NON HOMOGENEOUS SYSTEM OF EQUATIONS

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x, y, z

$2x + y - 3z = 9$
$x - y + z = 3$
$x + y + 2z = 7$

$$\underbrace{\begin{bmatrix} 2 & 1 & -3 \\ 1 & -1 & 1 \\ 1 & 1 & 2 \end{bmatrix}}_{A} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \\ 7 \end{bmatrix}$$

$AX = B$

A SYSTEM OF LINEAR EQUATIONS

Consider a system of m linear equations in n unknowns $x_1, x_2, x_3, \dots. x_n$

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n = b_3$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n = b_m$$

The system can be written compactly in matrix notation as $AX = B$

Where $A = [a_{ij}]$ is the matrix of order $(m \times n)$, called the matrix of coefficients.

$B = [b_1\ b_2\ b_3 \dots b_m]^T$ is the column vector of order $(m \times 1)$

and $X = [x_1\ x_2\ x_3 \dots x_n]^T$ is the column vector of order $(n \times 1)$

The matrix [A,B] i.e., the matrix formed by the coefficients and the constants is called the **augmented matrix.**

$$[A|B] = \left[\begin{array}{ccc|c} 2 & 1 & -3 & 9 \\ 1 & -1 & 1 & 3 \\ 1 & 1 & 2 & 7 \end{array}\right]$$

Any vector U such that AU = B is said to be a solutions of AX = B.

1. Non-Homogeneous system of equations

$AX = B$ where $B \neq 0$

2. Homogeneous system of equations

$AX = B$ where $B = 0$ ie $AX = 0$.

for
$$\left.\begin{array}{l} x + y + z = 0 \\ x - y + z = 0 \\ x + y - z = 0 \end{array}\right\}$$

Defn: A system is consistent if it has a solution otherwise we say that the system is inconsistent.

ex:- 1 $x + y + z = 2$ } → eqn of two parallel planes

ex :- ① $x+y+z=2$
$2x+2y+2z=5$ $\rightarrow$ eqn of two parallel planes
no soln.

② $x+y=3$
$x-y=1$ $\rightarrow (2,1)$ eqn of two intersecting lines
unique soln.

③ $x+y+z=3$
$x-2y+z=1$ $\rightarrow$ eqn of two intersecting planes
infinitely many soln.

A SYSTEM OF NON – HOMOGENEOUS LINEAR EQUATIONS:

SOLUTION OF n LINEAR EQUATIONS IN n UNKNOWN:

If we are given a system of equations $AX = B$, where A is a non – singular n – rowed square matrix, X is $n \times 1$ matrix and B is $n \times 1$ matrix then the system has **unique solution.**

We accept this theorem without proof and learn how to use it to solve the equations.

1. Write $AX = B$

2. Check that $|A| \neq 0$

3. Now find A^{-1} by any suitable method.

4. The solution is given by $X = A^{-1}B$.

$AX=B$

$A_{n\times n}$ $|A|\neq 0$

$A^{-1}(AX)=A^{-1}B$

$\boxed{X=A^{-1}B}$

Note: If A is singular matrix, then this inverse method fails. In that case the system may have infinitely many solutions or none at all.

SOLUTION OF m LINEAR EQUATIONS IN n UNKNOWN:

Working Rule:

① write the system in matrix form $AX=B$

② write the Augmented matrix $[A|B]$

③ Reduce the Augmented matrix to Echelon form

④ Get Rank of $[A|B]$ as well as Rank(A)

⑤ (a) If Rank $A \neq$ Rank $[A|B]$
then the system is inconsistent
ie no solution

(b) If Rank A = Rank $[A|B] = \underline{r}$

<u>consistent</u>.

If $r = \underline{n}$

ie rank = no of unknowns

there is unique <u>soln</u>

If $r < n$

ie rank < no of unknowns

there are infinitely many solutions.

no of parameters $= \underline{n - r}$

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SOME SOLVED EXAMPLES:

1. Test the consistency of the equations $x + y + z = 3, x + 2y + 3z = 4, x + 4y + 9z = 6$.

<u>Soln</u> :-

$$\underset{A}{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}}$$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 6 \end{array}\right]$$

Apply $R_2 - R_1$, $R_3 - R_1$

$$[A|B] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 3 & 8 & 3 \end{array}\right]$$

Apply $R_3 - 3R_2$

$$[A|B] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 2 & 0 \end{array}\right] \quad \text{———} \quad (1)$$

This is the echelon form

Rank $[A|B] = 3$ & Rank $[A] = 3$

$\therefore$ Rank $(A|B) =$ Rank (A) $\therefore$ System is consistent

Also $r = 3$ & $n = 3$

$\therefore r = n \Rightarrow$ The system has a unique soln.

writing the equations again using (1)

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$

$x + y + z = 3$ ——— (i)

$y + 2z = 1$ ——— (ii)

$2z = 0 \Rightarrow z = 0$

Sub $z = 0$ in (ii), $y = 1$

Sub $y = 1$ & $z = 0$ in (i) $\Rightarrow x = 2$

$\therefore$ The solution is $(x, y, z) = (2, 1, 0)$

2. Solve the following system of linear equations:

$$x - y + z = 2$$
$$3x - y + 2z = -6$$
$$3x + y + z = -18$$

Sol^n :-

$$\begin{bmatrix} 1 & -1 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \\ -18 \end{bmatrix}$$

$$A \quad X = B$$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 3 & -1 & 2 & -6 \\ 3 & 1 & 1 & -18 \end{array}\right]$$

Applying $R_2 - 3R_1$, $R_3 - 3R_1$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 2 & -1 & -12 \\ 0 & 4 & -2 & -24 \end{array}\right]$$

Applying $R_3 - 2R_2$

$$[A|B] \sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 2 & -1 & -12 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

This is the echelon form

$\therefore$ Rank $[A|B] = 2$ & Rank $[A] = 2$

The system is consistent.

Now $r = 2$ & $n = 3$

$\therefore$ [illegible] The system has infinitely

Now $r = 2$ & $n = 3$

$\therefore r < n \Rightarrow$ The system has infinitely many solutions

no. of parameters $= n - r = 3 - 2 = 1$

writing the equations again

$x - y + z = 2$ — (i)

$2y - z = -12$ — (ii)

let $z = t$ (t is a parameter)

(ii) $\Rightarrow y = \dfrac{-12 + t}{2} \Rightarrow y = -6 + \dfrac{t}{2}$

sub in (i)

$x + 6 - \dfrac{t}{2} + t = 2 \Rightarrow x = 2 - 6 + \dfrac{t}{2} - t = -4 - \dfrac{t}{2}$

$\therefore$ The solution is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 - \frac{t}{2} \\ -6 + \frac{t}{2} \\ t \end{bmatrix}$$

has infinite solutions as t varies.

3. Are the following equations consistent?

$$2x + y + z = 4$$
$$x + y + z = 2$$
$$5x + 3y + 3z = 6$$

Soln :

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 5 & 3 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccc:c} 2 & 1 & 1 & 4 \\ 1 & 1 & 1 & 2 \\ 5 & 3 & 3 & 6 \end{array}\right] \xrightarrow{R_{12}} \left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 2 & 1 & 1 & 4 \\ 5 & 3 & 3 & 6 \end{array}\right]$$

Applying $R_2 - 2R_1$, $R_3 - 5R_1$

$$[A|B] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -1 & -1 & 0 \\ 0 & -2 & -2 & -4 \end{array}\right]$$

Applying $R_3 - 2R_2$

$$[A|B] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & -4 \end{array}\right]$$

$\therefore$ Rank $[A|B] = 3$

& Rank $[A] = 2$

Rank $[A|B] \neq$ Rank $[A]$

$\therefore$ The given system is inconsistent.
has no solution.

4. Prove that the system of linear equations $\begin{aligned} x + y + z &= 5 \\ x + 2y + 3z &= 10 \\ x + 2y + 2z &= 8. \end{aligned}$ is consistent and find its solution

$(x, y, z) = (2, 1, 2)$

5. Solve the system of equations.

$$\begin{aligned} x_1 + x_2 - 2x_3 + x_4 + 3x_5 &= 1 \\ 2x_1 - x_2 + 2x_3 + 2x_4 + 6x_5 &= 2 \\ 3x_1 + 2x_2 - 4x_3 - 3x_4 - 9x_5 &= 3. \end{aligned}$$

Soln :-

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 \\ 2 & -1 & 2 & 2 & 6 \\ 3 & 2 & -4 & -3 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccccc|c} 1 & 1 & -2 & 1 & 3 & 1 \\ 2 & -1 & 2 & 2 & 6 & 2 \\ 3 & 2 & -4 & -3 & -9 & 3 \end{array}\right]$$

Applying $R_2 - 2R_1$, $R_3 - 3R_1$

$$[A|B] \sim \left[\begin{array}{ccccc|c} 1 & 1 & -2 & 1 & 3 & 1 \\ 0 & -3 & 6 & 0 & 0 & 0 \\ 0 & -1 & 2 & -6 & -18 & 0 \end{array}\right] \xrightarrow{-\frac{1}{3}R_2}$$

$R_3 - \frac{1}{3}R_2$

$$[A|B] \sim \left[\begin{array}{ccccc|c} 1 & 1 & -2 & 1 & 3 & 1 \\ 0 & -3 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & -6 & -18 & 0 \end{array}\right]$$

This is the echelon form

Rank $(A|B) = 3$ & Rank$(A) = 3$

$\therefore$ The system is consistent

Now, $r = 3$, $n = 5$

$\Rightarrow r < n \Rightarrow$ There are infinitely many solutions.

$\therefore$ no of parameters $= n - r = 5 - 3 = 2$

writing the reduced equations

$$x_1 + x_2 - 2x_3 + x_4 + 3x_5 = 1 \quad \text{—— (i)}$$
$$-3x_2 + 6x_3 = 0 \quad \text{—— (ii)}$$
$$-6x_4 - 18x_5 = 0 \quad \text{—— (iii)}$$

let $\boxed{x_3 = a}$ and $\boxed{x_5 = b}$ (a, b are parameters)

using (ii) $-3x_2 + 6a = 0 \Rightarrow \boxed{x_2 = 2a}$

Using (ii) $-3x_2 + 6a = 0 \Rightarrow \boxed{x_2 = 2a}$

Using (iii) $-6x_4 - 18b = 0 \Rightarrow \boxed{x_4 = -3b}$

Using (i) $x_1 + 2a - 2a - 3b + 3b = 1 \Rightarrow \boxed{x_1 = 1}$

$\therefore$ The $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 2a \\ a \\ -3b \\ b \end{bmatrix}$ is the infinite solutions as 'a' and 'b' vary "doubly infinite"

6. Investigate for what values of a, b the following linear equations $x + 2y + 3z = 4, x + 3y + 4z = 5, x + 3y + az = b$, have **(i)** no solution, **(ii)** a unique solution, **(iii)** An infinite number of solutions.

Soln:-

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 3 & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ b \end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 5 \\ 1 & 3 & a & b \end{array}\right]$$

$R_2 - R_1, \quad R_3 - R_1$

$$[A|B] \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & a-3 & b-4 \end{array}\right]$$

$R_3 - R_2$

$$[A|B] \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ & 1 & 1 & 1 \end{array}\right.$$

$$[A|B] \sim \begin{bmatrix} 1 & 2 & 3 & \vdots & 4 \\ 0 & 1 & 1 & \vdots & 1 \\ 0 & 0 & a-4 & \vdots & b-5 \end{bmatrix}$$

This is the echelon form

(i) No Solution :- The system has no solution if $\text{Rank}[A|B] \neq \text{Rank}[A]$

$\Rightarrow a-4=0$ & $b-5 \neq 0$

$\Rightarrow a=4$ & $b \neq 5$

$\rho(A)=2$ & $\rho[A|B]=3$

$\therefore$ No solution

(ii) Unique solution : The system has a unique solution if

$\text{Rank}(A) = \text{Rank}[A|B] = 3$ (no of unknown)

$\Rightarrow a-4 \neq 0$

$\Rightarrow a \neq 4$ & no condition on b

(iii) infinite no of solution : The system has infinite no of solutions if

$\text{Rank}(A) = \text{Rank}[A|B] < 3$

$\Rightarrow a-4=0$ & $b-5=0$

$\Rightarrow a=4$ & $b=5$

(i) No solution for $a=4$, $b \neq 5$

(ii) Unique solution for $a \neq 4$ & $b \in R$

(ii) Unique solution for $a \neq 4$ & $b \in R$

(iii) Infinitely many solutions for $a = 4$ & $b = 5$.

7. For what value of λ the following set of equations is consistent and solve them.

$x + 2y + z = 3, x + y + z = \lambda, 3x + y + 3z = \lambda^2$

Soln:

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 3 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ \lambda \\ \lambda^2 \end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 1 & 1 & 1 & \lambda \\ 3 & 1 & 3 & \lambda^2 \end{array}\right]$$

$R_2 - R_1, \quad R_3 - 3R_1$

$$[A|B] \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & \lambda - 3 \\ 0 & -5 & 0 & \lambda^2 - 9 \end{array}\right]$$

$R_3 - 5R_2$

$$[A|B] \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & \lambda - 3 \\ 0 & 0 & 0 & \underline{\lambda^2 - 5\lambda + 6} \end{array}\right] \quad \text{——— (1)}$$

Rank $[A] = 2$

$\therefore$ The system will be consistent if

Rank$[A|B]$ = Rank $[A] = 2$

$\text{Rank}[A|B] = \text{Rank}[A] = 2$

$\Rightarrow \lambda^2 - 5\lambda + 6 = 0$

$\Rightarrow \lambda = 2, 3$

$\therefore$ The system is consistent when $\lambda = 2$ or $\lambda = 3$

For $\lambda = 2$, using ①

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x + 2y + z = 3$

$-y = -1$

$\Rightarrow \boxed{y = 1}$

rank $= 2 < 3$

$\therefore$ infinitely many soln

no of parameters $= 3 - 2 = 1$

$x + 2 + z = 3$

$x + z = 1$

let $z = t \Rightarrow x = 1 - t$

$\therefore$ The soln is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1-t \\ 1 \\ t \end{bmatrix}$ is infinitely no of soln

For $\lambda = 3$, using ①

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

rank $= 2 < 3 \Rightarrow$ infinite no of soln

no of parameters $= n - r = 1$

$$x + 2y + z = 3$$
$$-y = 0 \Rightarrow \boxed{y = 0}$$
$$\Rightarrow x + z = 3 \quad \text{let } z = t \Rightarrow x = 3 - t$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3-t \\ 0 \\ t \end{bmatrix}$$ is the infinite no of solutions

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8. Show that the following system of equations is consistent if a, b, c are is A.P.

$3x + 4y + 5z = a, 4x + 5y + 6z = b, 5x + 6y + 7z = c.$

$$2b = a + c$$

Soln:-

$$\begin{bmatrix} 3 & 4 & 5 \\ 4 & 5 & 6 \\ 5 & 6 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccc|c} 3 & 4 & 5 & a \\ 4 & 5 & 6 & b \\ 5 & 6 & 7 & c \end{array}\right]$$

Apply $R_1 - R_2$

$$[A|B] \sim \left[\begin{array}{ccc|c} -1 & -1 & -1 & a-b \\ 4 & 5 & 6 & b \\ 5 & 6 & 7 & c \end{array}\right]$$

Apply $R_2 + 4R_1,\ R_3 + 5R_1$

$$[A|B] \sim \left[\begin{array}{ccc|c} -1 & -1 & -1 & a-b \\ 0 & 1 & 2 & 4a-3b \\ 0 & 1 & 2 & 5a-5b+c \end{array}\right]$$

Apply $R_3 - R_2$

$$[A|B] \sim \left[\begin{array}{ccc|c} -1 & -1 & -1 & a-b \\ 0 & 1 & 2 & 4a-3b \\ 0 & 0 & 0 & a-2b+c \end{array}\right]$$

The matrix is in echelon form

The system will be consistent if

$\text{Rank}[A|B] = \text{Rank}(A)$

$\Rightarrow \text{Rank}[A|B] = 2$

$\Rightarrow a - 2b + c = 0$

$\Rightarrow 2b = a + c$

$\Rightarrow a, b, c$ are in A.P.

find the solution when $a = 3, b = 5, c = 7$

9. Find the value of k for which the equations $x + y + z = 1, x + 2y + 3z = k, x + 5y + 9z = k^2$ has a solution. For these values of k, solve the system completely. (HW)

10. Show that if $\lambda \neq 0$, the system of equations $\begin{array}{c} 2x_1 + x_2 = a \\ x_1 + \lambda x_2 - x_3 = b \\ x_2 + 2x_3 = c \end{array}$ has a unique solution for every choice of a, b, c. If $\lambda = 0$, determine the relation satisfied by a, b, c such that the system is consistent. Find the general solution by taking $\lambda = 0$, a = 1, b = 1, c = −1.

Soln:

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & \lambda & -1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

For a unique solution,

$\rho(A) = \rho(A|B) = $ no. of unknowns $= 3$

$\rho(A) = 3$ if $|A| \neq 0$

$$|A| = \begin{vmatrix} 2 & 1 & 0 \\ 1 & \lambda & -1 \\ 0 & 1 & 2 \end{vmatrix} = 4\lambda$$

If $\lambda \neq 0 \Rightarrow |A| \neq 0 \Rightarrow \rho(A) = 3$

$\Rightarrow$ the system has unique soln for any value of a, b, c.

If $\lambda = 0$

$$[A|B] = \left[\begin{array}{ccc|c} 2 & 1 & 0 & a \\ 1 & 0 & -1 & b \\ 0 & 1 & 2 & c \end{array}\right]$$

Apply $2R_2 - R_1$

$$[A|B] \sim \left[\begin{array}{ccc|c} 2 & 1 & 0 & a \\ 0 & -1 & -2 & 2b-a \\ 0 & 1 & 2 & c \end{array}\right]$$

Apply $R_3 + R_2$

$$[A|B] \sim \left[\begin{array}{ccc|c} 2 & 1 & 0 & a \\ 0 & -1 & -2 & 2b-a \\ 0 & 0 & 0 & c+2b-a \end{array}\right]$$

This is the echelon form

The system will be consistent if

$\text{Rank}[A|B] = \text{Rank}(A)$

$= 2$

$\Rightarrow \boxed{c + 2b - a = 0}$

For $\lambda = 0$, $a = 1$, $b = 1$, $c = -1$

Now $c + 2b - a = -1 + 2 - 1 = 0$

$\therefore$ The system is consistent.

$$[A|B] \sim \left[\begin{array}{ccc|c} 2 & 1 & 0 & 1 \\ 0 & -1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore$ Rank $(A|B) =$ Rank$(A) = 2 < 3$

$\therefore$ There are infinitely many solutions.

no. of parameters $= 3 - 2 = 1$

writing the reduced equations

$$2x_1 + x_2 = 1 \quad \text{— (i)}$$
$$-x_2 - 2x_3 = 1 \quad \text{— (ii)}$$

let $\boxed{x_3 = t}$ ($t \rightarrow$ parameter)

(ii) $\Rightarrow -x_2 - 2t = 1 \Rightarrow \boxed{x_2 = -1 - 2t}$

(i) $\Rightarrow 2x_1 - 1 - 2t = 1 \Rightarrow \boxed{x_1 = 1 + t}$

$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1+t \\ -1-2t \\ t \end{bmatrix}$ is the infinite no of solutions as 't' varies.