Somaiya Vidyavihar University

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Pargat Singh Dhanjal	Pargat

Q1)

1.1) c) $\operatorname{cosech}^2 x - \coth^2 x - 1$

1.2) c) $2\sin(\alpha - \beta)$

1.3) b) $\pi/2$

1.4) b) 1

~~1.5) b) $i(A - A^\theta)$ is skew Hermitian.~~

1.5) c) For Hermitian Matrix A $\overline{iA} = -iA^t$

Q2)

b) $\cos(u + iv) = x + iy$ .... (given) $\rightarrow$ ①

$(1+x)^2 + y^2 = (\cosh v + \cos u)^2$ ... (To Prove)

$x + iy = \cos u \cos iv - \sin u \sin iv$ (From 1)

$x = \cos u \cosh v$ $(\because \cosh v = \cos iv)$

$y = -\sin u \sinh v$ $(\because \sin iv = i \sinh v)$

$\therefore (1+x)^2 + y^2$ .... (LHS)

$(1 + \cos u \cosh v)^2 + (-\sin u \sinh v)^2$

$1 + 2\cos u \cosh v + \cos^2 u \cosh^2 v + \sin^2 u \sinh^2 v$

$1 + 2\cos u \cosh v + \cos^2 u(1 + \sinh^2 v) + \sin^2 u \sinh^2 v$

$1 + \cos^2 u + \cos^2 u \sinh^2 v + \sin^2 u \sinh^2 v + 2\cos u \cosh v$

$1 + \cos^2 u + \sinh^2 v\,(\cancel{\cos^2 u + \sin^2 u})^{1} + 2\cos u \cosh v$

$1 + \cos^2 u + \sinh^2 v + 2\cos u \cosh v$

$1 + \sinh^2 v + \cos^2 u + 2\cos u \cosh v$

$\cosh^2 v + \cos^2 u + 2\cos u \cosh v$

LHS $\Rightarrow (\cosh v + \cos u)^2$

LHS = RHS (Hence Proved)

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Q2)

c) $a\cos\alpha + b\cos\beta + c\cos\gamma = a\sin\alpha + b\sin\beta + c\sin\gamma = 0$ (given)

$a^3\cos 3\alpha + b^3\cos 3\beta + c^3\cos 3\gamma = 3abc\cos(\alpha+\beta+\gamma)$ (To Prove)

Let $a(\cos\alpha + i\sin\alpha) = x$

$b(\cos\beta + i\sin\beta) = y$

$c(\cos\gamma + i\sin\gamma) = z$

$\because x + y + z = 0$ ($\because$ sum of real part = 0, sum of img part = 0)

$(x+y+z)^3 = 0$

$(x^3+y^3+z^3) + 3\cancel{(x+y+z)}^{0}(xy+yz+zx) - 3xyz = 0$

$x^3+y^3+z^3 - 3xyz = 0$

$x^3+y^3+z^3 = 3xyz$

inserting values

$a^3(\cos\alpha + i\sin\alpha)^3 + b^3(\cos\beta + i\sin\beta)^3 + c^3(\cos\gamma + i\sin\gamma)$

$= 3abc(\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta)(\cos\gamma + i\sin\gamma)$

By applying De-Moorvies Theorem

$a^3\cos 3\alpha + ia^3\sin 3\alpha + b^3\cos 3\beta + ib^3\sin 3\beta + c^3\cos 3\gamma + ic^3\sin 3\gamma$

$= 3abc\cos(\alpha+\beta+\gamma) + i\,3abc\sin(\alpha+\beta+\gamma)$

On comparing Real Parts.

$a^3\cos 3\alpha + b^3\cos 3\beta + c^3\cos 3\gamma = 3abc\cos(\alpha+\beta+\gamma)$

Hence Proved.

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Q3) a) $A = \frac{1}{9}\begin{bmatrix} a & 1 & b \\ c & b & 7 \\ 1 & a & c \end{bmatrix}$

orthogonal matrix $AA^T = A^TA = I$

$A^T = \frac{1}{9}\begin{bmatrix} a & c & 1 \\ 1 & b & a \\ b & 7 & c \end{bmatrix}$

$AA^T = \frac{1}{81}\begin{bmatrix} a & 1 & b \\ c & b & 7 \\ 1 & a & c \end{bmatrix}\begin{bmatrix} a & c & 1 \\ 1 & b & a \\ b & 7 & c \end{bmatrix}$

$= \frac{1}{81}\begin{bmatrix} a^2+1+b^2 & ac+8b & 2a+2bc \\ ac+8b & c^2+b^2+49 & 7c+ab \\ 2a+bc & 8c+ab & 1+a^2+c^2 \end{bmatrix}$

$AA^T = I$

$\therefore \quad a^2+b^2 = 80$

$ac = 8b$

$c^2+b^2 = 32$

$a^2+c^2 = 80$

on solving we get.

$a = \pm 8, \quad b = \pm 4, \quad c = \pm 4$

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Q3) b)

$X_1 = [1\ 2\ 1],\ X_2 = [2\ 1\ 4],\ X_3 = [4\ 5\ 6],\ X_4 = [1\ 8\ -3]$

Let, $K_1X_1 + K_2X_2 + K_3X_3 + K_4X_4 = 0 \rightarrow (A)$

$K_1[1\ 2\ 1] + K_2[2\ 1\ 4] + K_3[4\ 5\ 6] + K_4[1\ 8\ -3] = 0$

$[K_1\ 2K_1\ K_1] + [2K_2\ K_2\ 4K_2] + [4K_3\ 5K_3\ 6K_3] + [K_4\ 8K_4\ -3K_4] = 0$

$$\left.\begin{aligned} K_1 + 2K_2 + 4K_3 + K_4 &= 0 \\ 2K_1 + K_2 + 5K_3 + 8K_4 &= 0 \\ K_1 + 4K_2 + 6K_3 - 3K_4 &= 0 \end{aligned}\right]$$ Homogenous system of equations

$$\begin{bmatrix} 1 & 2 & 4 & 1 \\ 2 & 1 & 5 & 8 \\ 1 & 4 & 6 & -3 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \end{bmatrix} = 0$$

$R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 4 & 1 \\ 0 & -3 & -3 & 6 \\ 0 & 2 & 2 & -4 \end{bmatrix} \quad C_2 \rightarrow C_3 - C_3 \quad \begin{bmatrix} 1 & -2 & 4 & 1 \\ 0 & 0 & -3 & 6 \\ 0 & 0 & 2 & -4 \end{bmatrix}$$

$\therefore$ Rank $(A) = 3 < 4$

$\therefore$ There are infinitely many non trivial soln

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$K_1 - 2K_2 + 4K_3 + K_4 = 0 \quad \rightarrow (1)$

$-3K_3 + 6K_4 = 0 \quad \rightarrow (2)$

$2K_3 - 4K_4 = 0 \quad \rightarrow (3)$

putting $\boxed{K_4 = t}$ in equ (3)

$\boxed{K_3 = 2t}$

$K_1 - 2K_2 + 8t + t = 0$

$K_1 - 2K_2 = -9t$

~~[illegible] (A)~~

$\therefore$ The vectors are linearly dependent.