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Batch : A2
Tutorial : 4

1) $\dfrac{1+\cos 7\theta}{1+\cos\theta} = (x^3 - x^2 - 2x + 1)^2$ (To Prove)

Given $x = 2\cos\theta$

LHS: $\dfrac{1+\cos 7\theta}{1+\cos\theta} \Rightarrow \dfrac{2\cos^2\left(\frac{7\theta}{2}\right)}{2\cos^2(\theta/2)}$ $(\because 1+\cos\theta = 2\cos^2\theta/2)$

$\Rightarrow \left[\dfrac{\cos(7\theta/2)}{\cos(\theta/2)}\right]^2 \Rightarrow \dfrac{2\cos(7\theta/2)\sin(\theta/2)}{2\cos(\theta/2)\sin(\theta/2)}$

$\Rightarrow \left(\dfrac{\sin\left(\frac{7\theta}{2}+\frac{\theta}{c}\right) - \sin\left(\frac{7\theta}{2}-\frac{\theta}{2}\right)}{\sin^2\theta}\right)^2$ $\left(\because \sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}\right)$

$(\because \sin 2\theta = 2\sin\theta\cos\theta)$

$\Rightarrow \left(\dfrac{\sin 4\theta - \sin 3\theta}{\sin\theta}\right)^2 \rightarrow (1)$

- $(\cos\theta + i\sin\theta)^4 = \cos 4\theta + i\sin 4\theta$

Using Binomial Theorem.

$\cos^4\theta + i4\cos^3\theta\sin\theta - 6\cos^2\theta\sin^2\theta - i4\cos\theta\sin^3\theta + \sin^4\theta$

comparing imaginary parts.

$\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta \rightarrow (2)$

- $(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta$

$\cos^3\theta + 3i\cos^2\theta\sin\theta - 3\cos\theta\sin^2\theta - i\sin^3\theta$

comparing imaginary parts

$\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta \rightarrow (3)$

Inserting 2 & 3 in 1

$$\Rightarrow \frac{(\sin 4\theta - \sin 3\theta)^2}{(\sin\theta)^2}$$

$$\Rightarrow \left(\frac{4\cos^3\theta \sin\theta - 4\cos\theta\sin^3\theta - 3\cos^2\theta\sin\theta + \sin^3\theta}{\sin\theta}\right)^2$$

$$\Rightarrow \left(4\cos^3\theta - 4\cos\theta\sin^2\theta - 3\cos^2\theta + \sin^2\theta\right)^2$$

$$\Rightarrow \left(4\cos^3\theta - 4\cos\theta(1-\cos^2\theta) - 3\cos^2\theta + (1-\cos^2\theta)\right)^2$$

$$\Rightarrow \left(4\cos^3\theta - 4\cos\theta + 4\cos^3\theta - 3\cos^2\theta + 1 - \cos^2\theta\right)^2$$

$$\Rightarrow \left(8\cos^3\theta - 4\cos^2\theta - 4\cos\theta + 1\right)^2$$

$2\cos\theta = x$ (given)

$$\Rightarrow \left(x^3 - x^2 - 2x + 1\right)^2$$

$\therefore$ LHS = RHS

Hence Proved.

2) $\cos^8\theta = \frac{1}{2^7}\left[\cos 8\theta + 8\cos 6\theta + 28\cos 4\theta + 56\cos 2\theta + 35\right]$

To Prove.

Let $x = \cos\theta + i\sin\theta$; $\frac{1}{x} = \cos\theta - i\sin\theta$

$x + \frac{1}{x} = 2\cos\theta$ — A ; $x - \frac{1}{x} = 2i\sin\theta$

$x^n = (\cos n\theta + i\sin n\theta) = (\cos\theta + i\sin\theta)^n \rightarrow$ 1

$\frac{1}{x^n} = (\cos n\theta - i\sin n\theta) = (\cos\theta - i\sin\theta)^n \rightarrow$ 2

$$\therefore \quad x^n + \frac{1}{x^n} = 2\cos n\theta \rightarrow (3)$$

$$\left(x + \frac{1}{x}\right)^n = (2\cos\theta)^n \quad \text{(From A)}$$

For $n = 8$

$$(2\cos\theta)^8 = \left(x + \frac{1}{x}\right)^8$$

$$2^8\cos^8\theta = x^8 + 8x^6 + 28x^4 + 56x^2 + 70 + 56\frac{1}{x^2} + 28\frac{1}{x^4} + 8\frac{1}{x^6} + \frac{1}{x^8}$$

$$2^8\cos^8\theta \Rightarrow \left(x^8 + \frac{1}{x^8}\right) + 8\left(x^6 + \frac{1}{x^6}\right) + 28\left(x^4 + \frac{1}{x^4}\right) + 56\left(x^2 + \frac{1}{x^2}\right) + 70$$

From (3)

$$2^8\cos^8\theta \Rightarrow 2\cos 8\theta + 8(2\cos 6\theta) + 28(2\cos 4\theta) + 56(2\cos 2\theta) + 70.$$

dividing both sides by 2

$$2^7\cos^8\theta = \cos 8\theta + 8\cos 6\theta + 28\cos 4\theta + 56\cos 2\theta + 35$$

$$\cos^8\theta = \frac{1}{2^7}\left(\cos 8\theta + 8\cos 6\theta + 28\cos 4\theta + 56\cos 2\theta + 35\right)$$

~~LHS = RHS~~

Hence Proved.

3) $S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ $\quad |S| = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$

$\therefore\ |S| = 0 - 1(0-1) + 1(1-0)$

$|S| = 0 + 1 + 1$

$|S| = 2$

$\therefore$ The inverse of matrix S exists.

$a_{11} = 0 \quad M_{11} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1 \quad A_{11} = (-1)^{1+1}(-1) = -1$

$a_{12} = 1 \quad M_{12} = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -1 \quad A_{12} = (-1)^{1+2}(-1) = +1$

$a_{13} = 1 \quad M_{13} = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1 \quad A_{13} = (-1)^{1+3}(1) = 1$

$a_{21} = 1 \quad M_{21} = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -1 \quad A_{21} = (-1)^{2+1}(-1) = +1$

$a_{22} = 0 \quad M_{22} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1 \quad A_{22} = (-1)^{2+2}(-1) = -1$

$a_{23} = 1 \quad M_{23} = \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -1 \quad A_{23} = (-1)^{2+3}(-1) = +1$

$a_{31} = 1 \quad M_{31} = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1 \quad A_{31} = (-1)^{3+1}(1) = 1$

$a_{32} = 1 \quad M_{32} = \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -1 \quad A_{32} = (-1)^{3+2}(-1) = 1$

$a_{33} = 0 \quad M_{33} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1 \quad A_{33} = (-1)^{3+3}(-1)$

$= -1$

Hence cofactor matrix = $\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & +1 \\ 1 & 1 & -1 \end{bmatrix}$

$$\text{adj } S = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$S^{-} = \frac{1}{|S|} (\text{adj } S)$$

$$S^{-} = \frac{1}{2} \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$S^{-} = \begin{bmatrix} -1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \end{bmatrix} \rightarrow (1)$$

$$\therefore SA = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \times \frac{1}{2} \begin{bmatrix} b+c & c-a & b-a \\ c-b & c+a & a-b \\ b-c & a-c & a+b \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 0+c-b+b-c & 0+c+a+a-c & 0+a-b+a+b \\ b+c+0+b-c & c-a+0+a-c & b-a+0+a+b \\ b+c+c-d+0 & c-a+c+a+0 & b-a+a-b+0 \end{bmatrix}$$

$$\therefore SA = \frac{1}{2} \begin{bmatrix} 0 & 2a & 2a \\ 2b & 0 & 2b \\ 2c & 2c & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & a & a \\ b & 0 & b \\ c & c & 0 \end{bmatrix}$$

$$\therefore SAS^{-1} = \begin{bmatrix} 0 & a & a \\ b & 0 & b \\ c & c & 0 \end{bmatrix} \begin{bmatrix} -1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \end{bmatrix} \quad \text{(From 1)}$$

$$= \frac{1}{2} \begin{bmatrix} 0 & a & a \\ b & 0 & b \\ c & c & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 0+a+a & 0-a+a & 0+a-a \\ -b+0+b & b+0+b & b+0-b \\ -c+c+0 & c-c+0 & c+c+0 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 2a & 0 & 0 \\ 0 & 2b & 0 \\ 0 & 0 & 2c \end{bmatrix}$$

$$SAS^{-1} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$\therefore$ SAS^{-1} is a diagonal matrix

Hence Proved.