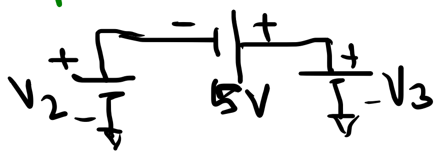
So $I_{\Omega} = \frac{V_2}{1} = 2.6A$

Ex. Find V_1, V_2 & V_3 in the following circuit using Nodal Analysis.



⇒ 5V source without series resistance between two non-reference nodes ② & ③ so supernode

→ Represent 5V in terms of V_2 & V_3



KVL $V_2 + 5 - V_3 = 0$

$V_2 - V_3 = -5$ --- ①

⇒ KCL at Supernode

$$\frac{V_2 - V_1}{0.33} + \frac{V_2}{0.2} + \frac{V_3}{1} + \frac{V_3 - V_1}{0.5} = 0$$

$3.03V_2 - 3.03V_1 + 5V_2 + V_3 + 2V_3 - 2V_1 = 0$

$-5.03V_1 + 8.03V_2 + 3V_3 = 0$ --- ②

⇒ KCL at node ①

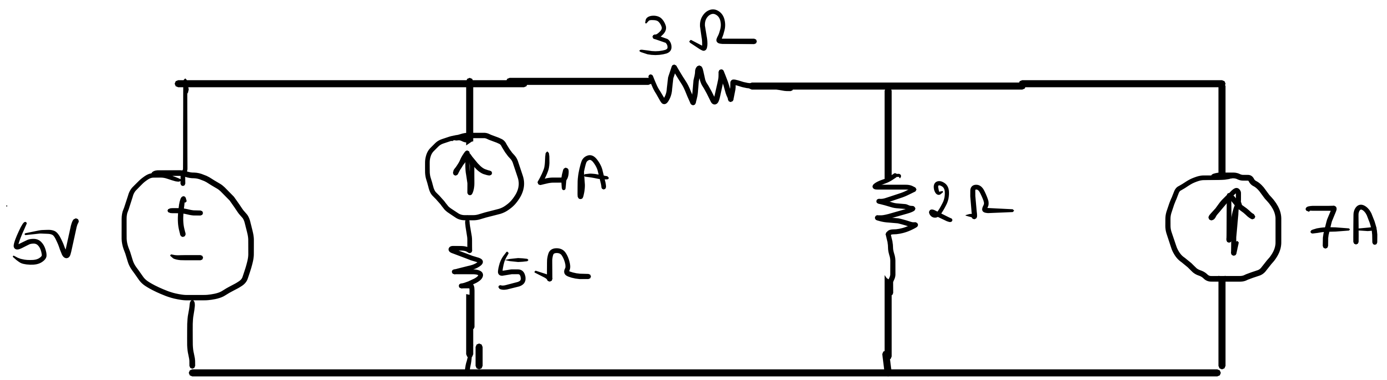
$$\frac{V_1 - V_2}{0.33} + \frac{V_1 - V_3}{0.5} = 4$$

$3.03V_1 - 3.03V_2 + 2V_1 - 2V_3 = 4$

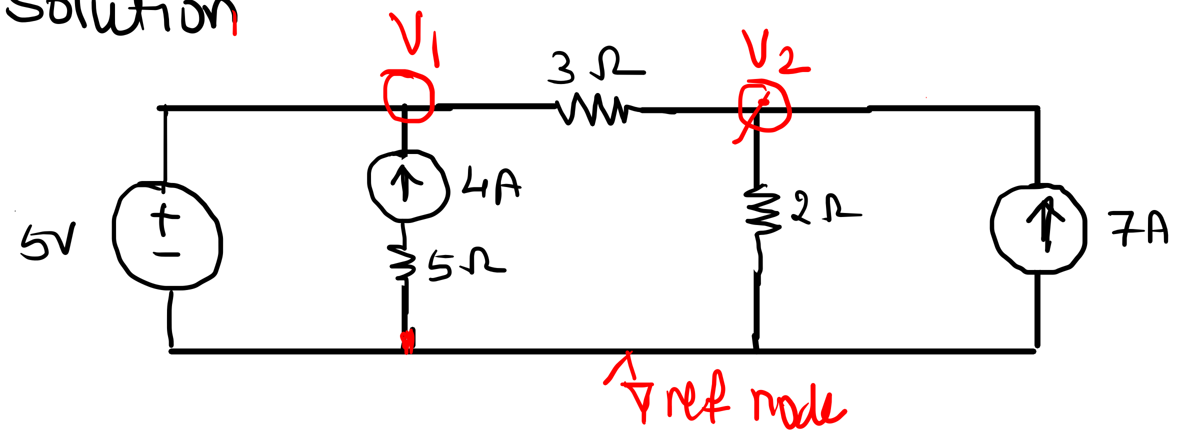
$5.03V_1 - 3.03V_2 - 2V_3 = 4$ --- ③

Solving ①, ② & ③ $V_1 = 2.63V$, $V_2 = -0.16V$, $V_3 = 4.83V$

Example (4) Find voltage across 2Ω resistor using nodal Analysis



⇒ Solution



⇒ 5V voltage source without series resistance appearing betn non-ref node (1) & ref. node.

So $V_1 = 5V$.

⇒ KCL at node (2)

$$\frac{V_2 - V_1}{3} + \frac{V_2}{2} = 7$$

$$\frac{2V_2 - 2V_1 + 3V_2}{6} = 7$$

$$-2V_1 + 5V_2 = 42$$

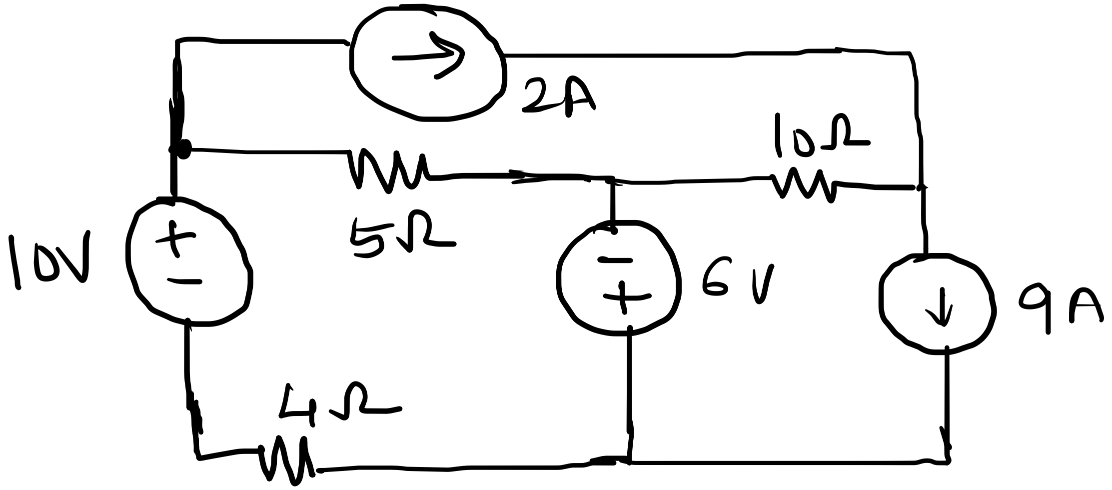
$$5V_2 = 42 + 2 \times 5$$

$$5V_2 = 52$$

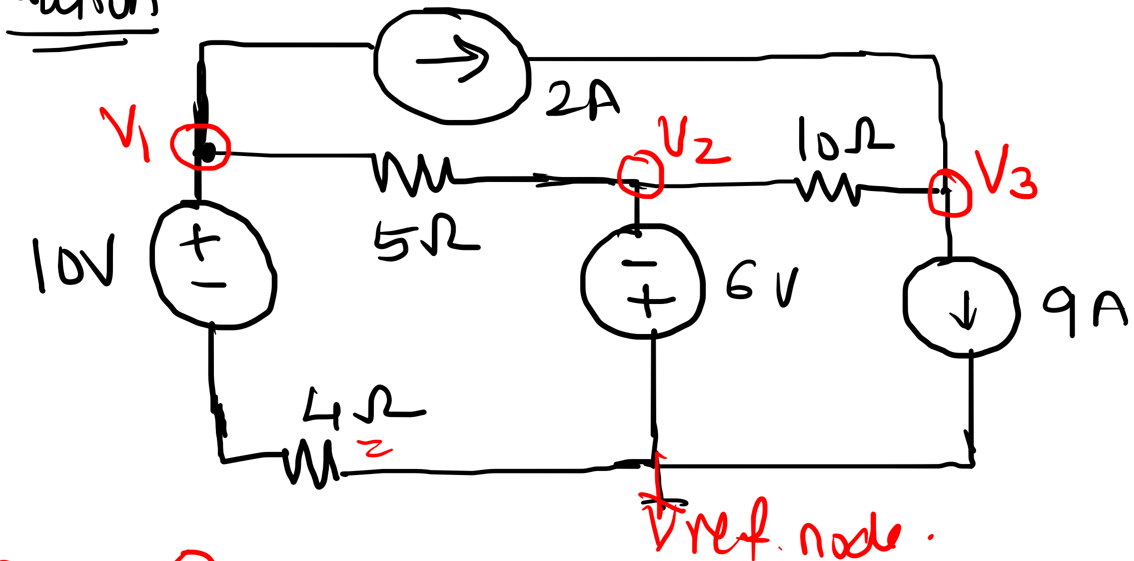
$$V_2 = \frac{52}{5} = 10.4V$$

$V_{2Ω} = V_2 = 10.4V$

Example (5) ⇒ Find power dissipated in 5Ω resistor using nodal Analysis



⇒ Solution:



from eqn (1)

$9V_1 - 4V_2 = 10$

$9V_1 = 10 + 4V_2 = 10 + 4 \times -6 =$

⇒ 6V source without series resistance between node (2) & ref node so

$$\boxed{V_2 = -6V}$$

⇒ KCL at node (1)

$$\frac{V_1 - V_2}{5} + \frac{V_1 - 10}{4} + 2 = 0$$

$$\frac{4V_1 - 4V_2 + 5V_1 - 50 + 40}{20} = 0$$

$$9V_1 - 4V_2 = 10 \quad \text{------ (1)}$$

⇒ KCL at node (3)

$$\frac{V_3 - V_2}{10} + 9 - 2 = 0$$

$$\frac{V_3 + 6}{10} = -7$$

$V_3 + 6 = -70$

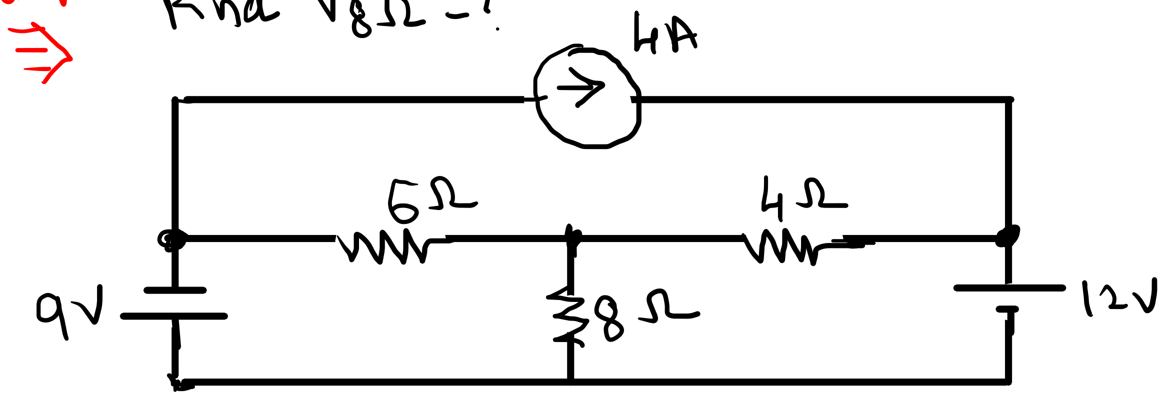
$V_3 = -76V$

$9V_1 = -14$

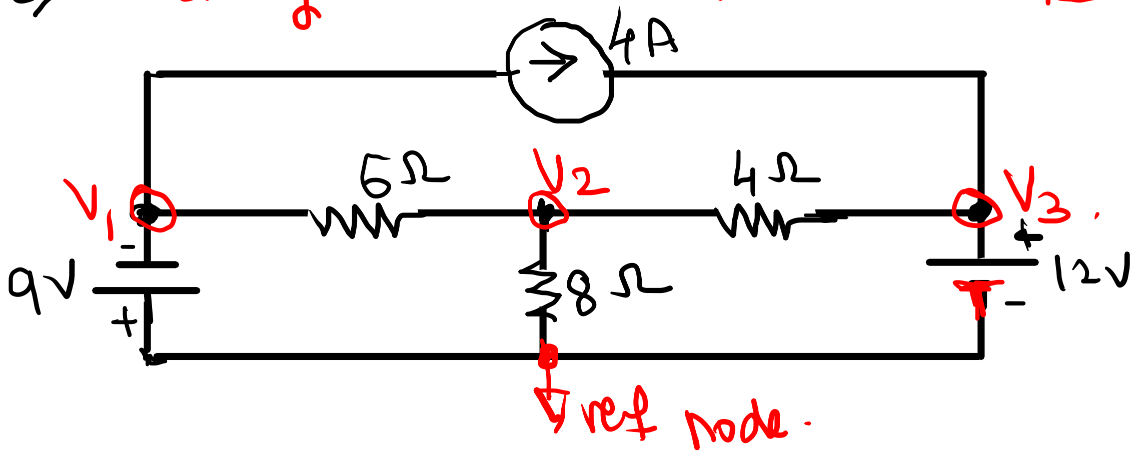
$V_1 = \frac{-14}{9}$

$I_{5Ω} = \frac{V_1 - V_2}{5} = \frac{-1.19 + 6}{5}$

Example ⇒ Find $V_{8\Omega} = ?$



⇒ Identify nodes & mark node voltages



⇒ 9V without series resistance betn node ① & ref node.

So $V_1 = -9V$.

⇒ 12V betn node ③ & ref node

So $V_3 = 12V$.

⇒ KCL to node ②

$$\frac{V_2 - V_1}{6} + \frac{V_2}{8} + \frac{V_2 - V_3}{4} = 0$$

$$\frac{4V_2 - 4V_1 + 3V_2 + 6V_2 - 6V_3}{24} = 0$$

$$13V_2 - 4V_1 - 6V_3 = 0$$

$$13V_2 = 4 \times -9 + 6 \times 12$$

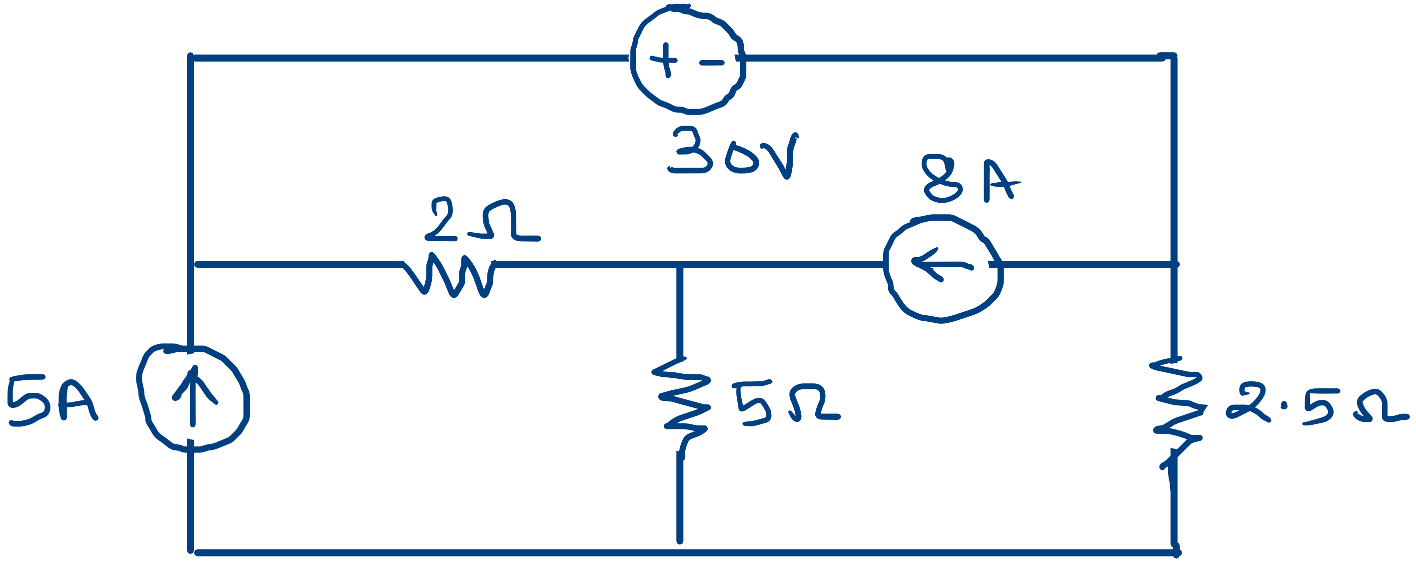
$$13V_2 = -36 + 72$$

$$13V_2 = 36 \qquad V_2 = \frac{36}{13}$$

$$V_{8\Omega} = V_2 = \frac{36}{13} V = 2.76V$$

Practice Numerical

① Find Current in 2Ω resistor Using Nodal Analysis.



Practice Numerical

(11) Find Current in 2Ω resistor Using Nodal Analysis.