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A2
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Q1)

$$A = \begin{bmatrix} 121045 & 0 & 0 \\ 0 & 121045 & 0 \\ 0 & 0 & 121045 \end{bmatrix}$$

1. $(121045 - \lambda)^3 = 0$

$121045 - \lambda = 0$

$\therefore \boxed{\lambda = 121045} \rightarrow$ eigen values

2. For $\lambda = 121045$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$0x_1 + 0x_2 + 0x_3 = 0$

Rank $= 0$

$\therefore$ no of eigen values $= 3 - 0 = 3$

$\therefore$ We have 3 linearly independent solutions.

Let parameters be α, β, γ.

$\therefore$ $\lambda = 121045$ the eigen values are $\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$

$$= \alpha \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \beta \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \gamma \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

To verify that the eigenvectors are linearly independent

$$K_1 X_1 + K_2 X_2 + K_3 X_3 = 0$$

$$K_1 [1\ 0\ 0] + K_2 [0\ 1\ 0] + K_3 [0\ 0\ 1] = 0$$

$$K_1 + 0 + 0 = 0 \rightarrow (1)$$
$$0 + K_2 + 0 = 0 \rightarrow (2)$$
$$0 + 0 + K_3 = 0 \rightarrow (3)$$

From the above equations

$$K_1 = K_2 = K_3 = 0$$

$\therefore$ The vectors are linearly independent

Q2)
ii)

$$P = \begin{bmatrix} 45 & 46 & 47 \\ 46 & 47 & 45 \\ 47 & 45 & 46 \end{bmatrix}$$

Finding eigen values of P

$$P - \lambda I = \begin{bmatrix} 45-\lambda & 46 & 47 \\ 46 & 47-\lambda & 45 \\ 47 & 45 & 46-\lambda \end{bmatrix}$$

$$|P - \lambda I| = 0$$

$$45-\lambda\left((47-\lambda)(46-\lambda)-(45)^2\right) - 46\left(46(46-\lambda) - 45(47)\right)$$
$$+ 47\left(46 \times 45 - 47(47-\lambda)\right) = 0$$

$$-\lambda^3 + 138\lambda^2 + 3\lambda - 414 = 0 \quad \rightarrow (A)$$

$$= -(\lambda - 138)(\lambda - \sqrt{3})(\lambda + \sqrt{3}) = 0$$

On solving we get

$$\lambda_1 = 138$$
$$\lambda_2 = \sqrt{3}$$
$$\lambda_3 = -\sqrt{3}$$

$\therefore$ Hence proved that the eigen values are equal to

$$\lambda_1 = a + b + c = 45 + 46 + 47 = 138$$
$$\lambda_2 = \sqrt{3}$$
$$\lambda_3 = -\sqrt{3}$$

iii) $\lambda^3 - 138\lambda^2 - 3\lambda + 414 = 0$ (From A)

Acc to calyel - Hamilton theorem

$$P^3 - 138P^2 - 3P + 414I = 0$$

$$P^2 = \begin{bmatrix} 45 & 46 & 47 \\ 46 & 47 & 45 \\ 47 & 45 & 46 \end{bmatrix} \begin{bmatrix} 45 & 46 & 47 \\ 46 & 47 & 45 \\ 47 & 45 & 46 \end{bmatrix} = \begin{bmatrix} 6350 & 6347 & 6347 \\ 6347 & 6350 & 6347 \\ 6347 & 6347 & 6350 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 45 & 46 & 47 \\ 46 & 47 & 45 \\ 47 & 46 & 46 \end{bmatrix} \begin{bmatrix} 6350 & 6347 & 6347 \\ 6347 & 6350 & 6347 \\ 6347 & 6347 & 6550 \end{bmatrix} = \begin{bmatrix} 876021 & 876024 & 876027 \\ 876024 & 876027 & 876021 \\ 876027 & 876021 & 876024 \end{bmatrix}$$

Now acc to equ.

$$\begin{bmatrix} 876021 & 876024 & 876027 \\ 876024 & 876027 & 876021 \\ 876027 & 876021 & 876024 \end{bmatrix} - \begin{bmatrix} 876300 & 875886 & 875886 \\ 875886 & 876300 & 875886 \\ 875886 & 875886 & 876300 \end{bmatrix}$$

$$- \begin{bmatrix} 135 & 138 & 141 \\ 138 & 141 & 135 \\ 141 & 135 & 138 \end{bmatrix} + \begin{bmatrix} 414 & 0 & 0 \\ 0 & 414 & 0 \\ 0 & 0 & 414 \end{bmatrix} = 0$$

Hence Proved.