PAGE NO.
DATE: / /

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1) $\begin{bmatrix} x-1 & x+1 & x \\ -1 & x & 0 \\ 0 & 1 & 1 \end{bmatrix}$ It is given that x is real.

$$|A| = \begin{vmatrix} x-1 & x+1 & x \\ -1 & x & 0 \\ 0 & 1 & 1 \end{vmatrix} = x^2 - x + x + 1 - x = x^2 - x + 1$$

If $|A| = 0$, then $x^2 - x + 1 = 0$

$$\therefore x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{-3}}{2}$$

$$= \frac{1}{2} \pm \frac{i\sqrt{3}}{2}$$

$\therefore$ It is given x is real

hence $|A| \neq 0$

$\therefore$ Rank$(A) = 3$

2) $$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 3 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ \lambda \\ \lambda^2 \end{bmatrix}$$

$$\therefore \quad [A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 1 & 1 & 1 & \lambda \\ 3 & 1 & 3 & \lambda^2 \end{array}\right]$$

$R_2 \to R_2 - R_1, \quad R_3 \to R_3 - 3R_1$

$$[A|B] \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & \lambda - 3 \\ 0 & -5 & 0 & \lambda^2 - 9 \end{array}\right]$$

$R_3 \to R_3 - 5R_2$

$$[A|B] \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & \lambda - 3 \\ 0 & 0 & 0 & \lambda^2 - 5\lambda + 6 \end{array}\right] \to (1)$$

$\text{Rank } [A] = 2$

$\therefore$ The system will be consistent if $\text{Rank } [A|B] = \text{Rank } [A] = 2$

$$\therefore \quad \lambda^2 - 5\lambda + 6 = 0$$
$$\lambda^2 - 3\lambda - 2\lambda + 6 = 0$$
$$(\lambda - 3)(\lambda - 2) = 0$$
$$\lambda = 2, 3$$

$\therefore$ The system is consistent when $\lambda = 2$ or $\lambda = 3$

For $\lambda = 2$; From ①

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore \; x_1 + 2x_2 + x_3 = 3 \rightarrow$ ②

$-x_2 = -1$

$\therefore \; \boxed{x_2 = 1}$

puting in equ ②

$$x_1 + x_3 = 1$$

let $x_3 = t \Rightarrow x_1 = 1 - t$

The soln is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1-t \\ 1 \\ t \end{bmatrix}$ no of parameters = 1

This will have infinitely no of soln

$\because$ rank $= 2 < 3$

For $\lambda = 3$, From ①

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 + 2x_2 + x_3 = 3 \rightarrow$ ③

$-x_2 = 0$

$\Rightarrow \boxed{x_2 = 0}$

putting in equ ③

$$x_1 + x_3 = 3$$

let $x_3 = t$ $\left(\begin{array}{c} n - r = 3 - 2 = 1 \\ \text{parameter} \end{array} \right)$

$\therefore$ $x_1 = 3 - t$

$\therefore$ The soln is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3-t \\ 0 \\ t \end{bmatrix}$ no of parameters is 1

will have infinitely no of soln

$\because$ rank $= 2 < 3$

3) Let $K_1 x_1 + K_2 x_2 + K_3 x_3 = 0 \rightarrow (A)$

$K_1 \begin{bmatrix} 2 & 3 & 4 & -2 \end{bmatrix} + K_2 \begin{bmatrix} -1 & -2 & -2 & 1 \end{bmatrix} + K_3 \begin{bmatrix} 1 & 1 & 2 & 1 \end{bmatrix} = 0$

$\begin{bmatrix} 2K_1 & 3K_1 & 4K_1 & -2K_1 \end{bmatrix} + \begin{bmatrix} -K_2 & -2K_2 & -2K_2 & K_2 \end{bmatrix}$
$+ \begin{bmatrix} K_3 & K_3 & 2K_3 & K_3 \end{bmatrix} = 0$

$$\left. \begin{array}{l} 2K_1 - K_2 + K_3 = 0 \\ 3K_1 - 2K_2 + K_3 = 0 \\ 4K_1 - 2K_2 + 2K_3 = 0 \\ -2K_1 + K_2 - K_3 = 0 \end{array} \right]$$ Homogenous system of equations

$$\begin{bmatrix} 2 & -1 & 1 \\ 3 & -2 & 1 \\ 4 & -2 & 2 \\ -2 & +1 & -1 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \rightarrow \frac{1}{2} R_1$

$$\begin{bmatrix} 1 & -1/2 & 1/2 \\ 3 & -2 & 1 \\ 4 & -2 & 2 \\ -2 & +1 & -1 \end{bmatrix} \xrightarrow[R_4 \rightarrow R_4 + 2R_1]{R_2 \rightarrow R_2 - 3R_1} \begin{bmatrix} 1 & -1/2 & 1/2 \\ 0 & -1/2 & -1/2 \\ 4 & -2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_3 \to R_3 - 4R_1 \quad \begin{bmatrix} 1 & -1/2 & 1/2 \\ 0 & -1/2 & -1/2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Rank (A) $= 2 < 3$

$\therefore$ There are infinitely many ~~soln~~ non-trivial soln.

Hence the given vectors are linearly dependent

$$K_1 - \frac{1}{2}K_2 + \frac{1}{2}K_3 = 0 \quad \to (1)$$

$$-\frac{1}{2}K_2 - \frac{1}{2}K_3 = 0 \quad \to (2)$$

let $\boxed{K_3 = t}$

putting in equ (2)

$$-\frac{1}{2}K_2 = \frac{1}{2}t \Rightarrow \boxed{K_2 = -t}$$

in equ (1) ; $K_1 + \frac{1}{2}t + \frac{1}{2}t = 0 \Rightarrow \boxed{K_1 = -t}$

substituting in (A)

$$-t x_1 + (-t)x_2 + t x_3 = 0$$

$$\therefore \ t x_3 = t x_1 + t x_2$$

$$x_3 = x_1 + x_2$$

$\therefore$ The relation is $x_3 = x_2 + x_1$