Phasor Representation of AC quantities

$\Rightarrow v(t) = V_m \sin(\omega t + \phi)$

$i(t) = I_m \sin(\omega t + \phi)$

$v(t)$, $i(t)$ are called instantaneous values

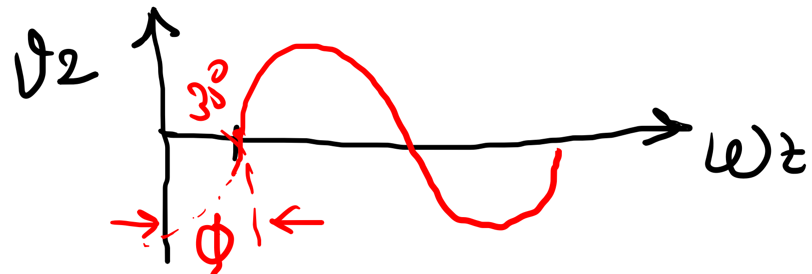
$\theta = \omega t \rightarrow \frac{rad}{\cancel{sec}} \times \cancel{sec}$

$V_m, I_m \Rightarrow$ peak values, Amplitude

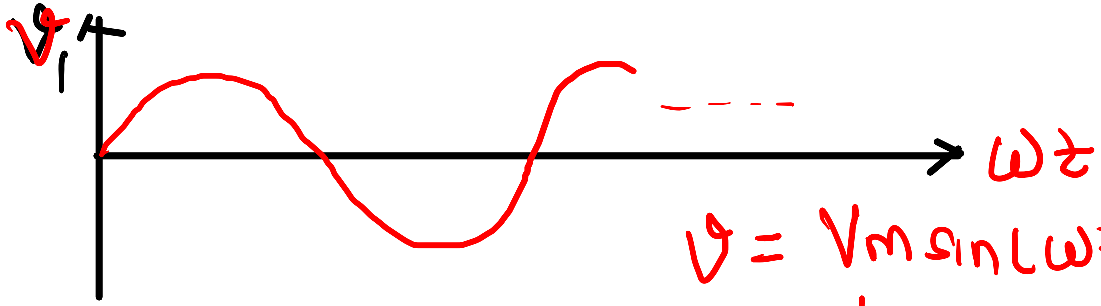
$\omega \rightarrow$ angular frequency radians/second

$t \rightarrow$ time

$\phi \rightarrow$ initial phase angle



$v_2 = V_m \sin(\omega t - \phi)$



$$v = V_m \sin(\omega t + 0^\circ)$$
$$= V_m \sin(\theta)$$

Phasor Representation of AC quantities

$\Rightarrow v(t) = V_m \sin(\omega t + \phi)$

$i(t) = I_m \sin(\omega t + \phi)$

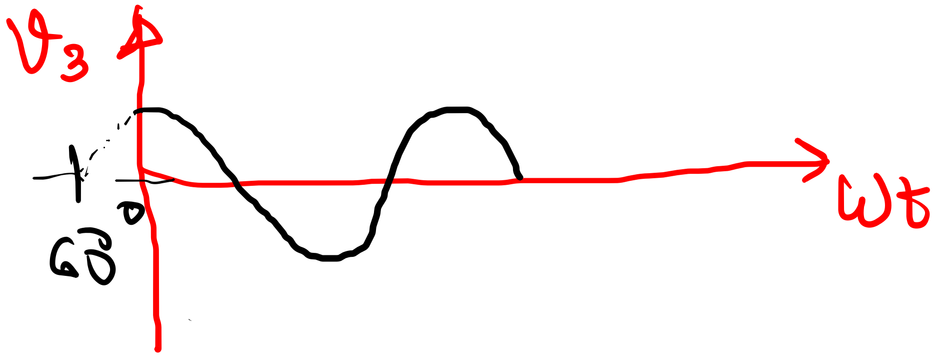
$v(t)$, $i(t)$ are called instanceous values $\quad \theta = \omega t \rightarrow \frac{rad}{\cancel{sec}} \times \cancel{sec}$

$V_m, I_m \Rightarrow$ peak values, Amplitude

$\omega \rightarrow$ angular frequency radians/second

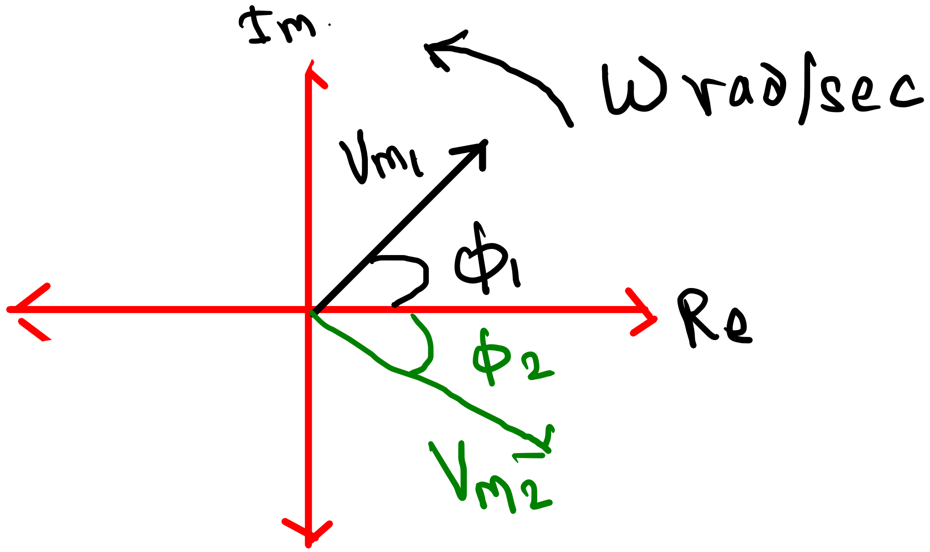
$t \rightarrow$ time

$\phi \rightarrow$ intial phase angle



$v_3 = V_m \sin(\omega t + 60^\circ)$

Phasor Representation of AC quantities



$$v_1 = V_{m_2} \sin(\omega t + \phi_1)$$

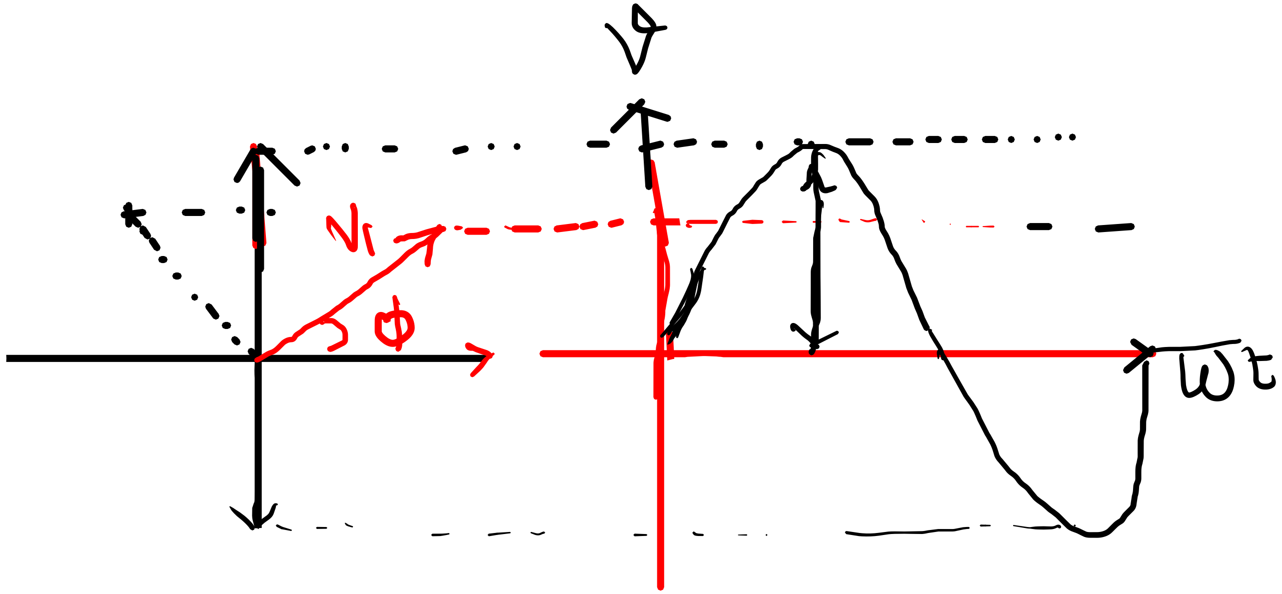
$$v_2 = V_{m_2} \sin(\omega t - \phi_2)$$

$v_1 + v_2$

$v_2 - v_1$

$v_1 * v_2$

$\dfrac{v_1}{v_2}$

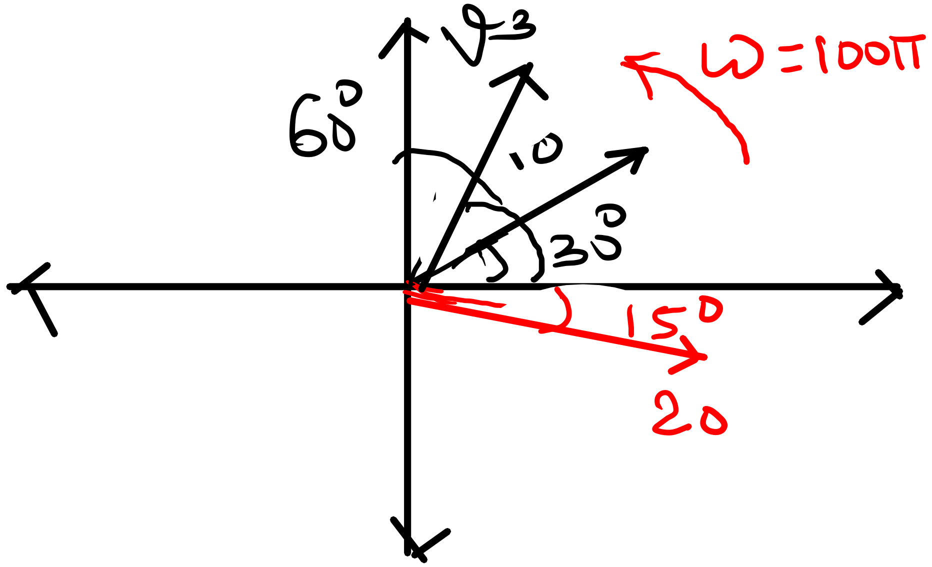


Normally rms values of ac quantities used in phasor representation

Phasor Representation of AC quantities

⇒ To draw phasor diagram of AC quantities they must have same frequency.

⇒ phasor diagram provides information about phase relation between two AC quantities.



$i_1 = 10\sin(100\pi t + 30^\circ)$

$v_1 = 20\sin(100\pi t - 15^\circ)$

⇒ Leading & Lagging phasors

→ i_1 is leading v_1 by 45°

v_1 is lagging i_1 by 45°

$v_3 = 5\sin(100\pi t + 60^\circ)$

→ v_3 leads both i_1 & v_1

Phasor Representation of AC quantities

⇒ Mathematical representation of phasors.

$$v = V_m \sin(\omega t + \phi)$$

$$v = V_m \angle \phi$$

$$i = I_m \sin(\omega t - \phi)$$

$$i = I_m \angle -\phi$$

Polar Form : $r \angle \phi$ $\quad r \rightarrow$ magnitude $\quad \phi =$ angle

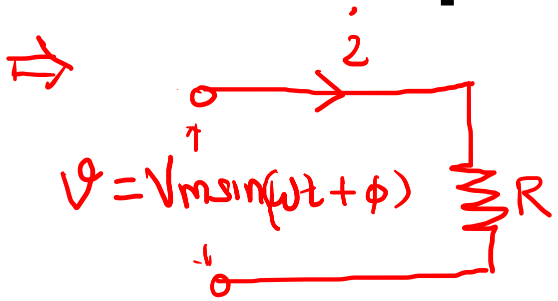
Rectangular form $\quad x + jy \quad r = \sqrt{x^2 + y^2}$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

Exponential form : $r e^{j\phi} \Rightarrow (\underbrace{r\cos\phi}_{x} + j\underbrace{r\sin\phi}_{y})$

$$x + jy$$

Phasor Representation of AC quantities

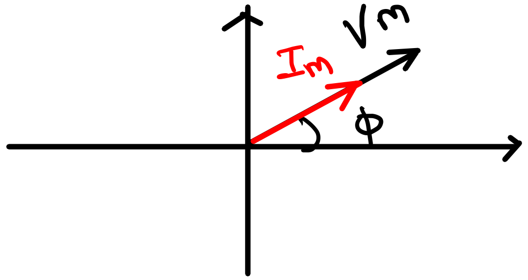


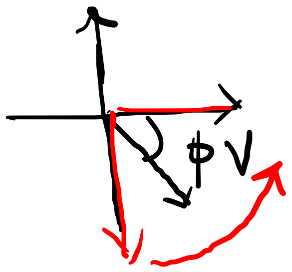
$i = \frac{v}{R}$ (Ohms Law)

$i = \frac{V_m \sin(\omega t + \phi)}{R}$

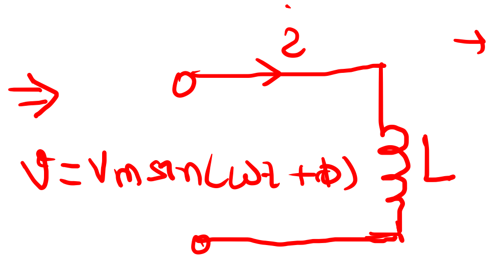
$i = \frac{V_m}{R} \sin(\omega t + \phi)$

$i = I_m \sin(\omega t + \phi)$





Vm & Im are inphase (No phase difference)



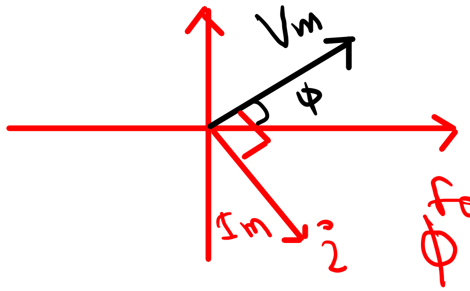
→ Voltage across (L) $V_L = L \cdot \frac{di}{dt}$

current flowing inductor $I_L = \frac{1}{L} \int V_L \, dt$

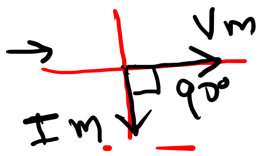
$i = \frac{1}{L} \int v \, dt = \frac{1}{L} \int V_m (\sin \omega t + \phi) \, dt$

$i = \frac{V_m}{L} \; \frac{-\cos(\omega t + \phi)}{\omega} = \frac{V_m}{\omega L} \left[-\cos(\omega t + \phi) \right]$

$i = \frac{V_m}{\omega L} \sin(\omega t + \phi - 90^\circ)$



for $\phi = 0$ →



Current is lagging voltage by 90°.

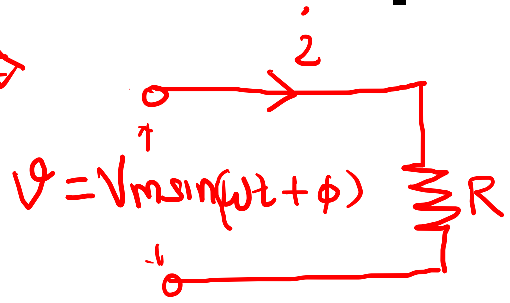
$\underline{\underline{\phi}}$

$P = VI \cos \phi$ Active power

$Q = VI \sin \phi$ Reactive power

Phasor Representation of AC quantities

⇒

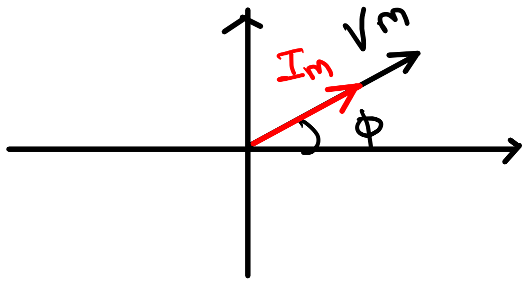


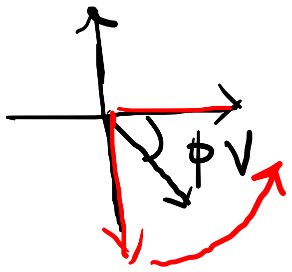
$$i = \frac{v}{R} \quad \text{(Ohms Law)}$$

$$i = \frac{V_m \sin(\omega t + \phi)}{R}$$

$$i = \frac{V_m}{R} \sin(\omega t + \phi)$$

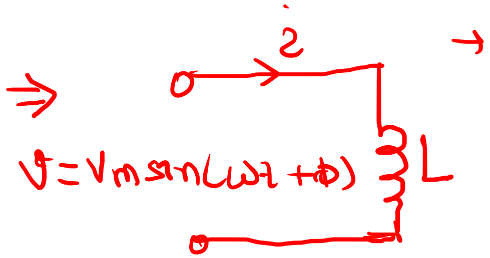
$$i = I_m \sin(\omega t + \phi)$$





V_m & I_m are inphase (No phase difference)

⇒



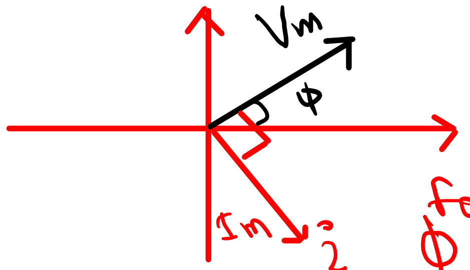
→ Voltage across (L) current flowing inductor

$$V_L = L \cdot \frac{di}{dt} \qquad I_L = \frac{1}{L} \int V_L \, dt$$

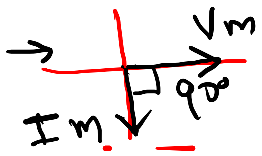
$$i = \frac{1}{L} \int v \, dt = \frac{1}{L} \int V_m (\sin\omega t + \phi) \, dt$$

$$i = \frac{V_m}{L} \; \frac{-\cos(\omega t + \phi)}{\omega} = \frac{V_m}{\omega L} [-\cos(\omega t + \phi)]$$

$$i = \frac{V_m}{\omega L} \sin(\omega t + \phi - 90^\circ)$$



for $\phi = 0$ →



Current is lagging voltage by 90°.

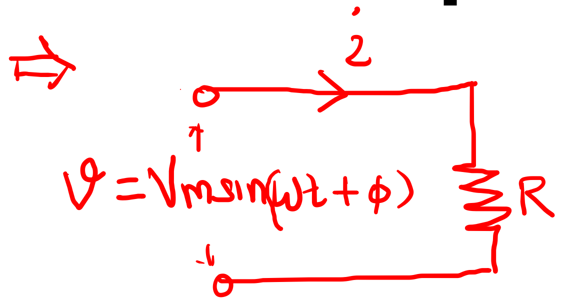
$$\underline{\underline{\phi}}$$

$P = V I \cos\phi$ Active power

$Q = V I \sin\phi$ Reactive power

Phasor Representation of AC quantities

$\Rightarrow$

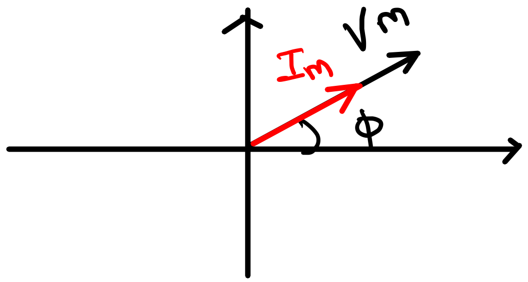


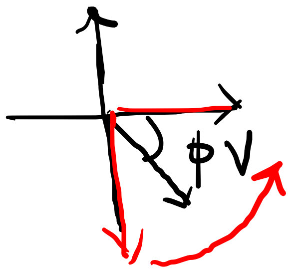
$$i = \frac{\vartheta}{R} \quad \text{(Ohms Law)}$$

$$i = \frac{V_m \sin(\omega t + \phi)}{R}$$

$$i = \frac{V_m}{R} \sin(\omega t + \phi)$$

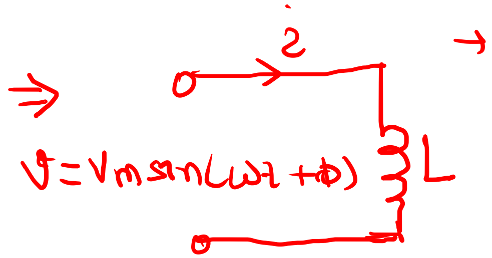
$$i = I_m \sin(\omega t + \phi)$$





V_m & I_m are inphase (No phase difference)

$\Rightarrow$



→ Voltage across (L)

$$V_L = L \cdot \frac{di}{dt}$$

Current flowing inductor

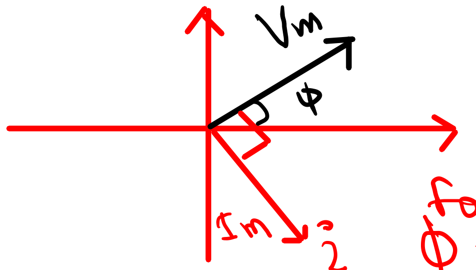
$$I_L = \frac{1}{L} \int V_L \, dt$$

$$i = \frac{1}{L} \int \vartheta \, dt = \frac{1}{L} \int V_m (\sin \omega t + \phi) \, dt$$

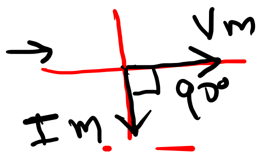
$$i = \frac{V_m}{L} \; \frac{-\cos(\omega t + \phi)}{\omega} = \frac{V_m}{\omega L} [-\cos(\omega t + \phi)]$$

$$i = \frac{V_m}{\omega L} \sin(\omega t + \phi - 90^\circ)$$

Current is lagging voltage by 90°.



for $\phi = 0$ →



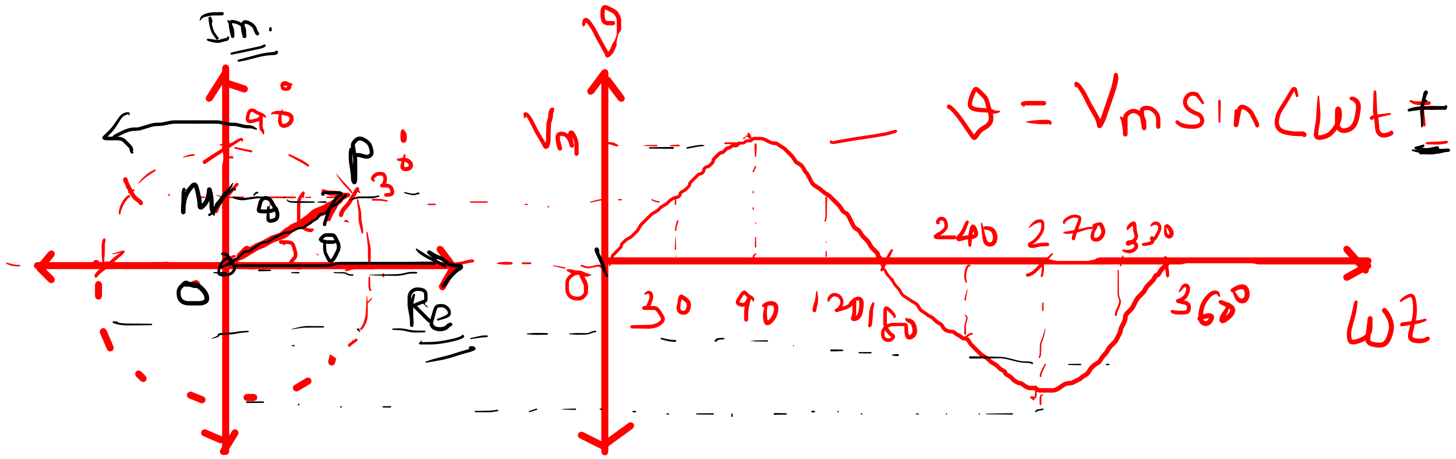
ϕ

$=$

$P = V I \cos \phi$ Active power

$Q = V I \sin \phi$ Reactive power

Phasor Representation of alternating quantities



$v = V_m \sin(\omega t \pm \phi)$

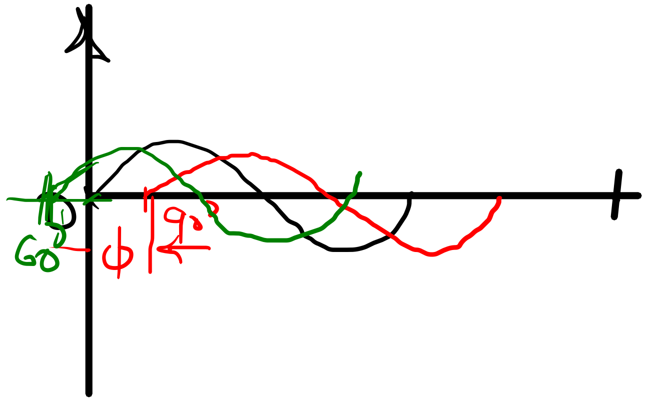
$v = V_m \sin\theta = V_m \sin(\omega t)$

$\sin\theta = \frac{OM}{OP}$ $\qquad$ $OM = OP \sin\theta$

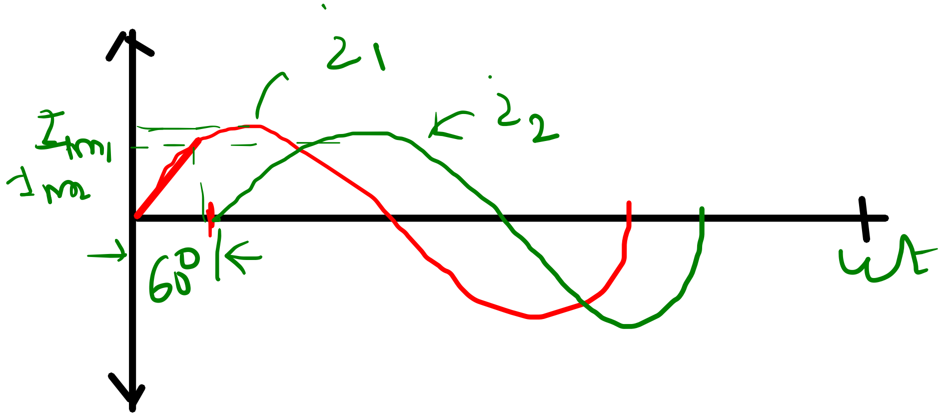
$$v = V_m \sin(\omega t \pm \phi)$$

Lagging ← $v_2 = V_m \sin(\omega t - 90^\circ)$

Leading $\quad v_3 = V_m \sin(\omega t + 60^\circ)$



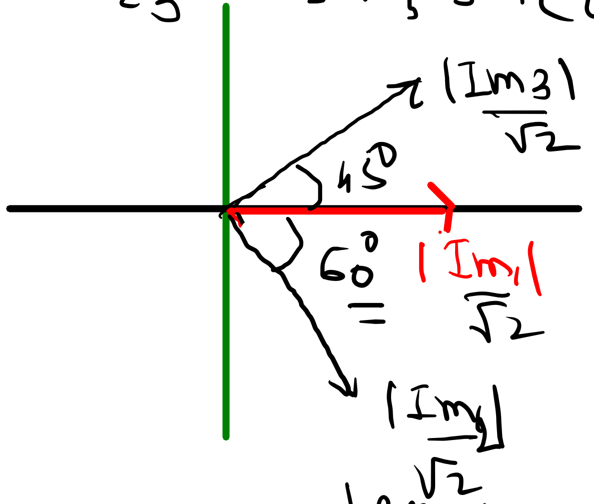
(1)



$$i_1 = I_{m1}\sin(\omega t) = |I_{m1}|\angle 0^\circ$$

$$i_2 = I_{m2}\sin(\omega t - 60^\circ) = |I_{m2}|\angle -60^\circ$$

$$i_3 = I_{m3}\sin(\omega t + 45^\circ) = |I_{m1}|\angle 45^\circ$$



✓ I_{m2} is lagging I_{m1} by 60°

✓ I_{m1} is leading I_{m2} by 60°

Length of phasor is Rms value of alternating quantity

① Two sinusiodal currents are given as

$$i_1 = 10\sqrt{2}\sin\omega t \quad \& \quad i_2 = 20\sqrt{2}\sin(\omega t + 90^\circ)$$

Find sum of the currents

$\Rightarrow$ $i_1 + i_2$

$$i_1 = \frac{10\sqrt{2}}{\sqrt{2}} \angle 0^\circ$$

$$i_2 = \frac{20\sqrt{2}}{\sqrt{2}} \angle 90^\circ$$

$$i_1 = 10\angle 0^\circ$$

$$i_2 = 20\angle 90^\circ$$

$$i_1 + i_2 = 10\angle 0^\circ + 20\angle 90^\circ$$

$$= 10\cos 0 + j10\sin 0 + 20\cos 90 + j20\sin 90$$

$$= 10 + j0 + 20(0) + j20(1)$$

$$i_1 + i_2 = 10 + j20$$

$$= \sqrt{(10)^2 + (20)^2} \angle \tan^{-1}\left(\frac{20}{10}\right)$$

$$= 22.36 \angle 63.43$$

$$\boxed{i_1 + i_2 = (22.36)\cdot\sqrt{2}\cdot\sin(\omega t + 63.43)}$$

① Two sinusiodal currents are given as

$$i_1 = 10\sqrt{2}\sin\omega t \quad \& \quad i_2 = 20\sqrt{2}\sin(\omega t + 90^\circ)$$

Find sum of the Currents

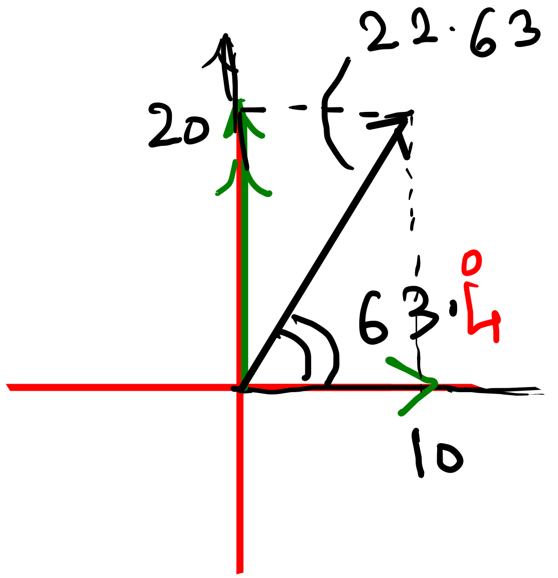
$\Rightarrow$ $i_1 + i_2$

$$i_1 = \frac{10\sqrt{2}}{\sqrt{2}} \angle 0^\circ$$

$$i_2 = \frac{20\sqrt{2}}{\sqrt{2}} \angle 90^\circ$$

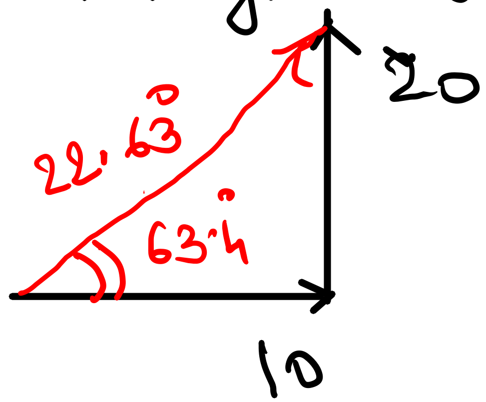
$$i_1 = 10 \angle 0^\circ$$

$$i_2 = 20 \angle 90^\circ$$

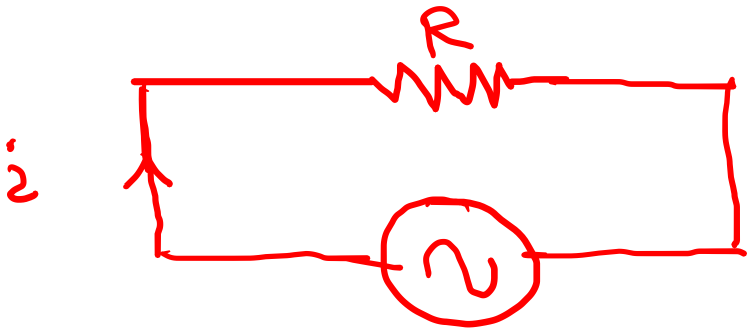


Law of parallelogram

Triangle Law



⇒ Response of resistor to AC input.



$v = V_m \sin\omega t$

⇒ Impedance of the circuit (Z)

$$Z = \frac{V}{I}, \quad Z = R.$$

⇒ $i = \frac{v}{R}$

$$i = \frac{V_m \sin\omega t}{R}$$

$$i = \frac{V_m}{R} \sin\omega t$$

$$i = I_m \cdot \sin\omega t$$

⇒ Instantaneous power

$$P_{inst} = v \cdot i$$

$$P_{inst} = V_m \sin\omega t \cdot I_m \sin\omega t$$

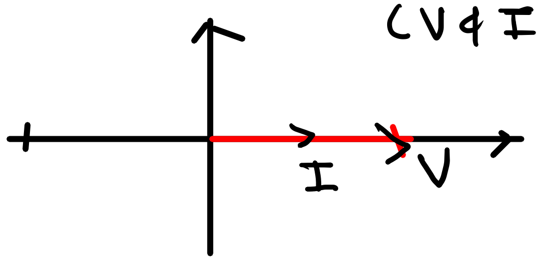
$$P_{inst} = V_m I_m \sin^2\omega t$$

$$P_{inst} = V_m I_m \left(\frac{1 - \cos 2\omega t}{2} \right)$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$$

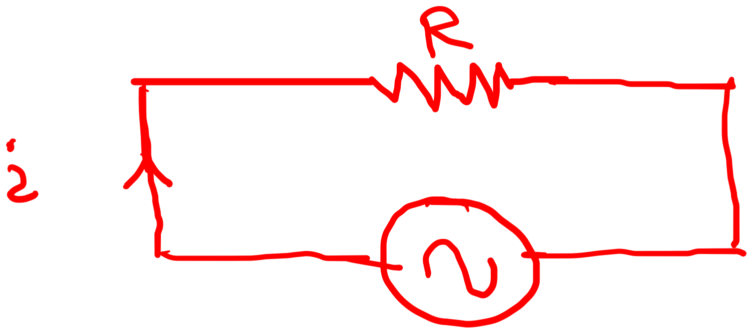
⇒ phasor diagram

(V & I are rms values)

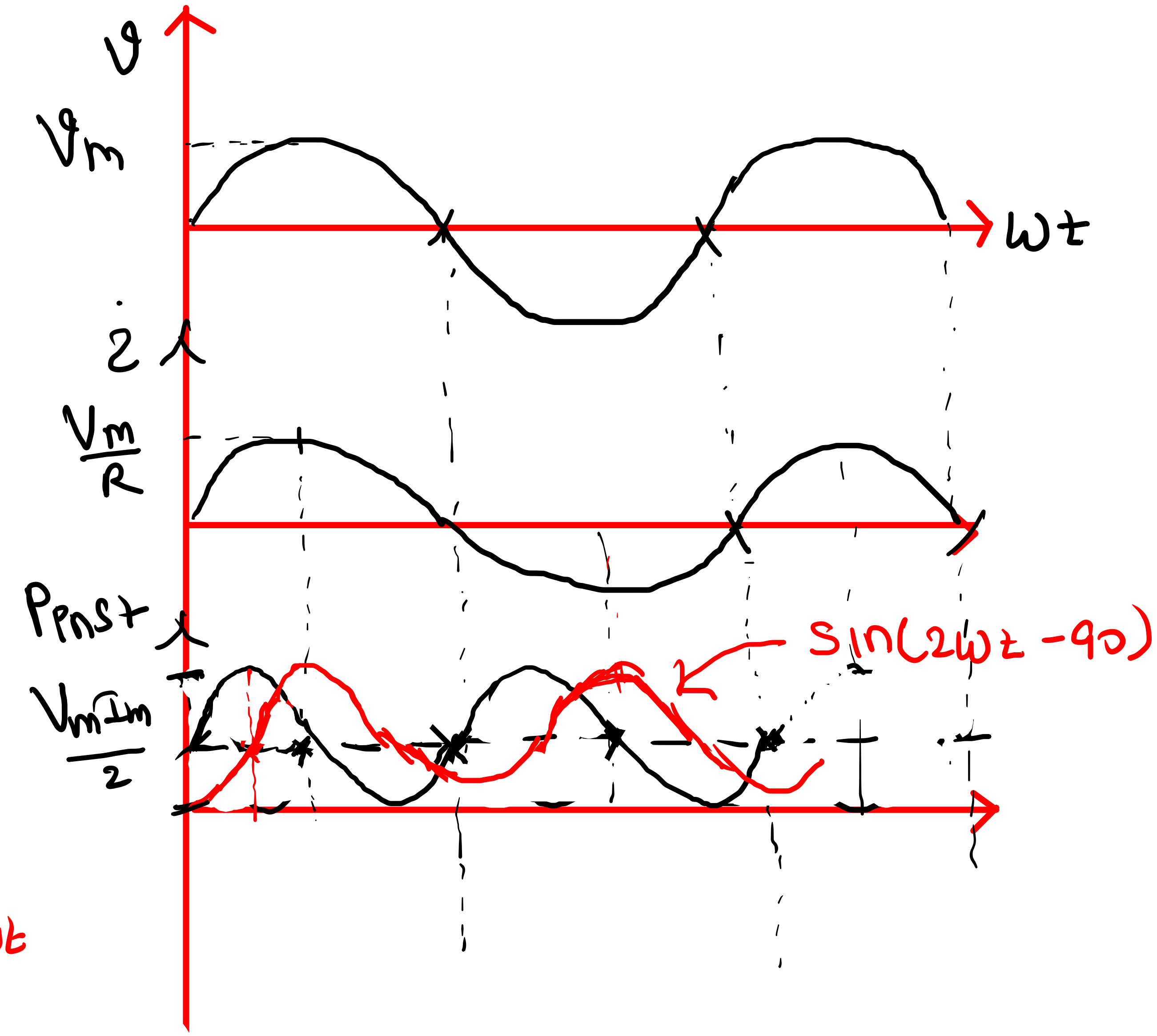


Voltage and current are in phase

⇒ Response of resistor to AC input.



$v = V_m \sin\omega t$



⇒ Instantaneous power

$$P_{inst} = v \cdot i$$

$$P_{inst} = V_m \sin\omega t \cdot I_m \sin\omega t$$

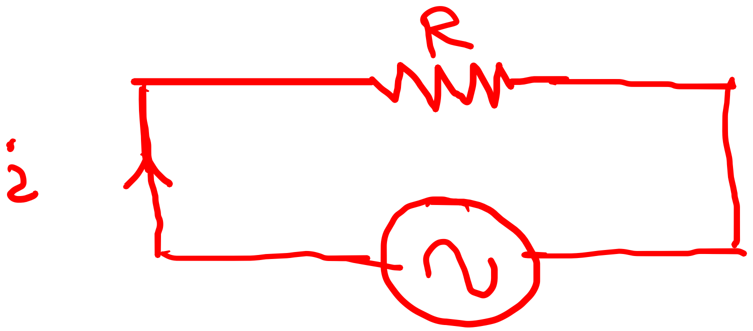
$$P_{inst} = V_m I_m \sin^2\omega t$$

$$P_{inst} = V_m I_m \left(\frac{1 - \cos 2\omega t}{2} \right)$$

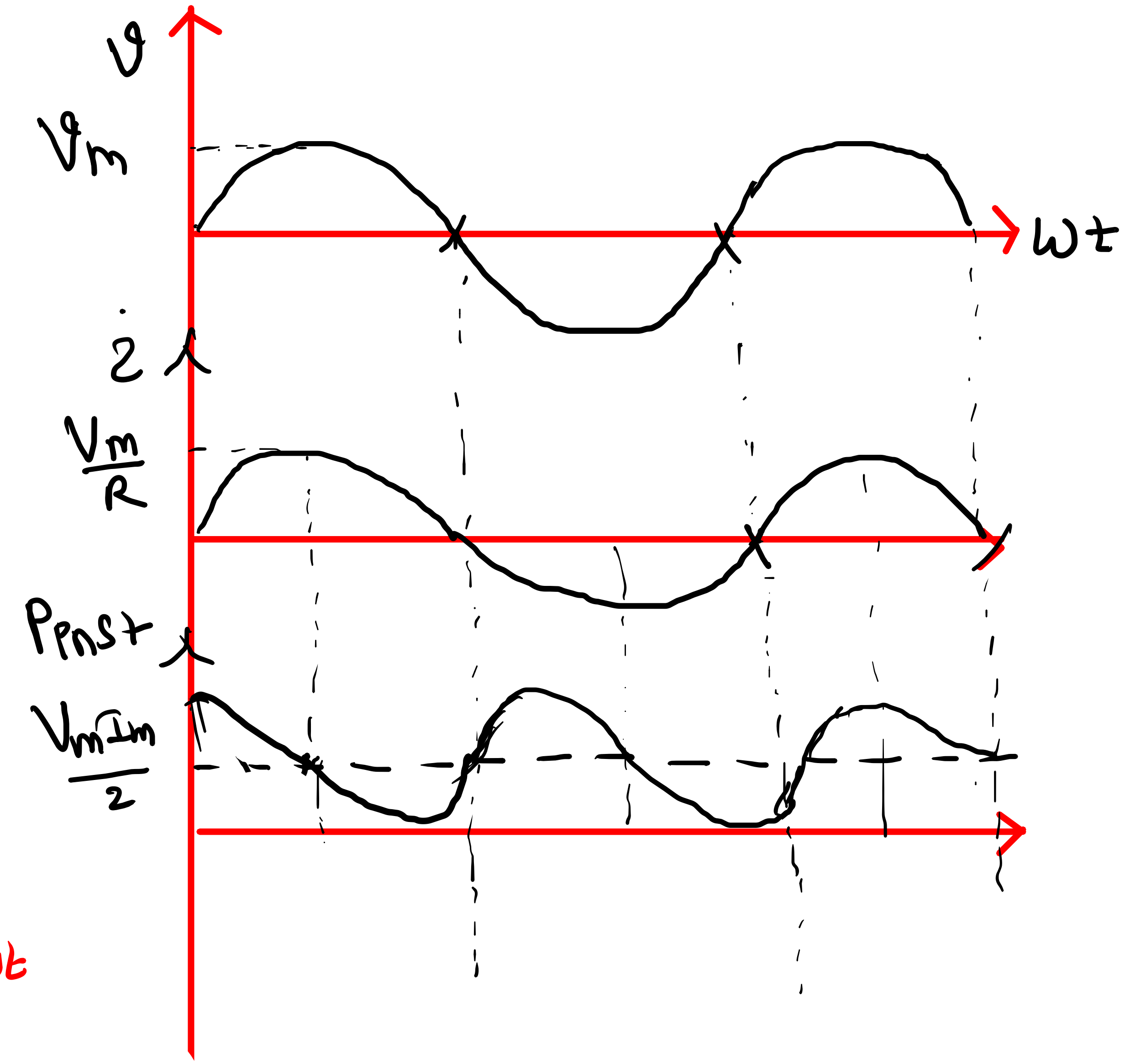
$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$$

$$= \frac{V_m I_m}{2} + \frac{V_m I_m}{2} \sin(2\omega t - 90)$$

⇒ Response of resistor to AC input.



$v = V_m \sin\omega t$



⇒ Instantaneous power

$P_{inst} = v \cdot i$

$P_{inst} = V_m \sin\omega t \cdot I_m \sin\omega t$

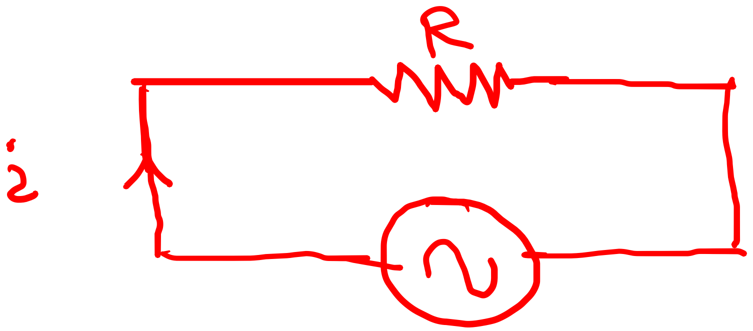
$P_{inst} = V_m I_m \sin^2\omega t$

$P_{inst} = V_m I_m \left(\frac{1 - \cos 2\omega t}{2}\right)$

$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$

$= \frac{V_m I_m}{2} + \frac{V_m I_m}{2} \sin(2\omega t - 90)$

⇒ Response of resistor to AC input.



$v = V_m \sin\omega t$

⇒ Instantaneous power

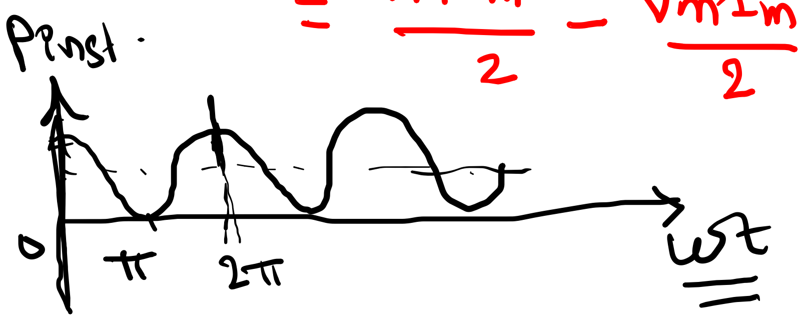
$$P_{inst} = v \cdot i$$

$$P_{inst} = V_m \sin\omega t \cdot I_m \sin\omega t$$

$$P_{inst} = V_m I_m \sin^2\omega t$$

$$P_{inst} = V_m I_m \left(\frac{1 - \cos 2\omega t}{2} \right)$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$$



⇒ Average power (P)

$$P_{av} = \frac{1}{2\pi} \int_0^{2\pi} P_{inst} \cdot d\omega t$$

$$= \frac{1}{2\pi} \int_0^{2\pi} V_m I_m \cdot \sin^2\omega t \, d\omega t$$

$$P_{av} = \frac{V_m I_m}{2\pi} \int_0^{2\pi} \left(\frac{1 - \cos 2\omega t}{2} \right) d\omega t$$

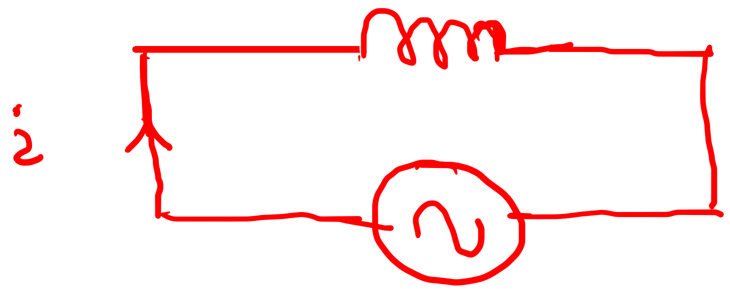
$$P_{av} = \frac{V_m I_m}{2\pi \cdot 2} \left[\omega t - \frac{\sin 2\omega t}{2} \right]_0^{2\pi}$$

$$P_{av} = \frac{V_m I_m}{2 \cdot 2\pi} \left[2\pi - \frac{\sin 4\pi}{2} - 0 - \sin 0 \right]$$

$$P_{av} = \frac{V_m I_m}{\cancel{2\pi} \cdot 2} \left[\cancel{2\pi} \right] = \frac{V_m I_m}{2}$$

$$P_{av} = \frac{V_m \cdot I_m}{\sqrt{2} \cdot \sqrt{2}} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = V_{rms} \cdot I_{rms}.$$

Reponse of Pure inductor to AC input



$v = V_m \sin\omega t$

$\Rightarrow \quad I_L = \frac{1}{L}\int V_L \, dt \quad , \quad V_L = L \cdot \frac{dI_L}{dt}$

$i = \frac{1}{L}\int v \cdot dt$

$= \frac{1}{L}\int V_m \sin\omega t \cdot dt$

$= \frac{V_m}{L}\left[-\frac{\cos\omega t}{\omega}\right]$

$i = \frac{V_m}{\omega L}(-\cos\omega t) = -I_m \cos\omega t$

$i = \frac{V_m}{\omega L}\sin(\omega t - 90^\circ)$

$i = I_m \cdot \sin(\omega t - 90^\circ)$

$I_m = \frac{V_m}{\omega L}$

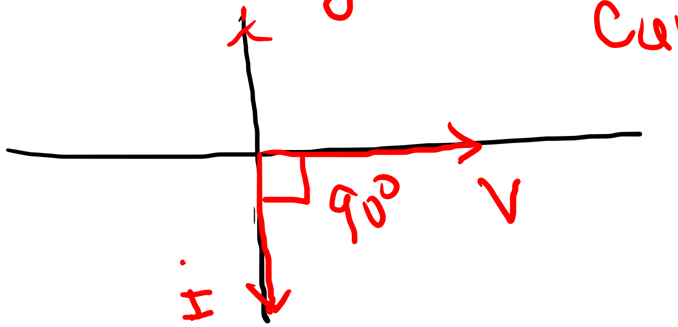
$\omega L = \frac{V_m}{I_m} = X_L$ (inductive reactance)

$\boxed{X_L = \omega L = 2\pi f L}$

for DC $f = 0$, $X_L = 0$

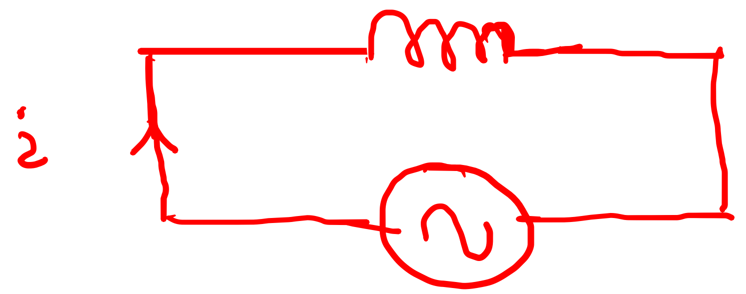
inductor short for DC

$\Rightarrow$ Phasor diagram



Current is lagging voltage by 90°.

Reponse of Pure inductor to AC input



$v = V_m \sin\omega t$

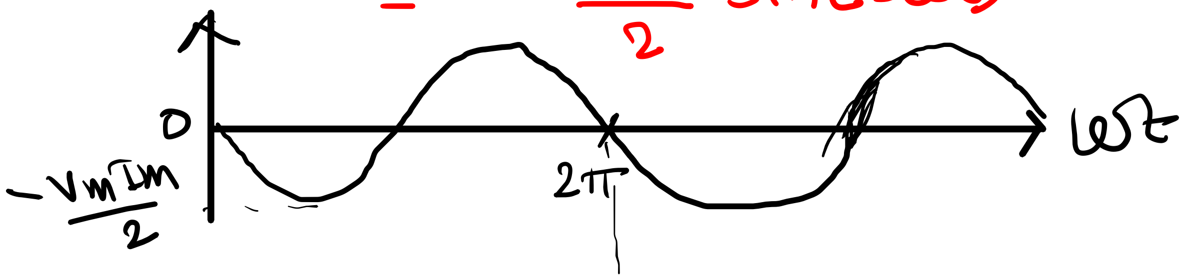
$$P_{inst} = v \cdot i$$

$$= (V_m \sin\omega t) \cdot (-I_m \cos\omega t)$$

$$= -V_m I_m \sin\omega t \cdot \cos\omega t$$

$$P_{inst} = -V_m I_m \frac{\sin(2\omega t)}{2}$$

$$= -\frac{V_m I_m}{2} \sin(2\omega t)$$



$$P_{av} = \frac{1}{2\pi} \int_0^{2\pi} P_{inst} \, d\omega t$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \left(-\frac{V_m I_m}{2}\right) \sin(2\omega t) \, d\omega t$$

$$= -\frac{V_m I_m}{4\pi} \left[-\frac{\cos(2\omega t)}{2}\right]_0^{2\pi}$$

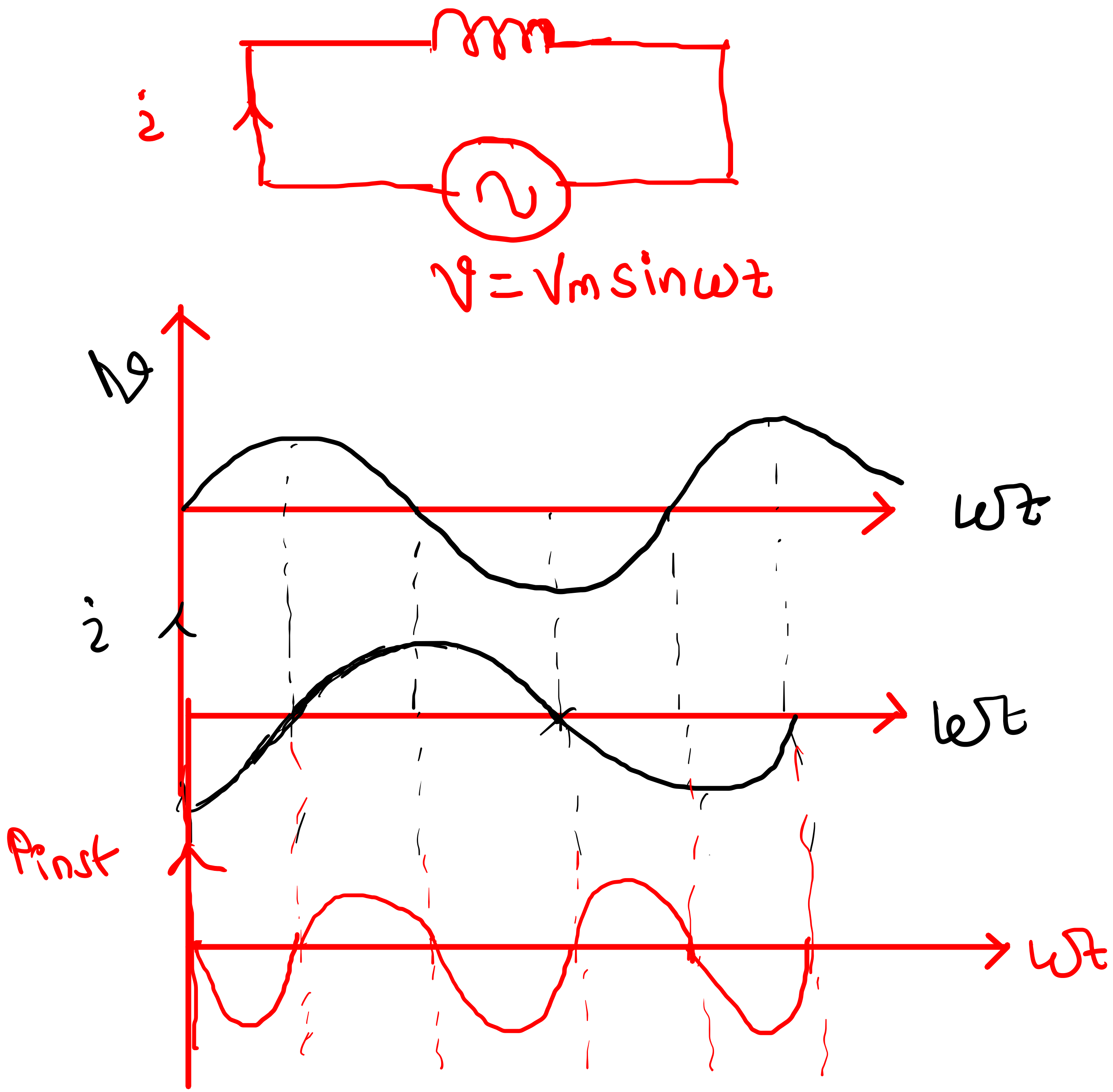
$$= -\frac{V_m I_m}{8\pi} \left[-\cos 4\pi + \cos 0\right]$$

$$= -\frac{V_m I_m}{8\pi} \left(-1 + 1\right)$$

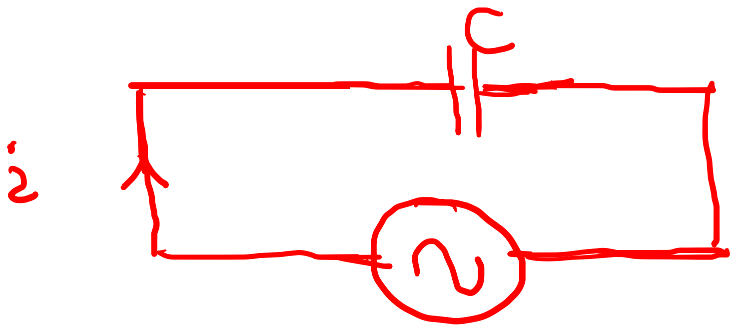
$$\boxed{P_{av} = 0}$$

Average power consumed by pure inductor is zero.

Reponse of Pure inductor to AC input



Reponse of Pure Capacitor to AC input



$v = V_m \sin\omega t$

$\Rightarrow$ For capacitor

$I_c = C \cdot \frac{dV_c}{dt}$, $V_c = \frac{1}{C}\int I_c\,dt$.

$i = C \cdot \frac{dv}{dt}$

$= C \cdot \frac{d(V_m \sin\omega t)}{dt}$

$= V_m \cdot C \cdot (\cos\omega t) \cdot \omega$.

$i = V_m \cdot \omega C \cos\omega t$.

$i = I_m \cos\omega t$

$I_m = V_m \cdot \omega C = \frac{V_m}{1/\omega C}$.

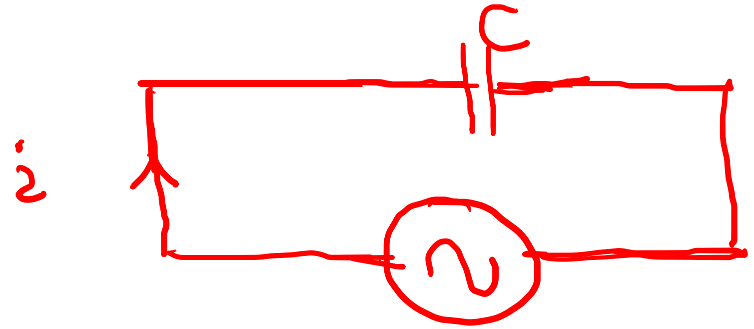
$\frac{V_m}{I_m} = \frac{1}{\omega C} \Rightarrow$ Capacitive Reactance.

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$f = 0 \Rightarrow$ For DC input

$X_C = \infty \rightarrow$ open circuit for DC.

Reponse of Pure Capacitor to AC input



$v = V_m \sin\omega t$

$\Rightarrow$ For capacitor

$I_c = C \cdot \frac{dV_c}{dt}$, $V_c = \frac{1}{C}\int I_c\,dt$

$$i = C \cdot \frac{dv}{dt}$$

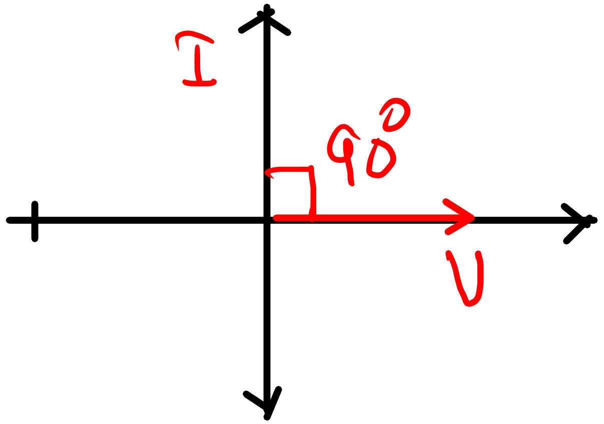
$$= C \cdot \frac{d}{dt}(V_m \sin\omega t)$$

$$= V_m \cdot C \cdot (\cos\omega t) \cdot \omega = I_m \sin(\omega t + 90)$$

$v = V_m \angle 0^\circ$

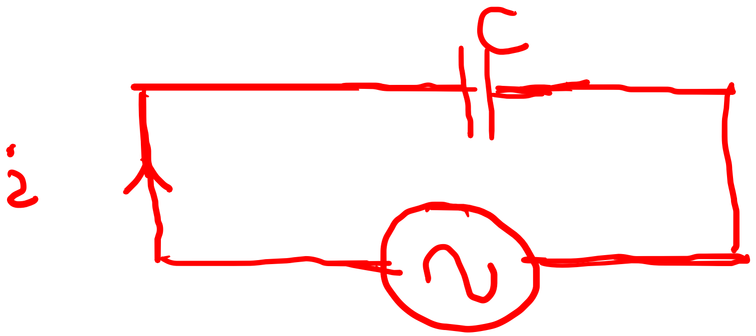
$i = I_m \angle +90^\circ$

$\Rightarrow$ phasor diagram



Current leads voltage by 90°.

Reponse of Pure Capacitor to AC input



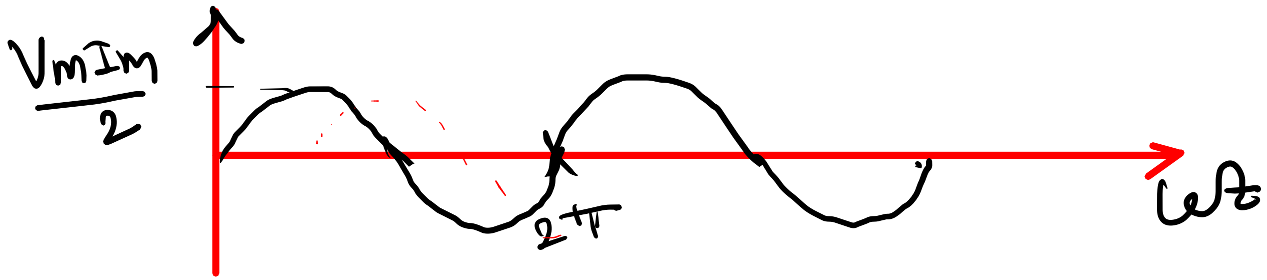
$$v = V_m \sin\omega t$$

$$P_{inst} = v \cdot i$$

$$= V_m \sin\omega t \cdot I_m \cdot \cos\omega t$$

$$= V_m I_m \frac{\sin(2\omega t)}{2}$$

$$P_{inst} = \frac{V_m I_m}{2} \sin(2\omega t)$$



$$P_{av} = \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} \sin(2\omega t)\, d\omega t$$

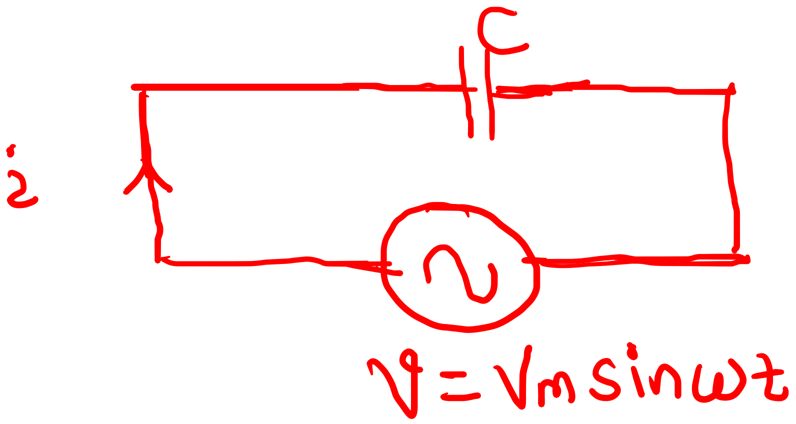
$$P_{av} = \frac{V_m I_m}{4\pi} \left[-\frac{\cos 2\omega t}{2} \right]_0^{2\pi}$$

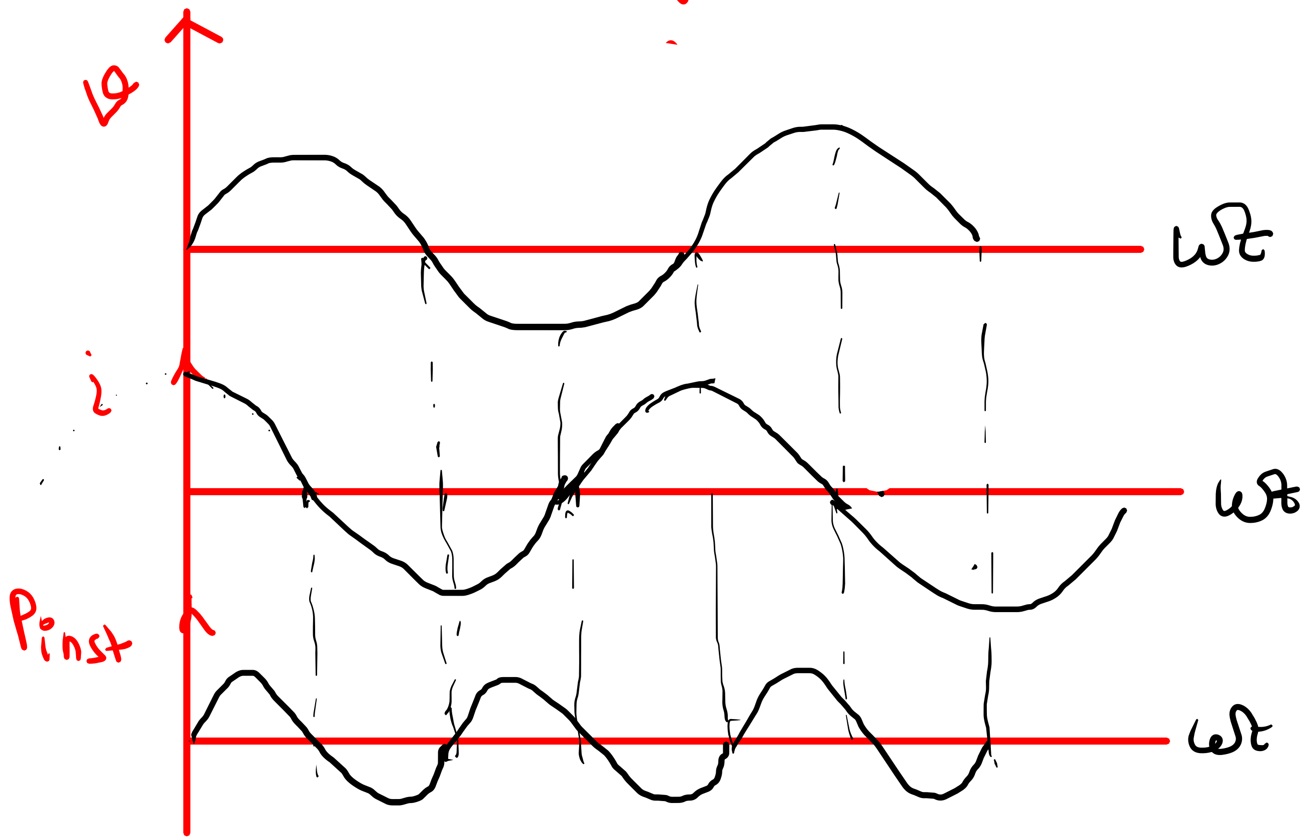
$$= \frac{-V_m I_m}{8\pi} \left[\cos 4\pi - \cos 0 \right]$$

$$P_{av} = \frac{-V_m I_m}{8\pi} [\underbrace{1 - 1}_{0}]$$

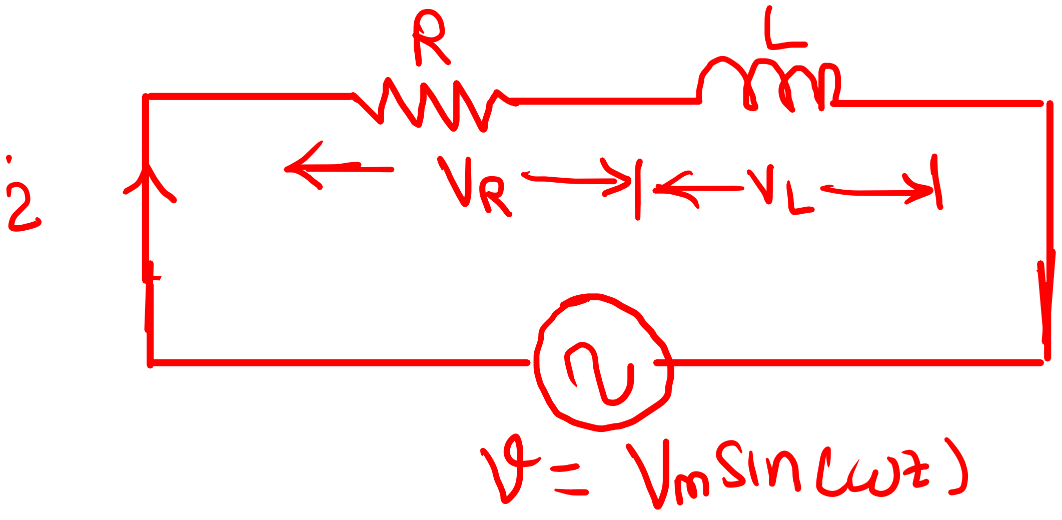
$$\boxed{P_{av} = 0}$$

Reponse of Pure Capacitor to AC input





Response of Resistor and Inductor series combination to ac input



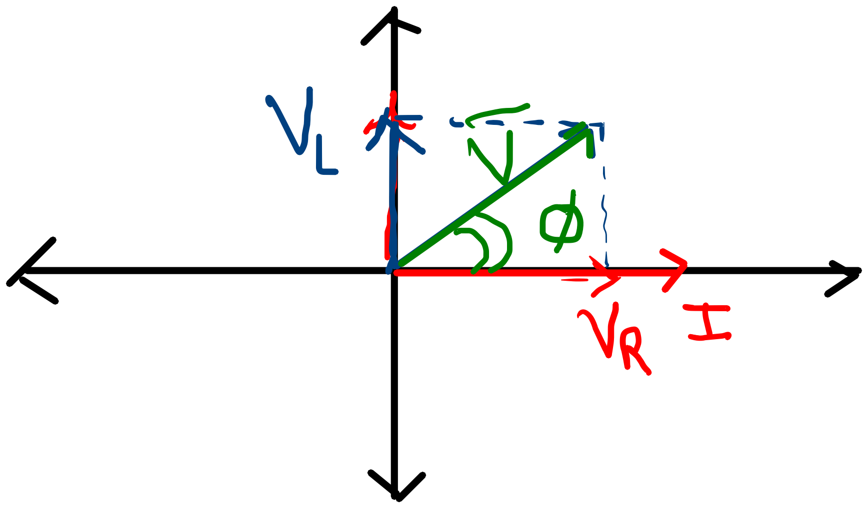
$$\vec{V_R} + \vec{V_L} = \vec{V}$$

⇒ angle between applied & resultant current is ϕ

⇒ voltage (V) leads the current I by ϕ.

⇒ V_R, V_L, V, I are rms values

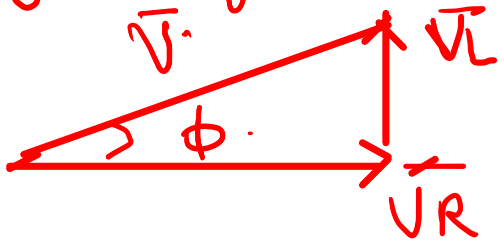
⇒ phasor diagram.



⇒ $v = V_m \sin\omega t$.

$i = I_m \sin(\omega t - \phi)$

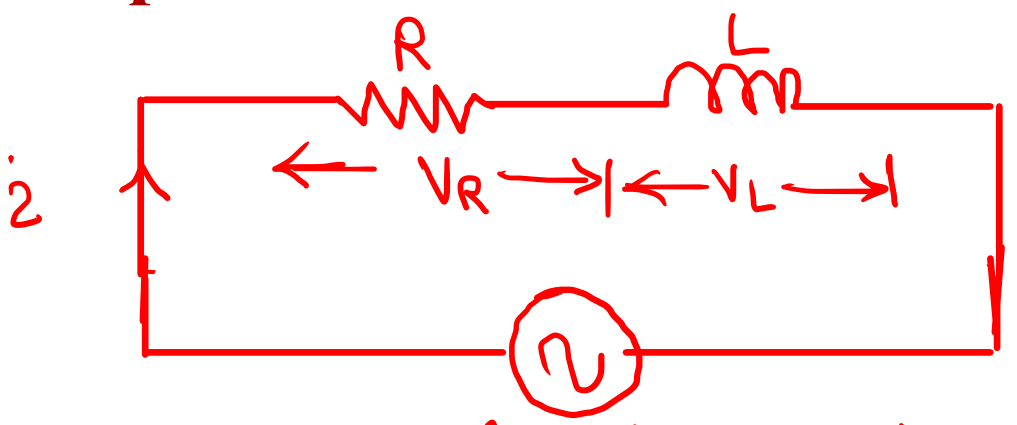
⇒ using triangle Law of vectors.



voltage triangle

Response of Resistor and Inductor series combination to ac input

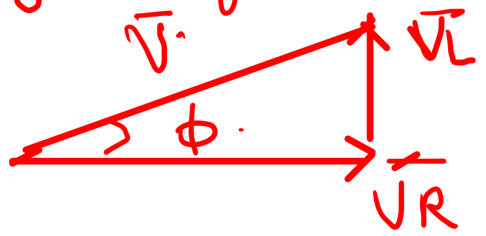
(R-L series Circuit)



$$v = V_m \sin(\omega t)$$

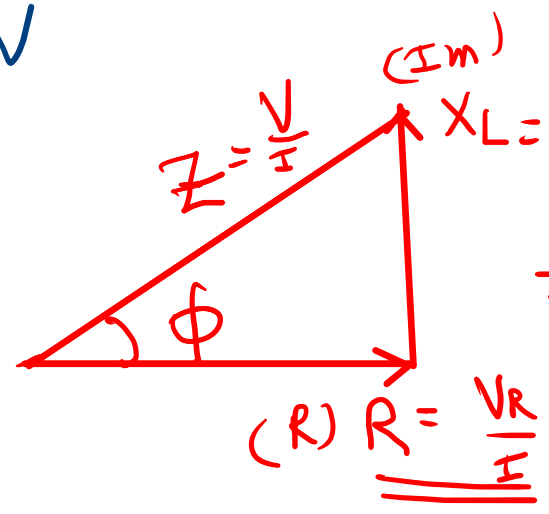
$$\bar{V}_R + \bar{V}_L = \bar{V}$$

⇒ using triangle Law of vectors.



Voltage triangle

⇒



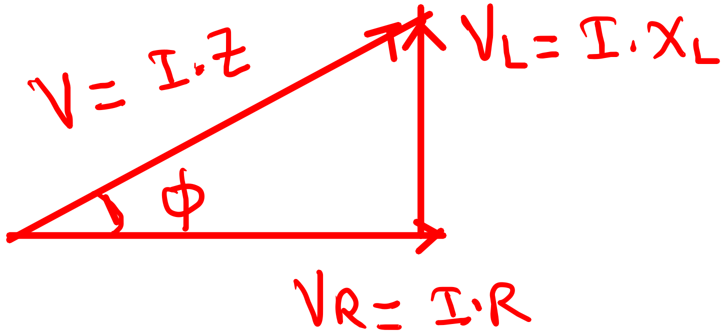
⇒ Impedance triangle

$$Z = R + jX_L$$ for R-L Series circuit

$$|Z| = \sqrt{R^2 + X_L^2}$$

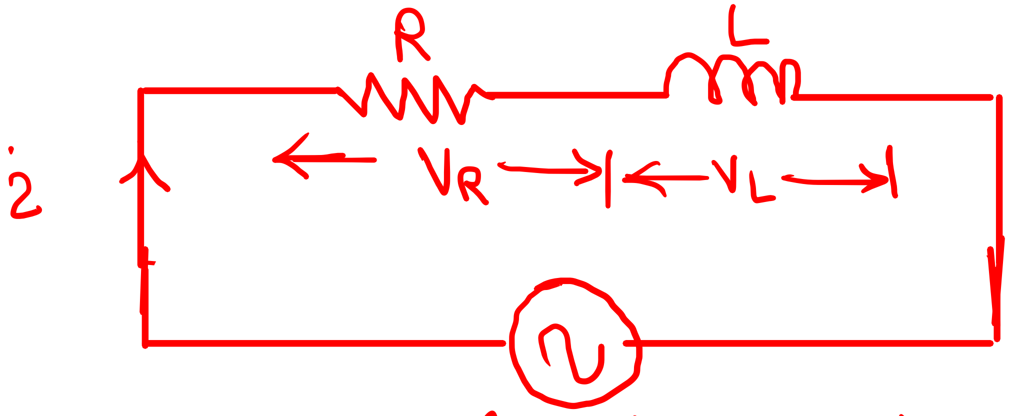
$$\angle Z = \tan^{-1}\left(\frac{X_L}{R}\right)$$

⇒



Response of Resistor and Inductor series combination to ac input

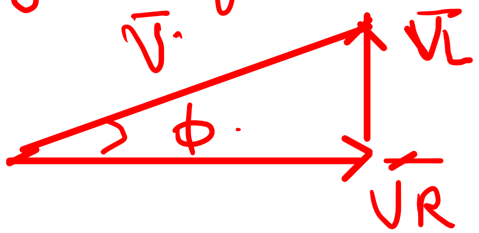
(R-L series Circuit)



$\vartheta = V_m \sin(\omega t)$

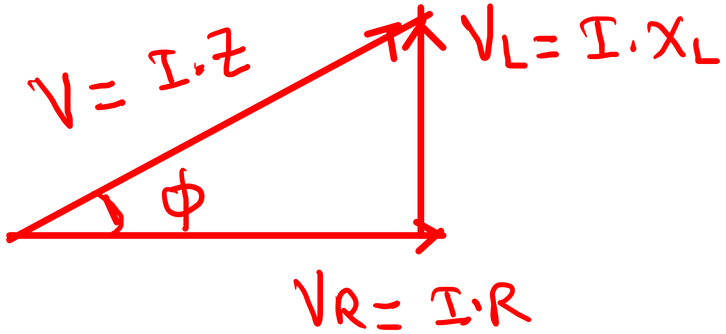
$$\bar{V}_R + \bar{V}_L = \bar{V}$$

$\Rightarrow$ using triangle Law of vectors.

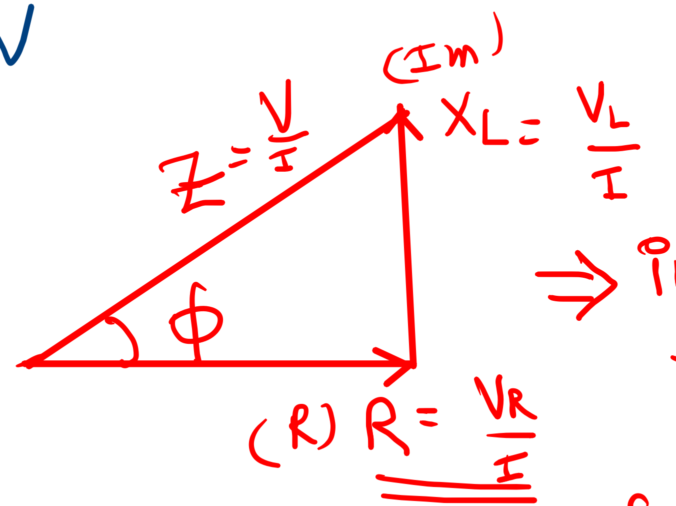


Voltage triangle

$\Rightarrow$



$\Rightarrow$



$\Rightarrow$ impedance triangle

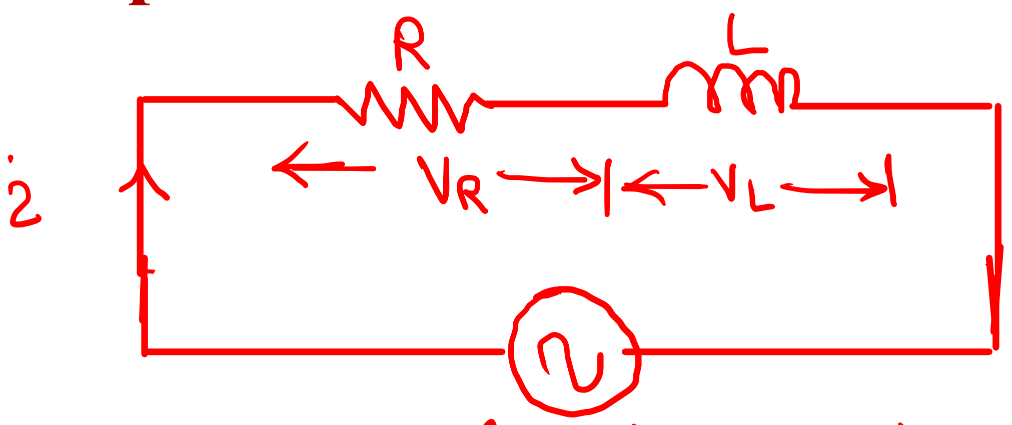
$$Z = R + jX_L$$ for R-L Series Circuit

$$|Z| = \sqrt{R^2 + X_L^2}$$

$$\measuredangle Z = \tan^{-1}\left(\frac{X_L}{R}\right)$$

Response of Resistor and Inductor series combination to ac input

(R-L series Circuit)



$v = V_m \sin(\omega t)$

$\Rightarrow P_{inst} = v \cdot i$

$= V_m \sin\omega t \cdot I_m \sin(\omega t - \phi)$

using $2\sin A \cdot \sin B = \cos(A-B) - \cos(A+B)$

$= V_m I_m \sin\omega t \cdot \sin(\omega t - \phi)$

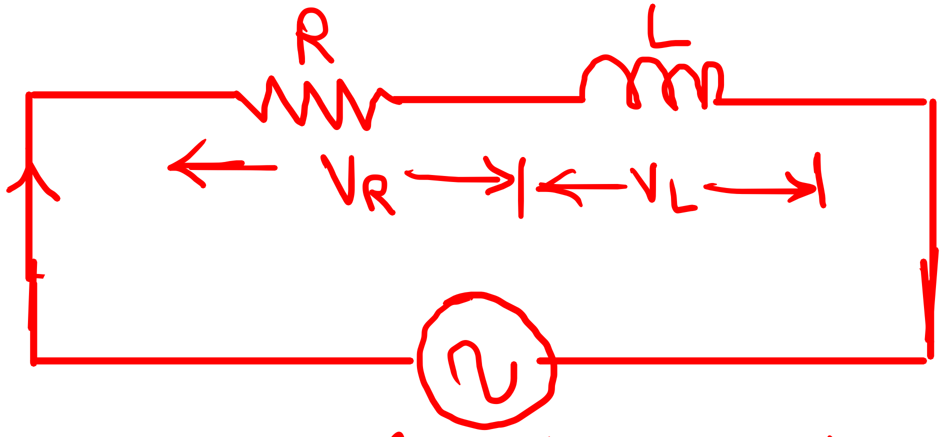
$= \frac{V_m I_m}{2}\left[\cos(\omega t - \omega t + \phi) - \cos(2\omega t - \phi)\right]$

$= \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t - \phi)\right]$

$P_{inst} = \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi)$

Response of Resistor and Inductor series combination to ac input

(R-L series Circuit)



2

$v = V_m \sin(\omega t)$

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} P_{inst} \cdot d\omega t$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi)\right] d\omega t$$

$$= \frac{1}{2\pi}\left[\frac{V_m I_m}{2}\cos\phi\,(\omega t) - \frac{V_m I_m}{2}\,\frac{\sin(2\omega t - \phi)}{2}\right]_0^{2\pi}$$

$$P_{av} = \frac{V_m I_m}{4\pi}\left[\cos\phi(2\pi) - \frac{\sin(-\phi)}{2} - 0 + \frac{\sin(-\phi)}{2}\right]$$

$$P_{av} = \frac{V_m I_m}{4\pi}\left[\cos\phi(2\pi) + \cancel{\frac{\sin\phi}{2}} - \cancel{\frac{\sin\phi}{2}}\right]$$

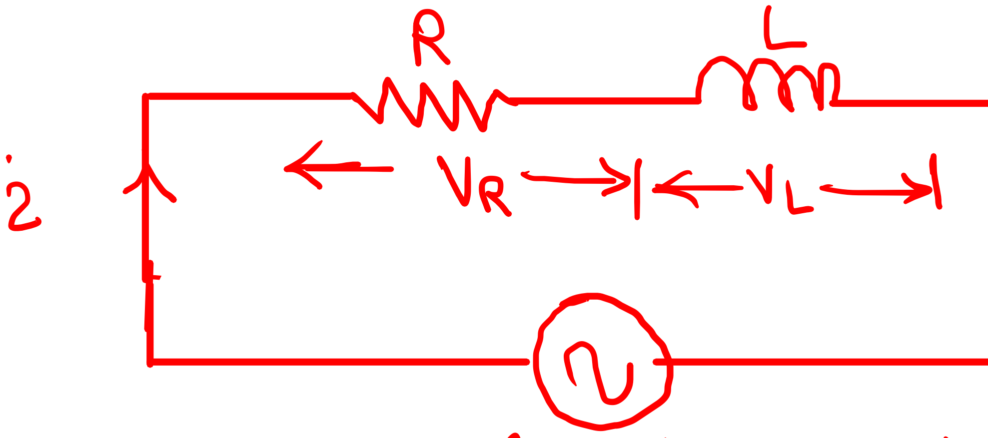
$$P_{av} = \frac{V_m I_m}{\cancel{4}\pi} \cdot 2\pi \cdot \cos\phi$$

$$P_{av} = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}\cos\phi$$

$$\boxed{P_{av} = V_{rms} \cdot I_{rms} \cos\phi}$$

Response of Resistor and Inductor series combination to ac input

(R-L series Circuit)



$v = V_m \sin(\omega t)$

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} P_{inst} \cdot d\omega t$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \left[\frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi)\right] d\omega t$$

$$= \frac{1}{2\pi}\left[\frac{V_m I_m}{2}\cos\phi\,(\omega t) - \frac{V_m I_m}{2}\frac{\sin(2\omega t - \phi)}{2}\right]_0^{2\pi}$$

$$P_{av} = \frac{V_m I_m}{4\pi}\left[\cos\phi(2\pi) - \frac{\sin(-\phi)}{2} - 0 + \frac{\sin(-\phi)}{2}\right]$$

$$P_{av} = \frac{V_m I_m}{4\pi}\left[\cos\phi(2\pi) + \cancel{\frac{\sin\phi}{2}} - \cancel{\frac{\sin\phi}{2}}\right]$$

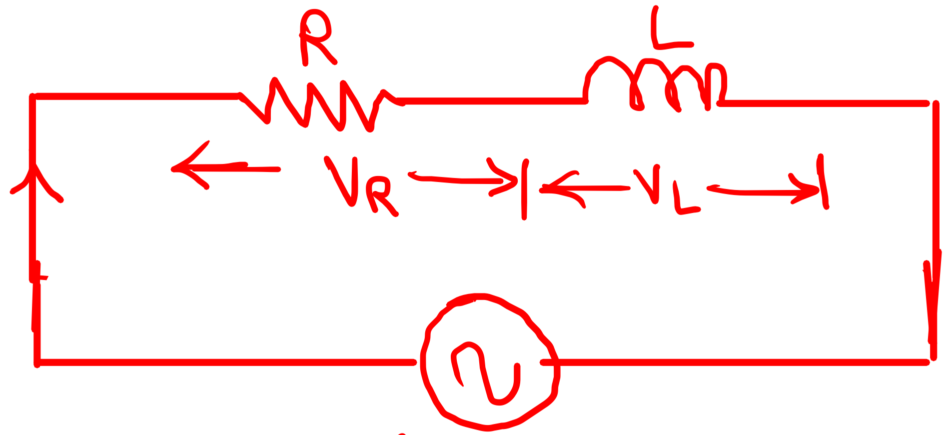
$$P_{av} = \frac{V_m I_m}{4\pi} \cdot 2\pi \cdot \cos\phi$$

$$P_{av} = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}\cos\phi$$

$$\boxed{P_{av} = V_{rms} \cdot I_{rms} \cos\phi}$$

Response of Resistor and Inductor series combination to ac input

(R-L series Circuit)



$$v = V_m \sin(\omega t)$$

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} P_{inst} \cdot d\omega t$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \left[\frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi)\right] d\omega t$$

$$= \frac{1}{2\pi}\left[\frac{V_m I_m}{2}\cos\phi\,(\omega t) - \frac{V_m I_m}{2}\,\frac{\sin(2\omega t - \phi)}{2}\right]_0^{2\pi}$$

$$P_{av} = \frac{V_m I_m}{4\pi}\left[\cos\phi\,(2\pi) - \frac{\sin(-\phi)}{2} - 0 + \frac{\sin(-\phi)}{2}\right]$$

$$P_{av} = \frac{V_m I_m}{4\pi}\left[\cos\phi\,(2\pi) + \cancel{\frac{\sin\phi}{2}} - \cancel{\frac{\sin\phi}{2}}\right]$$

$$P_{av} = \frac{V_m I_m}{4\pi} \cdot 2\pi \cdot \cos\phi$$

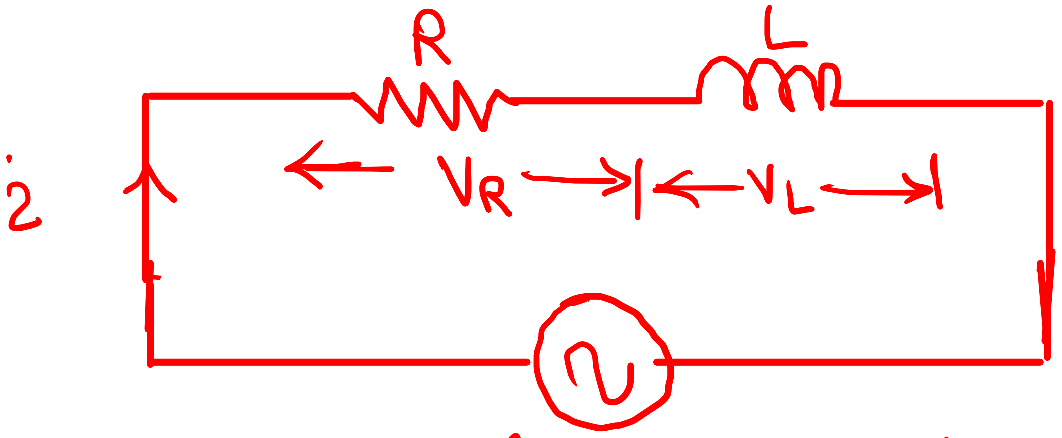
$$P_{av} = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}\cos\phi$$

$$\boxed{P_{av} = V_{rms} \cdot I_{rms} \cos\phi}$$

https://www.youtube.com/watch?v=OECFsfKxyYo

Response of Resistor and Inductor series combination to ac input

(R-L series circuit)



$v = V_m \sin(\omega t)$

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} P_{inst} \cdot d\omega t$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi)\right] d\omega t$$

$$= \frac{1}{2\pi}\left[\frac{V_m I_m}{2}\cos\phi\,(\omega t) - \frac{V_m I_m}{2}\frac{\sin(2\omega t - \phi)}{2}\right]_0^{2\pi}$$

$$P_{av} = \frac{V_m I_m}{4\pi}\left[\cos\phi(2\pi) - \frac{\sin(-\phi)}{2} - 0 + \frac{\sin(-\phi)}{2}\right]$$

$$P_{av} = \frac{V_m I_m}{4\pi}\left[\cos\phi(2\pi) + \cancel{\frac{\sin\phi}{2}} - \cancel{\frac{\sin\phi}{2}}\right]$$

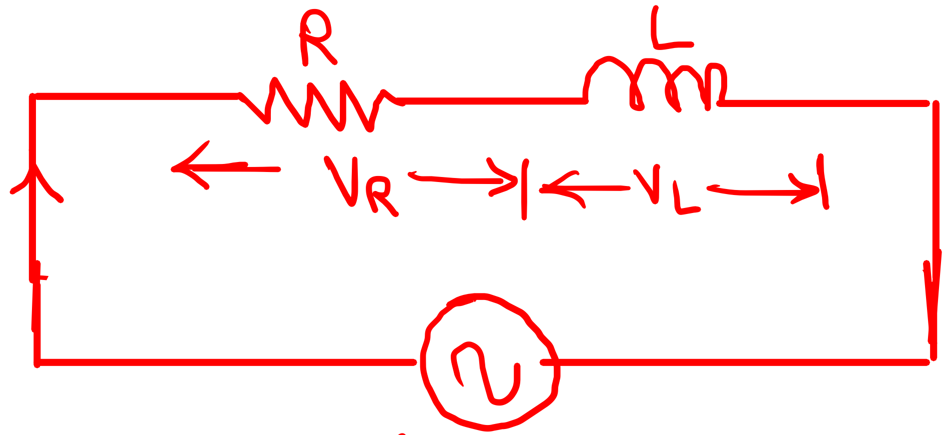
$$P_{av} = \frac{V_m I_m}{4\pi} \cdot 2\pi \cdot \cos\phi$$

$$P_{av} = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}\cos\phi$$

$$\boxed{P_{av} = V_{rms} \cdot I_{rms} \cos\phi}$$

Response of Resistor and Inductor series combination to ac input

2



$v = V_m \sin(\omega t)$

(R-L series Circuit)

⇒ Active power.

$P_{av} = P_{active} = V_{rms} I_{rms} \cos\phi$

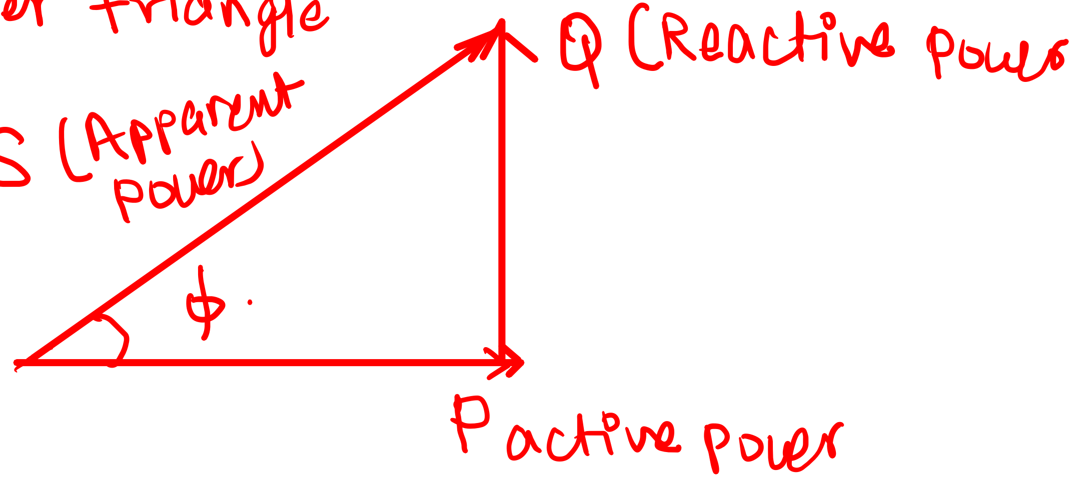
→ ϕ is angle between voltage & current.

$\cos\phi$ ⇒ power Factor.

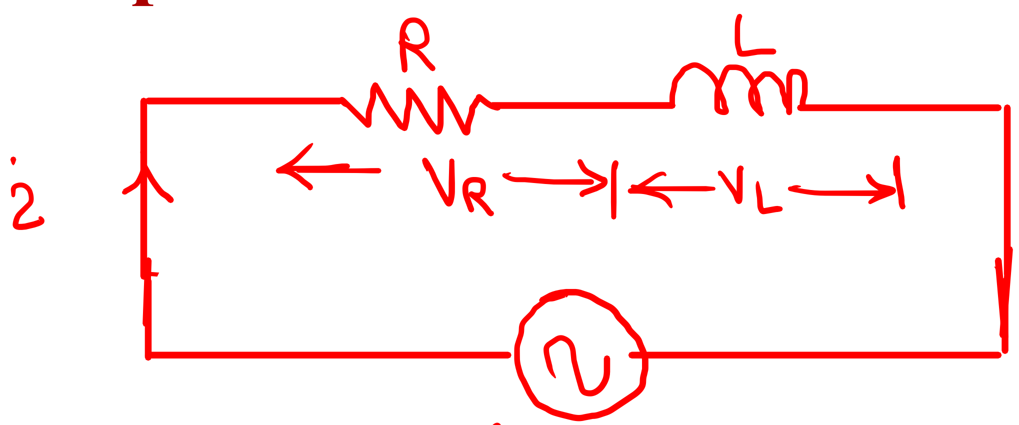
$P_{dc} = V \cdot I$

$$\boxed{\rightarrow P_{ac} = V \cdot I \cos\phi}$$

⇒ power triangle

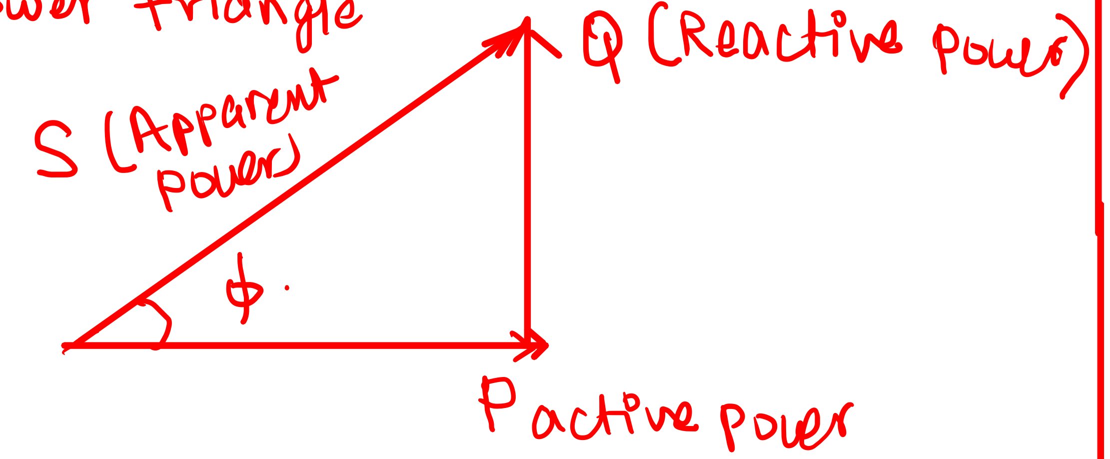


Response of Resistor and Inductor series combination to ac input



$v = V_m \sin(\omega t)$

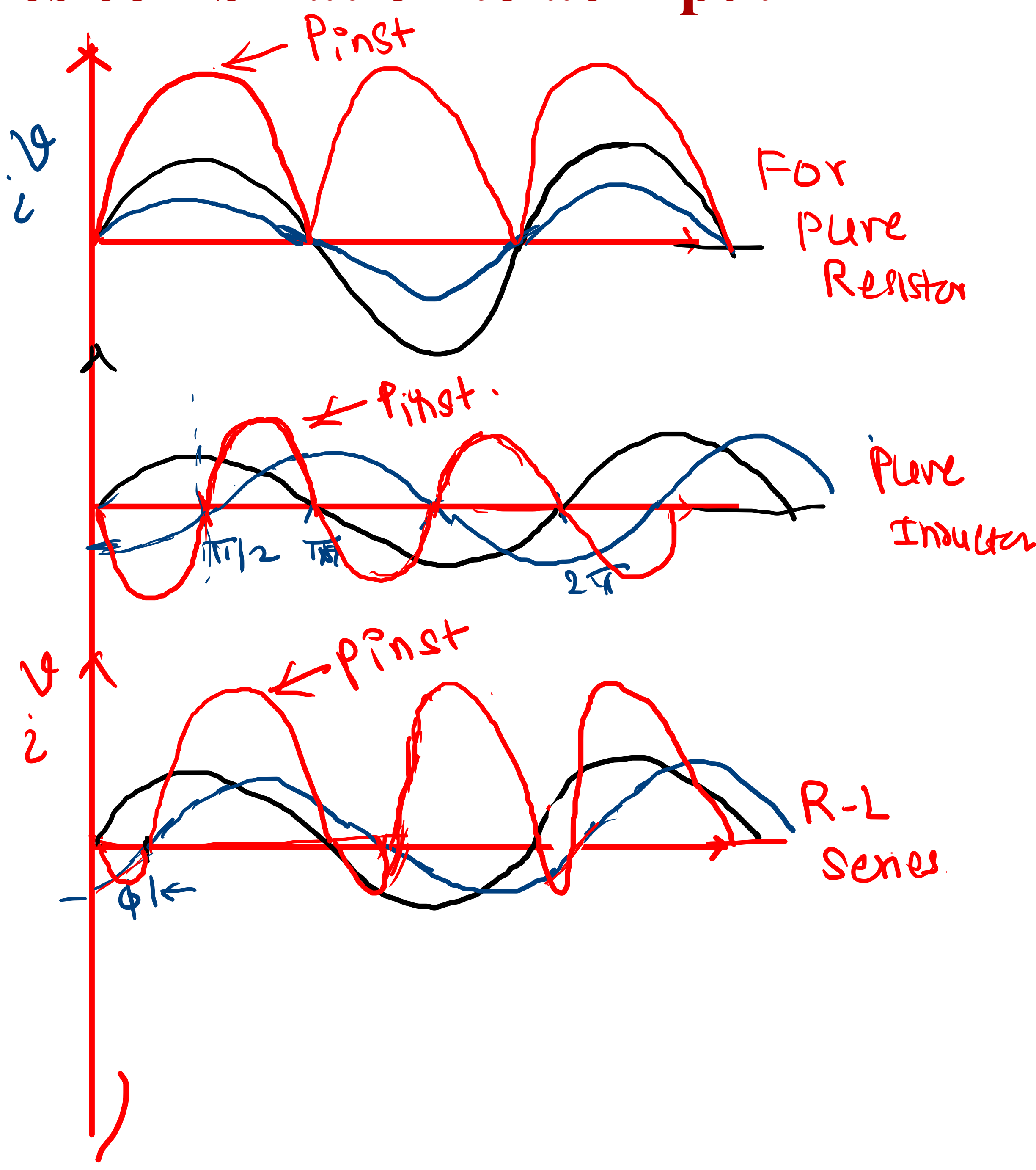
⇒ power triangle



$P_{active} = VI\cos\phi$ (Watts)

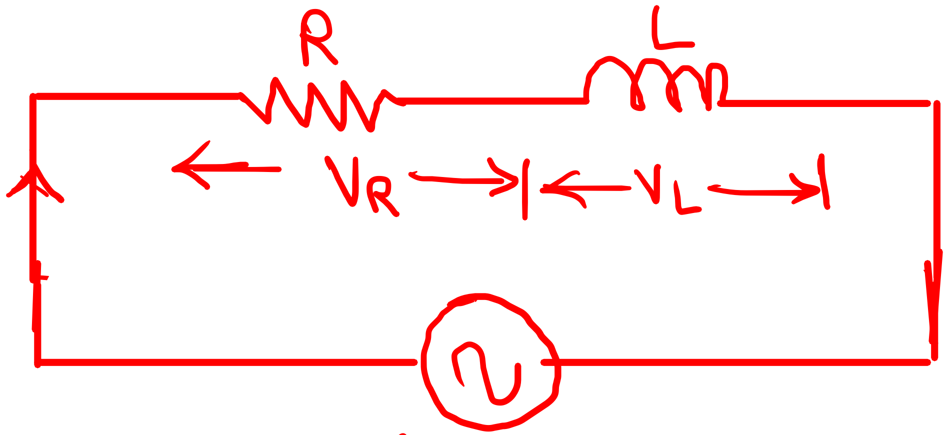
$Q = VI\sin\phi$ (VAR)

$S = \sqrt{P^2 + Q^2}$ (VA)



Response of Resistor and Inductor series combination to ac input

2



$v = V_m \sin(\omega t)$

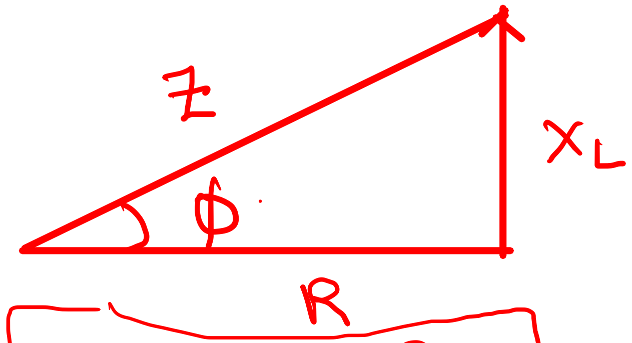
$\Rightarrow Z = R + jX_L$

$|Z| = \sqrt{R^2 + X_L^2}$

$\angle Z = \tan^{-1}\left(\frac{X_L}{R}\right) = \phi$

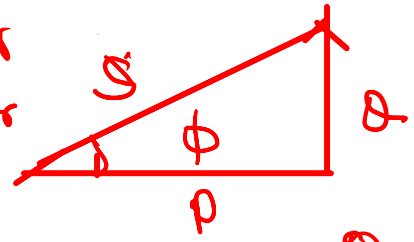
$PF = \cos\phi = \cos\left(\tan^{-1}\left(\frac{X_L}{R}\right)\right)$

$\Rightarrow$ power factor $\cos\phi$.



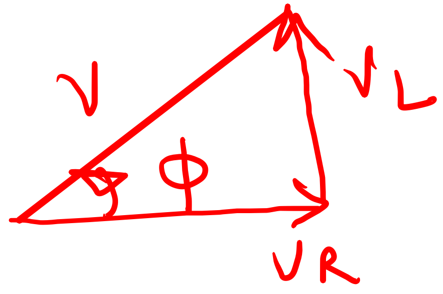
$$\boxed{\cos\phi = \frac{R}{Z}}$$

$\Rightarrow$ Power factor



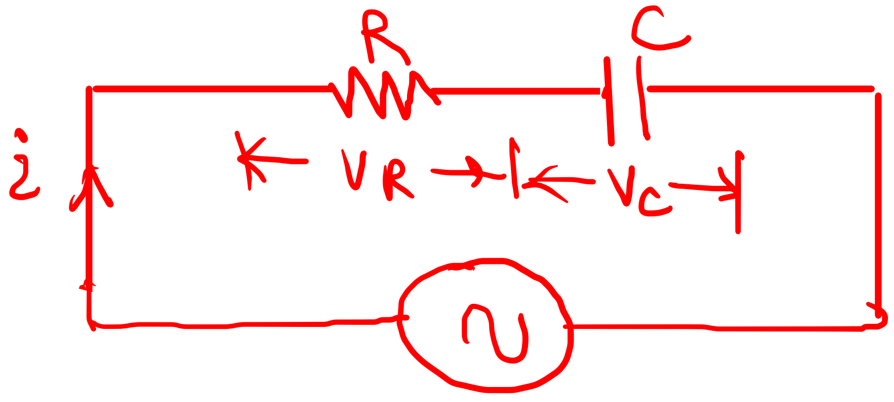
$\cos\phi = \frac{P}{S}$

$\Rightarrow$ Voltage triangle



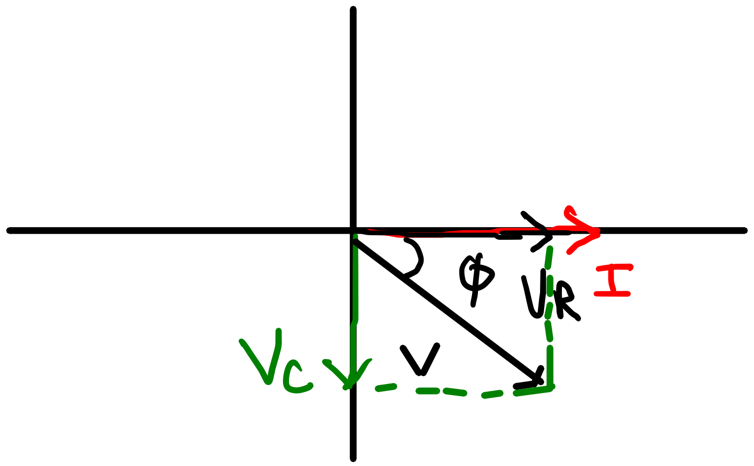
$\cos\phi = \frac{V_R}{V}$

Response of Resistor and capacitor series combination to ac input



$v = V_m \sin\omega t$

$\Rightarrow$ phasor diagram. $\bar{V} = \bar{V}_R + \bar{V}_C$



Current I leads V by ϕ°.
Leading power factor

$v = V_m \sin\omega t$

$i = I_m \sin(\omega t + \phi)$

$P_{inst} = V_m \sin\omega t \cdot I_m \sin(\omega t + \phi)$

$P_{inst} = V_m I_m \sin(\omega t) \cdot \sin(\omega t + \phi)$

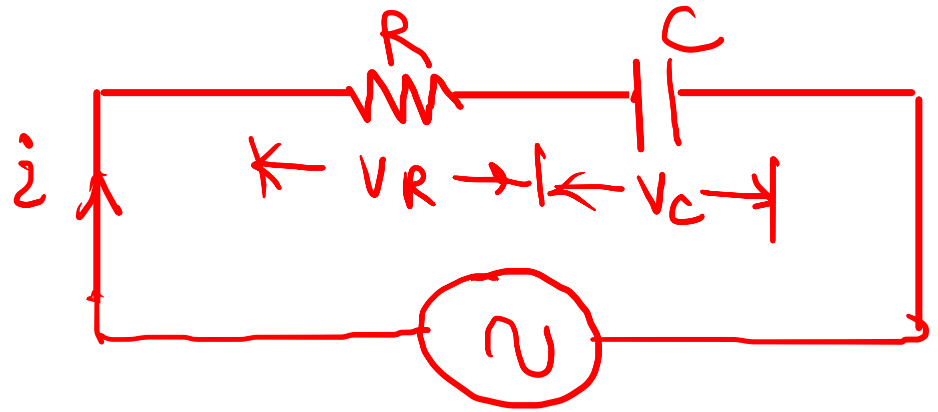
$P_{inst} = \frac{V_m I_m}{2}\left[\cos(\omega t - \omega t - \phi) - \cos(\omega t + \omega t + \phi)\right]$

$P_{inst} = \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t + \phi)$

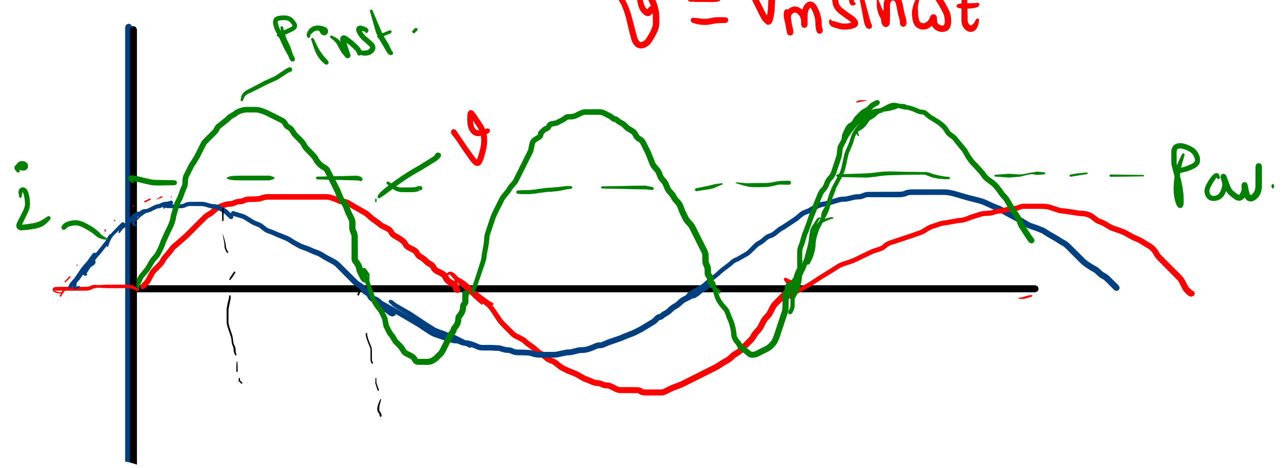
$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} P_{inst} \cdot d\omega t = \frac{V_m I_m}{2}\cos\phi$

$\boxed{P_{av} = V_{rms} \cdot I_{rms} \cos\phi}$

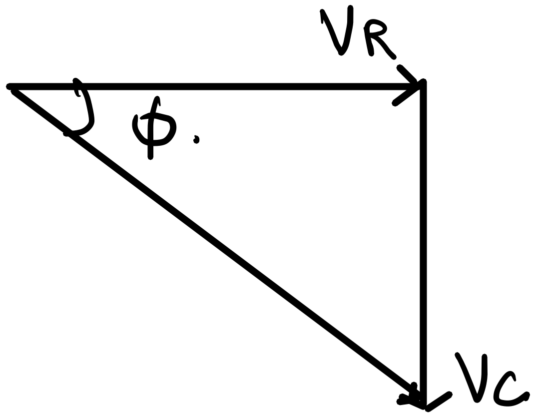
Response of Resistor and capacitor series combination to ac input



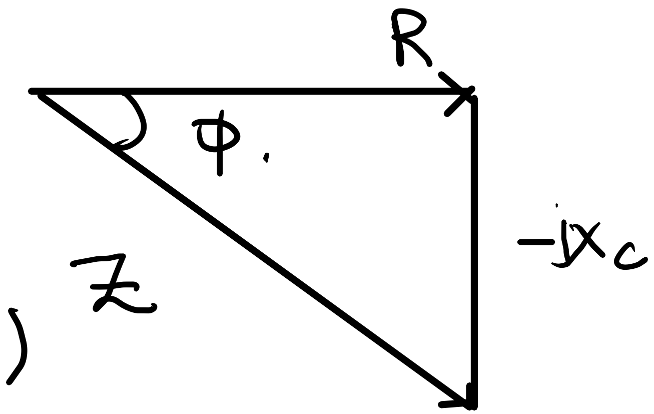
$v = V_m \sin\omega t$



⇒ Voltage triangle



⇒ Impedance triangle



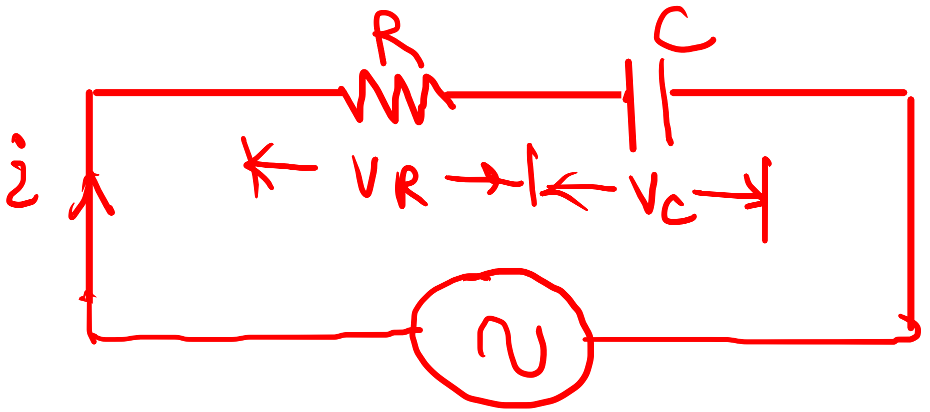
$$Z = R - jX_C$$

$$|Z| = \sqrt{R^2 + X_C^2}$$

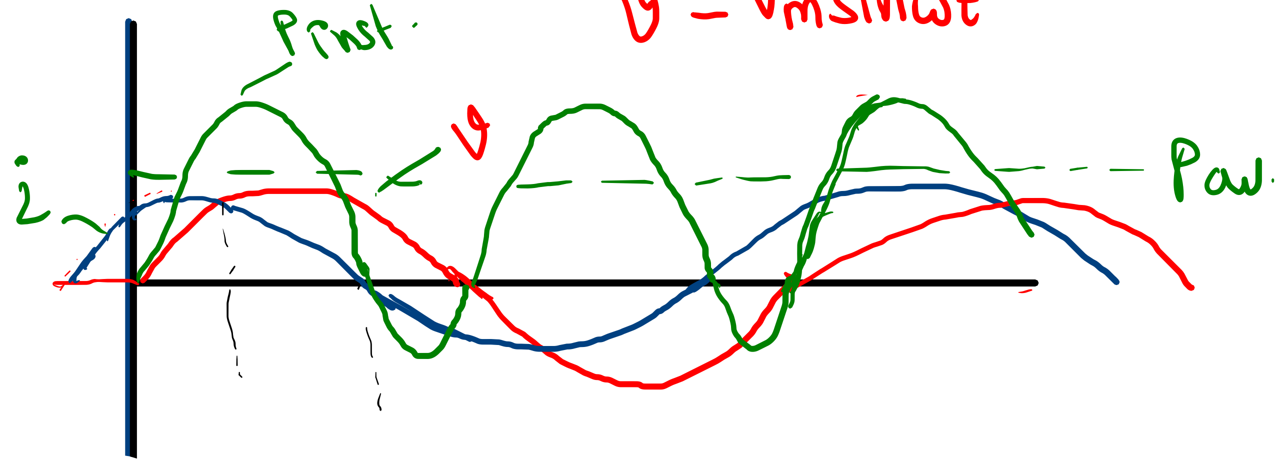
$$\measuredangle Z = \tan^{-1}\left(\frac{-X_C}{R}\right)$$

$\phi \rightarrow$ negative

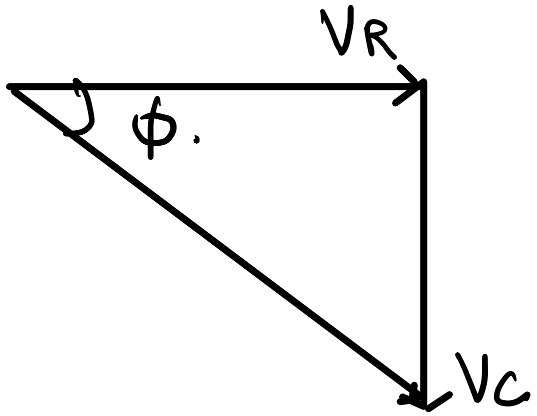
Response of Resistor and capacitor series combination to ac input



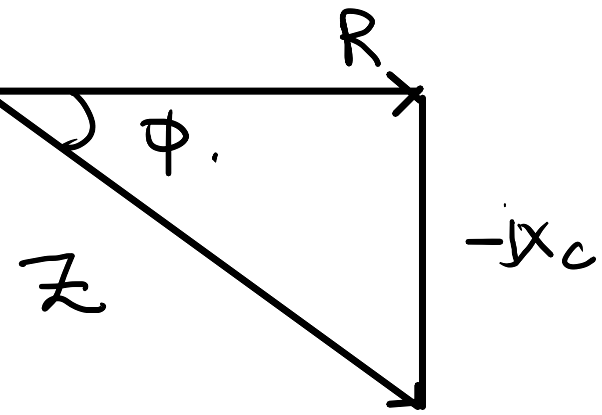
$v = V_m \sin \omega t$



⇒ Voltage triangle



⇒ Impedance triangle



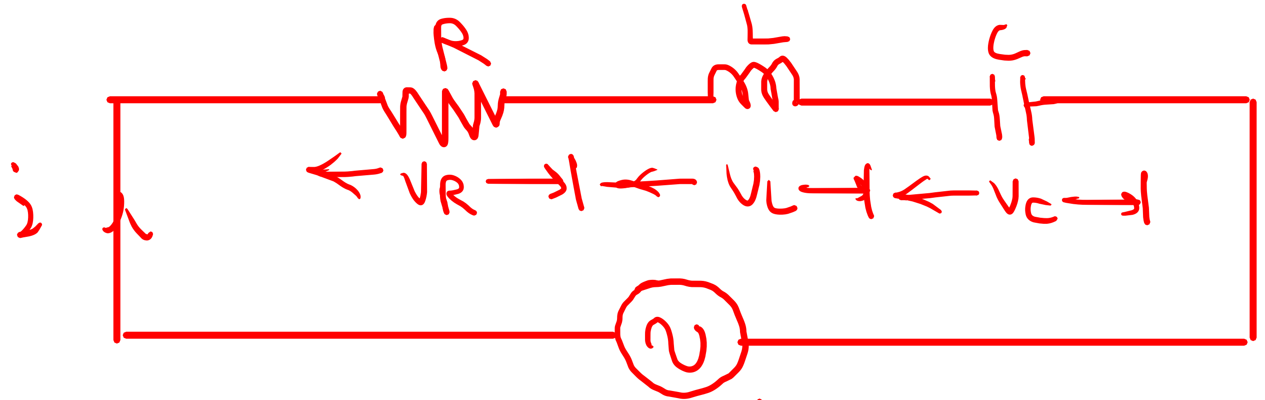
$$Z = R - jX_C$$

$$|Z| = \sqrt{R^2 + X_C^2}$$

$$\angle Z = \tan^{-1}\left(\frac{-X_C}{R}\right)$$

$\phi \rightarrow$ negative

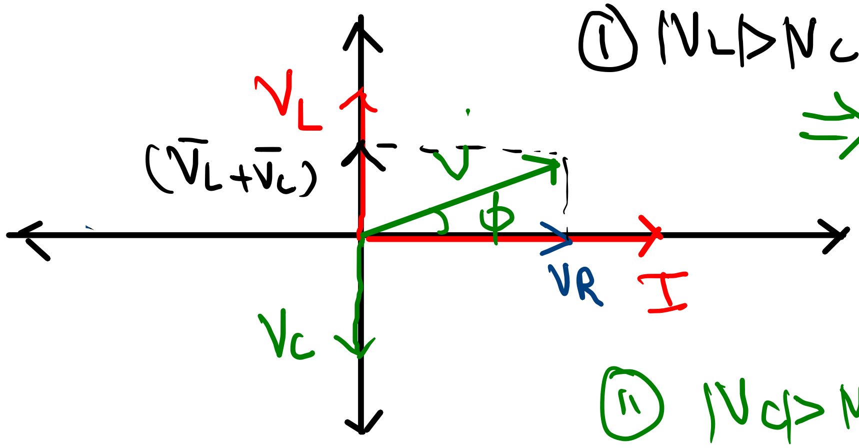
Response of Resistor , inductor and capacitor series combination to ac input



$v = V_m \sin \omega t$

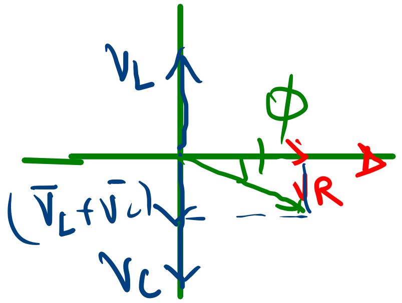
$\Rightarrow$ phasor diagram $\quad \bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C$

(i) $|V_L| > |V_C|$



$\Rightarrow$ I lags V by ϕ
so lagging pf

(ii) $|V_C| > |V_L|$



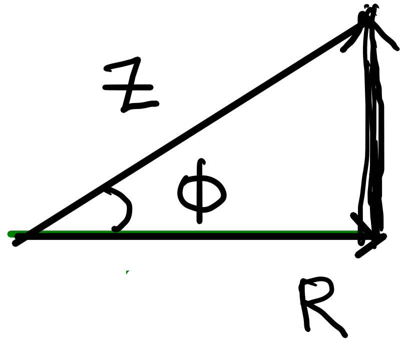
I lead V by ϕ
leading pf

$\Rightarrow \quad v = V_m \sin \omega t$

$i = I_m \sin(\omega t \pm \phi)$

$\Rightarrow$ Impedance triangle

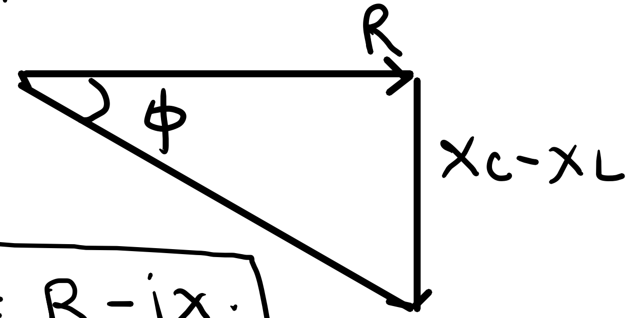
$\Rightarrow$



$X_L > X_C$

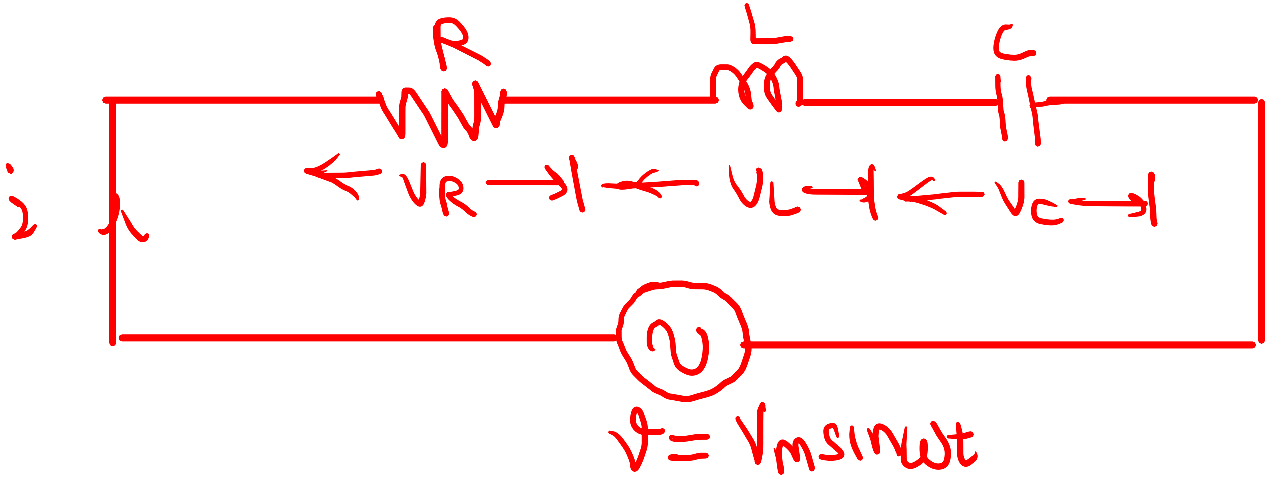
$Z = R + j(X_L - X_C) = \boxed{R + jX}$

$\Rightarrow \quad X_C > X_L$



$\boxed{Z = R - jX}$

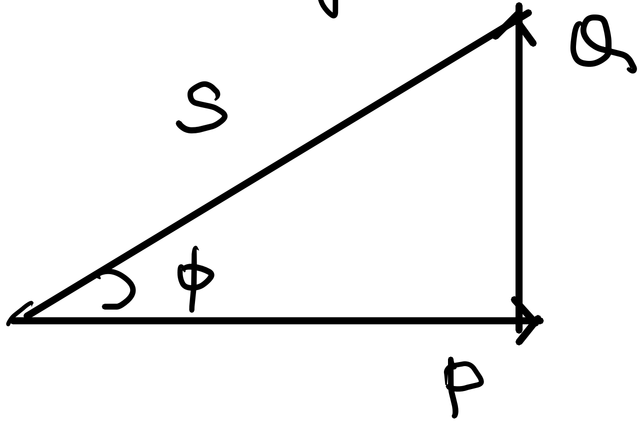
Response of Resistor , inductor and capacitor series combination to ac input



$\Rightarrow$ if V & I known
$f \rightarrow$ known

$Z = \dfrac{V}{I}$,

$\Rightarrow$ power triangle



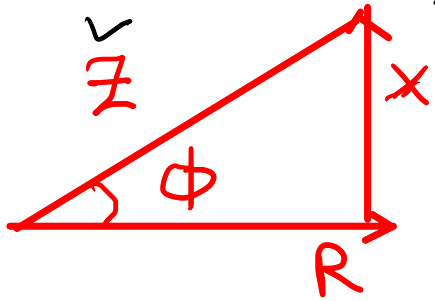
ϕ ✓

$PF = \cos\phi$ ✓

$\Rightarrow$ R, L & C is known
ω or f is known

$Z = R + j(X_L - X_C)$ ✓

$I = \dfrac{V}{Z}$ ✓



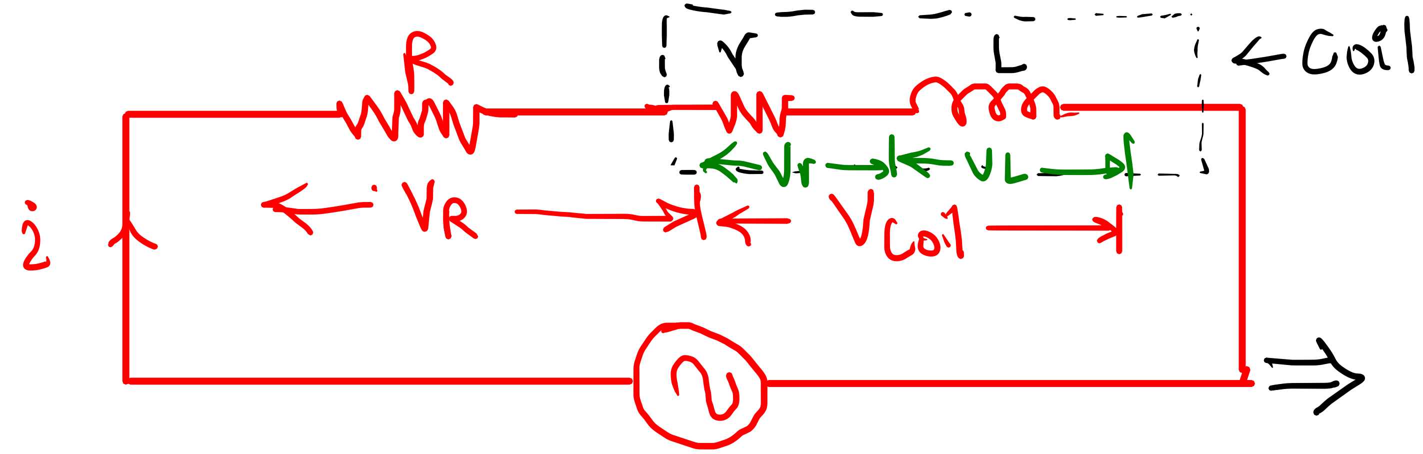
ϕ ✓

$PF = \cos\phi$

$R = Z\cos\phi \qquad X = Z\sin\phi$

R-L series circuit with AC input where inductor is Impure

⇒



$v = V_m \sin \omega t$

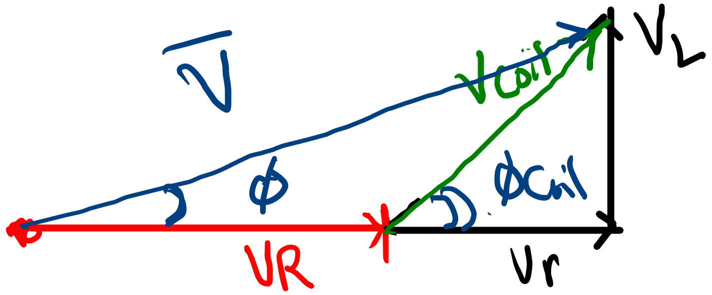
⇒ $\bar{V} = \bar{V}_R + \bar{V}_{coil}$

$\bar{V}_{coil} = \bar{V}_r + \bar{V}_L$

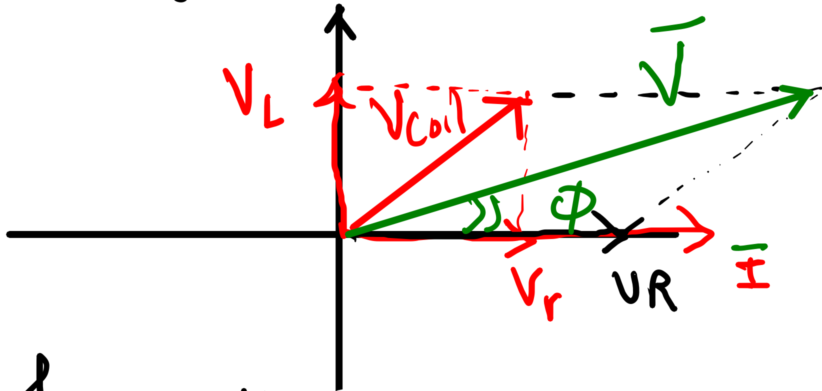
⇒ ϕ_{coil} power factor angle of coil

ϕ ⇒ over all pF angle of the circuit

⇒ Voltage triangle



⇒ phasor diagram.



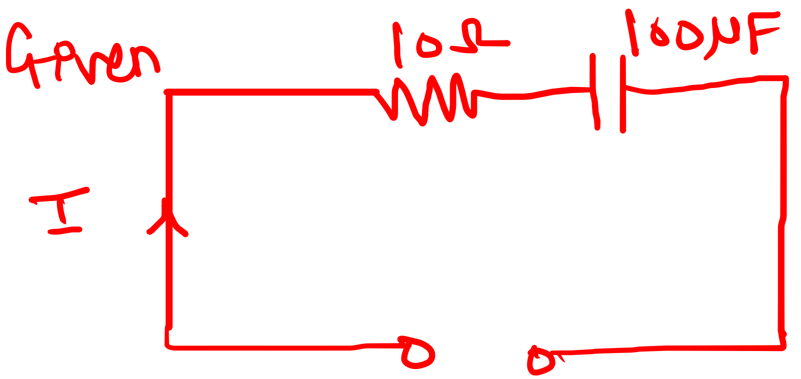
⇒ Law of parallelogram $V_1 - V_2$, ϕ

$\bar{V} = \bar{V}_1 + \bar{V}_2$

$$|V| = \sqrt{V_1^2 + V_2^2 + 2 V_1 V_2 \cos \phi}$$

Example ① ⇒ A capacitor which has an internal resistance of 10Ω & capacitance value of $100\mu F$ is connected to ac voltage $v(t) = 100\sin(314t)$. Calculate current flowing through the circuit & construct voltage triangle.

⇒ Given



$V(t) = 100\sin 314t$

$$I = \frac{V}{Z}$$

$$Z = R - jx_c$$

$R = 10\Omega$, $C = 100\mu F$, $V(t) = V_m \sin(\omega t)$

$\therefore \omega = 314$

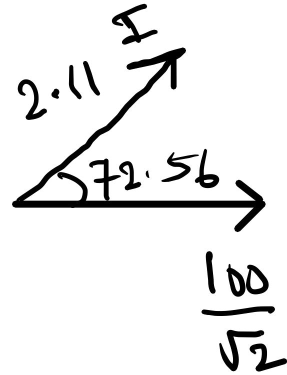
$$X_c = \frac{1}{\omega C}$$

$$x_c = \frac{1}{314 \times 100 \times 10^{-6}} = 31.84\,\Omega$$

$$Z = (10 - j31.84) = \sqrt{10^2 + 31.84^2}\angle \tan^{-1}\left(\frac{-31.84}{10}\right)$$

$$I = \frac{100/\sqrt{2}\angle 0^\circ}{(10 - j31.84)} = \frac{70.71\angle 0^\circ}{33.37\angle -72.56^\circ}$$

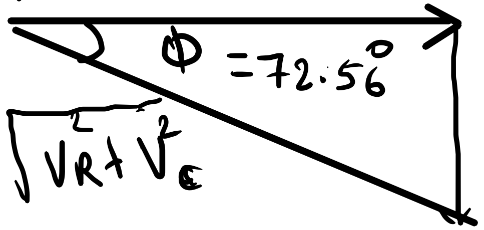
$$I = 2.11\angle 0 + 72.56^\circ = 2.11\angle +72.56^\circ$$



$I_m = 2.11 \times \sqrt{2} =$

$i = I_m \sin(\omega t + 72.56) = 2.98\sin(314t + 72.56)$

⇒ Voltage triangle



$VR = 21.1$

$V = \sqrt{V_R^2 + V_c^2}$

$V_C = 2.11 \times 31.84$

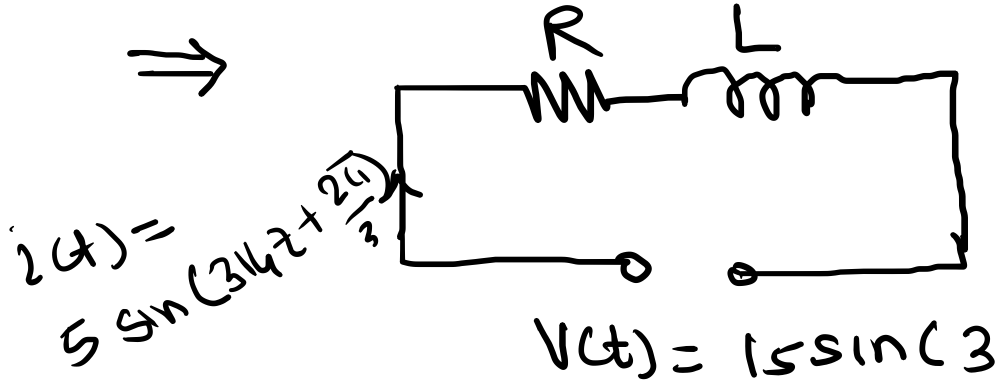
$=$

Example (2) In a series circuit containing a pure resistance and a pure inductor. The current & voltage are given as

$$i(t) = 5\sin\left(314t + \frac{2\pi}{3}\right) \quad \& \quad V(t) = 15\sin\left(314t + \frac{5\pi}{6}\right)$$

Find (i) Impedance of the circuit (ii) Value of resistance (iii) Value of inductor (iv) Active power (v) Power factor.

$\Rightarrow$



$V(t) = 15\sin\left(314t + \frac{5\pi}{6}\right)$

$$Z = \frac{V}{I} = \frac{\frac{15}{\sqrt{2}} \angle \frac{5\pi}{6}}{\frac{5}{\sqrt{2}} \angle \frac{2\pi}{3}}$$

$$Z = 3 \angle 150 - 120^\circ)$$

$Z = R + jX_L$

$Z = r\angle\phi$

$= r\cos\phi + jr\sin\phi$

$Z = 3\angle 30^\circ$

$Z = 3\cos 30^\circ + j3\sin 30^\circ$

$\phi = 30^\circ$

$Z = 2.59 + j1.5$

$\boxed{\therefore R = 2.59\Omega}$ $X_L = 1.5\Omega$

$X_L = \omega L \quad \therefore \boxed{L = \frac{X_L}{\omega} = \frac{1.5}{314} = 4.78\,mH}$

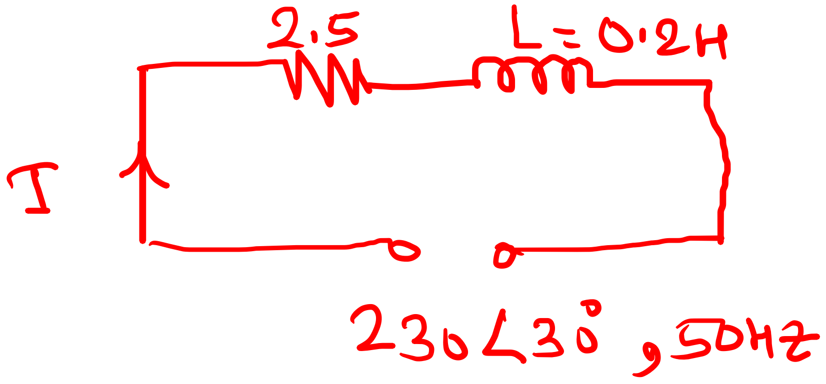
$= P = V_{rms} \cdot I_{rms} \cdot \cos\phi$

$$\boxed{P = \frac{15}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \cos 30^\circ = 37.5 \times \frac{\sqrt{3}}{2} = 32.47 \text{ Watts}}$$

$PF = \cos\phi = 0.86$ (lagging)

3). A R-L series circuit is connected across 230∠30°, 50Hz supply. The value of R is 2.5 ohms and inductor L=0.2 H. Find current flowing through the circuit and power factor of the circuit.

⇒



$\Rightarrow \quad Z = R + jX_L$

$Z = 2.5 + j(2\pi f L)$

$Z = 2.5 + j(2\pi \times 50 \times 0.2)$

$Z = 2.5 + j62.8$

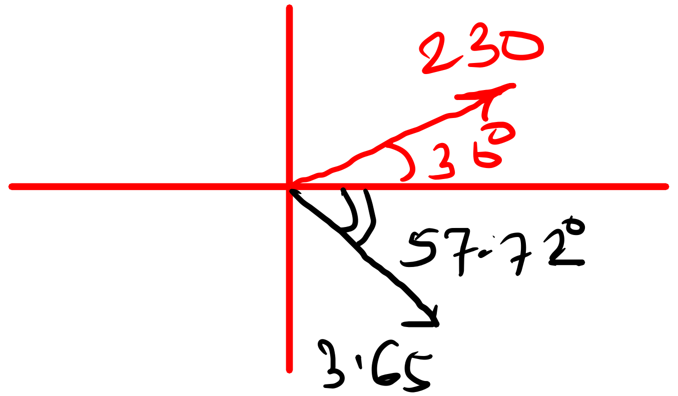
$Z = \sqrt{(2.5)^2 + (62.8)^2} \angle \tan^{-1}\left(\frac{62.8}{2.5}\right)$

$I = \frac{V}{Z} = \frac{230\angle 30°}{62.85\angle 87.72}$

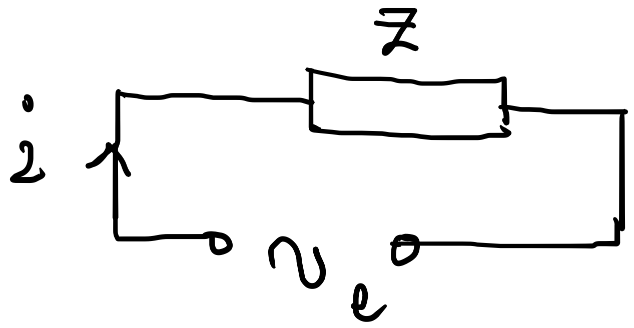
$I = \frac{230}{62.85} \angle (30° - 87.72°)$

$I = 3.65 \angle -57.72°)$

$PF = \cos\phi = \cos(87.72) = 0.03$ (lagging)



4). Two elements series circuit is connected across ac source $e=200\sqrt{2}\sin(314t + 20^\circ)$. The current flowing in the circuit is found to be $10\sqrt{2}\cos(314t-25^\circ)$. Determine the paramerters of the circuit.



$$e = 200\sqrt{2}\sin(314t+20^\circ)$$

$$i = 10\sqrt{2}\,\underline{\underline{\cos}}(314t-25^\circ)$$

$$i = 10\sqrt{2}\sin(314t-25^\circ+90^\circ)$$

$$i = 10\sqrt{2}\sin(314t+65^\circ)$$

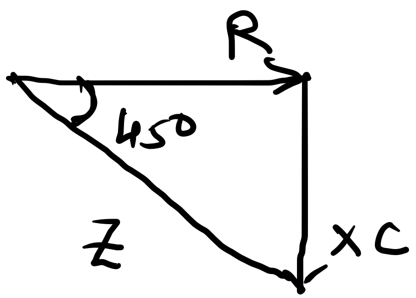
$$E = \frac{200\sqrt{2}}{\sqrt{2}}\angle 20^\circ$$

$$I = \frac{10\sqrt{2}}{\sqrt{2}}\angle 65^\circ$$

Since I leads E by $(65-20^\circ) = 45^\circ$

$$Z = \frac{E}{I} = \frac{200\angle 20^\circ}{10\angle 65^\circ}$$

$$Z = 20\angle -45^\circ$$



$$\cos 45^\circ = \frac{R}{Z}$$

$$R = Z\cos 45^\circ = 20\cos(45^\circ)$$

$$X_C = Z\sin 45^\circ = 20\sin(45^\circ)$$

$$X_C = \frac{1}{2\pi f c} = 14.14$$

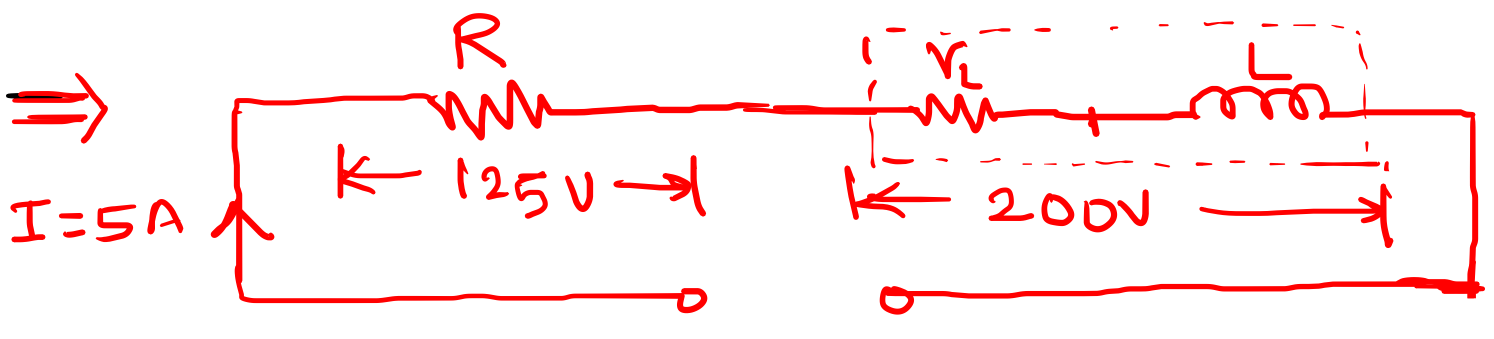
$$\therefore C = \frac{1}{\omega \cdot X_C} = \frac{1}{314\times 14.14}$$

$$C = 225\times 10^{-6} = 225\,\mu F$$

$$\underline{\underline{R = 14.14\,\Omega}}$$

5). A current of 5A flows through a non-inductive resistance in series with a choking coil when supplied at 250 V , 50Hz. If the voltage across the non-inductive resistance is 125 V and that of the coil 200 V. Calculate the impedance, reactance and resistance of the coil, power absorbed by the coil and total power. Draw the phasor diagram.

⇒



$Z_{coil} = ?$ $\quad r_L = ?$

$X_L = ?$ $\quad P_{coil} = ?$

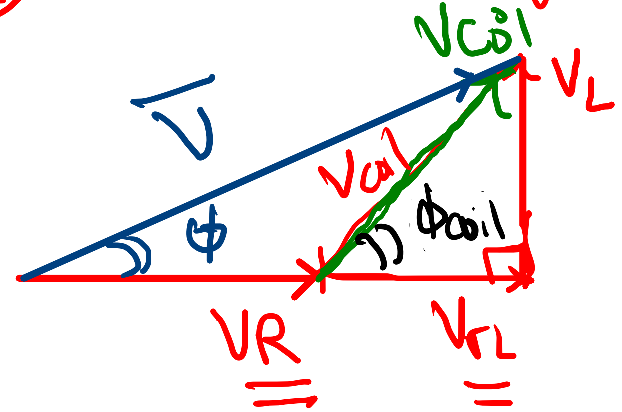
$P_{total} = ?$

$$\Rightarrow |Z_T| = \frac{250}{5} = 50\Omega$$

$$R = \frac{125}{5} = 25\Omega$$

$$|Z_{coil}| = \frac{200}{5} = 40\Omega$$

⇒ Voltage triangle.



$$\bar{V} = \bar{V}_R + \bar{V}_{coil}$$

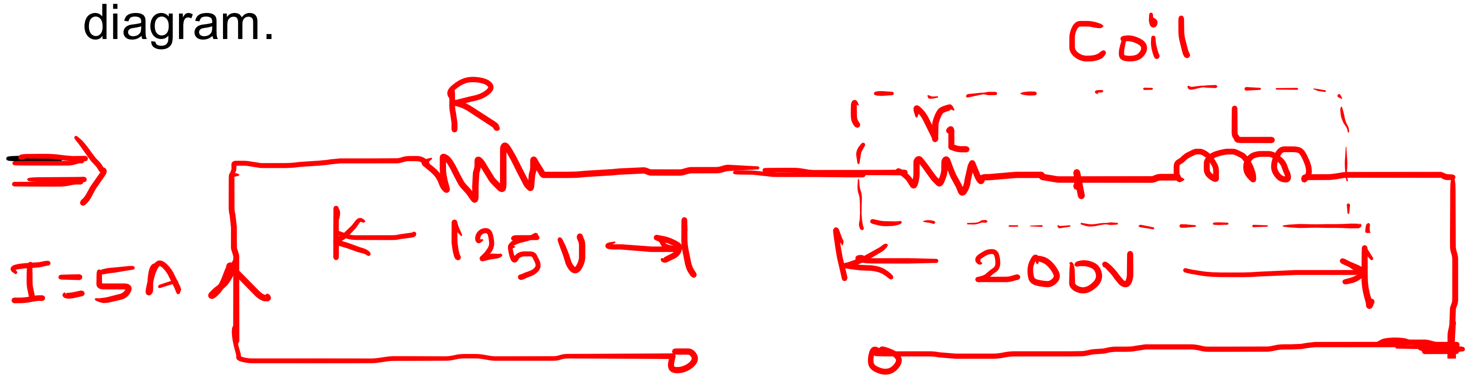
$$\bar{V}_{coil} = \bar{V}_{rL} + \bar{V}_L$$

$$V^2 = (V_R + V_{rL})^2 + (V_L)^2$$

$$V^2 = V_R^2 + 2V_R V_{rL} + \underline{V_{rL}^2 + V_L^2}$$

$$V^2 = V_R^2 + 2V_R V_{rL} + V_{coil}^2$$

5). A current of 5A flows through a non-inductive resistance in series with a choking coil when supplied at 250 V , 50Hz. If the voltage across the non-inductive resistance is 125 V and that of the coil 200 V. Calculate the impedance, reactance and resistance of the coil, power absorbed by the coil and total power. Draw the phasor diagram.



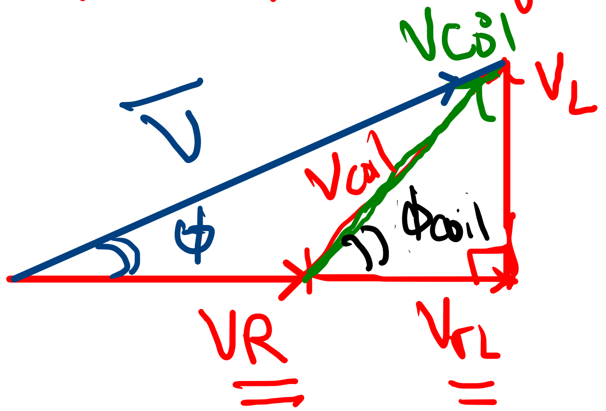
250V, 50HZ

$Z_{coil} = ?$ $r_L = ?$

$X_L = ?$ $P_{coil} = ?$

$P_{total} = ?$

voltage triangle.



$\bar{V} = \bar{V}_R + \bar{V}_{coil}$

$\bar{V}_{coil} = \bar{V}_{rL} + \bar{V}_L$

$V^2 = (V_R + V_{rL})^2 + (V_L)^2$

$V^2 = V_R^2 + 2V_R V_{rL} + V_{rL}^2 + V_L^2$

$V^2 = V_R^2 + 2V_R V_{rL} + V_{coil}^2$

$(250)^2 = (125)^2 + 2 \cdot V_{rL} \cdot 125 + (200)^2$

$250 V_{rL} = (250)^2 - (125)^2 - (200)^2$

$250 V_{rL} = 6875$

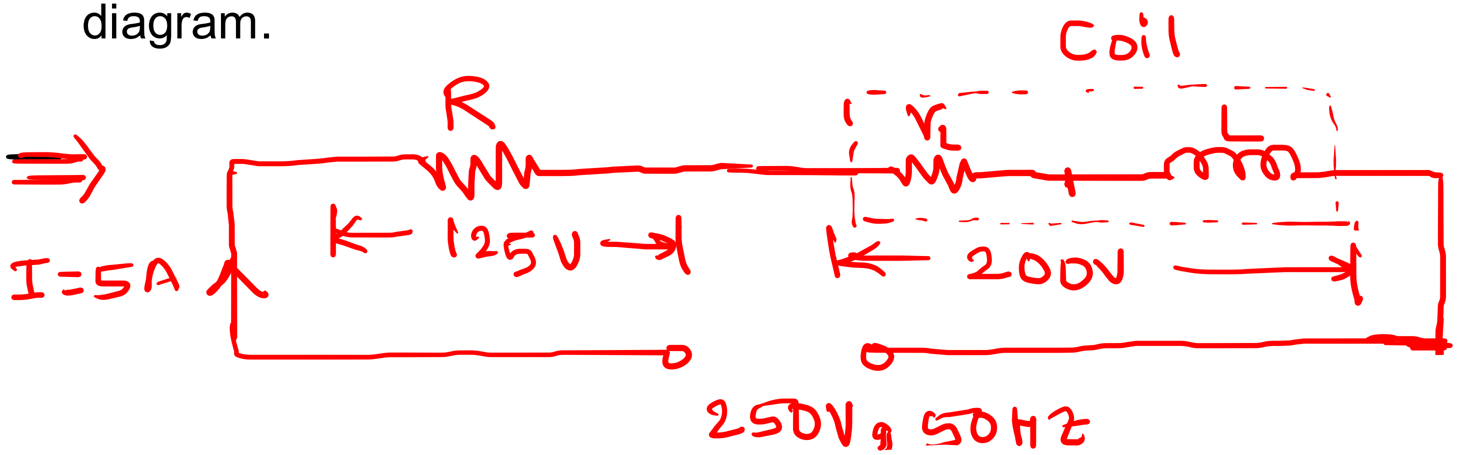
$\boxed{V_{rL} = 27.5V}$

$r_L = \frac{V_{rL}}{I} = \frac{27.5}{5} = 5.5\Omega$

$V_{coil}^2 = V_{rL}^2 + V_L^2$

$V_L^2 = V_{coil}^2 - V_{rL}^2 = (200)^2 - (27.5)^2$

5). A current of 5A flows through a non-inductive resistance in series with a choking coil when supplied at 250 V , 50Hz. If the voltage across the non-inductive resistance is 125 V and that of the coil 200 V. Calculate the impedance, reactance and resistance of the coil, power absorbed by the coil and total power. Draw the phasor diagram.



$Z_{coil} = ?$ $r_L = ?$

$X_L = ?$ $P_{coil} = ?$

$P_{total} = ?$

$$r_L = \frac{V_{rL}}{I} = \frac{27.5}{5} = 5.5\Omega$$

$$V_{coil}^2 = V_{rL}^2 + V_L^2$$

$$V_L^2 = V_{coil}^2 - V_{r}^2 = (200)^2 - (27.5)^2$$

$$V_L = 198.1V$$

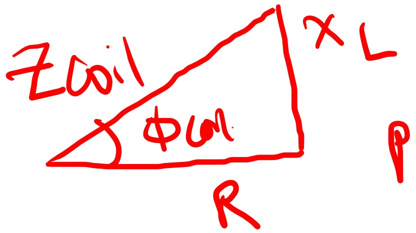
$$X_L = \frac{198.1}{5} = 39.63\Omega$$

$$P_{coil} = I^2 r_L = (5)^2 \times 5.5 = 137.5 \text{ watts}$$

$$\phi_{coil} = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{39.63}{5.5}\right) = 82.098$$

$$P_{coil} = V \times I \cdot \cos\phi$$

$$P_{coil} = 200 \times 5 \times \cos(82.98) = 137.47 \text{ watts}$$

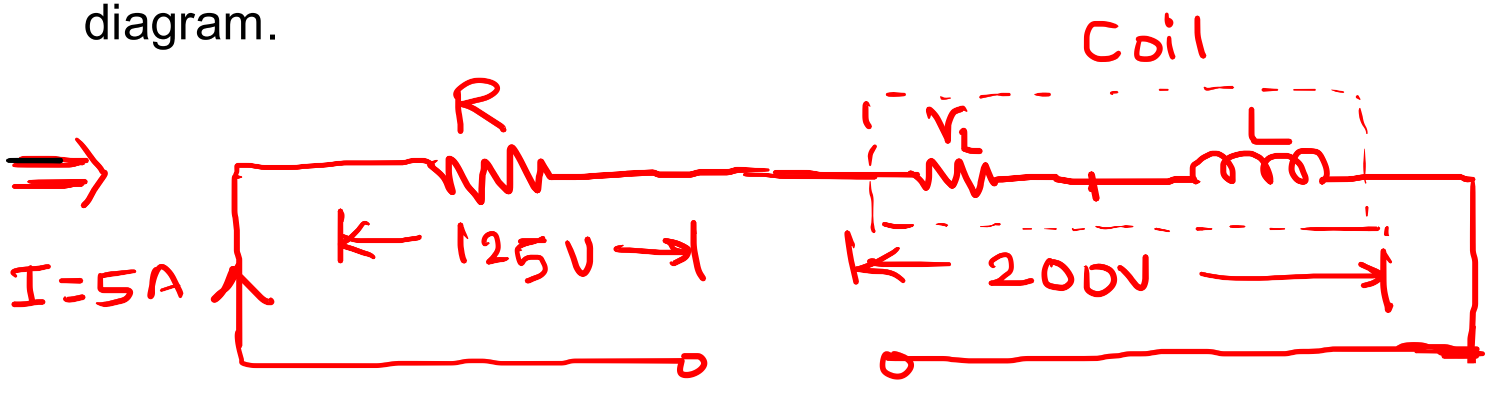


$$\cos\phi = \frac{R}{Z}$$

$$P_{coil} = V \times I \times \frac{R}{Z} = I \cdot \cancel{Z} \cdot I \frac{R}{\cancel{Z}}$$

$$= I^2 R$$

5). A current of 5A flows through a non-inductive resistance in series with a choking coil when supplied at 250 V , 50Hz. If the voltage across the non-inductive resistance is 125 V and that of the coil 200 V. Calculate the impedance, reactance and resistance of the coil, power absorbed by the coil and total power. Draw the phasor diagram.

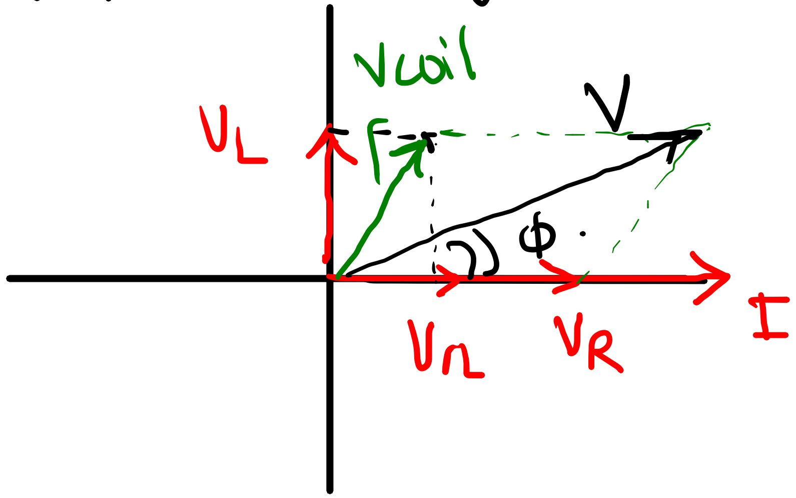


$Z_{coil} = ?$ $\quad r_L = ?$

$X_L = ?$ $\quad P_{coil} = ?$

$P_{total} = ?$

$\Rightarrow$ phasor diagram.



$P_{Total} = I^2 (R + r_L) = (5)^2 (25 + 5.5) =$

$P_{total} = 762.5$ Watts

$P_{Total} = V \cdot I \cdot \cos\phi = 250 \times 5 \times \cos(52.6^\circ)$

$P_{total} = 762.68$ Watts.

$Z = (R + r_L) + j(X_L)$

$= (25 + 5.5) + j(39.63)$

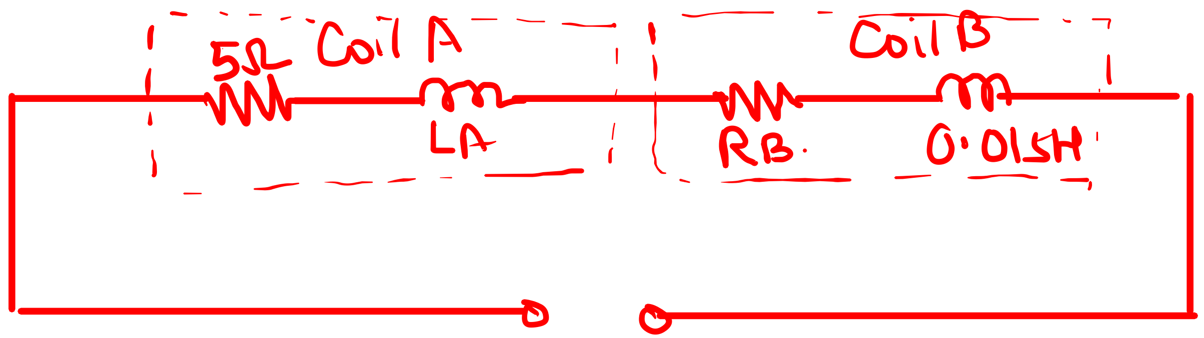
$Z = 30.5 + j\,39.63$

$\phi = \tan^{-1}\left(\frac{39.63}{30.5}\right)$

$\phi = 52.4^\circ$

6. Two coils A and B are connected in series across a 240 V, 50 Hz supply. The resistance of A is 5 Ω and inductance of B is 0.015 H. If the input from the supply is 3kWatts and 2kVAr. Find inductance of A and resistance of B. Calculate volatge across each coil.

$\Rightarrow$

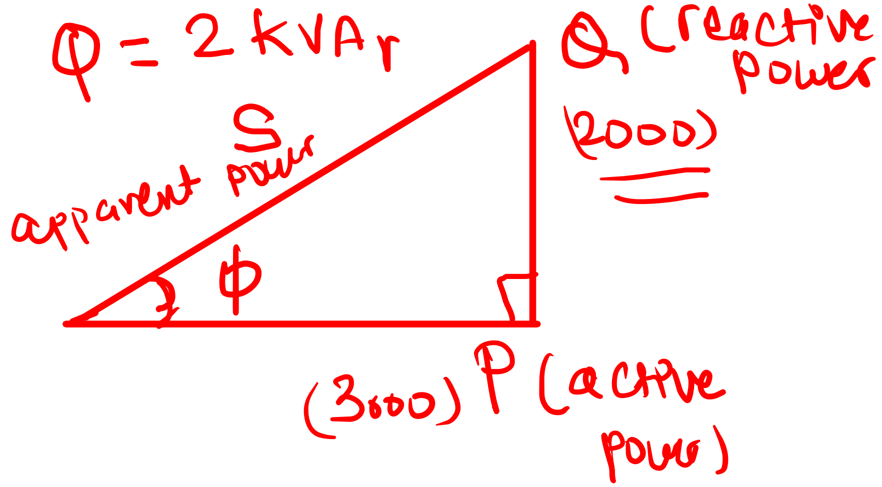


$L_A = ?$ $R_B = ?$

$V_{coil A} = ?$ $V_{coil B} = ?$

$P = 3$ kWatts

$Q = 2$ kVAr



$PF = \cos\phi = \frac{P}{S}$

$P^2 + Q^2 = S^2$

$S = \sqrt{(3000)^2 + (2000)^2}$

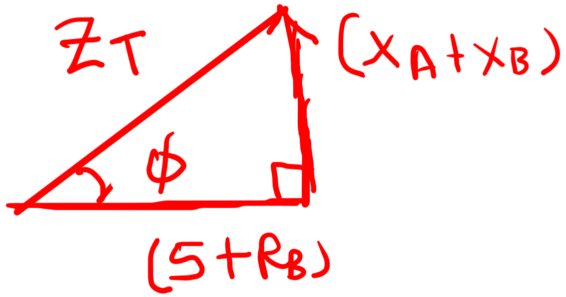
$S = 3605.5$ VA

$S = V \times I$

$I = \frac{S}{V} = \frac{3605.5}{240} =$

$I = 15.02A$

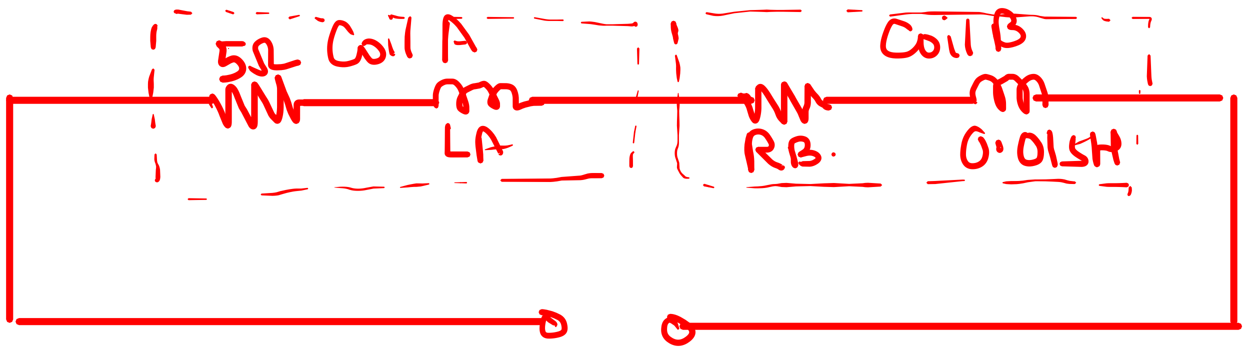
$pf = \cos\phi = 0.83, \phi = 33.9^\circ$



$Z_T = \frac{V}{I} \angle\phi = \frac{240}{15.05} \angle 33.9^\circ$

6. Two coils A and B are connected in series across a 240 V, 50 Hz supply. The resistance of A is 5 Ω and inductance of B is 0.015 H. If the input from the supply is 3kWatts and 2kVAr. Find inductance of A and resistance of B. Calculate volatge across each coil.

$\Rightarrow$



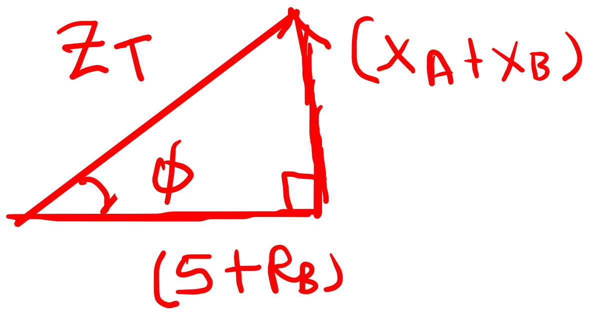
$L_A = ?$ $\quad R_B = ?$

$V_{coilA} = ?$ $\quad V_{coilB} = ?$

$I = \dfrac{S}{V} = \dfrac{3605.5}{240} =$

$I = 15.02A$

$pf = \cos\phi = 0.83, \; \phi = 33.9^\circ$



$Z_T = \dfrac{V}{I} \angle \phi = \dfrac{240}{15.05} \angle 33.9^\circ$

$\cos\phi = \dfrac{5+R_B}{Z_T}$

$0.83 \times \dfrac{240}{15.05} = 5 + R_B$

$\boxed{R_B = 8.23\,\Omega}$

OR $\rightarrow$ $P_{act} = I^2 (R_A + R_B)$

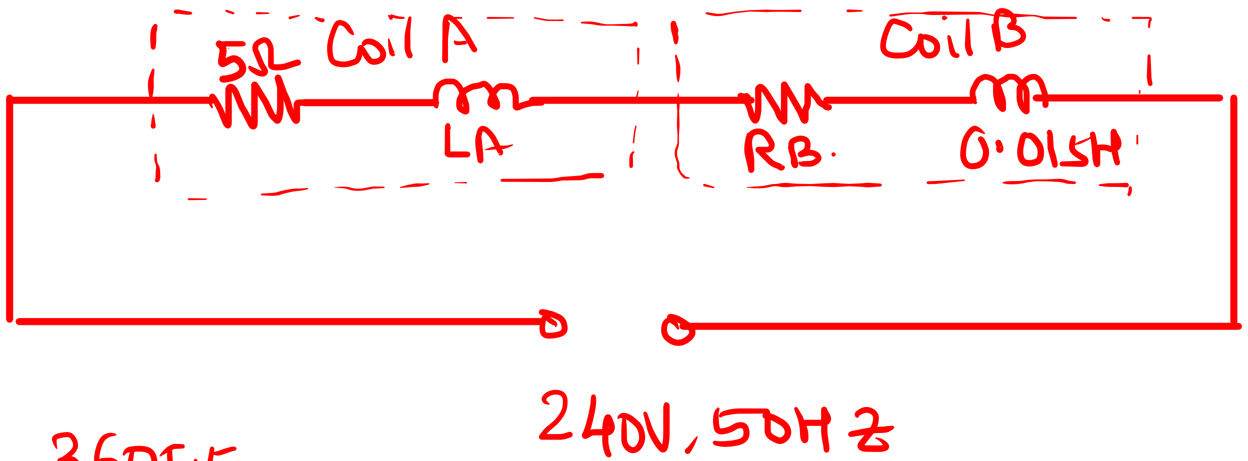
$3000 = (15.05)^2 \times (5 + R_B)$

$5 + R_B = \dfrac{3000}{(15.05)^2}$

$\boxed{R_B = 8.24\,\Omega}$

6. Two coils A and B are connected in series across a 240 V, 50 Hz supply. The resistance of A is 5 Ω and inductance of B is 0.015 H. If the input from the supply is 3kWatts and 2kVAr. Find inductance of A and resistance of B. Calculate volatge across each coil.

⇒



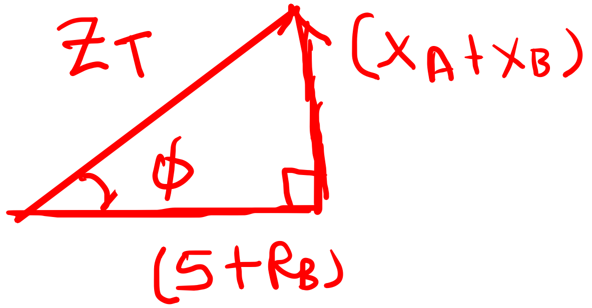
$L_A = ?$ $R_B = ?$

$V_{coil A} = ?$ $V_{coil B} = ?$

$I = \frac{S}{V} = \frac{3605.5}{240} =$

$I = 15.02A$

$pf = \cos\phi = 0.83$, $\phi = 33.9^\circ$



$Z_T = \frac{V}{I} \angle \phi = \frac{240}{15.05} \angle 33.9^\circ$

$\sin\phi = \frac{(X_A + X_B)}{Z_T}$

$X_B = 2\pi f \times 0.015$

$X_B = 2\pi \times 50 \times 0.015$

$X_B = 4.71\,\Omega$

$X_A + X_B = \frac{240}{15.05} \sin(33.9)$

$X_A + X_B = 8.89\,\Omega$

$X_A = 8.89 - 4.71$

$X_A = 4.18$

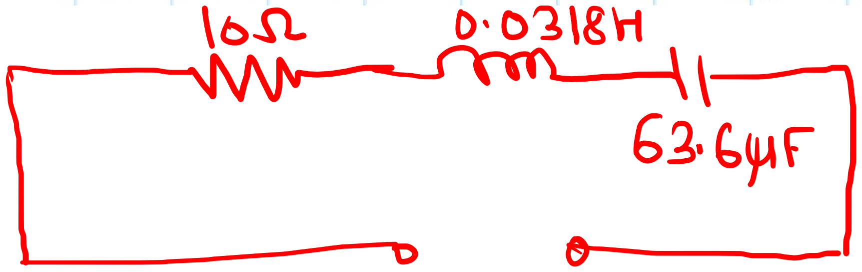
$2\pi f L_A = 4.18$

$L_A = \frac{4.18}{2\pi \cdot 50}$

$\boxed{L_A = 13.3\,mH}$

7)A voltage e(t)=100sin(314t) is applied to a series circuit consisting of 10 ohm resistance, a 0.0318 H inductor and 63.6 uF capacitor. Calculate (i) expression for current (ii) Power factor (iii) Active power.

$\Rightarrow$



$e(t) = 100 \sin(314t)$

$i(t) = ?$

$pf = \cos\phi = ?$

$P_{act} = ?$

$\Rightarrow$ $E = \frac{100}{\sqrt{2}} \angle 0$ $\omega = 314$ rad/sec

$Z = 10 + j\left(\omega L - \frac{1}{\omega C}\right)$

$\omega L = 314 \times 0.0318 = 9.98\,\Omega$

$\frac{1}{\omega C} = \frac{1}{314 \times 63.6 \times 10^{-6}} = 50.07\,\Omega$

$Z = 10 + j(9.98 - 50.07)$

$Z = (10 - j\,40.09)$

$Z = 41.31 \angle -75.99^\circ$

$\boxed{pf = \cos(-75.99^\circ) = 0.24 \text{ (leading)}}$

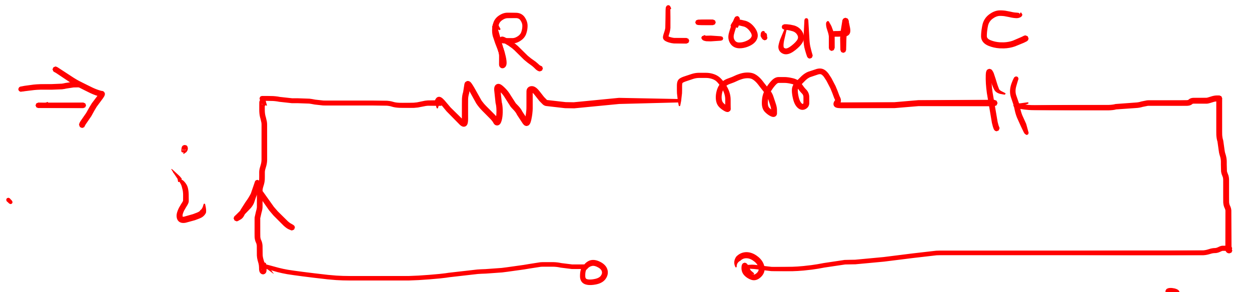
$I = \frac{V}{Z} = \frac{70.7 \angle 0}{41.31 \angle -75.99} = 1.71 \angle 75.99^\circ$

$\boxed{i(t) = 1.71 \times \sqrt{2} \sin(314t + 75.99^\circ)}$

$P_{act} = V_{rms} \cdot I_{rms} \cos\phi$

$= \frac{100}{\sqrt{2}} \times 1.71 \times 0.24 = 29.01$ Watts.

8). A resistance R, an inductor L =0.01 H and a capacitance C are connected in series when voltage v = 400 cos(3000t-10°) is applid to a series combination. if the current is i=10√2 cos(3000t-55°) . Find R and C.



$v = 400\cos(3000t - 10°)$

$i = 10\sqrt{2}\cos(3000t - 55°)$, $\boxed{\omega = 3000}$

$v = 400\sin(3000t - 10° + 90°)$

✓ $v = 400\sin(3000t + 80°)$

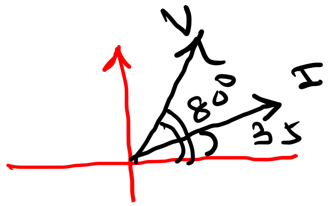
$i = 10\sqrt{2}\sin(3000t - 55 + 90°)$

✓ $i = 10\sqrt{2}\sin(3000t + 35°)$

$R = ?$ $C = ?$

rms values

$V = \frac{400}{\sqrt{2}}\angle 80°$, $I = \frac{10\sqrt{2}}{\sqrt{2}}\angle 35°$



$Z = \frac{V}{I} = \frac{400/\sqrt{2}\angle 80°}{10\angle 35°} = 28.3\angle 45°$

$Z = 20.01 + j\,20.01$

Comparing

$Z = R + jX$

$\boxed{R = 20.01\,\Omega}$

$X_L = \omega L = 3000 \times 0.01$

$X_L = 30\,\Omega$

$X = X_L - X_C$

$X_C = 30 - 20.1$

$\boxed{X_C = 9.99}$

$X_C = \frac{1}{\omega C}$

$C = \frac{1}{\omega \cdot X_C}$

$C = \frac{1}{3000 \times 9.99} = 33\,\mu F$

Concept Admittance (Y)

Admittance is Reciprocal of Impedance.

$$\bar{Y} = \frac{1}{Z} = \frac{1}{R \pm jX}$$

$$\bar{Y} = \frac{1}{R \pm jX} \times \frac{R \mp jX}{R \mp jX}$$

$$Y = \frac{R \mp jX}{R^2 + X^2}$$

$$Y = \frac{R}{R^2+X^2} \mp j\frac{X}{R^2+X^2}$$

$$Y = G \mp jB$$

Admittance = (℧) (mho) siemens

Conductance (℧) (mho) siemens

Suceptance (℧) (mho) simens

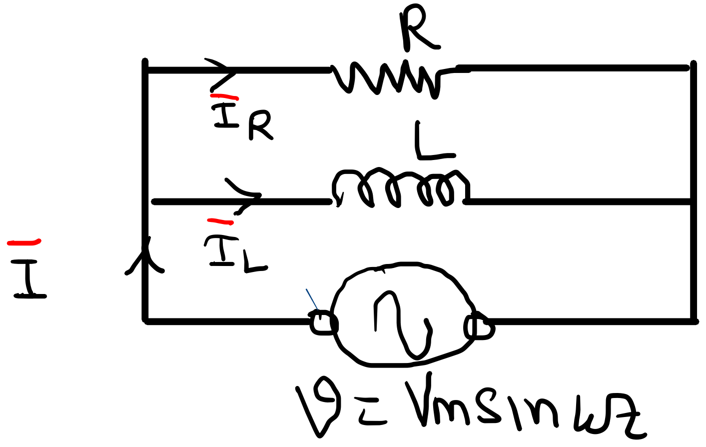
For R-L circuit

$Z = R + jX_L$ & $Y = G - jB$

for R-C circuit

$Z = R - jX_C$ & $Y = G + jB$

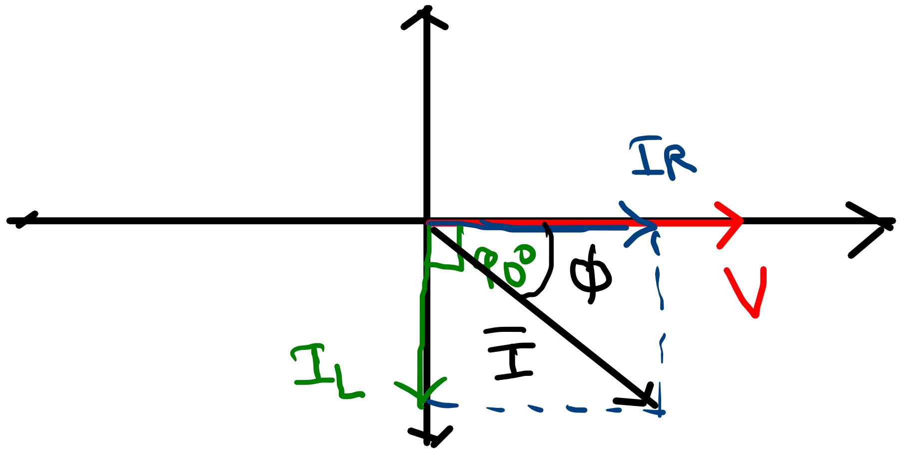
⇒ R-L Parallel Circuit



$\bar{I} = \bar{I}_R + \bar{I}_L$

⇒ Current is lagging voltage by an angle ϕ
So Lagging power factor

⇒ phasor diagram



⇒ since R & L are in parallel
So equivalent impedance

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{jX_L}$$

Admittance $\boxed{Y = G - j\frac{1}{X_L}}$

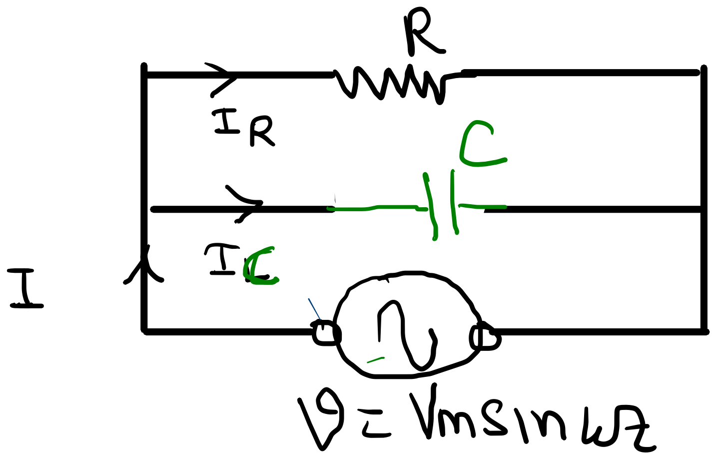
$$\frac{V}{I} = Z = \frac{1}{Y}$$

$P_{act} = V_{rms} I_{rms} \cos \phi$

$Q = V_{rm} I_{rms} \sin \phi$

$S = V_{rms} \times I_{rms}$

⇒ R-C Parallel Circuit



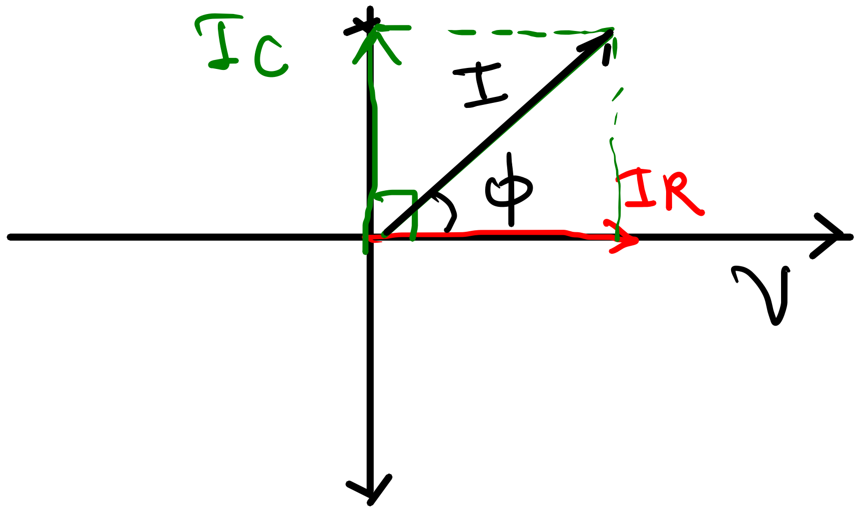
$$\bar{I} = \bar{I}_R + \bar{I}_C$$

overall impedance of the circuit

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{-j\left(\frac{1}{\omega C}\right)}$$

$$\boxed{Y = G + j\omega C}$$

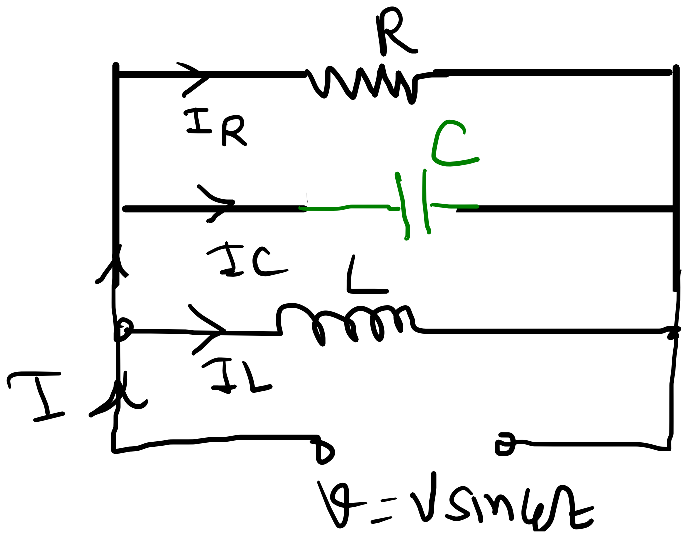
⇒ phasor diagram.



Current is leading voltage by $\phi°$

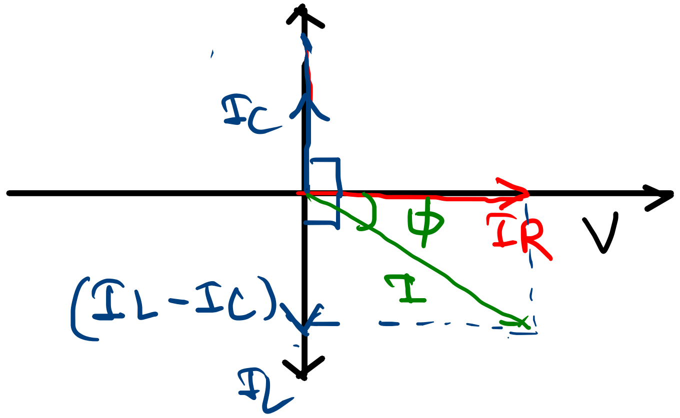
so $pf = \cos\phi$ (leading)

R-L-C Parallel Circuit



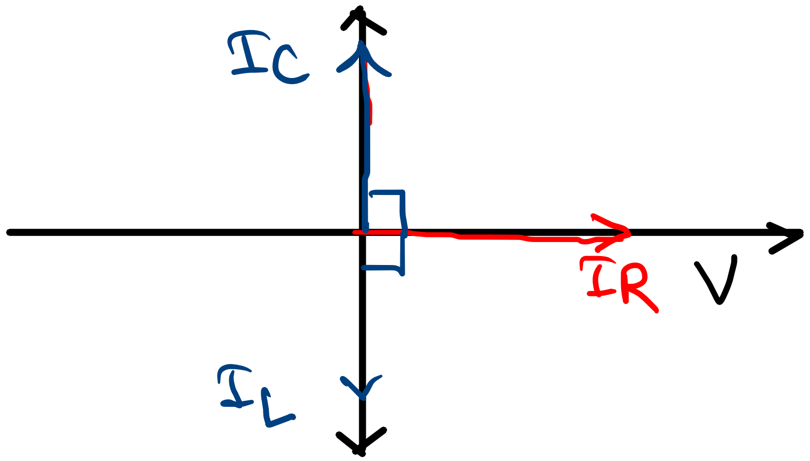
$$\bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_C$$

Case - I   If $I_L > I_C$ $(X_L < X_C)$

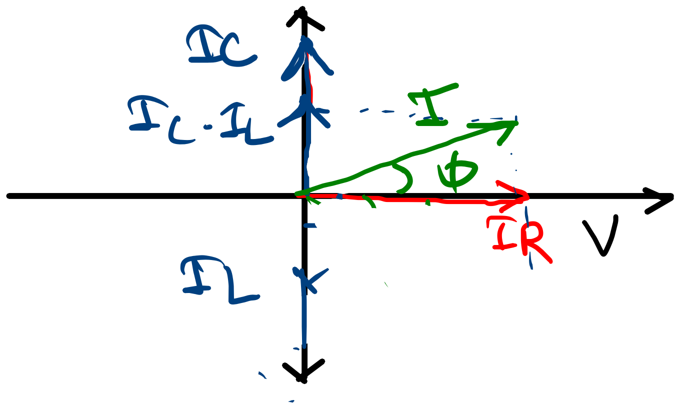


Lagging pf

⇒ phasor diagram.

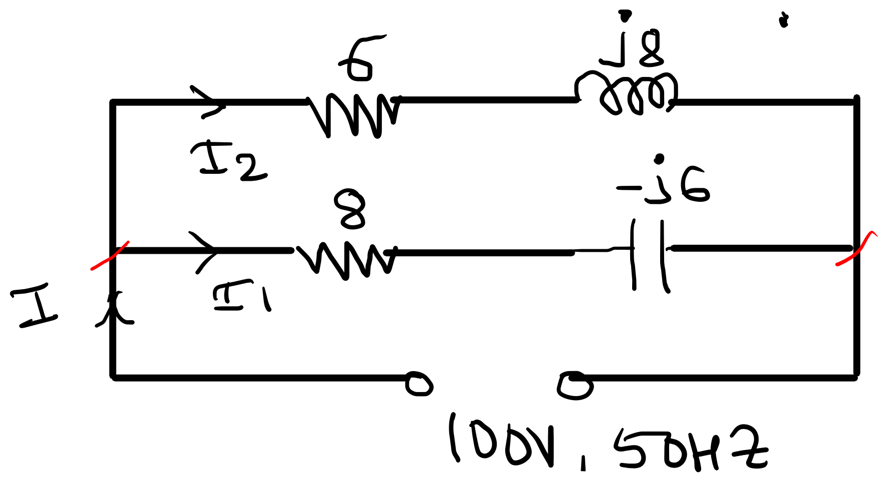


Case - II   If $I_C > I_L$ $(X_C < X_L)$



Leading pf.

1. Find current I1 and I2 in the following circuit. Find overall power factor of the circuit, Active power. Draw phasor diagram of the circuit.



$$\Rightarrow Z_1 = (8 - j6)$$

$$Z_2 = (6 + j8)$$

$$\bar{I} = \bar{I}_1 + \bar{I}_2$$

$$Z = (Z_1 \parallel Z_2)$$

OR

$$Y = Y_1 + Y_2$$

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$Z = \frac{(Z_1 \cdot Z_2)}{(Z_1 + Z_2)}$$

$$Z = \frac{(8-j6)(6+j8)}{(8-j6+6+j8)} = \frac{(10\angle -36.8)\times 10\angle 53.13}{14+j2}$$

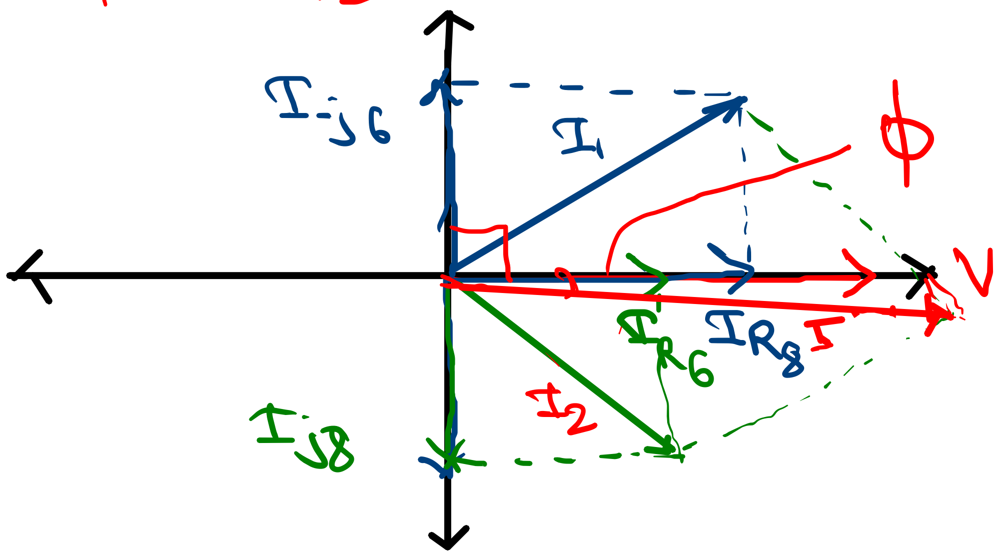
$$Z = \frac{100\angle 16.33}{14+j2} = \frac{(95.96+j28.11)}{14+j2}$$

$$Z = \frac{100\angle 16.33}{14.14\angle 8.13} = 7.02\angle 8.13^\circ$$

$$I = \frac{V}{Z} = \frac{100\angle 0}{7.02\angle 8.13} = 14.24\angle -8.13^\circ$$

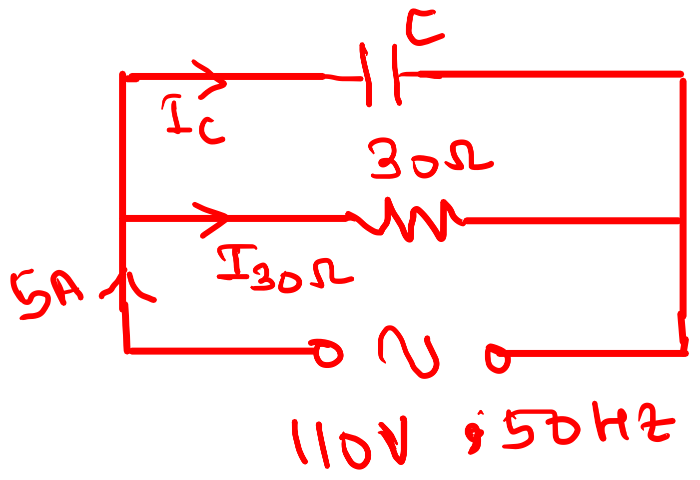
$$pf = \cos\phi = \cos(8.13) = 0.98\ (\text{lagging})$$

$$P_{act} = V_{rms} \times I_{rms} \cos\phi = 100 \times 14.24 \times 0.98 = 1395.5\ \text{Watts}$$



2. A resistor of 30 ohm and a capacitor of unknown value are connected in parallel across a 110 V, 50Hz supply. The combination draws a current of 5A from the supply. Find the value of unknown capacitance. The combination is connected across a 110 V supply of unknown frequency, it is observed that total current drawn from the mains falls to 4 A , determine frequency of the supply.

$\Rightarrow$



Case-I : $C = ?$ at $I = 5A$

Case-II $f = ?$ at $I = 4A$

$$I_{30\Omega} = \frac{110}{30} = 3.66\,A$$

$$\bar{I} = \bar{I}_C + \bar{I}_{30\Omega}$$

$$I^2 = I_C^2 + I_{30\Omega}^2$$

$$I_C^2 = I^2 - I_{30\Omega}^2$$

$$I_C^2 = (5)^2 - (3.66)^2$$

$$I_C = 3.4\,A$$

$$X_C = \frac{V}{I_C} = \frac{110}{3.4}$$

$$X_C = 32.35\,\Omega$$

$$X_C = \frac{1}{2\pi f C}$$

$$C = \frac{1}{2\pi \times 50 \times 32.35}$$

$$C = 98.3\,\mu F$$

Case II

$$I = 4A$$

$$I^2 = I_C^2 + I_{30\Omega}^2$$

$$I_C^2 = (4)^2 - (3.66)^2$$

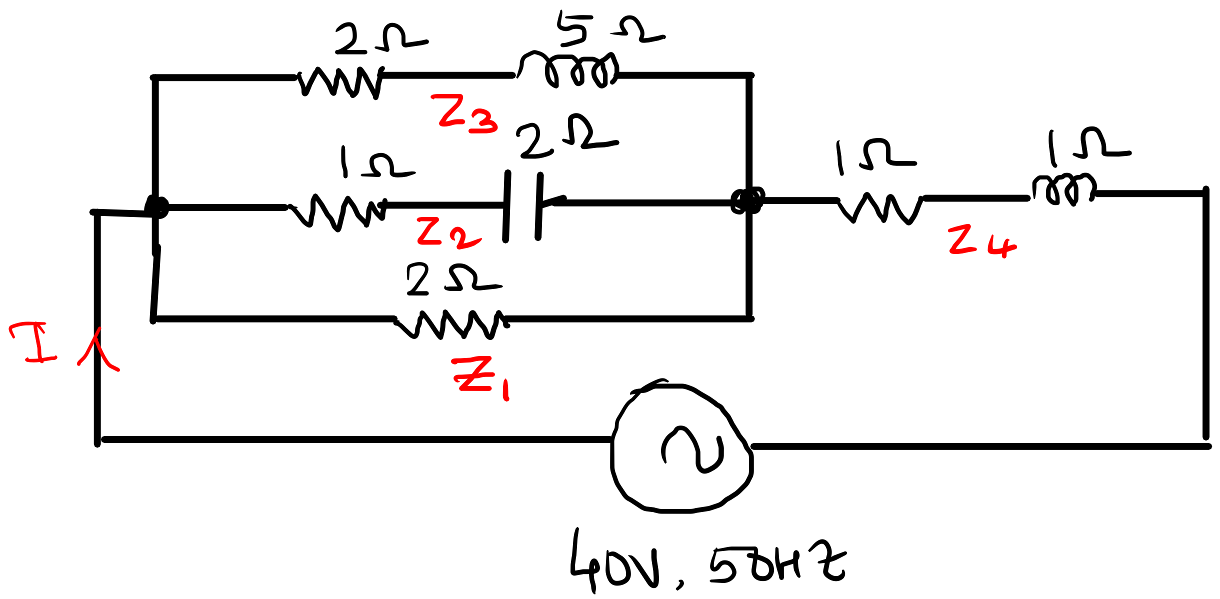
$$I_C = 1.61\,A$$

$$X_C = \frac{110}{1.61} = 68.32\,\Omega$$

$$X_C = \frac{1}{2\pi f C}, \quad f = \frac{1}{2\pi \times 68.32 \times 98.3 \times 10^{-6}}$$

$$f = 23.6\,Hz.$$

3. In the following circuit, calculate i) total impedance (ii) total current (iii) power factor (iv) Active and reactive power.



$\Rightarrow Z_1 = 2\,\Omega$, $Y_1 = \frac{1}{2} = 0.5\ ℧$

$Z_2 = 1 - j2$, $Y_2 = \frac{1}{1-j2} = 0.2 + j0.4$

$Z_3 = 2 + j5$, $Y_3 = \frac{1}{2+j5} = 0.068 - j0.17$

$Z_4 = 1 + j1$

$Y_{123} = Y_1 + Y_2 + Y_3$

$Y_{123} = 0.5 + 0.2 + j0.4 + 0.068 - j0.17$

$Y_{123} = 0.768 + j0.23$

$\therefore Z_{123} = \frac{1}{Y_{123}} = 1.19 - j0.35$

$Z = Z_{123} + Z_4$

$Z = 1.19 - j0.35 + 1 + j1$

$Z = 2.19 + j0.65$

$I = \frac{V}{Z} = \frac{40\angle 0}{(2.19 + j0.65)} = \frac{40\angle 0}{2.28\angle 16.5^\circ}$

$\boxed{I = 17.54\angle -16.5^\circ}$

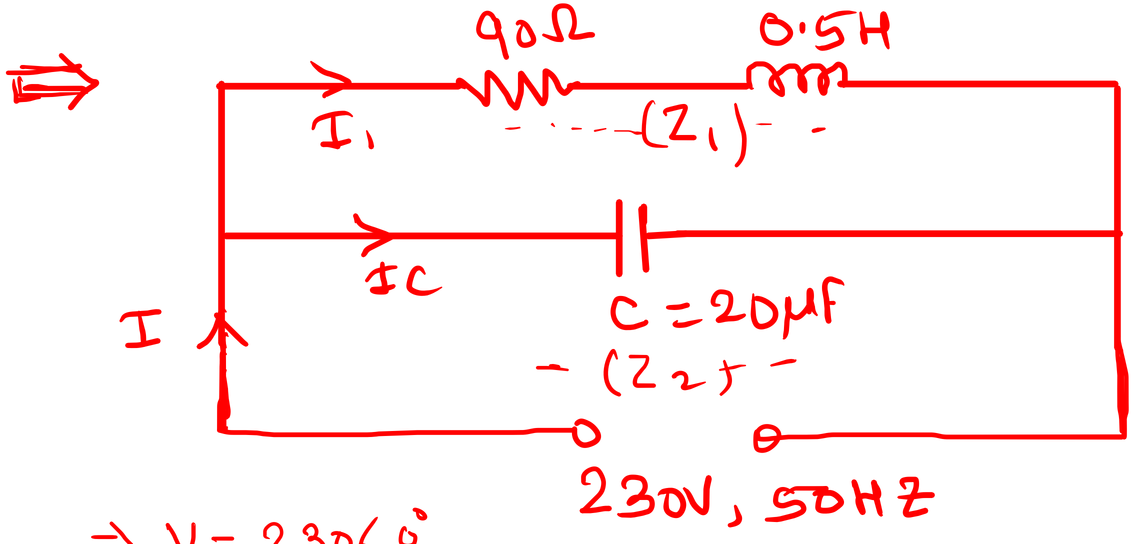
$PF = \cos\phi = \cos(16.5) = 0.95$ (lagging)

$P = V_{rms} I_{rms} \cos\phi = 40 \times 17.54 \times 0.95 = 666.5$ Watts

$Q = V_{rms} I_{rms} \sin\phi = 40 \times 17.54 \times \sin(16.5)$

$Q = 199.2$ VAr

4) A series combination of 0.5 H inductor and 90 ohm resistor are connected in parallel across 20 uF. Find (i) the total current (ii) power factor of the circuit (iii) total power taken from the source. Draw phasor diagram. A voltage of 230 V , 50 Hz is maintained across the circuit.



$I = ?$, $PF = \cos\phi = ?$

$P_{act} = ?$ phasor diagram

$\Rightarrow V = 230\angle 0^\circ$

$Z_1 = 90 + j\,2\pi f L$

$Z_1 = 90 + j\,2\pi \times 50 \times 0.5$

$Z_1 = 90 + j\,50\pi$

$Z_1 = 90 + j157$

$Z_2 = \frac{-j}{2\pi f C} = -j\frac{1}{2\pi \times 50 \times 20 \times 10^{-6}}$

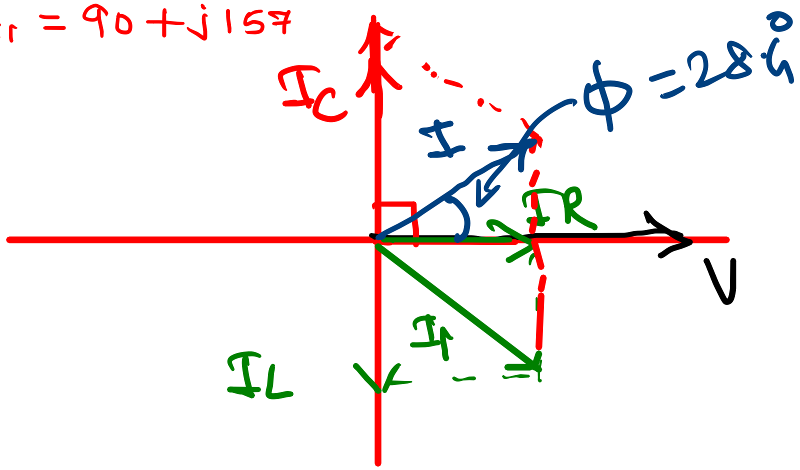
$Z_2 = (-j\,159.15)\,\Omega$

$Z = (Z_1) \parallel Z_2 = \frac{(90 + j157) \times (-j159.15)}{90 + j157 - j159.15}$

$Z = \frac{(24986.55 - j\,14323.5)}{(90 - j\,2.15)}$

$Z = (281.26 - j152.43)$

$I = \frac{V}{Z} = \frac{230\angle 0^\circ}{281.26 - j152.13} = 0.71\angle 28.4^\circ$



$PF = \cos\phi = \cos(28.4^\circ) = 0.87$ (leading)

$P_{act} = V_{rms} I_{rms} \cos\phi = 230 \times 0.71 \times 0.87 = 142.07$ Watts