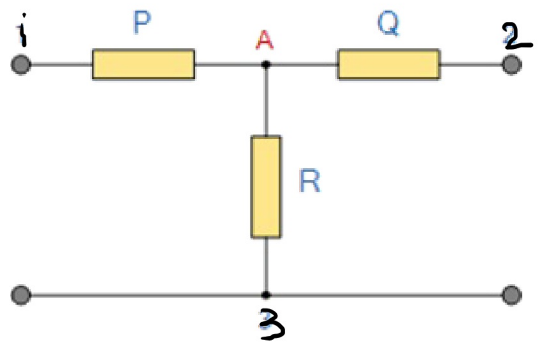
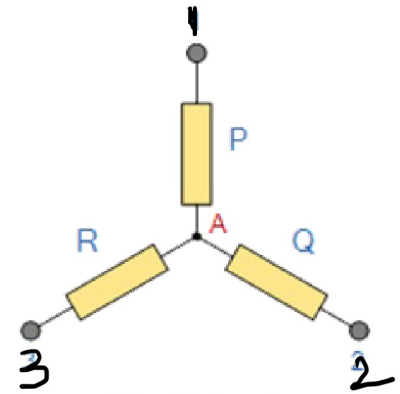
Star - Delta Network

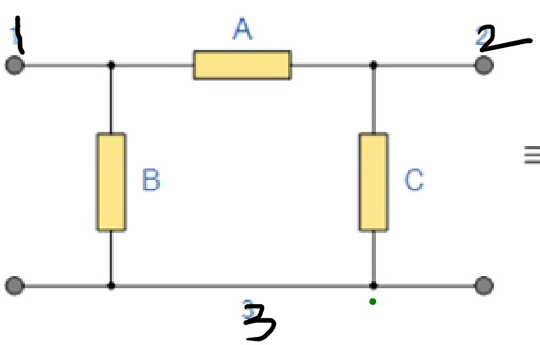


T-Network



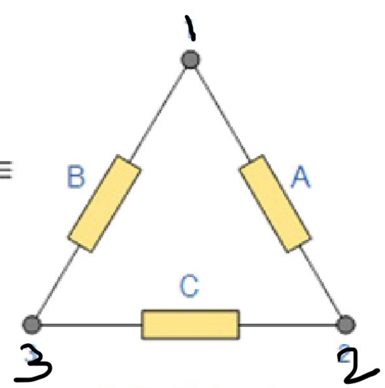
Star-Network

A resistive network consisting of three impedances can be connected together to form a T or “Tee” Star or Y type network.



Pi-Network

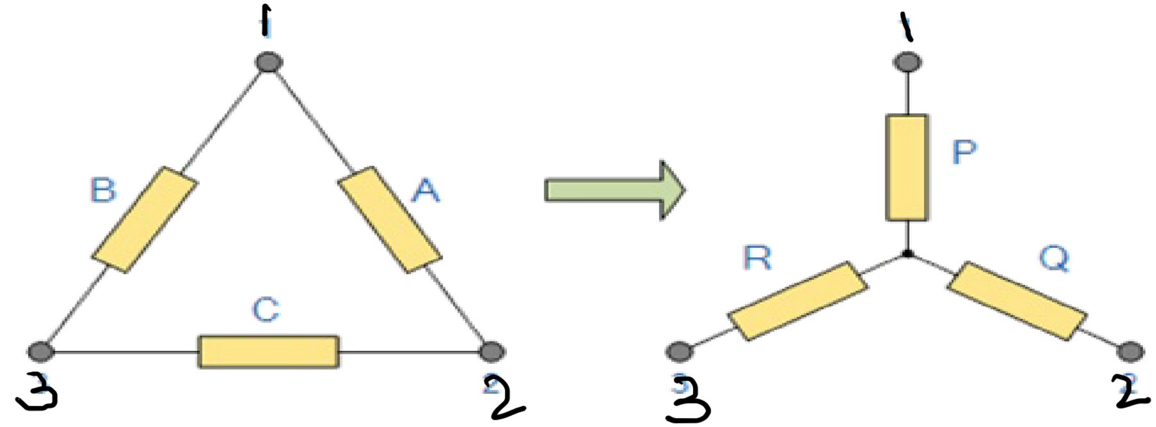
≡



Delta-Network

Pi or π type resistor network or Delta or Δ type network

Delta- Star Transformation



Resistances between terminals 1 and 2.

$$A \| (B+C) = P+Q$$

$$\frac{A(B+C)}{A+B+C} = P+Q \quad \text{-----} (1)$$

Resistances between terminals 2 and 3.

$$C \| (A+B) = Q+R \quad \text{--}$$

$$\frac{C(A+B)}{A+B+C} = Q+R \quad \text{---} (11)$$

Resistances between terminals 3 and 1.

$$B \| (A+C) = P+R$$

$$\frac{B(A+C)}{A+B+C} = P+R \quad \text{---} (111)$$

Delta- Star Transformation

$A \| (B+C) = P+Q$

$$\frac{A(B+C)}{A+B+C} = P+Q \text{ ------ (i)}$$

$C \| (A+B) = Q+R$ --

$$\frac{C(A+B)}{A+B+C} = Q+R \text{ ---- (ii)}$$

$B \| (A+C) = P+R$

$$\frac{B(A+C)}{A+B+C} = P+R \text{ ---- (iii)}$$

Adding (i) & (ii) & substract (iii)

eq^n (i) + eqn (ii) − eq^n (iii)

$$\frac{\cancel{AB}+AC+AC+\cancel{BC}-\cancel{AB}-\cancel{BC}}{A+B+C} = \cancel{P}+Q+Q+\cancel{R}-\cancel{P}-\cancel{R}$$

$$\frac{\cancel{2}AC}{A+B+C} = \cancel{2}Q$$

$$Q = \frac{AC}{A+B+C}$$

eq^n (i) + eq^n (iii) - eq^n (ii)

$$\frac{AB+\cancel{AC}+AB+\cancel{BC}-\cancel{AC}-\cancel{BC}}{A+B+C} = P+\cancel{Q}+P+\cancel{R}-\cancel{Q}-\cancel{R}$$

$$\frac{2AB}{A+B+C} = 2P \qquad \therefore P = \frac{AB}{A+B+C}$$

Similarly $R = \dfrac{BC}{A+B+C}$

Star-delta Transformation

$$P = \frac{AB}{A+B+C}$$

$$Q = \frac{AC}{A+B+C}$$

$$R = \frac{BC}{A+B+C}$$

$$PQ + QR + PR$$

$$PQ = \frac{A^2BC}{(A+B+C)^2} \qquad QR = \frac{ABC^2}{(A+B+C)^2} \qquad PR = \frac{AB^2C}{(A+B+C)^2}$$

$$PQ+QR+PR = \frac{A^2BC + ABC^2 + AB^2C}{(A+B+C)^2} = \frac{ABC\cancel{(A+B+C)}}{(A+B+C)^{\cancel{2}}}$$

$$PQ+QR+PR = \frac{ABC}{(A+B+C)}$$

$$\frac{PQ+QR+PR}{P} = \frac{\cancel{AB}C}{\cancel{(A+B+C)}} \times \frac{\cancel{(A+B+C)}}{\cancel{AB}} = C$$

$$C = \frac{PQ+QR+PR}{P}.$$

Star-delta Transformation

$$P = \frac{AB}{A+B+C}$$

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$$PQ+QR+PR = \frac{ABC}{(A+B+C)}$$

$$\frac{PQ+QR+PR}{Q} = \frac{\cancel{A}B\cancel{C}}{\cancel{(A+B+C)}} \times \frac{\cancel{(A+B+C)}}{\cancel{A}\cancel{C}}$$

$$B = \frac{PQ+QR+PR}{Q}$$

Star-delta Transformation

$$P = \frac{AB}{A+B+C}$$

$$Q = \frac{AC}{A+B+C}$$

$$R = \frac{BC}{A+B+C}$$

$$PQ + QR + PR$$

$$PQ = \frac{A^2BC}{(A+B+C)^2} \qquad QR = \frac{ABC^2}{(A+B+C)^2} \qquad PR = \frac{AB^2C}{(A+B+C)^2}$$

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$$PQ+QR+PR = \frac{ABC}{(A+B+C)}$$

$$\frac{PQ+QR+PR}{Q} = \frac{\cancel{A}B\cancel{C}}{\cancel{(A+B+C)}} \times \frac{\cancel{(A+B+C)}}{\cancel{A}\cancel{C}}$$

$$B = \frac{PQ+QR+PR}{Q}$$

Star-delta Transformation

$$P = \frac{AB}{A+B+C}$$

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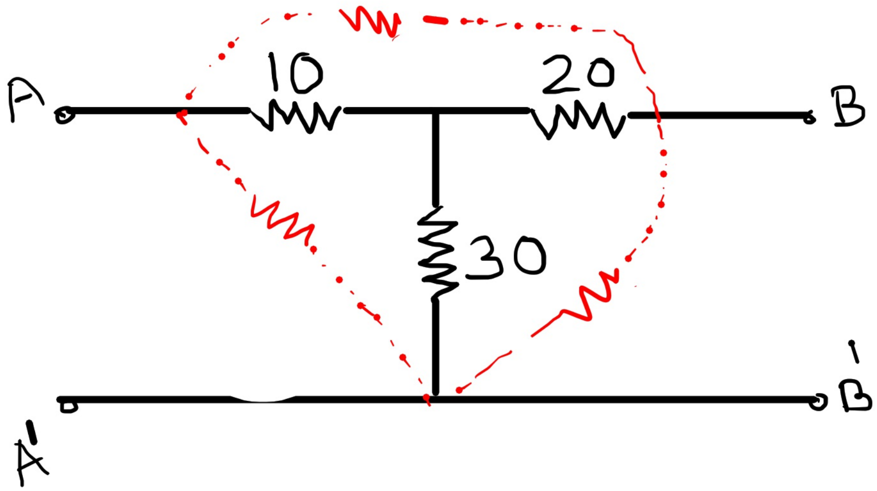
$$PQ+QR+PR = \frac{A^2BC + ABC^2 + AB^2C}{(A+B+C)^2} = \frac{ABC\cancel{(A+B+C)}}{(A+B+C)^{\cancel{2}}}$$

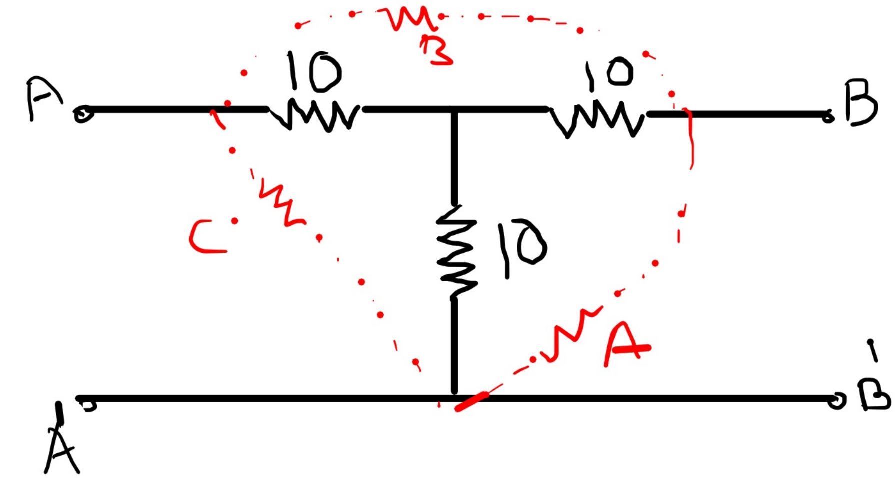
$$PQ+QR+PR = \frac{ABC}{(A+B+C)}$$

$$\frac{PQ+QR+PR}{R} = \frac{A\cancel{B}\cancel{C}}{\cancel{(A+B+C)}} \times \frac{\cancel{(A+B+C)}}{\cancel{B}\cancel{C}}$$

$$A = \frac{PQ+QR+PR}{R}$$

Example:1 Convert following star networks into equivalent delta network





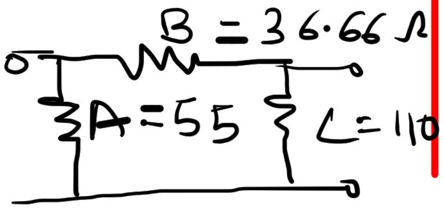
$$PQ+QR+PR = 10\times20+20\times30+30\times10$$

$$PQ+QR+PR = 200+600+300 = 1100$$

$$A = \frac{110\cancel{0}}{2\cancel{0}} = 55\,\Omega$$

$$B = \frac{110\cancel{0}}{3\cancel{0}} = \frac{110}{3} = 36.66\,\Omega$$

$$C = \frac{1100}{10} = 110\,\Omega$$

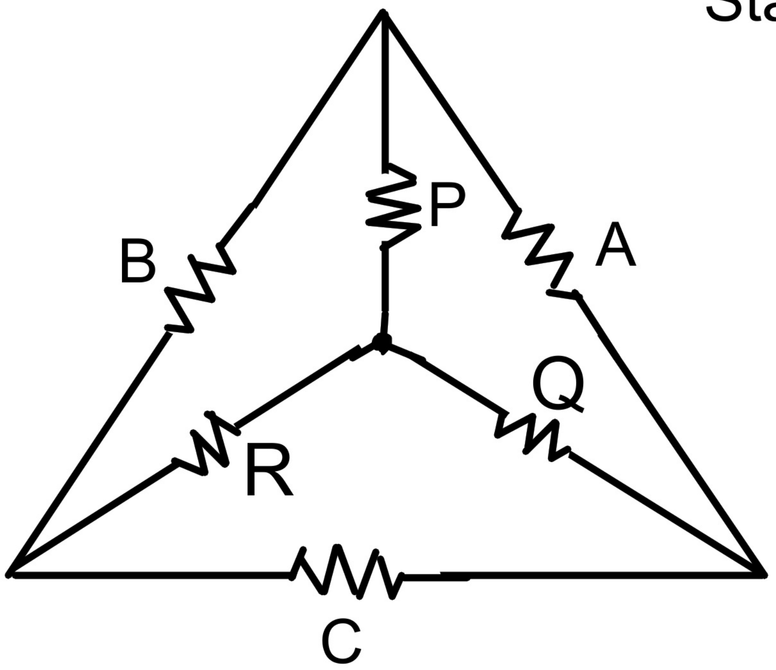


$$A = \frac{10\times10+10\times10+10\times10}{10}$$

$$A = \frac{300}{10} = 30\,\Omega$$

$$A = B = C = 30\,\Omega$$

Star-Delta Transformation



Delta - Star

$$P = \frac{AB}{A+B+C}, \quad Q = \frac{AC}{A+B+C}$$

$$R = \frac{BC}{A+B+C}$$

Star - Delta

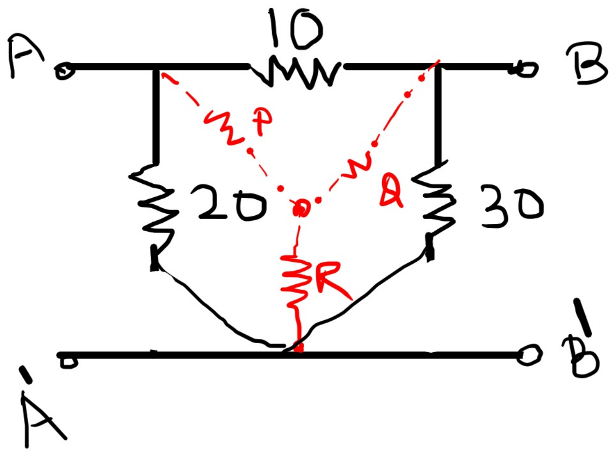
$$\Sigma R = PQ + QR + PR$$

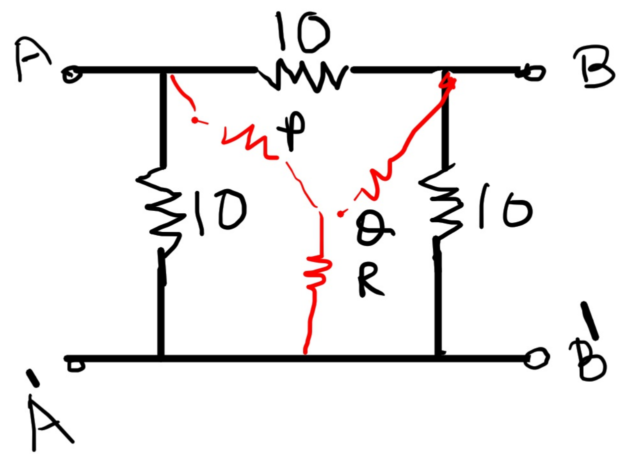
$$A = \frac{PQ+QR+PR}{R}$$

$$B = \frac{PQ+QR+PR}{Q}$$

$$C = \frac{PQ+QR+PR}{P}$$

Example:2 Convert following delta networks into equivalent star network





$$P = \frac{10 \times 20}{10+20+30} = \frac{20\cancel{0}}{6\cancel{0}} = \frac{20}{6}\,\Omega$$

$$Q = \frac{10 \times 30}{10+20+30} = \frac{300}{60} = 5\,\Omega$$

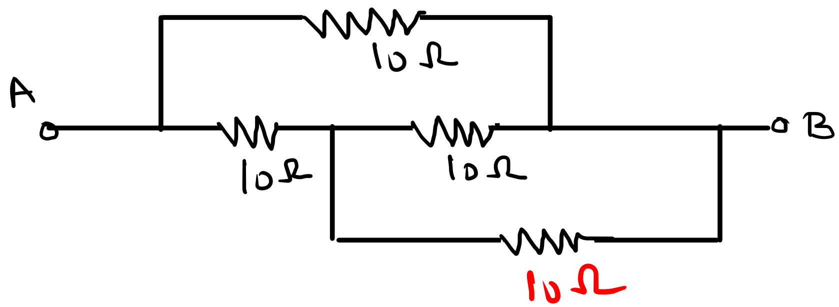
$$R = \frac{20 \times 30}{10+20+30} = \frac{600}{60} = 10\,\Omega$$

$$P = Q = R = \frac{10 \times 10}{10+10+0} = \frac{100}{30}$$

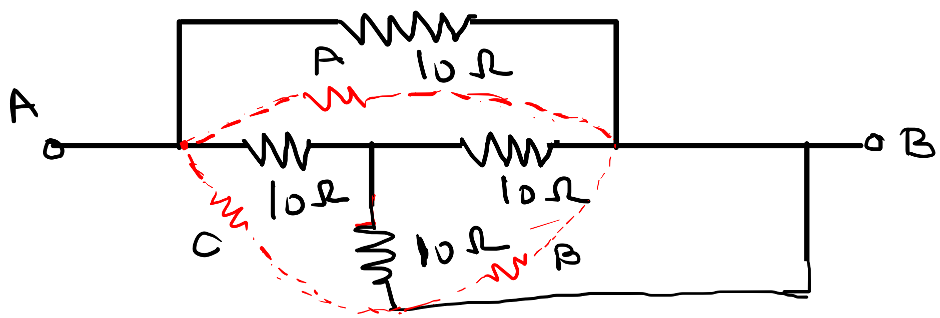
$$= 3.\underline{\underline{3}}3\,\Omega$$

Star - Delta Transformation

Ex. 1 Find Resistance between terminals A and B.



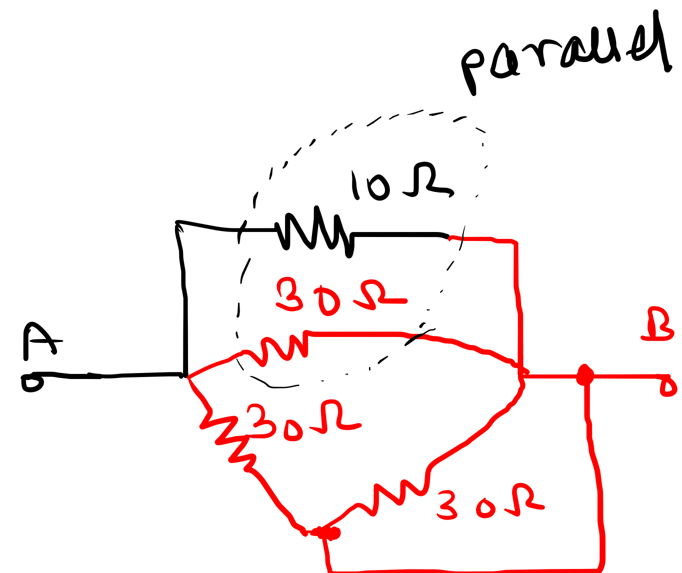
⇒



$$A = B = C = \frac{10\times10 + 10\times10 + 10\times10}{10}$$

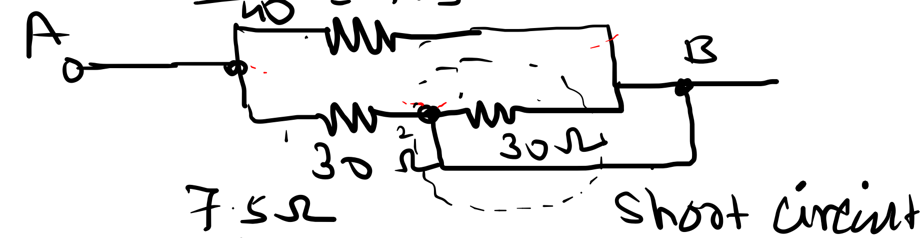
$$= \frac{300}{10} = 30\,\Omega$$

⇒



$$\frac{300}{40} = 7.5$$

⇒

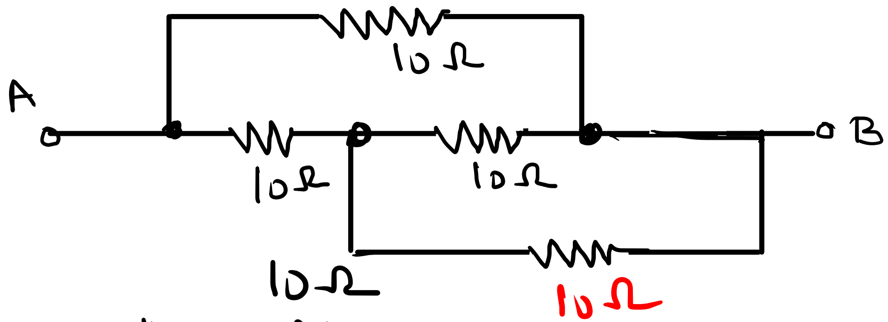


⇒

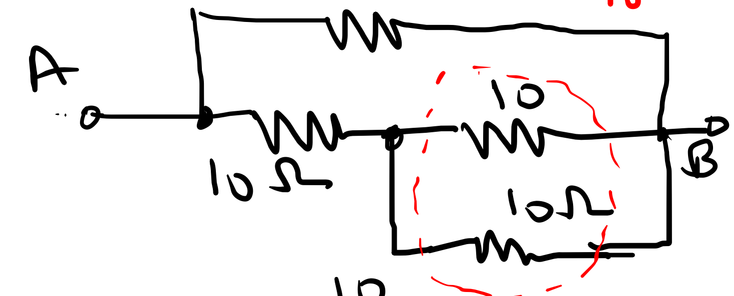
$$R_{AB} = \frac{7.5\times30}{7.5+30} = 6\,\Omega$$

Star - Delta Transformation.

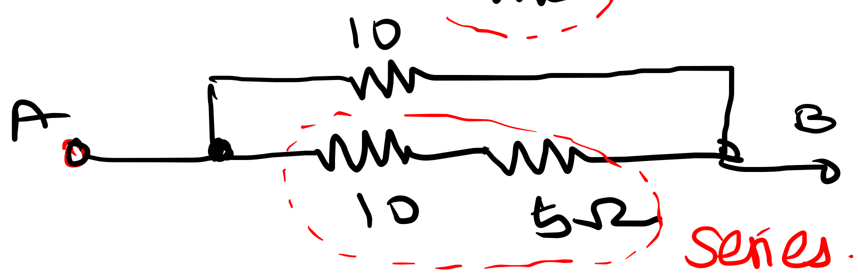
Ex. ① Find Resistance between terminals A and B.



⇒

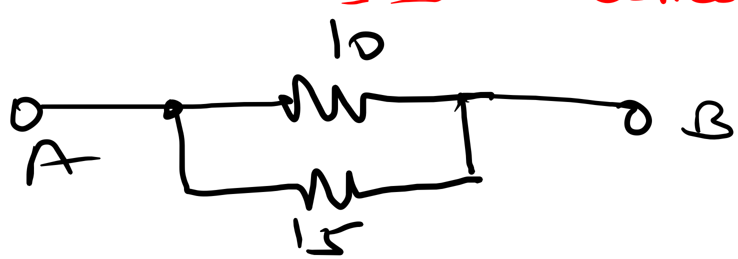


⇒



Series.

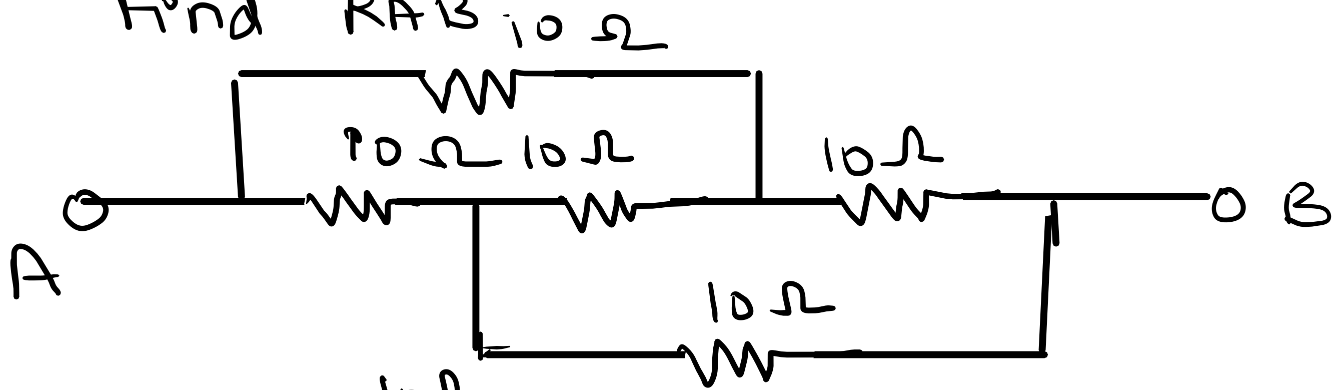
⇒

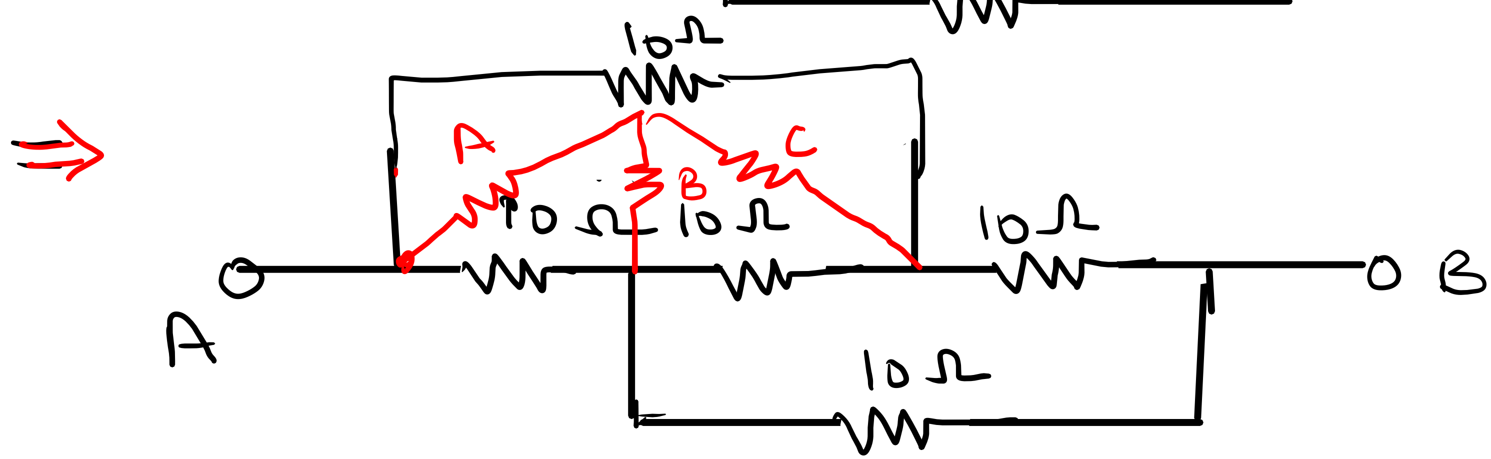


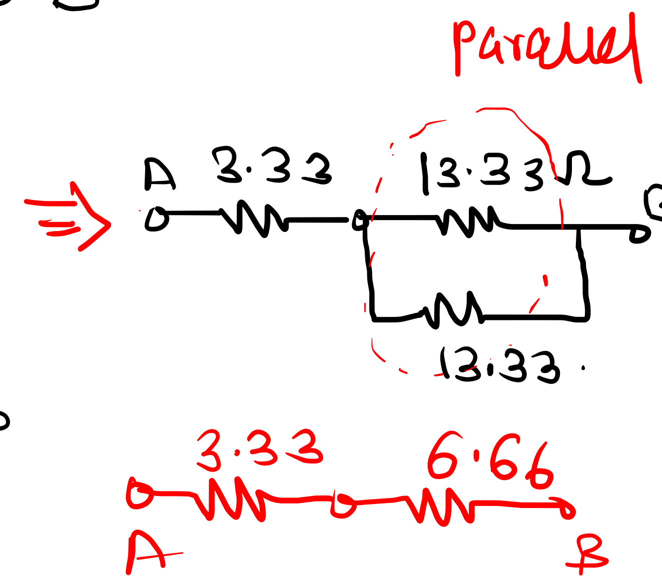
$$R_{AB} = \frac{10 \times 15}{10 + 15} = \frac{150}{25}$$

$$R_{AB} = 6\,\Omega$$

Ex. 11 Find R_{AB}



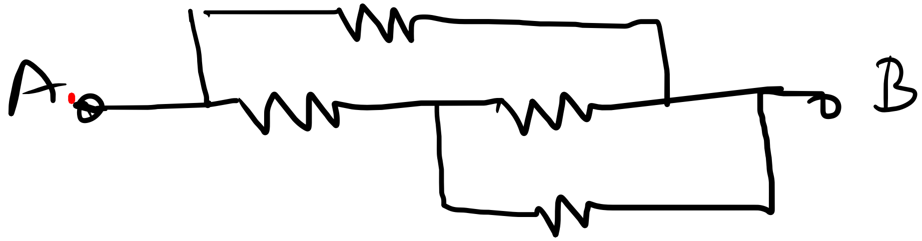




$$A = B = C = \frac{10 \times 10}{10+10+10} = \frac{100}{30} = 3.33\ \Omega$$

$$R_{AB} = 9.99 \simeq 10\ \Omega$$

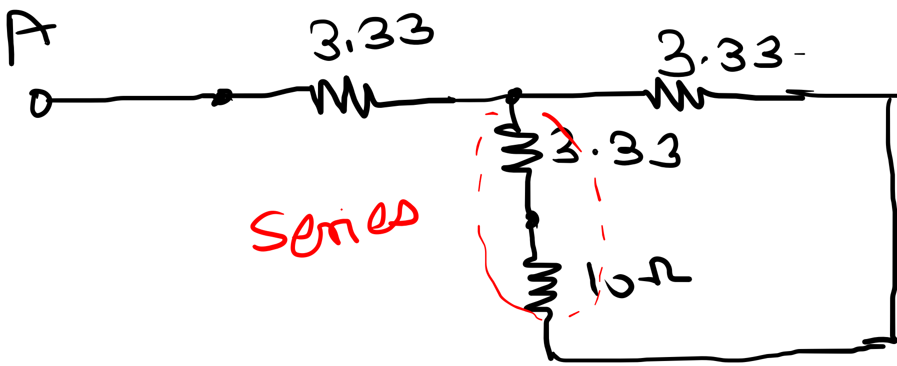
Star - Delta Transformation

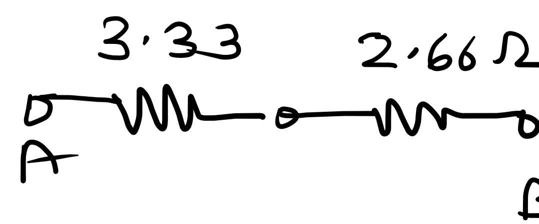


$\Rightarrow 13.33 \parallel 3.33$

$2.66\,\Omega$

$\Rightarrow$

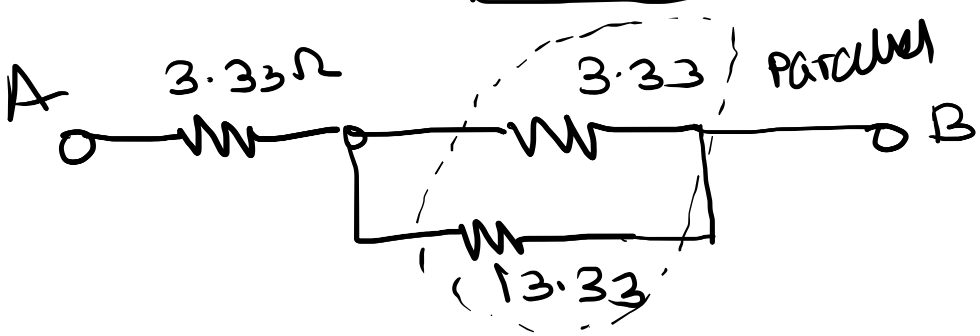




$R_{AB} = 3.33 + 2.66$

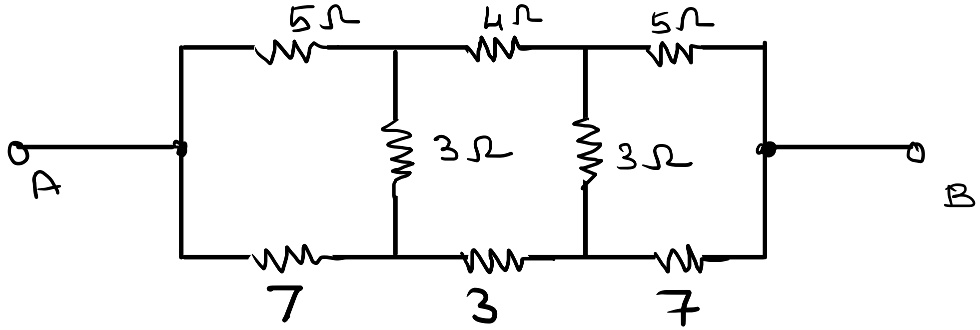
$R_{AB} = 5.99 \simeq 6\,\Omega$

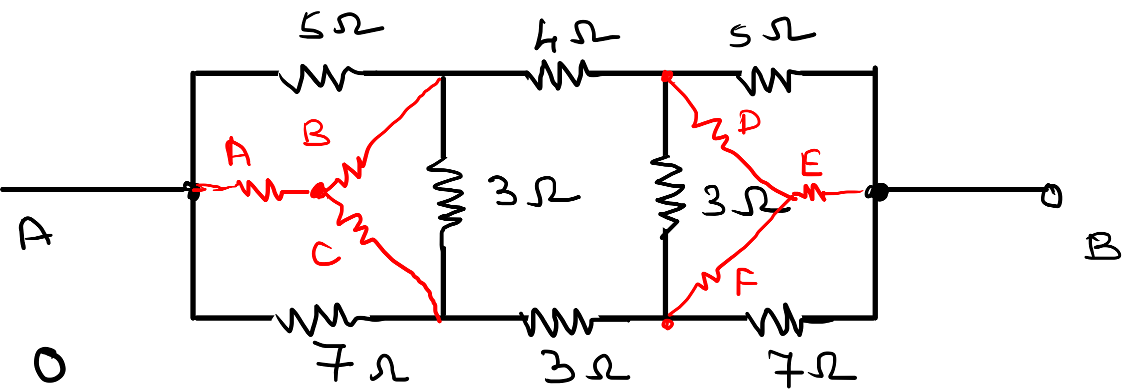
$\Rightarrow$



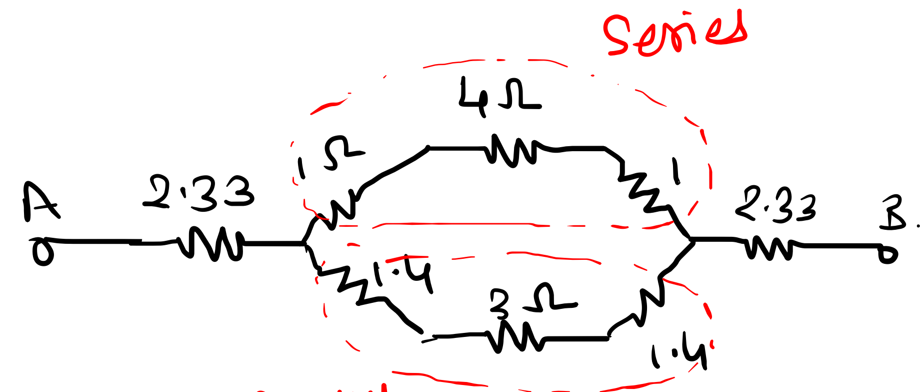
Star-Delta Transformation

Ex. (11) Find Resistance between terminals A and B.





$\Rightarrow$

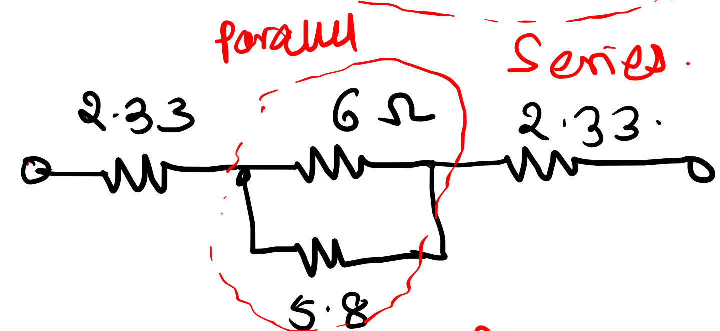


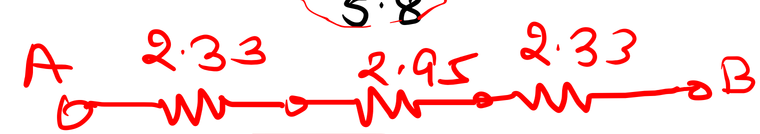
$$A = \frac{7\times5}{7+3+5} = \frac{35}{15} = 2.33\ \Omega \qquad B = \frac{5\times3}{15} = 1\Omega$$

$$D = \frac{15}{15} = 1\Omega, \quad E = \frac{35}{15} = 2.33, \quad F = \frac{21}{15} = 1.40\ \Omega$$

$$C = \frac{21}{15} = 1.4\Omega$$

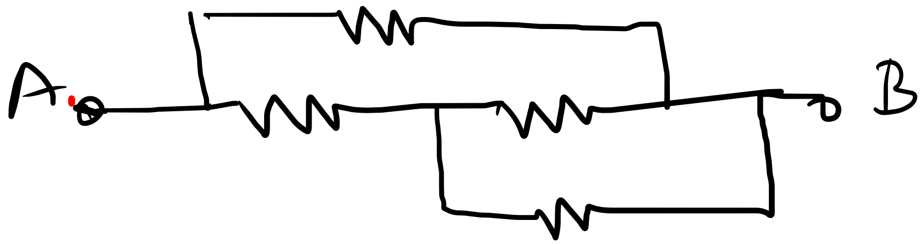
$\Rightarrow$



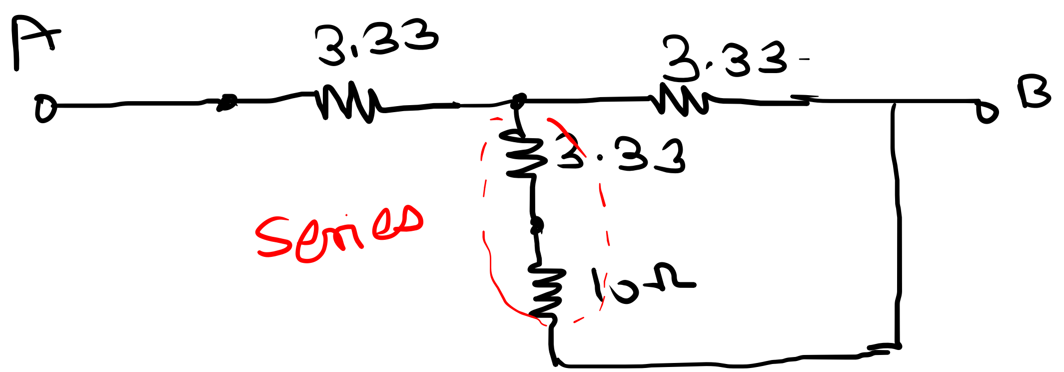


$$\boxed{R_{AB} = 7.61\Omega}$$

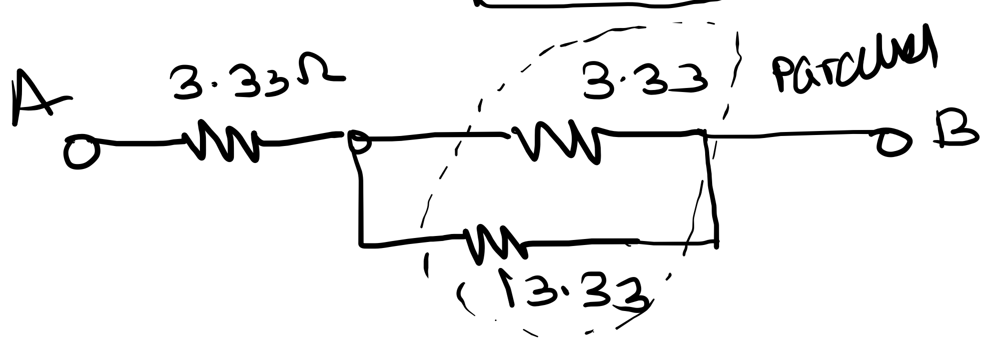
Star-Delta Transformation



$\Rightarrow$

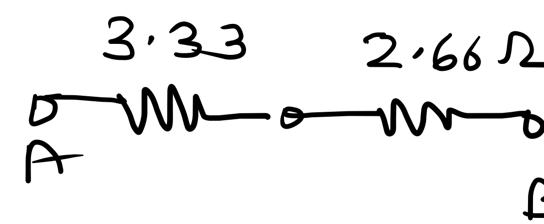


$\Rightarrow$



$\Rightarrow 13.33 \,||\, 3.33$

$2.66\,\Omega$

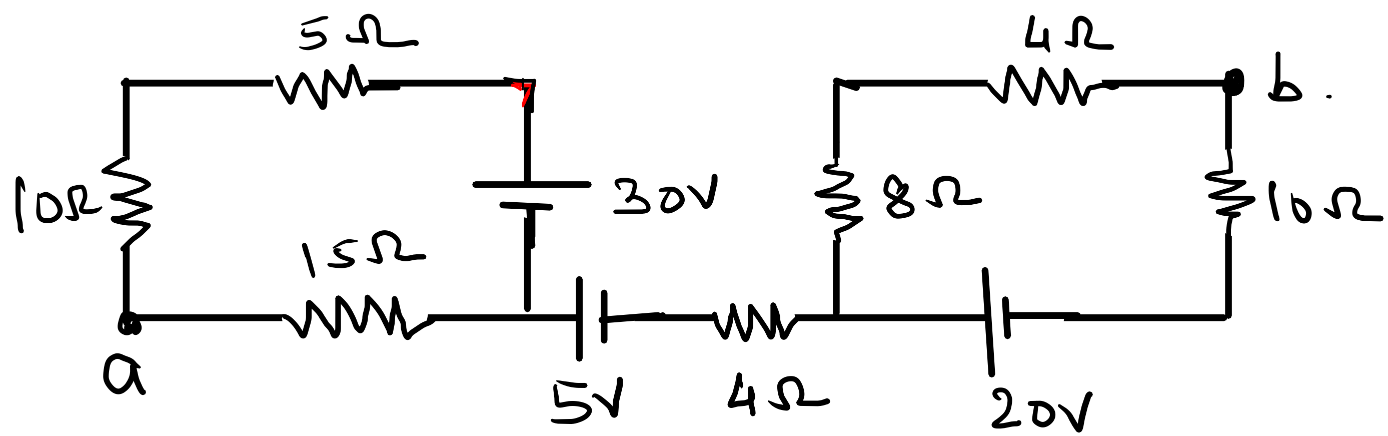


$R_{AB} = 3.33 + 2.66$

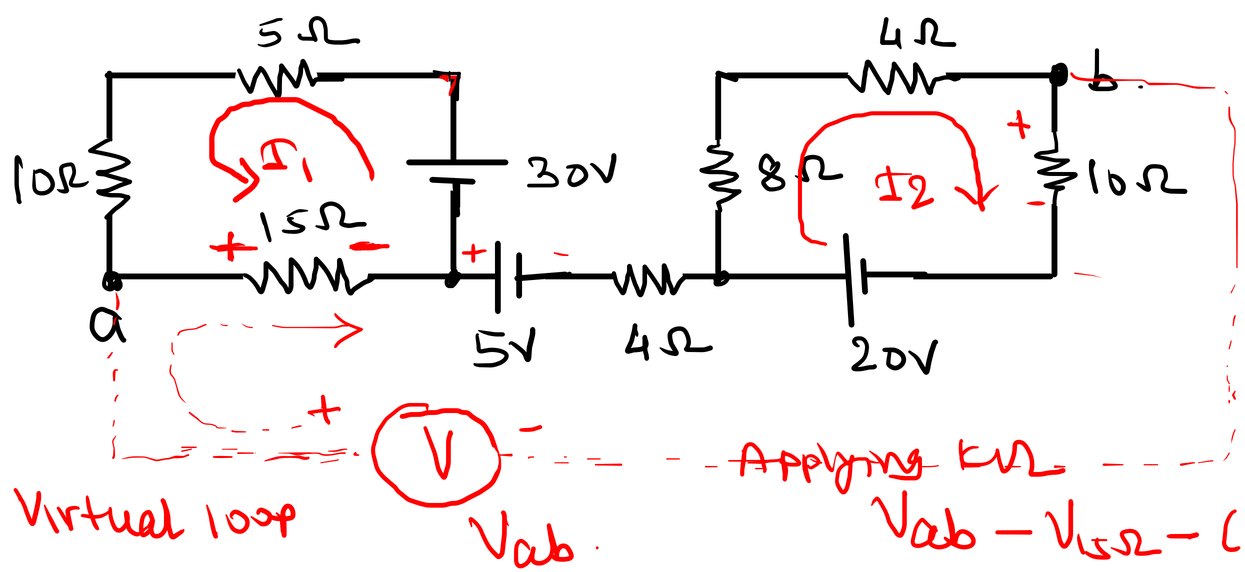
$R_{AB} = 5.99 \simeq 6\,\Omega$

⇒ Numericals Based on Kirchoff's Laws.

① Find voltage between terminals a and b.



⇒



$$I_1 = \frac{30}{5+10+15} = 1A$$

$$I_2 = \frac{20}{8+4+10} = \frac{20}{22} = 0.9A$$

Applying KVL

$$V_{ab} - V_{15\Omega} - (5V) - V_{4\Omega} - 20 - V_{10\Omega} = 0$$

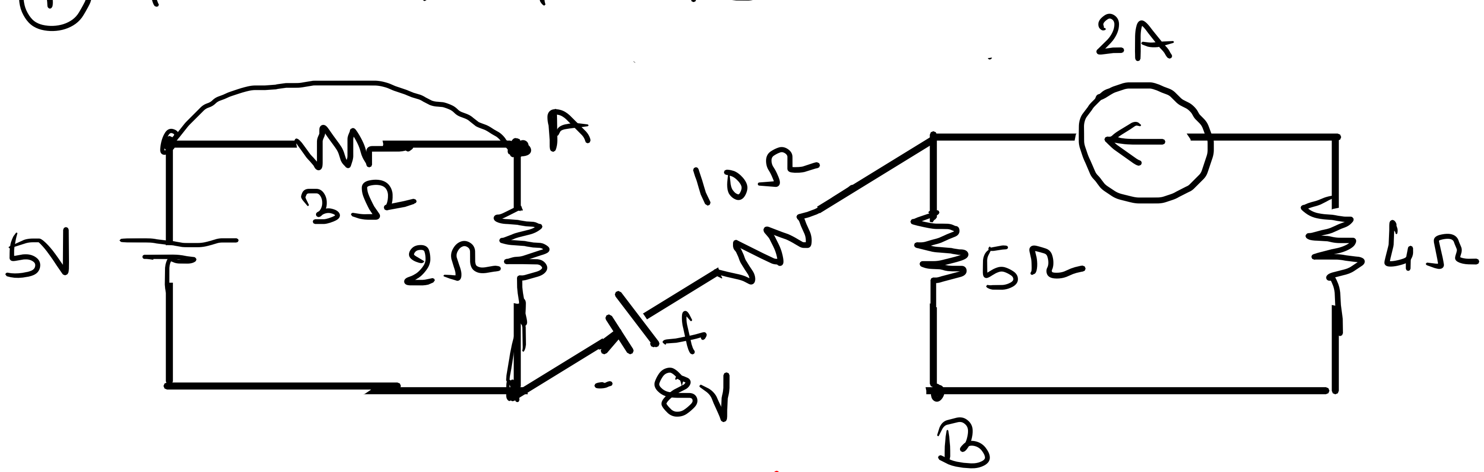
$$V_{ab} - (15\times 1) - 5 - 0 - 20 + (10 \times 0.9) = 0$$

$$V_{ab} - 15 - 5 - 20 + 9 = 0$$

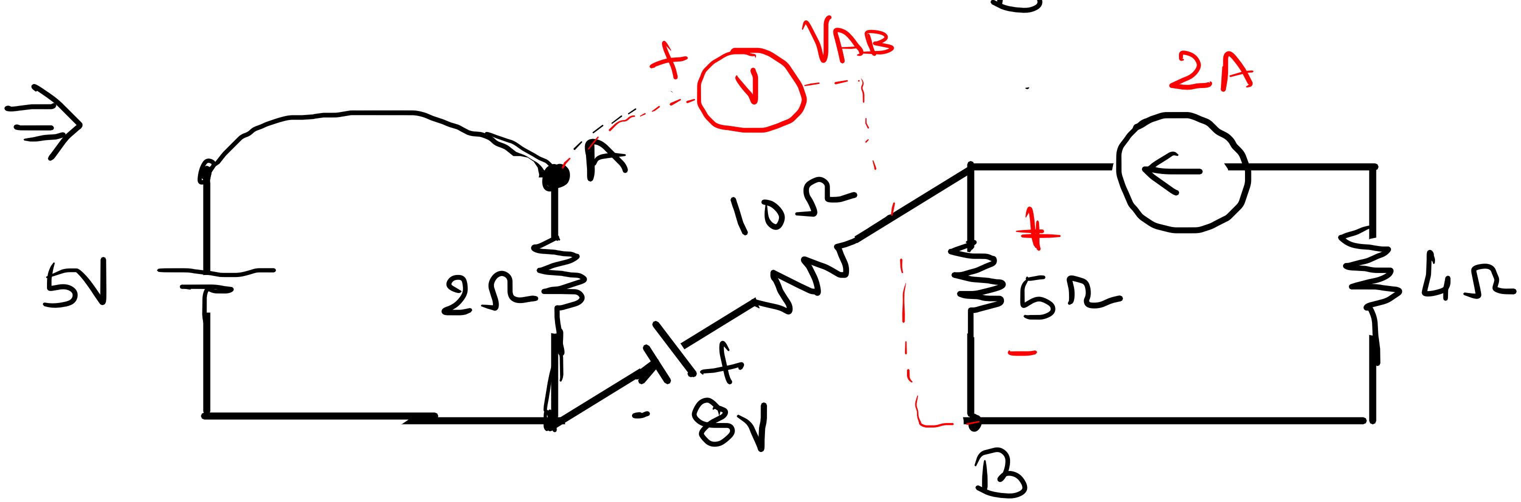
$$\boxed{V_{ab} = 31V}$$

⇒ Numericals Based on Kirchoff's Laws.

(1) Find voltage V_{AB}.



⇒



Applying KVL

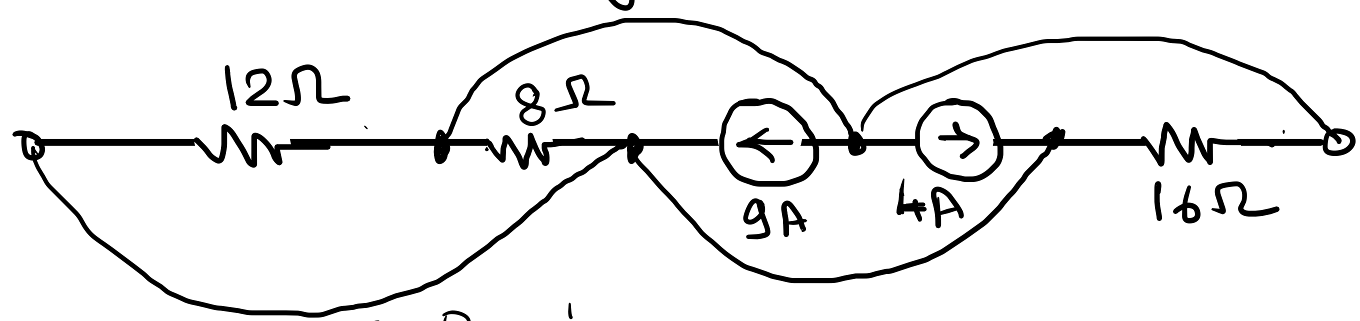
$$V_{AB} - V_{2\Omega} + 8 + V_{10\Omega} - V_{5\Omega} = 0$$

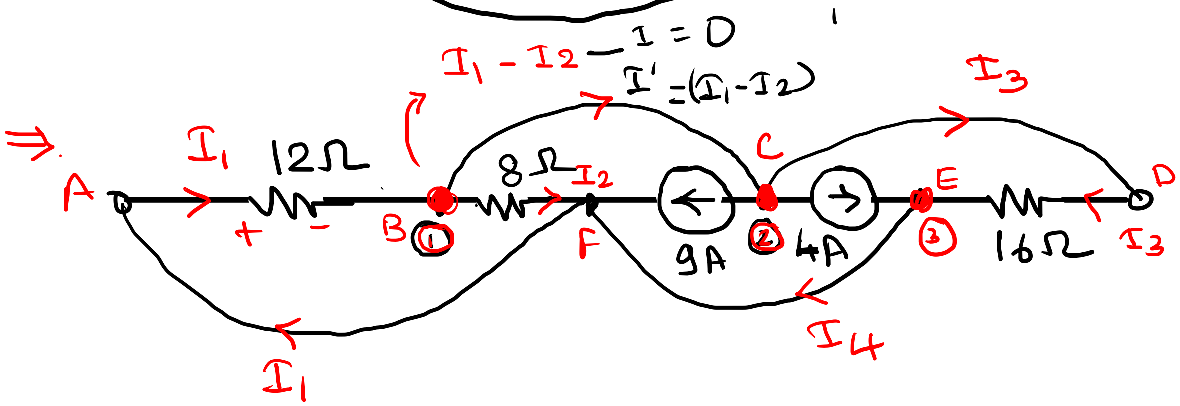
$$V_{AB} - 5 + 8 + 0 - 10 = 0$$

$$\boxed{V_{AB} = 7V}$$

⇒ Numericals Based on Kirchoff's Laws.

(11) Find current through 12Ω & 16Ω resistors.





KCL at node (2)

$(I_1 - I_2) - 9 - 4 - I_3 = 0$

$I_3 = (I_1 - I_2) - 13$ --- (1)

KCL at node (3)

$I_4 = I_3 + 4$ --- (2)

KVL to closed path (A-B-F-A)

$-12I_1 - 8I_2 = 0$

$3I_1 + 2I_2 = 0$ --- (3)

From equation (1) $I_1 - I_2 - I_3 = 13$ --- (5)

Solving (3), (4) and (5)

KVL to closed path (A-B-C-D-E-F-A)

$-12I_1 - 16I_3 = 0$

$3I_1 + 4I_3 = 0$ --- (4)

$$\boxed{I_1 = 4A\ (\rightarrow),\quad I_2 = -6A,\quad I_3 = -3A\ (\leftarrow)}$$