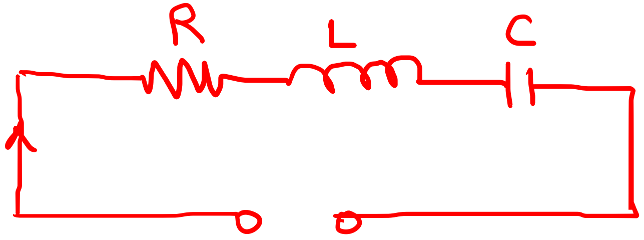
Resonance

Resonance is said to occur when the current is either maximum or minimum for a particular frquecncy of the AC input. At resonance circuit impedance becomes minimum or maximum. The power factor of the circuit beomes unity.

⇒ Series R-L-C circuit



$(v = V_m \sin\omega t)$, $\omega \updownarrow$

At some frequency (f_r)

$$X_L = X_C$$

$$Z = R + j(X_L - X_C) = R$$

Frequency at which $X_L = X_C$ is called resonant frequency.

$X_L = \omega L = 2\pi f L$ $\quad f = 0, X_L = 0$, $f = \infty$, $X_L = \infty$

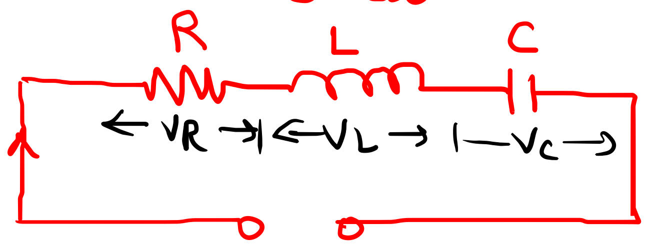
$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ $\quad f = 0$, $X_C = \infty$, $f = \infty$, $X_C = 0$

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Resonance

Resonance is said to occur when the current is either maximum or minimum for a particular frquecncy of the AC input. At resonance circuit impedance becomes minimum or maximum. The power factor of the circuit beomes unity.

$\Rightarrow$ Series R-L-C circuit



$(v = V_m \sin \omega t)$, $\omega \updownarrow$

At $f = f_r$ $\quad X_L = X_C$

$$2\pi f_r L = \frac{1}{2\pi f_r \cdot C}$$

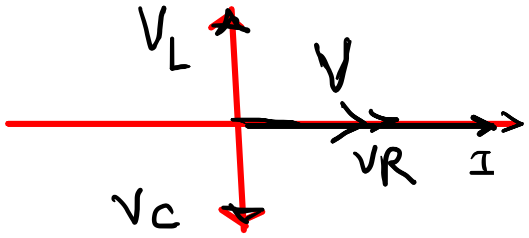
$$f_r^2 = \frac{1}{4\pi^2 LC}$$

$$\boxed{f_r = \frac{1}{2\pi \cdot \sqrt{LC}}} \quad Hz$$

$$2\pi f_r = \cancel{2\pi} \cdot \frac{1}{\cancel{2\pi}\sqrt{LC}}$$

$$\boxed{\omega_r = \frac{1}{\sqrt{LC}}} \quad rad/sec.$$

phasor diagram at $f = f_r$.



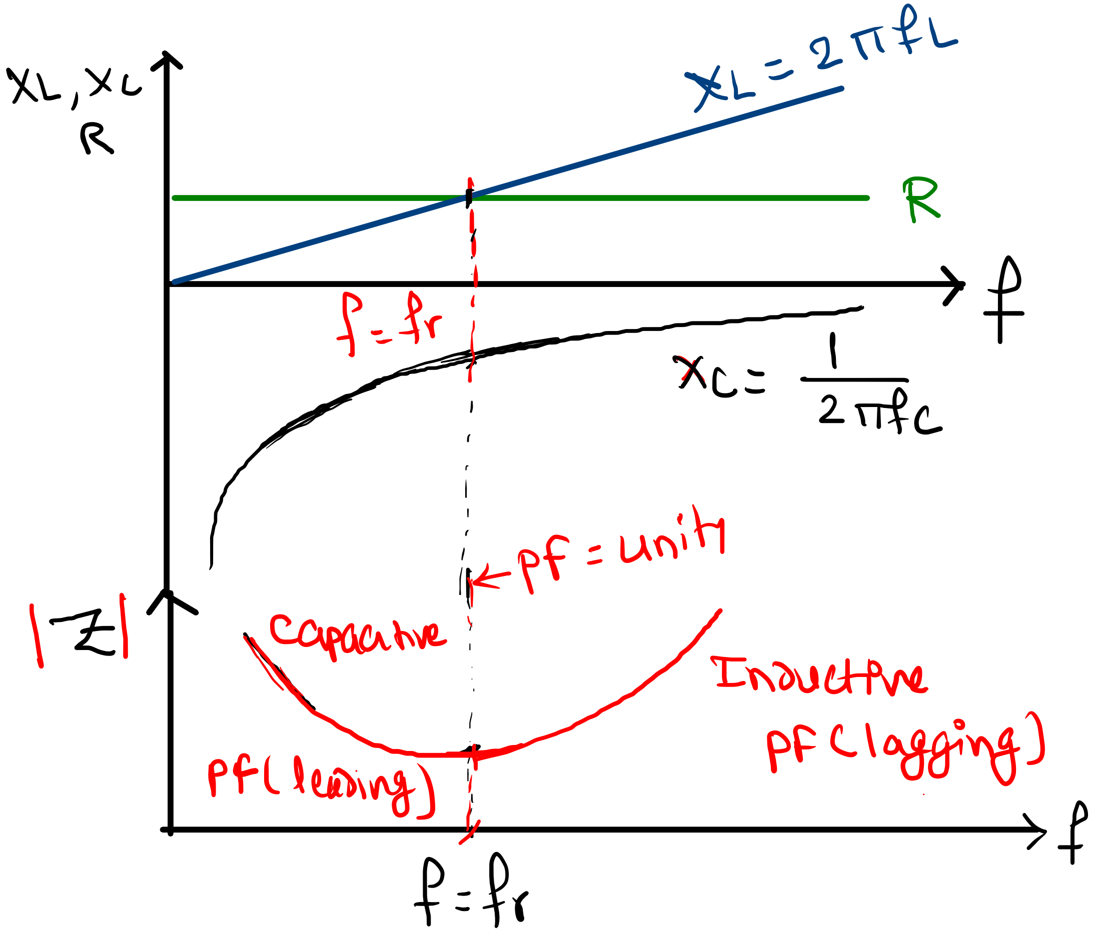
$V_L = V_C$

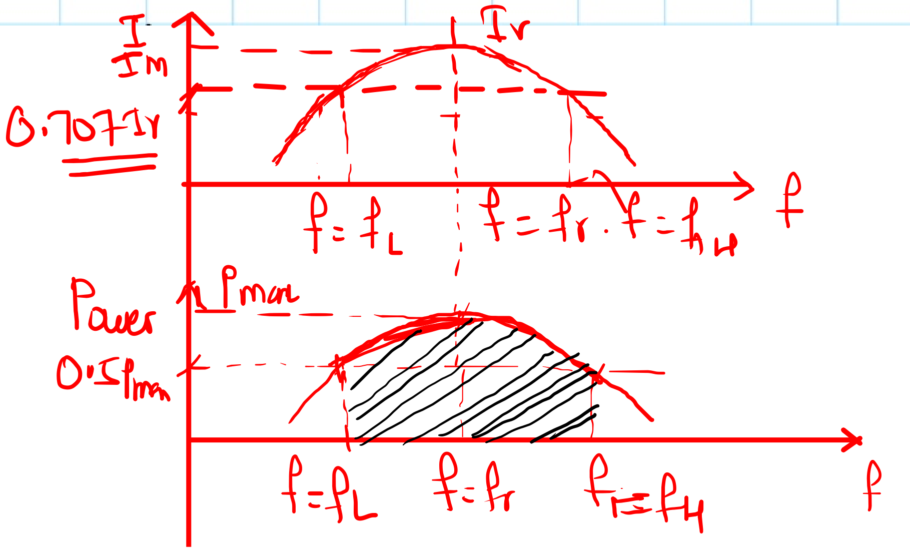
$pf = \cos\phi = 1$

$\phi = 0$

Resonance

Resonance is said to occur when the current is either maximum or minimum for a particular frquecncy of the AC input. At resonance circuit impedance becomes minimum or maximum. The power factor of the circuit beomes unity.





$BW = f_H - f_L \Rightarrow$ Bandwidth.

$f_L \rightarrow$ Lower cut-off freqn

$f_H \rightarrow$ Higher cut off freqn.

f_L, f_H are half power frequencies

$P = I_r^2 R$ at $f = f_r$ $I = I_r$

$P_r = I_r^2 R$

$$\frac{P_r}{2} = \frac{I_r^2 R}{2} = \left(\frac{I_r}{\sqrt{2}}\right)^2 R = (0.707 I_r)^2 R$$

Resonance

Resonance is said to occur when the current is either maximum or minimum for a particular frquecncy of the AC input. At resonance circuit impedance becomes minimum or maximum. The power factor of the circuit beomes unity.

⇒ R-L-C Series resonance circuit is called acceptor circuit

at $f = f_r$ $\quad I = I_r \rightarrow$ maximum.

⇒ Voltage magnification circuit :

$X_L = X_C \qquad I = I_r \rightarrow \text{maximum} = \frac{V}{R}$

$I_r \cdot X_L = I_r \cdot X_C$

$|V_L| = |V_C| >$ applied voltage (V)

$$V_L = I_r \cdot X_L = \frac{V}{R}\, \underline{\underline{\omega_r \cdot L}} = \frac{V}{R} \cdot \frac{1}{\sqrt{LC}} \cdot L = \underline{\underline{\frac{V}{R}\sqrt{\frac{L}{C}}}}$$

$$V_C = I_r \cdot X_C = \frac{V}{R} \cdot \frac{1}{\omega_r \cdot C} = \frac{V}{R} \cdot \frac{\sqrt{LC}}{C} = \frac{V}{R} \cdot \sqrt{\frac{LC}{C^2}} = \underline{\underline{\frac{V}{R}\sqrt{\frac{L}{C}}}}$$

Resonance

Resonance is said to occur when the current is either maximum or minimum for a particular frquecncy of the AC input. At resonance circuit impedance becomes minimum or maximum. The power factor of the circuit beomes unity.

At $f = f_r$ oR $\omega = \omega_r$ $(V_L = V_C) > V$

$V_C = I X_C$ or $V_L = I X_L$ $\quad Z = R + j(X_L - X_C)$ $\quad |Z| = \sqrt{R^2 + (X_L - X_C)^2}$

$$V_C = I \cdot X_C = \frac{V}{Z} X_C = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} \cdot X_C$$

$$V_C = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})}} \cdot \frac{1}{\omega C}$$

squaring both sides

$$V_C^2 = \frac{V^2}{[R^2 + (\omega L - \frac{1}{\omega C})^2 .]\,\omega^2 C^2}$$

$$V_C^2 = \frac{V^2}{\omega^2 R^2 C^2 + \left(\frac{\omega^2 LC - 1}{\omega C}\right)^2 \omega^2 C^2}$$

$$V_C^2 = \frac{V^2}{\omega^2 R^2 C^2 + (\omega^2 LC - 1)^2}$$

$$\frac{dV_C^2}{d\omega} = 0$$

Resonance

Resonance is said to occur when the current is either maximum or minimum for a particular frquecncy of the AC input. At resonance circuit impedance becomes minimum or maximum. The power factor of the circuit beomes unity.

$$V_c^2 = \frac{V^2}{\omega^2 R^2 C^2 + \left(\frac{\omega^2 LC - 1}{\omega C}\right)^2 \omega^2 C^2}$$

$$V_c^2 = \frac{V^2}{\omega^2 R^2 C^2 + (\omega^2 LC - 1)^2}$$

$$\frac{dV_c^2}{d\omega} = 0$$

$$0 = \frac{0 - V^2 \left(2\omega R^2 C^2 + 2(\omega^2 LC - 1) \cdot 2\omega LC\right)}{\left[\omega^2 R^2 C^2 + (\omega^2 LC - 1)^2\right]^2}$$

$V \neq 0$,

$$2\omega R^2 C^2 + 2(\omega^2 LC - 1) \cdot 2\omega LC = 0$$

$$2\omega C\left[R^2 C + 2(\omega^2 LC - 1)L\right] = 0$$

$\omega \neq 0 \quad c \neq 0$

$$R^2 C + 2\omega^2 L^2 C - 2L = 0$$

$$2\omega^2 L^2 C = 2L - R^2 C$$

$$\omega^2 = \frac{\cancel{2}\cancel{L}}{\cancel{2}L^{\cancel{2}}C} - \frac{R^2 \cancel{C}}{2L^2 \cancel{C}}$$

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{2L^2}$$

$$\frac{R^2}{2L^2} << \frac{1}{LC}$$

$$\omega^2 = \frac{1}{LC} \quad \& \quad \omega = \frac{1}{\sqrt{LC}} = \omega_r$$

Resonance

Resonance is said to occur when the current is either maximum or minimum for a particular frquecncy of the AC input. At resonance circuit impedance becomes minimum or maximum. The power factor of the circuit beomes unity.

$$\hat{V}_c^2 = \frac{V^2}{\omega^2R^2C^2 + \left(\frac{\omega^2LC-1}{\omega C}\right)^2\omega^2C^2}$$

$$V_c^2 = \frac{V^2}{\omega^2R^2C^2+(\omega^2LC-1)^2}$$

$$\frac{dV_c^2}{d\omega} = 0$$

$$0 = \frac{0 - V^2\left(2\omega R^2C^2 + 2(\omega^2LC-1)\cdot 2\omega LC\right)}{\left[\omega^2R^2C^2+(\omega^2LC-1)^2\right]^2}$$

$V \neq 0,$

$$2\omega R^2C^2 + 2(\omega^2LC-1)\cdot 2\omega LC = 0$$

$$2\omega C\left[R^2C + 2(\omega^2LC-1)L\right] = 0$$

$\omega \neq 0 \quad c \neq 0$

$$R^2C + 2\omega^2L^2C - 2L = 0$$

$$2\omega^2L^2C = 2L - R^2C$$

$$\omega^2 = \frac{\cancel{2}\cancel{L}}{\cancel{2}L^{\cancel{2}}C} - \frac{R^2\cancel{C}}{2L^2\cancel{C}}$$

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{2L^2}$$

$$\frac{R^2}{2L^2} \ll \frac{1}{LC}$$

$$\omega^2 = \frac{1}{LC} \quad \& \quad \omega = \frac{1}{\sqrt{LC}} = \omega_r$$

$\Rightarrow$ Bandwidth of R-L-C series resonant circuit

$\Rightarrow$ $$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad \text{----- (1)}$$

at resonance $I = I_r$ $\quad P = P_r$ (maximum)

at half power $I = \frac{I_r}{\sqrt{2}} = \frac{V}{R\sqrt{2}}$ ---- (2)

Subtitute I in eqn (1)

$$\frac{\cancel{V}}{R\sqrt{2}} = \frac{\cancel{V}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\sqrt{2} \cdot R = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

⇒ Bandwidth of R-L-C series resonant circuit

$$\sqrt{2}\cdot R = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

squaring both sides

$$2R^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

square root

$$\left(\omega L - \frac{1}{\omega C}\right) = \pm R$$

$$\left(\omega L - \frac{1}{\omega C}\right) = \pm R$$

$$\frac{\omega^2 LC - 1}{\omega C} = \pm R$$

$$\omega^2 LC - 1 = \pm R\cdot\omega C$$

$$\omega^2 LC \mp \omega RC - 1 = 0$$

$$\omega^2 \mp \omega\frac{R}{L} - \frac{1}{LC} = 0$$

$$ax^2 + bx + c$$

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

finding roots of quadratic equation

$$\omega_1, \omega_2 = \frac{\pm\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} + \frac{4}{LC}}}{2}$$

$$\omega_1, \omega_2 = \pm\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

$$\frac{R^2}{4L^2} << \frac{1}{LC}$$

⇒ Bandwidth of R-L-C series resonant circuit

$$\omega_1, \omega_2 = \pm \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

$$\frac{R^2}{4L^2} << \frac{1}{LC}$$

$$\omega_1, \omega_2 = \pm \frac{R}{2L} \pm \sqrt{\frac{1}{LC}}$$

$$\omega_1, \omega_2 = \pm \frac{R}{2L} \pm \omega_r$$

Since ω_r cannot be negative

$$\omega_1, \omega_2 = \omega_r \pm \frac{R}{2L}$$

$$\omega_1 = \omega_L = \omega_r - \frac{R}{2L}$$

$$\omega_2 = \omega_H = \omega_r + \frac{R}{2L}$$

$$BW = \omega_H - \omega_L$$

$$= \cancel{\omega_r} + \frac{R}{2L} - \cancel{\omega_r} + \frac{R}{2L}$$

$$BW = \cancel{2} \cdot \frac{R}{\cancel{2}L}$$

$$\boxed{BW = \frac{R}{L} \text{ rad/sec}}$$

⟹ Quality Factor (Q)

$$Q = \frac{\text{Potential drop across } L/C \text{ at resonance}}{\text{Potential drop across } R \text{ at resonance.}}$$

$$Q = \frac{I_r \times X_L \text{ OR } I_r \times X_C}{I_r \times R}$$

$$Q = \frac{X_L}{R} \text{ OR } \frac{X_C}{R}$$

$$Q = \frac{\omega_r \cdot L}{R} \text{ OR } \frac{1}{\omega_r \cdot C \cdot R}$$

$$\omega_r = \frac{1}{\sqrt{LC}} \text{ OR } \frac{\sqrt{L\cancel{C}}}{\sqrt{C\cancel{C}} \cdot R}$$

$$Q = \frac{1}{\sqrt{\cancel{L}C}} \sqrt{\frac{L^{\cancel{2}}}{R}} \text{ OR } \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Q = \omega_r \cdot \frac{L}{R}$$

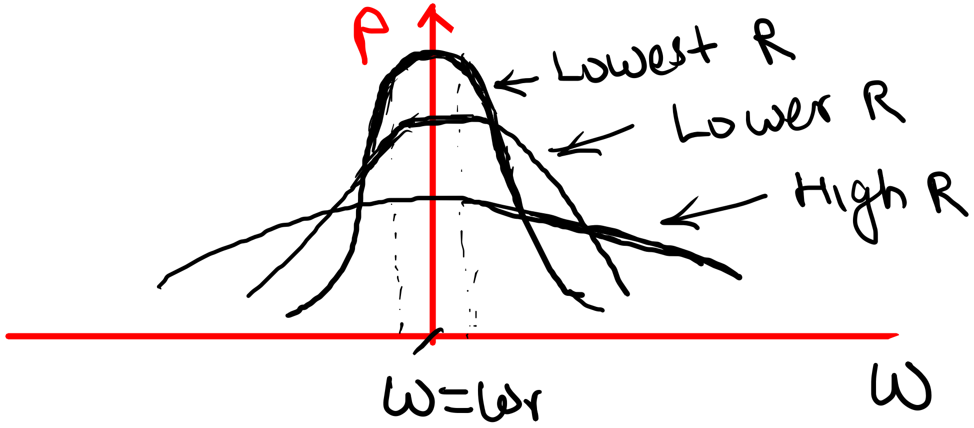
$$\because \frac{R}{L} = BW.$$

$$Q = \left(\frac{\omega_r}{BW}\right) \frac{rad/sec}{rad/sec}$$

$$Q = \frac{2\pi f_r}{2\pi (BW)} = \frac{f_r}{BW} \left(\frac{Hz}{Hz}\right)$$

$$\boxed{Q = \frac{f_r}{BW}}$$

⟹ Quality Factor (Q) & BW.



$$Q = \frac{X_L}{R}$$

R ↑ Q ↓

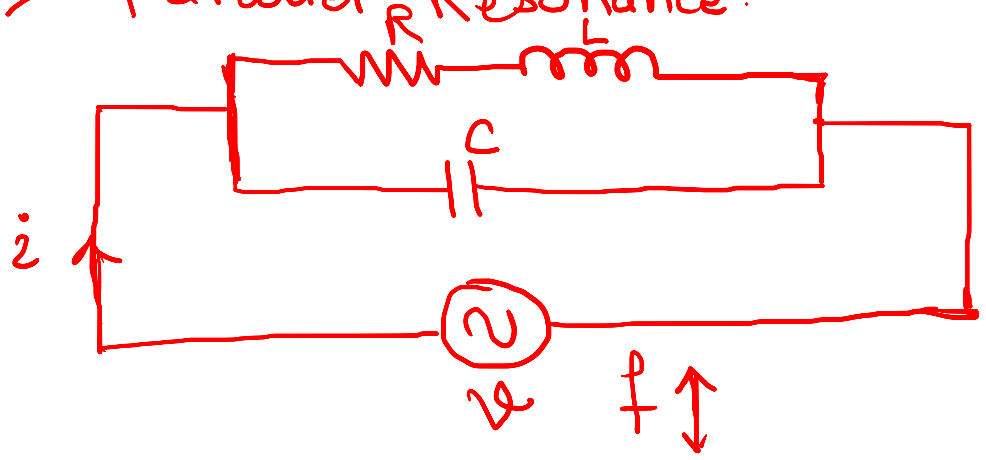
$$Q = \frac{\omega_r}{BW}$$

$$BW = \frac{\omega_r}{Q} \uparrow$$

→ Low R → high quality factor Q, Sharp Bandwidth

→ Circuit is selective

⟹ Parallel Resonance.



$$Z_1 = R + jX_L$$

$$Z_2 = -jX_C$$

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R+jX_L} \times \frac{R-jX_L}{R-jX_L} = \frac{R-jX_L}{R^2+X_L^2}$$

$$Y_2 = \frac{1}{-jX_C} = \frac{j}{X_C}$$

$$Y = Y_1 + Y_2$$

$$Y = \frac{R-jX_L}{R^2+X_L^2} + \frac{j}{X_C}$$

$$Y = \frac{R}{R^2+X_L^2} - j\left[\frac{X_L}{R^2+X_L^2} - \frac{1}{X_C}\right]$$

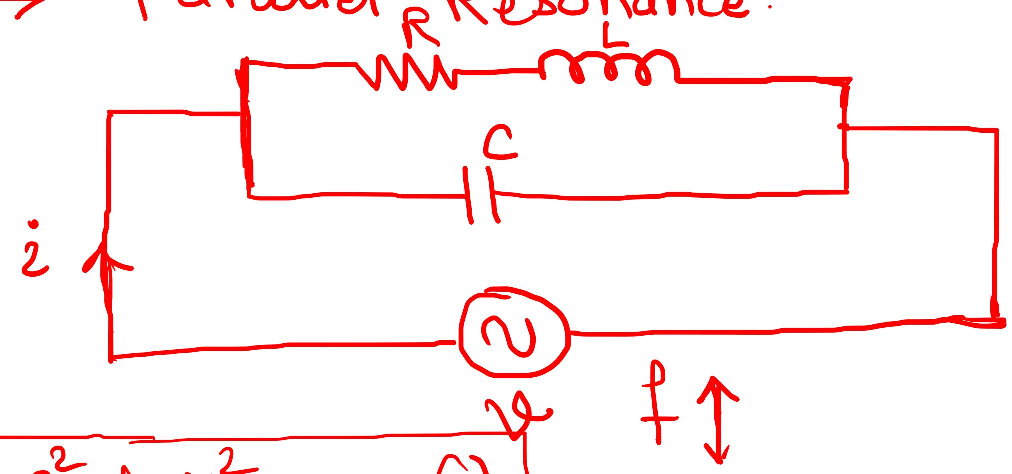
For resonance reactive term must be zero.

$$\therefore \frac{X_L}{R^2+X_L^2} - \frac{1}{X_C} = 0$$

$$\frac{X_L}{R^2+X_L^2} = \frac{1}{X_C}$$

$$\boxed{X_L \cdot X_C = R^2 + X_L^2 \quad \text{------} \; (1)}$$

⟹ Parallel Resonance.



$$\boxed{X_L \cdot X_C = R^2 + X_L^2 \quad \text{----} (1)}$$

at $\omega = \omega_r$.

$$\cancel{\omega_r} \cdot L \cdot \frac{1}{\cancel{\omega_r} \cdot C} = R^2 + (\omega_r L)^2$$

$$\frac{L}{C} = R^2 + \omega_r^2 \cdot L^2$$

$$\omega_r^2 L^2 = \frac{L}{C} - R^2$$

$$\omega_r^2 = \frac{\cancel{L}}{L^{\cancel{2}} \cdot C} - \frac{R^2}{L^2}$$

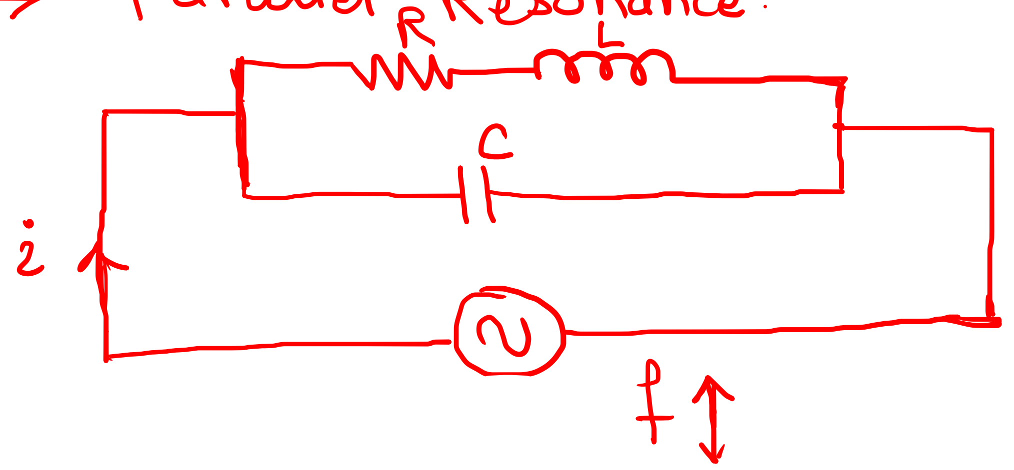
$$\omega_r^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\boxed{\omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

$$\frac{R^2}{L^2} << \frac{1}{LC}$$

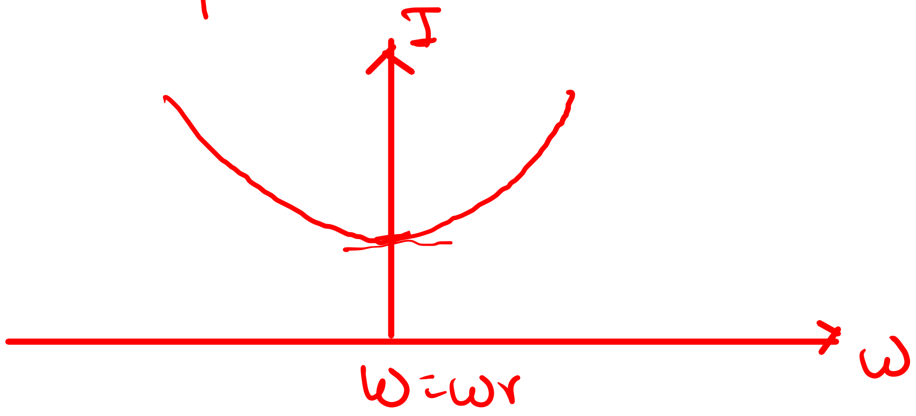
$$\boxed{\therefore \quad \omega_r = \frac{1}{\sqrt{LC}}}$$

⇒ Parallel Resonance

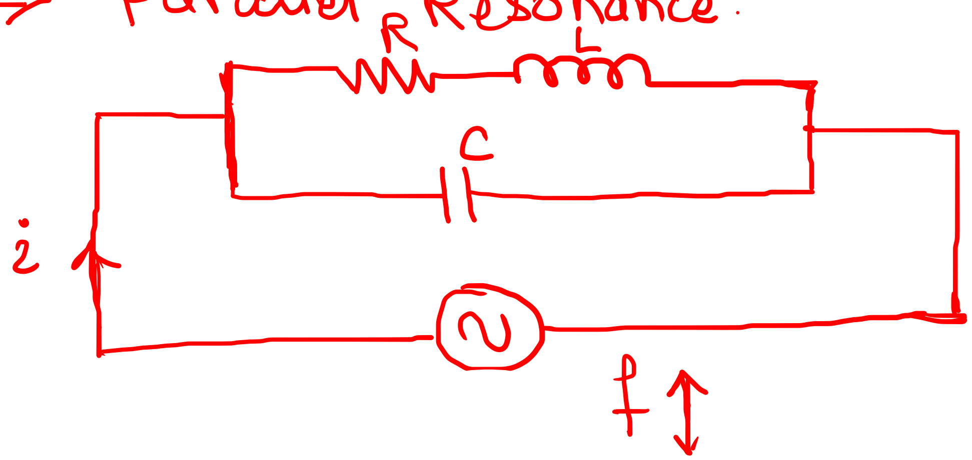


→ in parallel resonance circuit Admittance is minimum

$Z = \frac{1}{Y}$ ⇒ maximum. So I is minimum. So parallel resonant circuit rejector circuit.



⇒ Parallel Resonance.



⇒ Impedance at resonance

$$Y = \frac{R}{R^2+X_L^2} - j\left[\frac{X_L}{R^2+X_L^2} - \frac{1}{X_C}\right]$$

at resonance $Y = \dfrac{R}{R^2+X_L^2}$

impedance $Z = \dfrac{1}{Y} = \dfrac{R^2+X_L^2}{R}$

From equation (i)

$$R^2+X_L^2 = X_L \cdot X_C$$

$$Z = \frac{X_L \cdot X_C}{R}$$

$$Z = \frac{\cancel{\omega_r} \cdot L \cdot \frac{1}{\cancel{\omega_r} \cdot C}}{R}$$

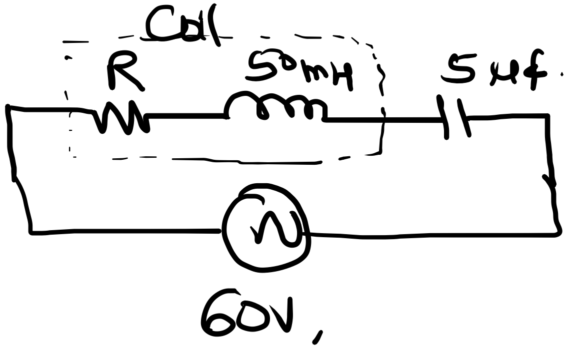
$$\boxed{Z = \frac{L}{CR}}$$ dynamic impedance of parallel resonant circuit.

⇒ Comparision of series / parallel resonant circuit

Parameter	series resonance	parallel Resonance.
Current	$I = \frac{V}{R}$ (maximum)	$I = \frac{V}{Z_D}$ (minimum)
Impedance	$Z = R$	$Z = Z_D = L/CR$
Power Factor	unity (1)	unity (1)
Resonant Frequency	$f_r = \frac{1}{2\pi\sqrt{LC}}$	$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
Quality factor	$Q = \frac{\omega_r L}{R}$	$Q = \frac{\omega_r \cdot L}{R}$
magnification	$(V_L, V_C) > V$ (voltage mag.) Acceptor circuit	$(I_L, I_C) > I$ (current magn.) rejector circuit.

Example

⇒ ① A 5 μF Capacitor is connected in series with the coil having an inductance of 50 mH. Calculate frequency of resonance & resistance of the coil if a 60V source operating at resonance causes current of 1.5 A. What is quality factor of coil?

⇒ 

at resonance $Z = R$

$$\Rightarrow \quad Z = R = \frac{60}{1.5} = 40\,\Omega$$

$$\rightarrow \quad f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$f_r = \frac{1}{2\pi\sqrt{50\times10^{-3}\times 5\times10^{-6}}}$$

$$\boxed{f_r = 318\ \text{Hz}}$$

$$\Rightarrow \quad Q = \frac{\omega_r L}{R}$$

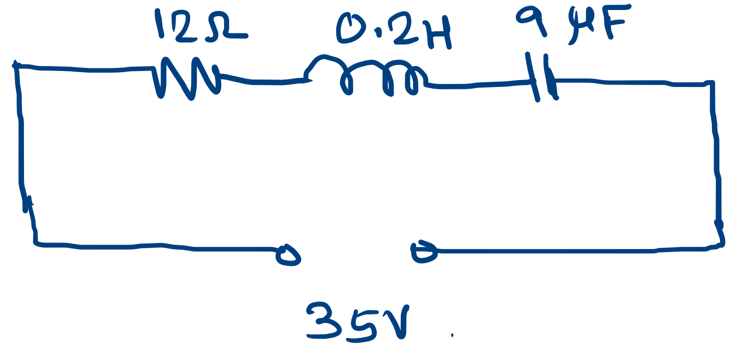
$$Q = \frac{2\pi \times 318 \times 50\times10^{-3}}{40}$$

$$\boxed{Q \simeq 2.5}$$

Example

⇒ (II) An non-inductive resistance of 12 Ω, an inductance of 0.2 H & capacitor of 9 μF are Connected in series. Calculate (i) current at resonance (ii) resonant frequency (iii) voltage across each Component when a voltage of 35V at resonance is applied to whole circuit.

⇒



1) Current $I_r = \frac{35}{12} = 2.9A$

ii) resonant freqn $f_r = \frac{1}{2\pi\sqrt{LC}}$

$$f_r = \frac{1}{2\pi\sqrt{0.2 \times 9 \times 10^{-6}}} = 119 HZ$$

(iii) $V_R = I_r \times R = 2.9 \times 12 = 34.8V$

$V_L = I_r \cdot X_L = 2.9 \times (2\pi \times 119 \times 0.2)$

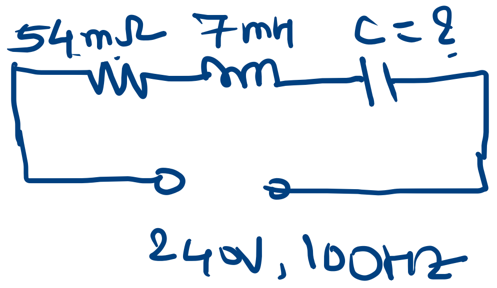
$$\boxed{V_L = 433.65 V}$$

$V_C = I_r \cdot X_C = 2.9 \times \frac{1}{2\pi \times 119 \times 9 \times 10^{-6}}$

$$\boxed{V_C = 430.95 V}$$

Example

⇒ (III) A series R-L-C circuit consisting of a coil & capacitor connected across 240V, 100Hz supply. If the coil has 54 mΩ resistance & 7 mH inductance. (i) Calculate value of capacitor at 100Hz resonant frequency (ii) Quality factor of the coil (iii) Half power frequencies

⇒ 

(i) → $f_r = 100\,Hz$.

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$f_r^2 = \frac{1}{4\pi^2 LC}$$

$$C = \frac{1}{4\pi^2 f_r^2 \times L} = \frac{1}{4\pi^2 (100)^2 \times 7\times10^{-3}}$$

$$C \simeq 362\,\mu F$$

(ii) $$Q = \frac{\omega_r L}{R} = \frac{2\pi\times100\times7\times10^{-3}}{54\times10^{-3}} = 81.45$$

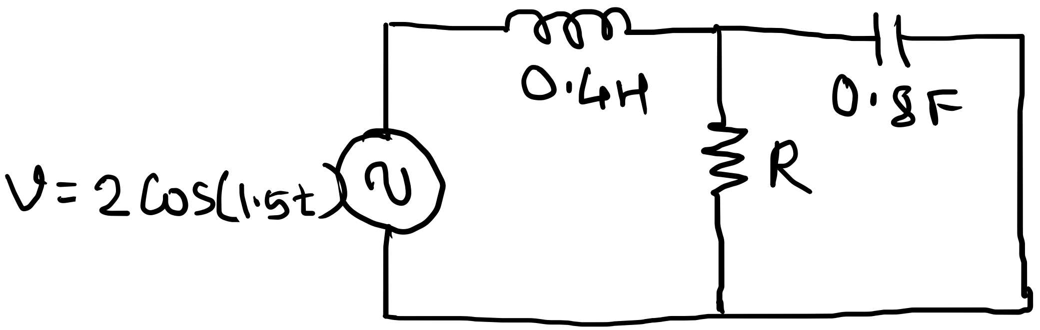
(iii) $$f_L = f_r - \frac{R}{4\pi L} = 100 - \frac{54\times10^{-3}}{4\pi\times7\times10^{-3}} = 99.38\,Hz$$

$$f_H = f_r + \frac{R}{4\pi L} = 100 + \frac{54\times10^{-3}}{4\pi\times7\times10^{-3}} = 100.61\,Hz$$

$$BW = f_H - f_L = 100.61 - 99.38 = 1.23\,Hz.$$

Example

⇒ (iv) Find value of R such that below circuit is at resonance.



$\omega = 1.5$

⇒ Impedance $(R \| c) + X_L$

$$Z = jX_L + \left(\frac{R \times -jX_C}{R - jX_C} \right)$$

$$Z = (j\,1.5 \times 0.4) + \left(\frac{R \times \frac{1}{j(1.5 \times 0.8)}}{R + \left(\frac{1}{j\,1.5 \times 0.8}\right)} \right)$$

$$Z = j0.6 + \frac{R \,/\, j(1.2)}{R + \frac{1}{1.2j}}$$

$$Z = j0.6 + \frac{R}{1 + j1.2R}$$

$$Z = j0.6 + \frac{R(1 - j1.2R)}{1 + (1.2R)^2}$$

$$Z = \frac{R}{1+(1.2R)^2} - j\left(\frac{1.2R^2}{1+(1.2R)^2} - 0.6\right)$$

At resonance reactive term $= 0$

$$\frac{1.2R^2}{1 + 1.44R^2} = 0.6$$

$$1.2R^2 = 0.6 + 0.864R^2$$

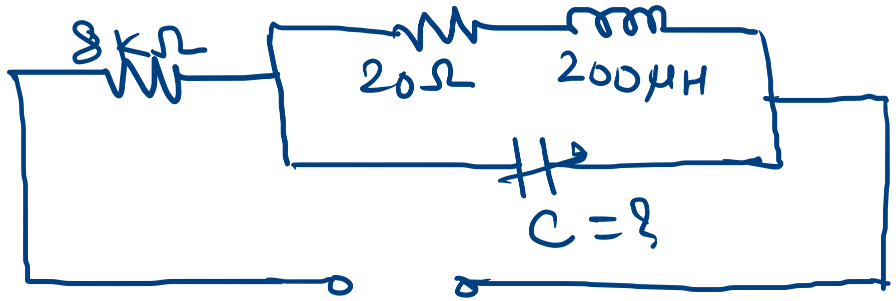
$$0.336R^2 = 0.6$$

$$R^2 = 1.78$$

$$\boxed{\therefore R = 1.33\,\Omega}$$

Example

⇒ (v) In the following network find (i) value of capacitor (ii) quality factor (iii) impedance at resonance (iv) total current, if the circuit is at resonance.

⇒ 

(i)

⇒ $f_r = 10^6$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f^2 = \frac{1}{4\pi^2}\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)$$

$$(10^6)^2 = \frac{1}{4\pi^2}\left[\frac{1}{200\times10^{-6}C} - \frac{20^2}{(200\times10^{-6})^2}\right]$$

$C \simeq 126.5\ pF$

(ii) $Q = \dfrac{2\pi f_r L}{R} = \dfrac{2\pi\times10^6\times200\times10^{-6}}{20}$

(iii) Impedance at resonance ($Z_T = 8k + Z_D$)

$$Z_D = \frac{L}{CR} = \frac{200\times10^{-6}}{126.5\times10^{-12}\times20}$$

$Z_D = 78.74\,k\Omega$

$Z_T = 8 + 78.74 = 86.74\,k\Omega$

(iv) $I = \dfrac{230}{Z_T} = \dfrac{230}{86.74k} = 2.65\ mA$