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Name: Pargat Singh
Roll no: 16010121045
Batch: A2
Tutorial 3

1) Let $w = \frac{-1+i\sqrt{3}}{2}$

then $w^2 = \frac{-1+i\sqrt{3}}{2} \times \frac{-1+i\sqrt{3}}{2} \Rightarrow \frac{+1-i\sqrt{3}-i\sqrt{3}-3}{4}$

$\Rightarrow \frac{-2-2i\sqrt{3}}{4} \Rightarrow \frac{-1-i\sqrt{3}}{4}$

we know,

$$\boxed{\begin{array}{l} w^{3n}=1 \\ w^{3n+1}=w \\ w^{3n+2}=w^2 \end{array}} \rightarrow (1) \qquad \boxed{1+w+w^2=0} \rightarrow (2)$$

$$\left(\frac{-1+i\sqrt{3}}{2}\right)^n + \left(\frac{-1-i\sqrt{3}}{2}\right)^n \Rightarrow w^n + (w^2)^n \rightarrow (3)$$

$\therefore$ if $n = 3k \pm 1$

$\Rightarrow w^{3k+1} + w^{6k+2}$

$\Rightarrow w^{3k} \cdot w + w^{6k} w^2$

$\Rightarrow w(1) + (1) w^2$ (From (1))

From 2

$\boxed{w + w^2 = -1}$ (similar ~~of~~ for $3k-1$)

$\therefore \left(\frac{-1+i\sqrt{3}}{2}\right)^n + \left(\frac{-1-i\sqrt{3}}{2}\right)^n = -1$ (when $n = 3k \pm 1$)

Hence Proved.

From (3)

$\Rightarrow \omega^n + \omega^{2n}$

if $n = 3k$

$\Rightarrow \omega^{3k} + \omega^{6k}$

$\Rightarrow (1) + (1)$ (From (1))

$\Rightarrow 2$

$\therefore \left(\frac{-1+i\sqrt{3}}{2}\right)^n + \left(\frac{-1-i\sqrt{3}}{2}\right)^n = 2$ (when $n = 3k$)

Hence Proved.

2) $x + iy = \cos(u + iv)$ ... (given) $\rightarrow$ (1)

i) $(1+x)^2 + y^2 = (\cosh v + \cos u)^2$ .... To Prove

$x + iy = \cos u \cos iv - \sin u \sin iv$ (From 1)

$x = \cos u \cosh v$ ($\because \cosh v = \cos iv$)

$y = -\sin u \sinh v$ ($\because \sin iv = i \sinh v$)

$\therefore (1+x)^2 + y^2$ .... (LHS)

$(1 + \cos u \cosh v)^2 + (-\sin u \sinh v)^2$

$1 + 2\cos u \cosh v + \cos^2 u \cosh^2 v + \sin^2 u \sinh^2 v$

$1 + 2\cos u \cosh v + \cos^2 u(1 + \sinh^2 v) + \sin^2 u \sinh^2 v$

$1 + \cos^2 u + \cos^2 u \sinh^2 v + \sin^2 u \sinh^2 v + 2\cos u \cosh v$

$1 + \cos^2 u + \sinh^2 v\underbrace{(\sin^2 u + \cos^2 u)}_{1} + 2\cos u \cosh v$

$1 + \sinh^2 v + \cos^2 u + 2\cos u \cosh v$

$\cosh^2 v + \cos^2 u + 2\cos u \cosh v$

LHS $\Rightarrow (\cosh v + \cos u)^2$

LHS = RHS (Hence Proved)

ii) $(1-x)^2 + y^2$

$(1 - \cos u \cosh v)^2 + (-\sin u \sinh v)^2$

$1 + \cos^2 hv \cos^2 u - 2\cos u \cosh v + \sin^2 u \sin^2 hv$

$\Rightarrow 1 + (1 + \sin^2 hv)\cos^2 u - 2\cos u \cosh v + \sin^2 u \sin^2 hv$

$\Rightarrow 1 + \cos^2 u + \sin^2 hv \cos^2 u + \sin^2 u \sin^2 hv$
$- 2\cos u \cosh v$

$\Rightarrow 1 + \cos^2 u + \sin^2 hv \underbrace{(\sin^2 u + \cos^2 u)}_{1} - 2\cos u \cosh v$

$\Rightarrow (1 + \sin^2 hv) + \cos^2 u - 2\cos u \cosh v$

$\Rightarrow \cos^2 hv + \cos^2 u - 2\cos u \cosh v$

LHS $\Rightarrow (\cosh v - \cos u)^2$

LHS = RHS

Hence Proved.

3) $\tan\left(\frac{\pi}{4} + iv\right) = re^{i\theta}$ (given)

$\therefore \tan\left(\frac{\pi}{4} + iv\right) = r(\cos\theta + i\sin\theta)$

$\frac{\tan(\pi/4) + \tan iv}{1 - \tan(\pi/4)\cdot\tan iv} = r(\cos\theta + i\sin\theta)$

$\frac{1 + \tan iv}{1 - \tan iv} = r(\cos\theta + i\sin\theta)$ (1)

$\frac{1 + i\tanh v}{1 - i\tanh v} \times \left(\frac{1 + i\tanh v}{1 + i\tanh v}\right) = r(\cos\theta + i\sin\theta)$

$\frac{(1 - \tan^2 hv) + 2i\tanh v}{1 + \tan^2 hv} = r(\cos\theta + i\sin\theta)$

$\frac{\sec^2 hv + 2i\tanh v}{1 + \tan^2 hv}$

$$\therefore\ r\cos\theta = \frac{\sec^2 h\omega}{1+\tan^2 h\omega} \rightarrow ①$$

$$r\sin\theta = \frac{(2\tan h\omega)}{1+\tan^2 h\omega} \rightarrow ②$$

$$①^2 + ②^2$$

$$r^2(\sin^2\theta+\cos^2\theta) = \left(\frac{\sec^2 h\omega}{1+\tan^2 h\omega}\right)^2 + \left(\frac{2\tan h\omega}{1+\tan^2 h\omega}\right)^2$$

$$r^2 = \frac{\sec^4 h\omega + 4\tan^2 h\omega}{(1+\tan^2 h\omega)^2}$$

$$r^2 = \frac{(1-\tan^2 h\omega)^2 + 4\tan^2 h\omega}{(1+\tan^2 h\omega)^2}$$

$$r^2 = \frac{(1+\tan^2 h\omega)^2}{(1+\tan^2 h\omega)^2} = 1$$

i) Hence Proved $r = 1$

ii) Dividing 2/1

$$\tan\theta = \frac{2\tan h\omega}{\sec^2 h\omega} = 2\tan h\omega\cos^2 h\omega$$

~~tan θ = [illegible] 2 tan~~

$$\tan\theta = \frac{2\sin h\omega}{\cos h\omega} \times \cos^2 h\omega$$

$$\tan\theta = \sin 2h\omega$$

Hence Proved.

iii) we know,

$$\frac{1 + i\tan hv}{1 - i\tan hv} = re^{i\theta} = e^{i\theta}$$

Taking compenendo dividendo

$$\therefore \frac{2}{2i\tan hv} = \frac{e^{i\theta}+1}{e^{i\theta}-1}$$

$$\Rightarrow \tan hv = \frac{e^{i\theta/2} + e^{-i\theta/2}}{e^{i\theta/2} - e^{-i\theta/2}}$$

$$\tan hv = \frac{2\cos(\theta/2)}{2i\sin(\theta/2)} = \tan(\theta/2)$$

$$\therefore \tan hv = \tan\theta/2$$

Hence Proved.