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Q1)

A) i) a) $\frac{3}{5}$

ii) b) $\frac{\partial f}{\partial x} = 3e^{3x}\cos yz^3$

iii) c) $-5, 10$

iv) c) (i) and (iii)

v) b) 0

B) i) $17\cosh x + 18\sinh x = 1$

$\because \cosh x = \frac{e^x + e^{-x}}{2}, \quad \sinh x = \frac{e^x - e^{-x}}{2}$

$\therefore 17\left(\frac{e^x + e^{-x}}{2}\right) + 18\left(\frac{e^x - e^{-x}}{2}\right) = 1$

$\Rightarrow 35e^x - e^{-x} = 2$

Multiplying both sides by $e^x$

$\Rightarrow 35e^{2x} - 1 = 2e^{x}$

$\Rightarrow 35e^{2x} - 2e^x - 1 = 0$

$e^x = \frac{2 \pm \sqrt{4 - 4(35)(-1)}}{2(35)} \Rightarrow \frac{2 \pm \sqrt{144}}{70}$

$\therefore e^x = \frac{14}{70} \Rightarrow \frac{1}{5}$ or $e^x = \frac{-10}{70} \Rightarrow -\frac{1}{7}$

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$\therefore \quad x = \ln\left(\frac{1}{5}\right) \quad$ or $\quad x = \ln\left(\frac{-1}{7}\right)$

$\because$ $x$ is real (given)

hence $x = \ln\left(\frac{1}{5}\right)$

ii) $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 1 \\ 4 & 3 & 2 \end{bmatrix}$

$|A| = 1(-2-3) - 2(4-4) + 0$
$= -5$

$\because$ $|A| \neq 0$, we can state

Rank of matrix A is 3.

iii) let $v_3$ be third unknown eigenvectors corresponding to the eigen value.

$\therefore \; 2x_1 + 2x_2 - x_3 = 0$
$2x_1 - x_2 + 2x_3 = 0$

$\therefore \; \dfrac{x_1}{\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}} = \dfrac{-x_2}{\begin{vmatrix} 0 & -1 \\ 2 & 2 \end{vmatrix}} = \dfrac{x_3}{\begin{vmatrix} 2 & 2 \\ 2 & -1 \end{vmatrix}}$

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$$\frac{x_1}{3} = \frac{-x_2}{6} = \frac{x_3}{-6}$$

$$\frac{x_1}{1} = \frac{-x_2}{2} = \frac{x_3}{-2}$$

$$\therefore X_3 = [1, -2, -2]$$

iv) $\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}$

$u_x = \frac{\partial(xy)}{\partial x} = y$ | $v_x = \frac{\partial(x+y)}{\partial x} = 1$

$u_y = \frac{\partial(xy)}{\partial y} = x$ | $v_y = \frac{\partial(x+y)}{\partial y} = 1$

$$\therefore \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} y & x \\ 1 & 1 \end{vmatrix}$$

$$\boxed{\therefore \frac{\partial(u,v)}{\partial(x,y)} = y - x}$$

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v) A, B are Hermitian matrices

$\therefore A^\theta = A$ & $B^\theta = B \rightarrow (1)$

TPT $AB+BA$ is hermitian, $(AB+BA)^\theta = AB+BA$

$(AB+BA)^\theta = (AB)^\theta + (BA)^\theta \Rightarrow B^\theta A^\theta + A^\theta B^\theta$

$= BA + AB$ (From (1))

$\Rightarrow (AB+BA)^\theta = AB+BA$

$\therefore$ $(AB+BA)$ is Hermitian matrix

Now, $(AB-BA)^\theta = (AB)^\theta - (BA)^\theta \Rightarrow B^\theta A^\theta - A^\theta B^\theta$

$= BA - AB$ (From (1))

$\therefore (AB-BA)^\theta = BA - AB$

Thus $(AB-BA)$ is skew-hermitian matrix

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Q2)

A) We have, $x^5 = 1+i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$\therefore x = 2^{1/10}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^{1/5}$ $\cancel{\Rightarrow 2^{1/10}\cos}$

$\Rightarrow 2^{1/10}\left(\cos\left(\frac{2k\pi+\pi}{4}\right)\frac{1}{5} + i\sin\left(\frac{2k\pi+\pi}{4}\right)\frac{1}{5}\right]$

$\Rightarrow 2^{1/10}\left(\cos(8k+1)\frac{\pi}{20} + i\sin(8k+1)\frac{\pi}{20}\right)$

The roots are obtained for $k = 0, 1, 2, 3, 4$.

The product of roots $= 2^{5/10}(\cos\phi + i\sin\phi)$

where $\phi = \underset{0}{\frac{\pi}{20}} + \underset{1}{\frac{9\pi}{20}} + \underset{2}{\frac{17\pi}{20}} + \underset{3}{\frac{25\pi}{20}} + \underset{4}{\frac{33\pi}{20}} \Rightarrow \frac{85\pi}{20} \Rightarrow \frac{17\pi}{4}$

$\Rightarrow 4\pi + \frac{\pi}{4}$

$\therefore$ The product of roots is $= \sqrt{2}\left(\cos(4\pi+\pi/4) + i\sin(4\pi+\pi/4)\right)$

$= \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$\boxed{\Rightarrow 1+i}$

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B) $28x + 4y - z = 32$

$2x + 17y + 4z = 35$

$x + 3y + 10z = 24$ (given eqn)

$x = \frac{1}{28}(32 - 4y + z)$

$y = \frac{1}{17}(35 - 2x - 4z)$

$z = \frac{1}{10}(24 - x - 3y)$

Putting $y = 0$ & $z = 0$

1st iteration

$x' = \frac{1}{28}(32 - 0 + 0) = 1.1429$

$y' = \frac{1}{17}(35 - 2(1.1429) - 0) = 1.9244$

$z' = \frac{1}{10}(24 - 1.1429 - 3(1.9244)) = 1.8084$

2nd iteration

$x^2 = \frac{1}{28}(32 - 4(1.9244) + 1.8084) = 0.9325$

$y^2 = \frac{1}{17}(35 - 2(0.9325) - 4(1.8084)) = 1.5236$

$z^2 = \frac{1}{10}(24 - 0.9325 - 3(1.5236)) = 1.8497$

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3rd iteration

$x^3 = \frac{1}{28}\left(32 - 4(1.5236) + (1.8497)\right) = 0.9913$

$y^3 = \frac{1}{17}\left(35 - 2(0.9913) - 4(1.8497)\right) = 1.5070$

$z^3 = \frac{1}{10}\left(24 - 0.9913 - 3(1.5070)\right) = 1.8488$

4th iteration

$x^4 = \frac{1}{28}\left(32 - 4(1.5070) + 1.8488\right) = 0.9936$

$y^4 = \frac{1}{17}\left(35 - 2(0.9936) - 4(1.8488)\right) = 1.5069$

$z^4 = \frac{1}{10}\left(24 - 0.9936 - 3(1.5069)\right) = 1.8486$

we get the values for $x, y, z$

$x = 0.9936$ $y = 1.5069$ $z = 1.8486$

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Q3)

A)

$$\begin{bmatrix} -9-\lambda & 4 & 4 \\ -8 & 3-\lambda & 4 \\ -16 & 8 & 7-\lambda \end{bmatrix}$$

$$\therefore (-9-\lambda)\left[(3-\lambda)(7-\lambda) - 32\right] - 4\left[-8(7-\lambda) + 64\right] + 4\left[-64 + 48 - 16\lambda\right] = 0$$

$$\Rightarrow (-9-\lambda)\left[21 - 3\lambda - 7\lambda + \lambda^2 - 32\right] - 4\left[-56 + 8\lambda + 64\right] + 4\left[-16 - 16\lambda\right] = 0$$

$$\Rightarrow -9\lambda^2 + 90\lambda + 99 - \lambda^3 + 10\lambda^2 + 11\lambda - 32 - 32\lambda - 64 - 64\lambda = 0$$

$$\Rightarrow -\lambda^3 + \lambda^2 + 5\lambda + 3 = 0$$

$$\therefore \lambda = -1, -1, 3$$

For $\lambda = -1$ $[A - \lambda_1 I]X = 0$

$$\begin{bmatrix} -8 & 4 & 4 \\ -8 & 4 & 4 \\ -16 & 8 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

On expanding $R_1$

$$-8x_1 + 4x_2 + 4x_3 = 0$$

$$-2x_1 + x_2 + x_3 = 0$$

Put $x_2 = 0$, $x_3 = 1$ we get $x_1 = \frac{1}{2}$

$$X_1 = \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

Put $x_1 = -$

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For $\lambda = -1$ again for $2^{nd}$ eigen value

similarly, we get $-2x_1 + x_2 + x_3 = 0$ ; where $x_2 = 1$; $x_3 = 0$

we get $x_1 = 1/2$

$$X_2 \begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix} \text{ or } \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$\therefore$ $X_1 = [1, 0, 2]$ $\quad$ $X_2 = [1, 2, 0]$

For $\lambda = 3$ $[A - \lambda_2 I] X = 0$

$$\begin{bmatrix} -12 & 4 & 4 \\ -8 & 0 & 4 \\ -16 & 8 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

expanding $R_1$ & $R_2$

$$-12x_1 + 4x_2 + 4x_3 = 0$$
$$-8x_1 + 0x_2 + 4x_3 = 0$$

By Cramer's rule

$$\frac{x_1}{16} = \frac{-x_2}{-16} = \frac{x_3}{32}$$

$$\frac{x_1}{1} = \frac{+x_2}{1} = \frac{x_3}{2}$$

$\therefore$ $X_3 = [1, 1, 2]$

$\therefore \quad X_1 = [1, 0, 2] \qquad X_2 = [1, 2, 0] \qquad X_3 = [1, 1, 2]$

Although the eigen values are not distinct the geometric multiplicity of each value is equal to its multiplicity.

As is diagnalizable. $\because$ AM = GM for all the eigen values

$$\therefore A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$ will be diagnalised

to diagnol form $D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

by transforming matrix $M = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 0 & 2 & 2 \end{bmatrix}$

such that $M^{-1}AM = D$

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Q3)b) $2A^5 - 3A^4 + A^2 - 4I$

$$|A - \lambda| = \begin{vmatrix} 3-\lambda & 1 \\ -1 & 2-\lambda \end{vmatrix} = (3-\lambda)(2-\lambda) + 1$$
$$= \lambda^2 - 5\lambda + 7$$

The characteristic equ$^n$

$$\lambda^2 - 5\lambda + 7 = 0 \rightarrow (1)$$

By Cayley - Hamilton theorem the matrix A must satisfy (1)

$$\therefore \quad A^2 - 5A + 7I = 0$$

From (2) $\quad A^2 = 5A - 7I$

Multiplying by 3

$$A^3 = 5A^2 - 7A$$

$$\therefore \quad A^5 = 5A^4 - 7A^3$$

Q4)

a) $U = \frac{1}{3} \log\left(\frac{x^3+y^3}{x^2+y^2}\right)$

$\therefore 3U = \ln\left(\frac{x^3+y^3}{x^2+y^2}\right)$

$\therefore e^{3U} = \frac{x^3+y^3}{x^2+y^2}$

$\because$ U is not a homogenous function but

$f(U) = e^{3U} = \left(\frac{x^3+y^3}{x^2+y^2}\right)$

putting $x = xt$ & $y = yt$

$= \frac{x^3t^3 + y^3t^3}{x^2t^2 + y^2t^2}$

$= t\left(\frac{x^3+y^3}{x^2+y^2}\right)$

$\therefore$ degree $n = 1$

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Using corollary 2 we get

$$\frac{x\partial u}{\partial x} + \frac{y\partial u}{\partial y} = \frac{n f(u)}{f'(u)}$$

$f(u) = e^{3u}$ & $f'(u) = 3e^{3u}$ & $n = 1$

$$\therefore \frac{x\partial u}{\partial y} + \frac{y\partial u}{\partial y} = \frac{1\cancel{(e^{3u})}}{3\cancel{(e^{3u})}} \Rightarrow \frac{1}{3} \rightarrow 1$$

Using corollary 3 we get.

$$\frac{x^2\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + \frac{y^2\partial^2 u}{\partial y^2} = g(u)(g'(u)-1)$$

$$g(u) = \frac{n f(u)}{f'(u)} = \frac{1}{3} \quad \text{(From 1)}$$

$\therefore g'(u) = 0$

$$\therefore \frac{x^2\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + \frac{y^2\partial^2 u}{\partial y^2} = \frac{1}{3}(0-1) \Rightarrow \frac{-1}{3} \rightarrow 2$$

$\therefore$ i) 1/3 (From 1)

ii) −1/3 (From 2)